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General Aspects of Evolutionary Structural Optimization: A Review

Evrimsel Yapı Optimizasyonuna Genel Bir Bakış

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ABSTRACT

Evolutionary structural optimization (ESO) method has been presented by Xie and Steven in 1993 to deal with numerical structural topology optimization problems. Although ESO has appeared on a simple foundation, many contributions have been made by many researchers up to now. ESO has an algorithm -sometimes defined as intuitive- which is running by removing the elements that have lower design values (stress, strain energy, i.e.) from the design domain and attaining the shape and topology of a more fully stressed structure. Such a process is carried out repeatedly until an optimum design is achieved and final decision is made by evaluating the applicability of the last design formed after the process. This study is based on the general aspects of ESO to give information about the development and a clear explanation of the ESO procedure. Also two examples that have part in the literature have been presented to demonstrate the capability of this method.

Keywords : *Structural optimization, Finite element methods, Evolutionary method, Computer applications.*

ÖZET

Sayısal tabanlı yapısal topoloji optimizasyonu problemlerinin üstesinden gelmek için 1993 yılında Xie ve Steven tarafından evrimsel yapı optimizasyonu (ESO) yöntemi geliştirilmiştir. Yöntem, esasında basit bir temel üzerine oturtulmuş olmasına rağmen günümüze dek birçok araştırmacı tarafından katkı sunulmuştur. Zaman zaman sezgisel olarak tanımlansa da düşük tasarım değerlerine (gerilme, şekil değiştirme enerjisi, v.s.) sahip elemanların tasarım alanından kaldırılması ile çalışan ve nispeten tam gerilmeli duruma sahip şekil ve topolojileri hedefleyen bir algoritmaya sahiptir. Bu işlem, uygun bir tasarım elde edilene dek tekrarlı olarak gerçekleştirilmekte ve işlem sonucunda ulaşılan nihai tasarımın uygulanabilirliğini değerlendirerek gerçekleştirilmektedir. Bu çalışma ile evrimsel yapı optimizasyonu, genel bir bakış açısı ile incelenmekte ve söz konusu yöntemin bugüne dek gelişimi ortaya konmaktadır. Ayrıca söz konusu yöntemin başarısını göstermek için literatürde yer alan iki örnek çalışma da sunulmaktadır.

Anahtar Kelimeler : *Yapısal optimizasyon, Sonlu elemanlar yöntemi, Evrimsel yöntem, Bilgisayar uygulamaları.*

1. INTRODUCTION

Structural optimization is a fusion in the areas of engineering, mathematics, science and technology that has the goal of achieving the best performance for a structure, be it a bridge or a space vehicle or a spectacle frame (Querin et al., 2000). In the past two decades, significant progress has been made in the area of structural optimization, which aims at achieving the best structural performance by appropriate material distribution (Li et al., 1999).

Mathematical methods for structural optimization and shape optimization have been developed within

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the last 30 years (Uysal et al., 2007). To avoid the complexities of traditional continuous and discrete methods and to perform a simultaneous size, shape and topology optimization, evolutionary structural optimization (ESO) has been developed by Y.M. Xie and G.P. Steven in 1993. Figure 1 (Lee and Hinton, 2000) which illustrates the types of structural optimization will be helpful to understand the benefits of such a design method. ESO method offers a new approach to structural optimization, which overcomes most of the problems associated with traditional techniques. ESO is based on the simple idea that the optimal structure (maximum stiffness, minimum weight, i.e.) can be produced by gradually removing the ineffectively used material from the design domain. The design domain is constructed by the FE method, and furthermore, external loads and support conditions are applied to the element model. Considering the engineering aspects, ESO seems to have some attractive features: the ESO method is very simple to program via the finite element analysis (FEA) packages and requires a relatively small amount of FEA time. Additionally, the ESO topologies have been compared with analytical ones, e.g. Michell trusses, and so far the results are quite promising (Chu et al., 1997).

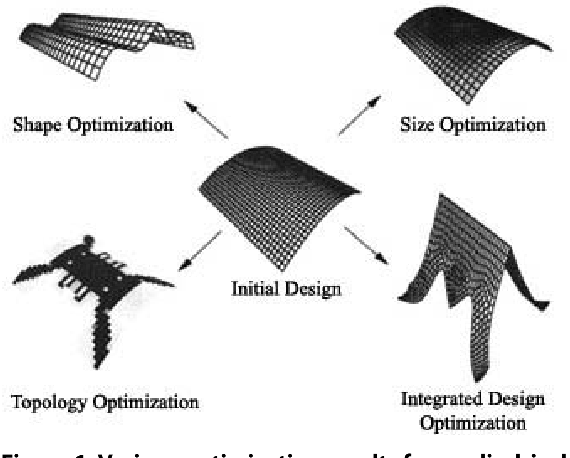

Figure 1. Various optimization results for a cylindrical shell.

