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On $(1,2)^{\star}$-$g^{\#}$-Continuous Functions

O. Ravi[1,*](siingam@yahoo.com)
I. Rajasekaran[1] (rajasekarani@yahoo.com)
A. Pandi[1] (pandi2085@yahoo.com)

[1]*Department of Mathematics, P. M. Thevar College, Usilampatti, Madurai District, Tamil Nadu, India.*

Abstract – The aim of this paper is to study and characterize $(1,2)^{\star}$-$g^{\#}$-continuous functions and $(1,2)^{\star}$-$g^{\#}$-irresolute functions formed with the help of $(1,2)^{\star}$-$g^{\#}$-closed sets.

Keywords – *Bitopological space, $(1,2)^{\star}$-$g^{\#}$-closed set, $(1,2)^{\star}$-$g^{\#}$-continuous function, $(1,2)^{\star}$-$g^{\#}$-irresolute function.*

1 Introduction

Several authors ([1, 4, 5, 19]) working in the field of general topology have shown more interest in studying the concepts of generalizations of continuous functions. A weak form of continuous functions called g-continuous functions were introduced by Balachandran et al [3]. Recently Sheik John [18] have introduced and studied another form of generalized continuous functions called ω-continuous functions.

In this paper, we first study $(1,2)^{\star}$-$g^{\#}$-continuous functions and investigate their relations with various generalized $(1,2)^{\star}$-continuous functions. We also discuss some properties of $(1,2)^{\star}$-$g^{\#}$-continuous functions. We also introduce $(1,2)^{\star}$-$g^{\#}$-irresolute functions and study some of its applications. Finally using $(1,2)^{\star}$-$g^{\#}$-continuous function we obtain a decomposition of $(1,2)^{\star}$-continuity.

2 Preliminary

Throughout this paper, X, Y and Z denote bitopological spaces (X, τ_1, τ_2), (Y, σ_1, σ_1) and (Z, η_1, η_2) respectively.

Definition 2.1. *Let A be a subset of a bitopological space X. Then A is called $\tau_{1,2}$-open [9] if $A = P \cup Q$, for some $P \in \tau_1$ and $Q \in \tau_2$. The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.*

The family of all $\tau_{1,2}$-open (resp. $\tau_{1,2}$-closed) sets of X is denoted by $(1,2)^{\star}$-O(X) (resp. $(1,2)^{\star}$-C(X)).

Definition 2.2. *Let A be a subset of a bitopological space X. Then*

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[*]*Corresponding Author.*

1. the $\tau_{1,2}$-interior of A, denoted by $\tau_{1,2}$-int(A), is defined by $\cup$ { $U : U \subseteq A$ and U is $\tau_{1,2}$-open};
2. the $\tau_{1,2}$-closure of A, denoted by $\tau_{1,2}$-cl(A), is defined by $\cap$ { $U : A \subseteq U$ and U is $\tau_{1,2}$-closed}.

Remark 2.3. *Notice that $\tau_{1,2}$-open subsets of X need not necessarily form a topology.*

Definition 2.4. *Let A be a subset of a bitopological space X is called*

1. $(1,2)^\star$-semi-open set [9] if $A \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(A)).
2. $(1,2)^\star$-preopen set [9] if $A \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl(A)).
3. $(1,2)^\star$-α-open set [9] if $A \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(A))).
4. $(1,2)^\star$-β-open set [12] if $A \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(A))).
5. $(1,2)^\star$-regular open set [13] if $A = \tau_{1,2}$-int($\tau_{1,2}$-cl(A)).

The complements of the above mentioned open sets are called their respective closed sets.

The $(1,2)^\star$-preclosure [11] (resp. $(1,2)^\star$-semi-closure [11], $(1,2)^\star$-α-closure [11], $(1,2)^\star$-β-closure [16]) of a subset A of X, denoted by $(1,2)^\star$-pcl(A) (resp. $(1,2)^\star$-scl(A), $(1,2)^\star$-αcl(A), $(1,2)^\star$-βcl(A)) is defined to be the intersection of all $(1,2)^\star$-preclosed (resp. $(1,2)^\star$-semi-closed, $(1,2)^\star$-α-closed, $(1,2)^\star$-β-closed) sets of X containing A. It is known that $(1,2)^\star$-pcl(A) (resp. $(1,2)^\star$-scl(A), $(1,2)^\star$-αcl(A), $(1,2)^\star$-βcl(A)) is a $(1,2)^\star$-preclosed (resp. $(1,2)^\star$-semi-closed, $(1,2)^\star$-α-closed, $(1,2)^\star$-β-closed) set. For any subset A of an arbitrarily chosen bitopological space, the $(1,2)^\star$-semi-interior [11] (resp. $(1,2)^\star$-α-interior [11], $(1,2)^\star$-preinterior [11]) of A, denoted by $(1,2)^\star$-sint(A) (resp. $(1,2)^\star$-αint(A), $(1,2)^\star$-pint(A)), is defined to be the union of all $(1,2)^\star$-semi-open (resp. $(1,2)^\star$-α-open, $(1,2)^\star$-preopen) sets of X contained in A.

Definition 2.5. *Let A be a subset of a bitopological space X is called*

1. a $(1,2)^\star$-generalized closed (briefly, $(1,2)^\star$-g-closed) set [17] if $\tau_{1,2}$-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X.
   The complement of $(1,2)^\star$-g-closed set is called $(1,2)^\star$-g-open set.
2. a $(1,2)^\star$-$g^\star$-closed set [17] if $\tau_{1,2}$-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^\star$-g-open in X.
   The complement of $(1,2)^\star$-$g^\star$-closed set is called $(1,2)^\star$-$g^\star$-open set.
3. a $(1,2)^\star$-semi-generalized closed (briefly, $(1,2)^\star$-sg-closed) set [2] if $(1,2)^\star$-scl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^\star$-semi-open in X.
   The complement of $(1,2)^\star$-sg-closed set is called $(1,2)^\star$-sg-open set.
4. a $(1,2)^\star$-generalized semi-closed (briefly, $(1,2)^\star$-gs-closed) set [2] if $(1,2)^\star$-scl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X.
   The complement of $(1,2)^\star$-gs-closed set is called $(1,2)^\star$-gs-open set.
5. an $(1,2)^\star$-α-generalized closed (briefly, $(1,2)^\star$-αg-closed) set [6] if $(1,2)^\star$-αcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X.
   The complement of $(1,2)^\star$-αg-closed set is called $(1,2)^\star$-αg-open set.
6. a $(1,2)^\star$-generalized semi-preclosed (briefly, $(1,2)^\star$-gsp-closed) set [6] if $(1,2)^\star$-βcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X.
   The complement of $(1,2)^\star$-gsp-closed set is called $(1,2)^\star$-gsp-open set.
7. a $(1,2)^\star$-gα-closed set [15] if $(1,2)^\star$-αcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^\star$-α-open in X.
   The complement of $(1,2)^\star$-gα-closed set is called $(1,2)^\star$-gα-open set.