By observing the evolution of naturally occurring structures such as shells, bones and trees, it becomes obvious that the topology and the shape of such structures achieve their optimum over a long evolutionary period and adapt to whatever environment they find themselves in (Xie and Steven, 1993). By some researchers, it has been assumed that ESO works by attempting to imitate the growth of biological structures in nature. It is observed that naturally occurring species tend to achieve shapes that are close to 'fully stressed' configurations as this leads to optimum material utilization. This tendency may be realized by some of the topologies which have been optimized by ESO. ESO removes inefficient material from the structure based on certain predefined criteria. Here the term 'inefficient' means that the material is not contributing effectively to the overall performance of the structure. ESO can be used with various design objective functions and constraints such as stress, stiffness, displacement, frequency, buckling load, moment of inertia and thermal parameters may be imposed upon a structure and finite element method (FEM) is generally used for evaluating the structural response (Das et al., 2005).

This paper presents a general definition of evolutionary structural optimization which aims to reach the most acceptable topology while seeking a lighter and relatively a fully stressed design. ESO has been improved too much for the last 10 years and a work summing this progress up is a necessity.

2. EVOLUTIONARY STRUCTURAL OPTIMIZATION

At the end of the 19th century and the turn of the 20th century, came the capability of engineers to combine optimization principles and analytical prowess (Proos, 2002). Topology optimization was pioneered by Michell, who studied statically determinate trusses for a number of loading and support conditions. His analytical results, so-called Michell trusses, have an infinite number of members of varying length. In Michell trusses, each bar is subjected to a constant strain (stress) (Tanskanen, 2002). Michell theory plays an important role in structural topology optimization. Most numerical studies of topological optimization like Rozvany's paper (1998) use these classical Michell trusses for verifying their results (Zhou et al., 2004).

Considerable attention has recently been paid to the work of Bendsøe and Kikuchi who first introduced the so-called homogenization method (Bendsøe and Kikuchi, 1988). Here the design domain is constructed from a finite number of cells, each of which can have individual microstructure, and furthermore, each cell can have either material or a rectangular void (Tanskanen, 2002). Aim for the optimization problem is seeking optimal porosity of the structure. Homogenization and ESO methods have a kind of partnership on the focus of this porosity idea.

ESO is an effective tool that is capable of handling topology optimization. It is a heuristic process that uses discrete finite elements as its foundation. It uses the finite element method as its analysis engine. Its approach to optimizing a structure is to remove inefficient elements iteratively, which has been set up in a particular environment of loads, constraints and/or restraints. Here "inefficiency" is a very general term, meaning the sensitivity of the alteration of an element in a finite element mesh to the optimality criterion. This sensitivity can be a composite of several performance measures and the optimality criterion can be a composite of several individual physical criteria. Much work has been done on ESO where many detailed studies have established systematic rules that make the method work for a full range of structural situations (Proos, 2002).

Also, in many design assignments, internal cavities are not allowed and the designer can only modify the structural boundaries. These are traditionally classified as shape optimization problems (Xie and Steven, 1997). This implies that the topology of the structure is given and only the shape of the boundaries is varied (boundary variation method) in order to obtain an optimized design which is an optimum only for the given topology (Hinton and Sienz 1995). Although the idea of material removal has been tried by other researchers including Maier (1973), Rodriguez-Velazquez and Seireg (1985) and Atrek (1989), these studies have not resulted in a generalized method (Chu et al., 1997).

2. 1. Basic Principles of ESO

The aim of topology optimization is to find a conceptual layout of a design by distributing a given amount of material in a domain thereby achieving the lightest and stiffest structure while satisfying certain specified design constraints (Guan et al., 2003). A reliable sign of potential structure failure is excessive stress or strain. Inversely, a reliable sign of inefficient material use is low stress or strain. Ideally the stress in every part of a structure is near the same safe level.

This concept leads to the rejection criterion based on local stress level, where lowly stressed material is assumed to be under-utilized and will be removed subsequently. By gradually removing material with lower stress, the stress level in the new designs becomes more and more uniform (Xie and Steven, 1997).

In most initial designs, element removal occurs without any problem but some situations cause the structure to be unstable because of the loading and support conditions. Some elements near the loading and support points may have the least stress values; therefore, in order to prevent that failure, these elements should not be included in the removing process in someway. A work on the failure possibility of ESO has been done by Zhou and Rozvany (Zhou and Rozvany, 2001) and this subject should be considered before the optimization process begins.

After finite element analysis, the stress distribution throughout the structure is found. Often it happens that part of the material is not effectively used. Using some criterion for rejection, a rejection criterion, such as the von Mises stress, this unneeded part of the material can be eliminated. For example, elements are deleted where the von Mises stress in the element is less than a rejection ratio (RR) times the maximum von Mises stress over the structure (Xie and Steven, 1993). Such a finite element analysis and element removal cycle is repeated using the same RR until a steady state is reached. At this stage an evolution rate (ER) is introduced and added to the RR. The iteration takes place again until a new steady state is reached (Steven et al., 2001).

This evolution process continues until a desired optimum is reached, for example, until all stress levels are within 25% of the maximum. It might not be the absolute best result but such an evolutionary optimization procedure offers the possibility of knowing every stage of the shape and layout path towards the true optimum (Xie and Steven, 1993). Consequently, the shape and topology at each steady state may be chosen as the final design. Ideally the final structure becomes a fully stressed design where the material at each point of the structure is stressed to its full strength. However only in a few special cases can a fully stressed structure be possible (Xie and Steven, 1997) and some methods offered by researchers are useful to determine the best design.

During the evolutionary process, it is not necessary to generate a new mesh. Instead, the material property number of the rejected elements may be assigned to zero and ignore these elements when the global stiffness matrix is assembled (Xie and Steven, 1993). Also it is possible to reduce one of the characteristics of the elements (elasticity modulus, thickness, density, i.e.) to be removed (Tanskanen, 2002). Indeed anything that affects the performance of the design can potentially be included (Steven et al., 2001). Also the study by Tanskanen (2002) that presents the theoretical basis of ESO, is a mathematical explanation for this optimization procedure.

2. 2. Objectives and Constraints

The main idea of ESO is to obtain an optimal shape and topology of a structure by gradual removal of unnecessary elements from the structure by working out an appropriate criterion which allows assessing the contribution of each element to the specified behavior (response) of the structure and elements with the least contribution are subsequently removed (Chu et al., 1997). Many results are available within optimal shape design, but a number of important issues still need to be addressed (Pedersen, 2000). The design constraints and also objective functions or constraints can be any of the following responses: volumes or weights of structural parts, compliance, eigen frequencies, displacements, and stresses, i.e. (Zhou et al., 2004).

ESO was developed by Xie and Steven for shape and layout problems under stress consideration and then for frequency optimization. The ESO method for shape and topology problems with displacement and stiffness constraints has been presented recently by Chu et al. (1996). An evolutionary procedure for sizing members to increase the buckling load factor has been proposed by Manickarajah et al. (1998) and a procedure to increase the inertia moment has been presented by Proos (2002). Also a method has been built up to incorporate nonstructural constraints such as the number of cavities in the final topology and manufacturing constraints by Kim et al. (2000). However as a different field, ESO has been used for problems in thermal environments by some researchers like Li et al. (2000, 2004) and Steven et al. (2000).

Commonly, there are two types of objective functions for structural problems. One is the stress and the other is the weight of the structure (Pourazady and Fu, 1996). For most structural situations, stiffness and strength are the main concerns of design engineers. Often the designer endeavors to balance the two design objectives of the stiffness maximization (stiffest) and the maximum stress minimization (strongest) (Steven et al., 2002).

Based on the results of some publications, it was assumed that the ESO method minimizes the compliance-volume product of a structure or a finite element model. The minimum compliance topology optimization can be expressed as

$$\begin{array}{ll} \text{minimize} & C(x) \quad x \quad \Omega \\ \text{subject to} & V(x)/V_0 \leq f \end{array}$$

where $C(x)$ is the compliance of the topology, $V(x)$ and V_0 the material volume and the design domain volume, respectively, and f the prescribed volume fraction (Wang et al., 2006).

2. 3. Simple Procedure of ESO

The stress based early version of ESO method generally uses the von Mises stress to guide removal as preferred as the optimality criteria for this study. This initial ESO concept of removing low stressed elements can be shown to be equivalent to changing an optimality sensitivity to remove material. This said sensitivity being the change in compliance with respect to the removal of material (Steven et al., 2001).