Remark 2.6. *The collection of all $(1,2)^\star$-$g^\star$-closed (resp. $(1,2)^\star$-g-closed, $(1,2)^\star$-gs-closed, $(1,2)^\star$-gsp-closed, $(1,2)^\star$-αg-closed, $(1,2)^\star$-sg-closed, $(1,2)^\star$-α-closed, $(1,2)^\star$-semi-closed) sets of X is denoted by $(1,2)^\star$-$G^\star C(X)$ (resp. $(1,2)^\star$-GC(X), $(1,2)^\star$-GSC(X), $(1,2)^\star$-GSPC(X), $(1,2)^\star$-$\alpha GC(X)$, $(1,2)^\star$-SGC(X), $(1,2)^\star$-$\alpha C(X)$, $(1,2)^\star$-SC(X)).*

The collection of all $(1,2)^\star$-$g^\star$-open (resp. $(1,2)^\star$-g-open, $(1,2)^\star$-gs-open, $(1,2)^\star$-gsp-open, $(1,2)^\star$-αg-open, $(1,2)^\star$-sg-open, $(1,2)^\star$-α-open, $(1,2)^\star$-semi-open) sets of X is denoted by $(1,2)^\star$-$G^\star O(X)$ (resp. $(1,2)^\star$-GO(X), $(1,2)^\star$-GSO(X), $(1,2)^\star$-GSPO(X), $(1,2)^\star$-$\alpha GO(X)$, $(1,2)^\star$-SGO(X), $(1,2)^\star$-$\alpha O(X)$, $(1,2)^\star$-SO(X)).

We denote the power set of X by P(X).

Definition 2.7. *[10] Let A be a subset of a bitopological space X. Then A is called*

1. *$(1,2)^\star$-$g^\#$-closed set if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $(1,2)^\star$-αg-open in X. The family of all $(1,2)^\star$-$g^\#$-closed sets in X is denoted by $(1,2)^\star$-$G^\# C(X)$.*
2. *$(1,2)^\star$-$g^\#_\alpha$-closed set if $(1,2)^\star$-$\alpha cl(A) \subseteq U$ whenever A $\subseteq$ U and U is $(1,2)^\star$-αg-open in X. The family of all $(1,2)^\star$-$g^\#_\alpha$-closed sets in X is denoted by $(1,2)^\star$-$G^\#_\alpha C(X)$.*

Definition 2.8. *A function $f: X \to Y$ is called:*

1. *$(1,2)^\star$-$g^\star$-continuous [7] if $f^{-1}(V)$ is a $(1,2)^\star$-$g^\star$-closed set in X for every $\sigma_{1,2}$-closed set V of Y.*
2. *$(1,2)^\star$-g-continuous [7] if $f^{-1}(V)$ is a $(1,2)^\star$-g-closed set in X for every $\sigma_{1,2}$-closed set V of Y.*
3. *$(1,2)^\star$-αg-continuous [16] if $f^{-1}(V)$ is an $(1,2)^\star$-αg-closed set in X for every $\sigma_{1,2}$-closed set V of Y.*
4. *$(1,2)^\star$-gs-continuous [16] if $f^{-1}(V)$ is a $(1,2)^\star$-gs-closed set in X for every $\sigma_{1,2}$-closed set V of Y.*
5. *$(1,2)^\star$-gsp-continuous [16] if $f^{-1}(V)$ is a $(1,2)^\star$-gsp-closed set in X for every $\sigma_{1,2}$-closed set V of Y.*
6. *$(1,2)^\star$-sg-continuous [14] if $f^{-1}(V)$ is a $(1,2)^\star$-sg-closed set in X for every $\sigma_{1,2}$-closed set V of Y.*
7. *$(1,2)^\star$-semi-continuous [11] if $f^{-1}(V)$ is a $(1,2)^\star$-semi-open set in X for every $\sigma_{1,2}$-open set V of Y.*
8. *$(1,2)^\star$-α-continuous [11] if $f^{-1}(V)$ is an $(1,2)^\star$-α-closed set in X for every $\sigma_{1,2}$-closed set V of Y.*

Definition 2.9. *A function $f: X \to Y$ is called:*

1. *$(1,2)^\star$-αg-irresolute [16] if the inverse image of every $(1,2)^\star$-αg-closed (resp. $(1,2)^\star$-αg-open) set in Y is $(1,2)^\star$-αg-closed (resp. $(1,2)^\star$-αg-open) in X.*
2. *$(1,2)^\star$-gc-irresolute [7] if the inverse image of every $(1,2)^\star$-g-closed set in Y is $(1,2)^\star$-g-closed in X.*
3. *$(1,2)^\star$-sg-irresolute [16] if the inverse image of every $(1,2)^\star$-sg-closed (resp. $(1,2)^\star$-sg-open) set in Y is $(1,2)^\star$-sg-closed (resp. $(1,2)^\star$-sg-open) in X.*

Definition 2.10. *[16] A function $f: X \to Y$ is called pre-$(1,2)^\star$-αg-closed if f(U) is $(1,2)^\star$-αg-closed in Y, for each $(1,2)^\star$-αg-closed set U in X.*

Definition 2.11. *A bitopological space X is called:*

1. *$(1,2)^\star$-$T_{1/2}$-space [14] if every $(1,2)^\star$-g-closed set in it is $\tau_{1,2}$-closed.*
2. *$(1,2)^\star$-$T_{\star 1/2}$-space [12] if every $(1,2)^\star$-$^\star g$-closed set in it is $\tau_{1,2}$-closed.*
3. *$(1,2)^\star$-$^\star T_{1/2}$-space [12] if every $(1,2)^\star$-g-closed set in it is $(1,2)^\star$-$g^\star$-closed.*
4. *$(1,2)^\star$-T_b-space [12] if every $(1,2)^\star$-gs-closed set in it is $\tau_{1,2}$-closed.*

5. $(1,2)^\star$-$_\alpha T_b$-*space [16] if every* $(1,2)^\star$-αg-*closed set in it is* $\tau_{1,2}$-*closed.*
6. $(1,2)^\star$-T_d-*space [16] if every* $(1,2)^\star$-αg-*closed set in it is* $(1,2)^\star$-g-*closed.*
7. $(1,2)^\star$-α-*space [11] if every* $(1,2)^\star$-α-*closed set in it is* $\tau_{1,2}$-*closed.*
8. $(1,2)^\star$-$T_{\#g}$-*space [10] if every* $(1,2)^\star$-$g^\#$-*closed set in it is* $\tau_{1,2}$-*closed.*

Theorem 2.12. *[10] A set A of X is* $(1,2)^\star$-$g^\#$-*open if and only if* $F \subseteq \tau_{1,2}$-*int(A) whenever F is* $(1,2)^\star$-αg-*closed and* $F \subseteq A$.