First a piece of material which is large enough to cover the area of the final design is divided into a fine mesh of finite elements. Loads and boundary conditions are applied and a stress analysis is carried out using a finite element program. Since the structure has been divided into many small elements, the removal of material from the structure can be conveniently represented by any method.

The stress level at each point can be measured by some sort of average of all the stress components. For this purpose, the von Mises stress has been one of the most frequently used criteria for isotropic materials. For plane stress problems, the von Mises stress σ^{vm} is defined as

$$\sigma^{vm} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2}\ , \qquad (1)$$

where σ_x and σ_y are normal stresses in x and y directions, respectively, and τ_{xy} is the shear stress. The stress level of each element is determined by comparing the von Mises stress of the element σ_e^{vm} to the maximum von Mises stress of the whole structure $\sigma_{\max}^{vm}$. At the end of each finite element analysis, all the elements which satisfy the following condition are deleted from the model:

$$\frac{\sigma_e^{vm}}{\sigma_{\max}^{vm}} < RR_i\ , \qquad (2)$$

where RRi is the current rejection ratio (RR). According to the examples presented in various papers, the limit value for RR is found nearly 25%.

Also for example, in order to design compression-only structures, the tension-dominant elements are improper for the design condition, and therefore are first removed. The elements under compression but at low stress levels are considered as inefficient, and should be gradually deleted as well (Xie et al., 2005).

The cycle of finite element analysis and element removal is repeated using the same value of RRi until a steady state is reached, by which it means that there are no more elements being deleted at the current iteration. At this stage an evolutionary rate (ER) is introduced and added to the rejection ratio, i.e.

$$RR_{i+1} = RR_i + ER \qquad i = 0,1,2,3,.... \qquad (3)$$

With this increased rejection ratio, the cycle of finite element analysis and element removal takes place again until a new steady state is reached.

In finite element analysis, the element absence or presence can be simply represented by a property of type 0 or 1 (Li et al., 1999). Another way of 'removing' an element is to reduce its elasticity modulus or dimensions such as element thickness to a very small value. For example, Hinton and Sienz (1995) reduce the elasticity modulus of the elements to be removed by a factor of 10^{-5} or 10^{-6} etc. However, the most suitable way is modifying the element properties (elasticity modulus, thickness, density, i.e.). Because of removing the elements by assigning the material property number to 0, some elements having insufficient connection to other elements by only one of its nodes may cause singularity of the stiffness matrix in subsequent analyzes (Özkal, 2006; Özkal and Uysal, 2008).

The evolutionary procedure for optimization (as seen in Figure 2) with von Mises stress constraints can be summarized as follows:

Step 1	:	Discretize the structure using a fine mesh of finite elements.
Step 2	:	Analyze the **structure** under the support and loading conditions
Step 3	:	Calculate the maximum von Mises stress of the whole structure and for each elements.
Step 4	:	Remove a number of elements which have the lowest stress.
Step 5	:	Repeat steps 2 to 4 until desired stress distribution, structure weight or topology.

2. 4. Parameters for the Procedure

In addition to the parameters of ESO such as RR, ER and ERR, because finite element models are used to represent the structure, the influences of mesh size and element type should also be examined carefully. In more recent works, it is more convenient to use a reliable finite element package program for structural analysis (Uysal et al., 2004). Weight, shape and topology of the final designs and naturally the computation time make difference by varying one of these factors.

2. 4. 1. Rejection Ratio and Evolutionary Rate

The evolutionary procedure requires two parameters to be prescribed. The first is the initial rejection ratio RR0 and the second is the evolutionary rate ER. Typical values of RR0=1% and ER=0.5-1% have been used for many test examples. But for some problems, much lower values need to be used (Xie and Steven, 1997). In standard ESO, a steady state is reached when no elements have criteria less than the deletion criterion. When a topology reaches a steady state, the deletion criterion (RR) is increased (ER) to further optimizing the structure if desired (Kim et al., 2003). For any new model, after a few trials, it is not difficult to choose suitable values for these parameters. For example, if too much material has been removed from the structure within one iteration or one steady state, it indicates that smaller values should be used for RR0 or ER. If the evolution rate ER is too high, then over rejection occurs and the structure becomes singular. When this happens, it is necessary that the software steps back and starts of the current evolutionary cycle with a halved ER to try again (Xie and Steven, 1993).