Theorem 2.13. *[10] For a space X, the following properties are equivalent:*

1. *X is a* $(1,2)^\star$-$T_g^\#$-*space.*
2. *Every singleton subset of X is either* $(1,2)^\star$-αg-*closed or* $\tau_{1,2}$-*open.*

3 $(1,2)^\star$-$g^\#$-Continuous Functions

We introduce the following definitions:

Definition 3.1. *A function* $f : X \to Y$ *is called:*

1. $(1,2)^\star$-$g^\#$-*continuous if the inverse image of every* $\sigma_{1,2}$-*closed set in Y is* $(1,2)^\star$-$g^\#$-*closed set in X.*
2. $(1,2)^\star$-$g_\alpha^\#$-*continuous if* $f^{-1}(V)$ *is an* $(1,2)^\star$-$g_\alpha^\#$-*closed set in X for every* $\sigma_{1,2}$-*closed set V of Y.*
3. *strongly* $(1,2)^\star$-$g^\#$-*continuous if the inverse image of every* $(1,2)^\star$-$g^\#$-*open set in Y is* $\tau_{1,2}$-*open in X.*

Example 3.2. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{c\}, X\}$ *and* $\tau_2 = \{\phi, \{a, c\}, X\}$. *Then the sets in* $\{\phi, \{c\}, \{a, c\}, X\}$ *are called* $\tau_{1,2}$-*open and the sets in* $\{\phi, \{b\}, \{a, b\}, X\}$ *are called* $\tau_{1,2}$-*closed. Let* $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ *and* $\sigma_2 = \{\phi, \{c\}, Y\}$. *Then the sets in* $\{\phi, \{c\}, Y\}$ *are called* $\sigma_{1,2}$-*open and the sets in* $\{\phi, \{a, b\}, Y\}$ *are called* $\sigma_{1,2}$-*closed. We have* $(1,2)^\star$-$G^\# C(X) = \{\phi, \{b\}, \{a, b\}, X\}$. *Let* $f : X \to Y$ *be the identity function. Then f is* $(1,2)^\star$-$g^\#$-*continuous.*

Proposition 3.3. *Every* $(1,2)^\star$-*continuous function is* $(1,2)^\star$-$g^\#$-*continuous but not conversely.*

Example 3.4. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ *and* $\tau_2 = \{\phi, \{a, b\}, X\}$. *Then the sets in* $\{\phi, \{a, b\}, X\}$ *are called* $\tau_{1,2}$-*open and the sets in* $\{\phi, \{c\}, X\}$ *are called* $\tau_{1,2}$-*closed. Let* $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, \{b\}, Y\}$ *and* $\sigma_2 = \{\phi, Y\}$. *Then the sets in* $\{\phi, \{b\}, Y\}$ *are called* $\sigma_{1,2}$-*open and the sets in* $\{\phi, \{a, c\}, Y\}$ *are called* $\sigma_{1,2}$-*closed. We have* $(1,2)^\star$-$G^\# C(X) = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$. *Let* $f : X \to Y$ *be the identity function. Then f is* $(1,2)^\star$-$g^\#$-*continuous but not* $(1,2)^\star$-*continuous, since* $f^{-1}(\{a, c\}) = \{a, c\}$ *is not* $\tau_{1,2}$-*closed in X.*

Proposition 3.5. *Every* $(1,2)^\star$-$g^\#$-*continuous function is* $(1,2)^\star$-$g_\alpha^\#$-*continuous but not conversely.*

Example 3.6. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ *and* $\tau_2 = \{\phi, \{b\}, X\}$. *Then the sets in* $\{\phi, \{b\}, X\}$ *are called* $\tau_{1,2}$-*open and the sets in* $\{\phi, \{a, c\}, X\}$ *are called* $\tau_{1,2}$-*closed. Let* $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ *and* $\sigma_2 = \{\phi, \{b, c\}, Y\}$. *Then the sets in* $\{\phi, \{b, c\}, Y\}$ *are called* $\sigma_{1,2}$-*open and the sets in* $\{\phi, \{a\}, Y\}$ *are called* $\sigma_{1,2}$-*closed. We have* $(1,2)^\star$-$G^\# C(X) = \{\phi, \{a, c\}, X\}$ *and* $(1,2)^\star$-$G_\alpha^\# C(X) = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$. *Let* $f : X \to Y$ *be the identity function. Then f is* $(1,2)^\star$-$g_\alpha^\#$-*continuous but not* $(1,2)^\star$-$g^\#$-*continuous, since* $f^{-1}(\{a\}) = \{a\}$ *is not* $(1,2)^\star$-$g^\#$-*closed in X.*

Proposition 3.7. *Every* $(1,2)^\star$-$g^\#$-*continuous function is* $(1,2)^\star$-$g^\star$-*continuous but not conversely.*

Example 3.8. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{c\}, X\}$ *and* $\tau_2 = \{\phi, \{a, c\}, X\}$. *Then the sets in* $\{\phi, \{c\}, \{a, c\}, X\}$ *are called* $\tau_{1,2}$-*open and the sets in* $\{\phi, \{b\}, \{a, b\}, X\}$ *are called* $\tau_{1,2}$-*closed. Let* $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ *and* $\sigma_2 = \{\phi, \{a\}, Y\}$. *Then the sets in* $\{\phi, \{a\}, Y\}$ *are called* $\sigma_{1,2}$-*open and the sets in* $\{\phi, \{b, c\}, Y\}$ *are called* $\sigma_{1,2}$-*closed. We have* $(1,2)^\star$-$G^\# C(X) = \{\phi, \{b\}, \{a, b\}, X\}$ *and* $(1,2)^\star$-$G^\star C(X) = \{\phi, \{b\}, \{a, b\}, \{b, c\}, X\}$. *Let* $f : X \to Y$ *be the identity function. Then f is* $(1,2)^\star$-$g^\star$-*continuous but not* $(1,2)^\star$-$g^\#$-*continuous, since* $f^{-1}(\{b, c\}) = \{b, c\}$ *is not* $(1,2)^\star$-$g^\#$-*closed in X.*

Proposition 3.9. *Every $(1,2)^{\star}$-$g^{\#}$-continuous function is $(1,2)^{\star}$-g-continuous but not conversely.*

Example 3.10. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{c\}, Y\}$. Then the sets in $\{\phi, \{c\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{a, b\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^{\star}$-$G^{\#}C(X) = \{\phi, \{a\}, \{b, c\}, X\}$ and $(1,2)^{\star}$-$GC(X) = P(X)$. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^{\star}$-g-continuous but not $(1,2)^{\star}$-$g^{\#}$-continuous, since $f^{-1}(\{a, b\}) = \{a, b\}$ is not $(1,2)^{\star}$-$g^{\#}$-closed in X.*

Proposition 3.11. *Every $(1,2)^{\star}$-$g^{\#}$-continuous function is $(1,2)^{\star}$-αg-continuous but not conversely.*

Example 3.12. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{b\}, Y\}$. Then the sets in $\{\phi, \{b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{a, c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^{\star}$-$G^{\#}C(X) = \{\phi, \{a\}, \{b, c\}, X\}$ and $(1,2)^{\star}$-$\alpha GC(X) = P(X)$. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^{\star}$-αg-continuous but not $(1,2)^{\star}$-$g^{\#}$-continuous, since $f^{-1}(\{a, c\}) = \{a, c\}$ is not $(1,2)^{\star}$-$g^{\#}$-closed in X.*

Proposition 3.13. *Every $(1,2)^{\star}$-$g^{\#}$-continuous function is $(1,2)^{\star}$-gs-continuous but not conversely.*

Example 3.14. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{a\}, X\}$. Then the sets in $\{\phi, \{a\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{a, b\}, Y\}$. Then the sets in $\{\phi, \{a, b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^{\star}$-$G^{\#}C(X) = \{\phi, \{b, c\}, X\}$ and $(1,2)^{\star}$-$GSC(X) = \{\phi, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^{\star}$-gs-continuous but not $(1,2)^{\star}$-$g^{\#}$-continuous, since $f^{-1}(\{c\}) = \{c\}$ is not $(1,2)^{\star}$-$g^{\#}$-closed in X.*

Proposition 3.15. *Every $(1,2)^{\star}$-$g^{\#}$-continuous function is $(1,2)^{\star}$-gsp-continuous but not conversely.*

Example 3.16. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{b\}, X\}$. Then the sets in $\{\phi, \{b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{a, b\}, Y\}$. Then the sets in $\{\phi, \{a, b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^{\star}$-$G^{\#}C(X) = \{\phi, \{a, c\}, X\}$ and $(1,2)^{\star}$-$GSPC(X) = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^{\star}$-gsp-continuous but not $(1,2)^{\star}$-$g^{\#}$-continuous, since $f^{-1}(\{c\}) = \{c\}$ is not $(1,2)^{\star}$-$g^{\#}$-closed in X.*

Proposition 3.17. *Every $(1,2)^{\star}$-$g^{\#}$-continuous function is $(1,2)^{\star}$-sg-continuous but not conversely.*

Example 3.18. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{a, b\}, Y\}$. Then the sets in $\{\phi, \{a, b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^{\star}$-$G^{\#}C(X) = \{\phi, \{a\}, \{b, c\}, X\}$ and $(1,2)^{\star}$-$SGC(X) = P(X)$. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^{\star}$-sg-continuous but not $(1,2)^{\star}$-$g^{\#}$-continuous, since $f^{-1}(\{c\}) = \{c\}$ is not $(1,2)^{\star}$-$g^{\#}$-closed in X.*

Remark 3.19. *The following examples show that $(1,2)^{\star}$-$g^{\#}$-continuity is independent of $(1,2)^{\star}$-α-continuity and $(1,2)^{\star}$-semi-continuity.*

Example 3.20. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{a, b\}, X\}$. Then the sets in $\{\phi, \{a, b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{a\}, Y\}$. Then the sets in $\{\phi, \{a\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{b, c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^{\star}$-$G^{\#}C(X) = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$ and $(1,2)^{\star}$-$\alpha C(X) = (1,2)^{\star}$-$SC(X) = \{\phi, \{c\}, X\}$. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^{\star}$-$g^{\#}$-continuous but it is neither $(1,2)^{\star}$-α-continuous nor $(1,2)^{\star}$-semi-continuous, since $f^{-1}(\{b, c\}) = \{b, c\}$ is neither $(1,2)^{\star}$-α-closed nor $(1,2)^{\star}$-semi-closed in X.*

Example 3.21. *In Example 3.14, we have $(1,2)^{\star}$-$G^{\#}C(X) = \{\phi, \{b, c\}, X\}$ and $(1,2)^{\star}$-$\alpha C(X) = (1,2)^{\star}$-$SC(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, X\}$. Let $f : X \to Y$ be the identity function. Then f is both $(1,2)^{\star}$-α-continuous and $(1,2)^{\star}$-semi-continuous but it is not $(1,2)^{\star}$-$g^{\#}$-continuous, since $f^{-1}(\{c\}) = \{c\}$ is not $(1,2)^{\star}$-$g^{\#}$-closed in X.*

Proposition 3.22. *A function $f : X \to Y$ is $(1,2)^{\star}$-$g^{\#}$-continuous if and only if $f^{-1}(U)$ is $(1,2)^{\star}$-$g^{\#}$-open in X for every $\sigma_{1,2}$-open set U in Y.*

Proof. Let $f : X \to Y$ be $(1,2)^{\star}$-$g^{\#}$-continuous and U be an $\sigma_{1,2}$-open set in Y. Then U^c is $\sigma_{1,2}$-closed in Y and since f is $(1,2)^{\star}$-$g^{\#}$-continuous, $f^{-1}(U^c)$ is $(1,2)^{\star}$-$g^{\#}$-closed in X. But $f^{-1}(U^c) = (f^{-1}(U))^c$ and so $f^{-1}(U)$ is $(1,2)^{\star}$-$g^{\#}$-open in X.

Conversely, assume that $f^{-1}(U)$ is $(1,2)^{\star}$-$g^{\#}$-open in X for each $\sigma_{1,2}$-open set U in Y. Let F be a $\sigma_{1,2}$-closed set in Y. Then F^c is $\sigma_{1,2}$-open in Y and by assumption, $f^{-1}(F^c)$ is $(1,2)^{\star}$-$g^{\#}$-open in X. Since $f^{-1}(F^c) = (f^{-1}(F))^c$, we have $f^{-1}(F)$ is $(1,2)^{\star}$-$g^{\#}$-closed in X and so f is $(1,2)^{\star}$-$g^{\#}$-continuous.

Remark 3.23. *The composition of two $(1,2)^{\star}$-$g^{\#}$-continuous functions need not be a $(1,2)^{\star}$-$g^{\#}$-continuous function as is shown in the following example.*

Example 3.24. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{a, c\}, X\}$ and $\tau_2 = \{\phi, \{a, b\}, X\}$. Then the sets in $\{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{b\}, \{c\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{a, b\}, Y\}$. Then the sets in $\{\phi, \{a, b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{c\}, Y\}$ are called $\sigma_{1,2}$-closed. Let $Z = \{a, b, c\}$, $\eta_1 = \{\phi, Z\}$ and $\eta_2 = \{\phi, \{b\}, Z\}$. Then the sets in $\{\phi, \{b\}, Z\}$ are called $\eta_{1,2}$-open and the sets in $\{\phi, \{a, c\}, Z\}$ are called $\eta_{1,2}$-closed. Let $f : X \to Y$ and $g : Y \to Z$ be the identity functions. Then f and g are $(1,2)^{\star}$-$g^{\#}$-continuous but their $g \ o \ f : X \to Z$ is not $(1,2)^{\star}$-$g^{\#}$-continuous, since for the set $V = \{a, c\}$ is $\eta_{1,2}$-closed in Z, $(g \ o \ f)^{-1}(V) = f^{-1}(g^{-1}(V)) = f^{-1}(g^{-1}(\{a, c\})) = f^{-1}(\{a, c\}) = \{a, c\}$ is not $(1,2)^{\star}$-$g^{\#}$-closed in X.*

Proposition 3.25. *Let X and Z be bitopological spaces and Y be a $(1,2)^{\star}$-$T_g^{\#}$-space. Then the composition $g \ o \ f : X \to Z$ of the $(1,2)^{\star}$-$g^{\#}$-continuous functions $f : X \to Y$ and $g : Y \to Z$ is $(1,2)^{\star}$-$g^{\#}$-continuous.*

Proof. Let F be any $\eta_{1,2}$-closed set of Z. Then $g^{-1}(F)$ is $(1,2)^{\star}$-$g^{\#}$-closed in Y, since g is $(1,2)^{\star}$-$g^{\#}$-continuous. Since Y is a $(1,2)^{\star}$-$T_g^{\#}$-space, $g^{-1}(F)$ is $\sigma_{1,2}$-closed in Y. Since f is $(1,2)^{\star}$-$g^{\#}$-continuous, $f^{-1}(g^{-1}(F))$ is $(1,2)^{\star}$-$g^{\#}$-closed in X. But $f^{-1}(g^{-1}(F)) = (g \ o \ f)^{-1}(F)$ and so $g \ o$ f is $(1,2)^{\star}$-$g^{\#}$-continuous.

Proposition 3.26. *Let X and Z be bitopological spaces and Y be a $(1,2)^{\star}$-$T_{1/2}$-space (resp. $(1,2)^{\star}$-T_b-space, $(1,2)^{\star}$-${}_{\alpha}T_b$-space). Then the composition $g \ o \ f : X \to Z$ of the $(1,2)^{\star}$-$g^{\#}$-continuous function f $: X \to Y$ and the $(1,2)^{\star}$-g-continuous (resp. $(1,2)^{\star}$-gs-continuous, $(1,2)^{\star}$-αg-continuous) function $g : Y \to Z$ is $(1,2)^{\star}$-$g^{\#}$-continuous.*

Proof. Similar to Proposition 3.25.

Proposition 3.27. *If $f : X \to Y$ is $(1,2)^{\star}$-$g^{\#}$-continuous and $g : Y \to Z$ is $(1,2)^{\star}$-continuous, then their composition $g \ o \ f : X \to Z$ is $(1,2)^{\star}$-$g^{\#}$-continuous.*

Proof. Let F be any $\eta_{1,2}$-closed set in Z. Since $g : Y \to Z$ is $(1,2)^{\star}$-continuous, $g^{-1}(F)$ is $\sigma_{1,2}$-closed in Y. Since $f : X \to Y$ is $(1,2)^{\star}$-$g^{\#}$-continuous, $f^{-1}(g^{-1}(F)) = (g \ o \ f)^{-1}(F)$ is $(1,2)^{\star}$-$g^{\#}$-closed in X and so $g \ o$ f is $(1,2)^{\star}$-$g^{\#}$-continuous.

Proposition 3.28. *Let A be $(1,2)^{\star}$-$g^{\#}$-closed in X. If $f : X \to Y$ is $(1,2)^{\star}$-αg-irresolute and $(1,2)^{\star}$-closed, then f(A) is $(1,2)^{\star}$-$g^{\#}$-closed in Y.*

Proof. Let U be any $(1,2)^{\star}$-αg-open in Y such that $f(A) \subseteq U$. Then $A \subseteq f^{-1}(U)$ and by hypothesis, $\tau_{1,2}$-cl(A) $\subseteq f^{-1}(U)$. Thus $f(\tau_{1,2}\text{-cl}(A)) \subseteq U$ and $f(\tau_{1,2}\text{-cl}(A))$ is a $\sigma_{1,2}$-closed set. Now, $\sigma_{1,2}\text{-cl}(f(A)) \subseteq \sigma_{1,2}\text{-cl}(f(\tau_{1,2}\text{-cl}(A))) = f(\tau_{1,2}\text{-cl}(A)) \subseteq U$. i.e., $\sigma_{1,2}\text{-cl}(f(A)) \subseteq U$ and so f(A) is $(1,2)^{\star}$-$g^{\#}$-closed in Y.

Theorem 3.29. *Let $f : X \to Y$ be a pre-$(1,2)^{\star}$-αg-closed and $(1,2)^{\star}$-open bijection. If X is a $(1,2)^{\star}$-$T_{g\#}$-space, then Y is also a $(1,2)^{\star}$-$T_{g\#}$-space.*

Proof. Let $y \in Y$. Since f is bijective, $y = f(x)$ for some $x \in X$. Since X is a $(1,2)^{\star}$-$T_{g\#}$-space, {x} is $(1,2)^{\star}$-αg-closed or $\tau_{1,2}$-open by Theorem 2.13. If {x} is $(1,2)^{\star}$-αg-closed then $\{y\} = f(\{x\})$ is $(1,2)^{\star}$-αg-closed, since f is pre-$(1,2)^{\star}$-αg-closed. Also {y} is $\sigma_{1,2}$-open if {x} is $\tau_{1,2}$-open since f is $(1,2)^{\star}$-open. Therefore by Theorem 2.13, Y is a $(1,2)^{\star}$-$T_{g\#}$-space.

Theorem 3.30. *If f : X → Y is $(1,2)^\star$-$g^\#$-continuous and pre-$(1,2)^\star$-αg-closed and if A is an $(1,2)^\star$-$g^\#$-open (or $(1,2)^\star$-$g^\#$-closed) subset of Y, then $f^{-1}(A)$ is $(1,2)^\star$-$g^\#$-open (or $(1,2)^\star$-$g^\#$-closed) in X.*

Proof. Let A be an $(1,2)^\star$-$g^\#$-open set in Y and F be any $(1,2)^\star$-αg-closed set in X such that F ⊆ $f^{-1}(A)$. Then f(F) ⊆ A. By hypothesis, f(F) is $(1,2)^\star$-αg-closed and A is $(1,2)^\star$-$g^\#$-open in Y. Therefore, f(F) ⊆ $\sigma_{1,2}$-int(A) by Theorem 2.12, and so F ⊆ $f^{-1}(\sigma_{1,2}\text{-int(A)})$. Since f is $(1,2)^\star$-$g^\#$-continuous and $\sigma_{1,2}$-int(A) is $\sigma_{1,2}$-open in Y, $f^{-1}(\sigma_{1,2}\text{-int(A)})$ is $(1,2)^\star$-$g^\#$-open in X. Thus F ⊆ $\tau_{1,2}$-int($f^{-1}(\sigma_{1,2}$-int(A))) ⊆ $\tau_{1,2}$-int(f^{-1}(A)). i.e., F ⊆ $\tau_{1,2}$-int(f^{-1}(A)) and by Theorem 2.12, f^{-1}(A) is $(1,2)^\star$-$g^\#$-open in X. By taking complements, we can show that if A is $(1,2)^\star$-$g^\#$-closed in Y, f^{-1}(A) is $(1,2)^\star$-$g^\#$-closed in X.

Corollary 3.31. *If f : X → Y is $(1,2)^\star$-continuous and pre-$(1,2)^\star$-αg-closed and if B is a $(1,2)^\star$-$g^\#$-closed (or $(1,2)^\star$-$g^\#$-open) subset of Y, then $f^{-1}(B)$ is $(1,2)^\star$-$g^\#$-closed (or $(1,2)^\star$-$g^\#$-open) in X.*

Proof. Follows from Proposition 3.3, and Theorem 3.30.

Corollary 3.32. *Let X, Y and Z be any three bitopological spaces. If f : X → Y is $(1,2)^\star$-$g^\#$-continuous and pre-$(1,2)^\star$-αg-closed and g : Y → Z is $(1,2)^\star$-$g^\#$-continuous, then their composition g o f : X → Z is $(1,2)^\star$-$g^\#$-continuous.*

Proof. Let F be any $\eta_{1,2}$-closed set in Z. Since g : Y → Z is $(1,2)^\star$-$g^\#$-continuous, g^{-1}(F) is $(1,2)^\star$-$g^\#$-closed in Y. Since f : X → Y is $(1,2)^\star$-$g^\#$-continuous and pre-$(1,2)^\star$-αg-closed, by Theorem 3.30, $f^{-1}(g^{-1}(F))$ = (g *o* f) $^{-1}$(F) is $(1,2)^\star$-$g^\#$-closed in X and so *g o* f is $(1,2)^\star$-$g^\#$-continuous.

4 $(1,2)^\star$-$g^\#$-Irresolute Functions

We introduce the following definition.

Definition 4.1. *A function f : X → Y is called an $(1,2)^\star$-$g^\#$-irresolute if the inverse image of every $(1,2)^\star$-$g^\#$-closed set in Y is $(1,2)^\star$-$g^\#$-closed in X.*

Remark 4.2. *The following examples show that the notions of $(1,2)^\star$-sg-irresolute functions and $(1,2)^\star$-$g^\#$-irresolute functions are independent.*

Example 4.3. *Let X = {a, b, c}, τ_1 = {φ, X} and τ_2 = {φ, {a, b}, X}. Then the sets in {φ, {a, b}, X} are called $\tau_{1,2}$-open and the sets in {φ, {c}, X} are called $\tau_{1,2}$-closed. Let Y = {a, b, c}, σ_1 = {φ, {a}, {a, b}, Y} and σ_2 = {φ, {b}, Y}. Then the sets in {φ, {a}, {b}, {a, b}, Y} are called $\sigma_{1,2}$-open and the sets in {φ, {c}, {a, c}, {b, c}, Y} are called $\sigma_{1,2}$-closed. We have $(1,2)^\star$-$G^\# C(X)$ = {φ, {c}, {a, c}, {b, c}, X}, $(1,2)^\star$-SGC(X) = {φ, {c}, {a, c}, {b, c}, X}, $(1,2)^\star$-$G^\# C(Y)$ = {φ, {c}, {a, c}, {b, c}, Y} and $(1,2)^\star$-SGC(Y) = {φ, {a}, {b}, {c}, {a, c}, {b, c}, Y}. Let f : X → Y be the identity function. Then f is $(1,2)^\star$-$g^\#$-irresolute but it is not $(1,2)^\star$-sg-irresolute, since $f^{-1}(\{b\}) = \{b\}$ is not $(1,2)^\star$-sg-closed in X.*

Example 4.4. *Let X = {a, b, c}, τ_1 = {φ, {a}, {a, b}, X} and τ_2 = {φ, {b}, X}. Then the sets in {φ, {a}, {b}, {a, b}, X} are called $\tau_{1,2}$-open and the sets in {φ, {c}, {a, c}, {b, c}, X} are called $\tau_{1,2}$-closed. Let Y = {a, b, c}, σ_1 = {φ, {b}, Y} and σ_2 = {φ, {b, c}, Y}. Then the sets in {φ, {b}, {b, c}, Y} are called $\sigma_{1,2}$-open and the sets in {φ, {a}, {a, c}, Y} are called $\sigma_{1,2}$-closed. We have $(1,2)^\star$-$G^\# C(X)$ = {φ, {c}, {a, c}, {b, c}, X} and $(1,2)^\star$-SGC(X) = {φ, {a}, {b}, {c}, {a, c}, {b, c}, X}, $(1,2)^\star$-$G^\# C(Y)$ = {φ, {a}, {a, c}, Y} and $(1,2)^\star$-SGC(Y) = {φ, {a}, {c}, {a, c}, Y}. Let f : X → Y be the identity function. Then f is $(1,2)^\star$-sg-irresolute but it is not $(1,2)^\star$-$g^\#$-irresolute, since $f^{-1}(\{a\}) = \{a\}$ is not $(1,2)^\star$-$g^\#$-closed in X.*

Proposition 4.5. *A function f : X → Y is $(1,2)^\star$-$g^\#$-irresolute if and only if the inverse of every $(1,2)^\star$-$g^\#$-open set in Y is $(1,2)^\star$-$g^\#$-open in X.*

Proof. Similar to Proposition 3.22.

Proposition 4.6. *If a function f : X → Y is $(1,2)^\star$-$g^\#$-irresolute then it is $(1,2)^\star$-$g^\#$-continuous but not conversely.*

Example 4.7. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{b\}, X\}$. Then the sets in $\{\phi, \{b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{a, b\}, Y\}$. Then the sets in $\{\phi, \{a, b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^\star$-$G^\# C(X) = \{\phi, \{a, c\}, X\}$ and $(1,2)^\star$-$G^\# C(Y) = \{\phi, \{c\}, \{a, c\}, \{b, c\}, Y\}$. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^\star$-$g^\#$-continuous but it is not $(1,2)^\star$-$g^\#$-irresolute, since $f^{-1}(\{a\}) = \{a\}$ is not $(1,2)^\star$-$g^\#$-open in X.*

Proposition 4.8. *Let X be any bitopological space, Y be a $(1,2)^\star$-$T_{g^\#}$-space and $f : X \to Y$ be a function. Then the following are equivalent:*

1. *f is $(1,2)^\star$-$g^\#$-irresolute.*
2. *f is $(1,2)^\star$-$g^\#$-continuous.*

Proof. (1) ⇒ (2) Follows from Proposition 4.6.

(2) ⇒ (1) Let F be a $(1,2)^\star$-$g^\#$-closed set in Y. Since Y is a $(1,2)^\star$-$T_{g^\#}$-space, F is a $\sigma_{1,2}$-closed set in Y and by hypothesis, $f^{-1}(F)$ is $(1,2)^\star$-$g^\#$-closed in X. Therefore f is $(1,2)^\star$-$g^\#$-irresolute.

Definition 4.9. *A function $f : X \to Y$ is called pre-$(1,2)^\star$-αg-open if f(U) is $(1,2)^\star$-αg-open in Y, for each $(1,2)^\star$-αg-open set U in X.*

Proposition 4.10. *If $f : X \to Y$ is bijective pre-$(1,2)^\star$-αg-open and $(1,2)^\star$-$g^\#$-continuous then f is $(1,2)^\star$-$g^\#$-irresolute.*

Proof. Let A be $(1,2)^\star$-$g^\#$-closed set in Y. Let U be any $(1,2)^\star$-αg-open set in X such that $f^{-1}(A) \subseteq U$. Then $A \subseteq f(U)$. Since A is $(1,2)^\star$-$g^\#$-closed and f(U) is $(1,2)^\star$-αg-open in Y, $\sigma_{1,2}$-cl(A) $\subseteq$ f(U) holds and hence $f^{-1}(\sigma_{1,2}\text{-cl}(A)) \subseteq U$. Since f is $(1,2)^\star$-$g^\#$-continuous and $\sigma_{1,2}$-cl(A) is $\sigma_{1,2}$-closed in Y, $f^{-1}(\sigma_{1,2}\text{-cl}(A))$ is $(1,2)^\star$-$g^\#$-closed and hence $\tau_{1,2}\text{-cl}(f^{-1}(\sigma_{1,2}\text{-cl}(A))) \subseteq U$ and so $\tau_{1,2}\text{-cl}(f^{-1}(A)) \subseteq U$. Therefore, $f^{-1}(A)$ is $(1,2)^\star$-$g^\#$-closed in X and hence f is $(1,2)^\star$-$g^\#$-irresolute.

The following examples show that no assumption of Proposition 4.10 can be removed.

Example 4.11. *The identity function defined in Example 4.7 is $(1,2)^\star$-$g^\#$-continuous and bijective but not pre-$(1,2)^\star$-αg-open and so f is not $(1,2)^\star$-$g^\#$-irresolute.*

Example 4.12. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{a, b\}, X\}$ and $\tau_2 = \{\phi, \{b\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, \{a\}, Y\}$ and $\sigma_2 = \{\phi, \{b, c\}, Y\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{a\}, \{b, c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^\star$-$G^\# C(X) = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$ and $(1,2)^\star$-$SGC(X) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$, $(1,2)^\star$-$G^\# C(Y) = \{\phi, \{a\}, \{b, c\}, Y\}$ and $(1,2)^\star$-$SGC(Y) = P(Y)$. Let $f : X \to Y$ be the identity function. Then f is bijective and pre-$(1,2)^\star$-αg-open but not $(1,2)^\star$-$g^\#$-continuous and so f is not $(1,2)^\star$-$g^\#$-irresolute, since $f^{-1}(\{a\}) = \{a\}$ is not $(1,2)^\star$-$g^\#$-closed in X.*

Proposition 4.13. *If $f : X \to Y$ is bijective $(1,2)^\star$-closed and $(1,2)^\star$-αg-irresolute then the inverse function $f^{-1} : Y \to X$ is $(1,2)^\star$-$g^\#$-irresolute.*

Proof. Let A be $(1,2)^\star$-$g^\#$-closed in X. Let $(f^{-1})^{-1}(A) = f(A) \subseteq U$ where U is $(1,2)^\star$-αg-open in Y. Then $A \subseteq f^{-1}(U)$ holds. Since $f^{-1}(U)$ is $(1,2)^\star$-αg-open in X and A is $(1,2)^\star$-$g^\#$-closed in X, $\tau_{1,2}$-cl(A) $\subseteq f^{-1}(U)$ and hence $f(\tau_{1,2}\text{-cl}(A)) \subseteq U$. Since f is $(1,2)^\star$-closed and $\tau_{1,2}$-cl(A) is $\tau_{1,2}$-closed in X, $f(\tau_{1,2}\text{-cl}(A))$ is $\sigma_{1,2}$-closed in Y and so $f(\tau_{1,2}\text{-cl}(A))$ is $(1,2)^\star$-$g^\#$-closed in Y. Therefore $\sigma_{1,2}\text{-cl}(f(\tau_{1,2}\text{-cl}(A))) \subseteq U$ and hence $\sigma_{1,2}\text{-cl}(f(A)) \subseteq U$. Thus f(A) is $(1,2)^\star$-$g^\#$-closed in Y and so f^{-1} is $(1,2)^\star$-$g^\#$-irresolute.

5 Applications

To obtain a decomposition of $(1,2)^\star$-continuity, we introduce the notion of $(1,2)^\star$-$\alpha glc^\#$-continuous function in bitopological spaces and prove that a function is $(1,2)^\star$-continuous if and only if it is both $(1,2)^\star$-$g^\#$-continuous and $(1,2)^\star$-$\alpha glc^\#$-continuous.

Definition 5.1. *A subset A of a bitopological space X is called $(1,2)^\star$-$\alpha glc^\star$-set if $A = M \cap N$, where M is $(1,2)^\star$-αg-open and N is $\tau_{1,2}$-closed in X.*

The family of all $(1,2)^{\star}$-$\alpha glc^{\star}$-sets in a space X is denoted by $(1,2)^{\star}$-$\alpha glc^{\star}$(X).

Example 5.2. *Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\phi,\ X\}$ and $\tau_2 = \{\phi,\ \{c\},\ X\}$. Then the sets in $\{\phi,\ \{c\},\ X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi,\ \{a,\ b\},\ X\}$ are called $\tau_{1,2}$-closed. Then $\{a\}$ is $(1,2)^{\star}$-$\alpha glc^{\star}$-set in X.*

Remark 5.3. *Every $\tau_{1,2}$-closed set is $(1,2)^{\star}$-$\alpha glc^{\star}$-set but not conversely.*

Example 5.4. *Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\phi,\ X\}$ and $\tau_2 = \{\phi,\ \{a\},\ X\}$. Then the sets in $\{\phi,\ \{a\},\ X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi,\ \{b,\ c\},\ X\}$ are called $\tau_{1,2}$-closed. Then $\{a,\ b\}$ is $(1,2)^{\star}$-$\alpha glc^{\star}$-set but not $\tau_{1,2}$-closed in X.*

Remark 5.5. *$(1,2)^{\star}$-$g^{\#}$-closed sets and $(1,2)^{\star}$-$\alpha glc^{\star}$-sets are independent of each other.*

Example 5.6. *Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\phi,\ X\}$ and $\tau_2 = \{\phi,\ \{a,\ c\},\ X\}$. Then the sets in $\{\phi,\ \{a,\ c\},\ X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi,\ \{b\},\ X\}$ are called $\tau_{1,2}$-closed. Then $\{b,\ c\}$ is a $(1,2)^{\star}$-$g^{\#}$-closed set but not $(1,2)^{\star}$-$\alpha glc^{\star}$-set in X.*

Example 5.7. *Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\phi,\ X\}$ and $\tau_2 = \{\phi,\ \{b\},\ X\}$. Then the sets in $\{\phi,\ \{b\},\ X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi,\ \{a,\ c\},\ X\}$ are called $\tau_{1,2}$-closed. Then $\{a,\ b\}$ is an $(1,2)^{\star}$-$\alpha glc^{\star}$-set but not $(1,2)^{\star}$-$g^{\#}$-closed set in X.*

Proposition 5.8. *Let X be a bitopological space. Then a subset A of X is $\tau_{1,2}$-closed if and only if it is both $(1,2)^{\star}$-$g^{\#}$-closed and $(1,2)^{\star}$-$\alpha glc^{\star}$-set.*

Proof. Necessity is trivial. To prove the sufficiency, assume that A is both $(1,2)^{\star}$-$g^{\#}$-closed and $(1,2)^{\star}$-$\alpha glc^{\star}$-set. Then A = M ∩ N, where M is $(1,2)^{\star}$-αg-open and N is $\tau_{1,2}$-closed in X. Therefore, A ⊆ M and A ⊆ N and so by hypothesis, $\tau_{1,2}$-cl(A) ⊆ M and $\tau_{1,2}$-cl(A) ⊆ N. Thus $\tau_{1,2}$-cl(A) ⊆ M ∩ N = A and hence $\tau_{1,2}$-cl(A) = A i.e., A is $\tau_{1,2}$-closed in X.

We introduce the following definition.

Definition 5.9. *A function $f : X \to Y$ is said to be $(1,2)^{\star}$-$\alpha glc^{\#}$-continuous if for each $\sigma_{1,2}$-closed set V of Y, $f^{-1}(V)$ is an $(1,2)^{\star}$-$\alpha glc^{\star}$-set in X.*

Example 5.10. *Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\phi,\ X\}$ and $\tau_2 = \{\phi,\ \{a\},\ X\}$. Then the sets in $\{\phi,\ \{a\},\ X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi,\ \{b,\ c\},\ X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a,\ b,\ c\}$, $\sigma_1 = \{\phi,\ \{a\},\ Y\}$ and $\sigma_2 = \{\phi,\ \{b,\ c\},\ Y\}$. Then the sets in $\{\phi,\ \{a\},\ \{b,\ c\},\ Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi,\ \{a\},\ \{b,\ c\},\ Y\}$ are called $\sigma_{1,2}$-closed. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^{\star}$-$\alpha glc^{\#}$-continuous function.*

Remark 5.11. *From the definitions it is clear that every $(1,2)^{\star}$-continuous function is $(1,2)^{\star}$-$\alpha glc^{\#}$-continuous but not conversely.*

Example 5.12. *Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\phi,\ X\}$ and $\tau_2 = \{\phi,\ \{b\},\ X\}$. Then the sets in $\{\phi,\ \{b\},\ X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi,\ \{a,\ c\},\ X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a,\ b,\ c\}$, $\sigma_1 = \{\phi,\ \{b\},\ Y\}$ and $\sigma_2 = \{\phi,\ \{a,\ c\},\ Y\}$. Then the sets in $\{\phi,\ \{b\},\ \{a,\ c\},\ Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi,\ \{b\},\ \{a,\ c\},\ Y\}$ are called $\sigma_{1,2}$-closed. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^{\star}$-$\alpha glc^{\#}$-continuous function but not $(1,2)^{\star}$-continuous. Since for the $\sigma_{1,2}$-closed set $\{b\}$ in Y, $f^{-1}(\{b\}) = \{b\}$, which is not $\tau_{1,2}$-closed in X.*

Remark 5.13. *$(1,2)^{\star}$-$g^{\#}$-continuity and $(1,2)^{\star}$-$\alpha glc^{\#}$-continuity are independent of each other.*

Example 5.14. *Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\phi,\ X\}$ and $\tau_2 = \{\phi,\ \{a,\ b\},\ X\}$. Then the sets in $\{\phi,\ \{a,\ b\},\ X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi,\ \{c\},\ X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a,\ b,\ c\}$, $\sigma_1 = \{\phi,\ Y\}$ and $\sigma_2 = \{\phi,\ \{a\},\ Y\}$. Then the sets in $\{\phi,\ \{a\},\ Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi,\ \{b,\ c\},\ Y\}$ are called $\sigma_{1,2}$-closed. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^{\star}$-$g^{\#}$-continuous function but not $(1,2)^{\star}$-$\alpha glc^{\#}$-continuous.*

Example 5.15. *Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\phi,\ X\}$ and $\tau_2 = \{\phi,\ \{a\},\ X\}$. Then the sets in $\{\phi,\ \{a\},\ X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi,\ \{b,\ c\},\ X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a,\ b,\ c\}$, $\sigma_1 = \{\phi,\ Y\}$ and $\sigma_2 = \{\phi,\ \{b,\ c\},\ Y\}$. Then the sets in $\{\phi,\ \{b,\ c\},\ Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi,\ \{a\},\ Y\}$ are called $\sigma_{1,2}$-closed. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^{\star}$-$\alpha glc^{\#}$-continuous function but not $(1,2)^{\star}$-$g^{\#}$-continuous.*

We have the following decomposition for $(1,2)^{\star}$-continuity.

Theorem 5.16. *A function $f : X \rightarrow Y$ is $(1,2)^{\star}$-continuous if and only if it is both $(1,2)^{\star}$-$g^{\#}$-continuous and $(1,2)^{\star}$-$\alpha glc^{\#}$-continuous.*

Proof. Assume that f is $(1,2)^{\star}$-continuous. Then by Proposition 3.3 and Remark 5.11, f is both $(1,2)^{\star}$-$g^{\#}$-continuous and $(1,2)^{\star}$-$\alpha glc^{\#}$-continuous.

Conversely, assume that f is both $(1,2)^{\star}$-$g^{\#}$-continuous and $(1,2)^{\star}$-$\alpha glc^{\#}$-continuous. Let V be a $\sigma_{1,2}$-closed subset of Y. Then $f^{-1}(V)$ is both $(1,2)^{\star}$-$g^{\#}$-closed set and $(1,2)^{\star}$-$\alpha glc^{\star}$-set. By Proposition 5.8, $f^{-1}(V)$ is a $\tau_{1,2}$-closed set in X and so f is $(1,2)^{\star}$-continuous.

6 Conclusion

The notions of the sets, functions and spaces in bitopological spaces are highly developed and used extensively in many practical and engineering problems, computational topology for geometric design, computer-aided geometric design, engineering design research and mathematical sciences. Also, topology plays a significant role in space time geometry and high-energy physics. Thus generalized continuity is one of the most important subjects on topological spaces. Hence we studied new types of generalizations of non-continuous functions, obtained some of their properties in bitopological spaces.

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