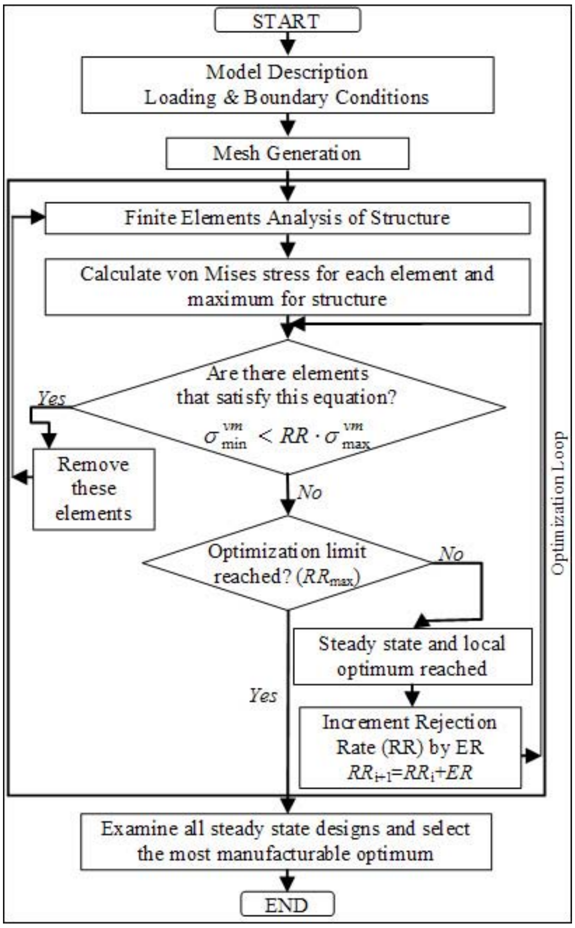

Figure 2. Flow chart depicting the logical steps of the ESO process.

For most of the examples, it has been noted that the models can reach close to their optimum configurations with the RR round about 25 % as presented in the examples of this paper. This indicates that there is still a wide range of stress between the lowest and highest stressed elements using the element von Mises stress compared with the maximum von Mises stress as the rejection criterion. Because of the presence of rigid joints and fixed support joints, there can be significant stress raises at these locations which can explain the low terminal RR value (Xie and Steven, 1993).

Based on the paper of Abolbashari and Keshavarzmanesh (2006), it is seen that the minimum stresses increase slightly by increasing RR and there is usually no significant difference for various evolutionary rates. Also, the maximum von Mises stresses remain constant whereas the minimum von Mises stresses are slightly increased smoothly as rejection ratios increased. Therefore, it may be concluded that for a same mesh size, up to rejection ratio of 15%, the minimum stresses are more sensitive to the rejection ratios than the maximum stresses.

2. 4. 2. Element Removal Ratio

The numbers of elements to be removed at each iteration is determined by a prescribed element removal ratio (ERR), which is defined as the ratio of the number of elements to be removed at each iteration to the total number of elements in the initial or current FEA model. The removal ratio is an important parameter, which plays a similar role as the move limit or step size in mathematical programming and optimality criteria methods (Chu et al., 1996). Typical values for the element removal ratio are 1% and 2%. Further discussions on the influence of the amount of material removed at each iteration on the final solutions can be found in the study of Chu et al. (1997). But ERR should be rounded off to the nearest integer. In the case where the symmetry of a structure needs to be maintained, an even number of elements should be removed at each iteration. Before the main constraint reaches its limit, the evolutionary procedure can also be terminated when a prescribed percentage of volume has been eliminated from the structure.

It is expected that, the smaller the value of the element removal ratio used, the more accurate is the final design, at the expense of larger computation time. Supported by the examples of Chu et al. (1997), when the element removal ratio varies from 1% to 4%, it has little effect on the weight and the outer shape of the optimal design. The element removal ratio affects the details of the inner parts;

however the main pattern and orientation of these inner parts are similar. In that paper, it is suggested that one could use an element removal ratio as high as 4% to obtain optimal shape and topology with sufficient accuracy and significant time saving but it can not be generalized for all of the models.

2. 4. 3. Element (Mesh) Size and Type

Unlike many other FEA based structural optimization techniques, the ESO method does not require re-generating new finite element meshes even when the final structure has departed substantially from the initial design. This is a great advantage of the ESO method. The use of a fixed FEA model for the design domain by the ESO method results in non-smooth boundaries, but it avoids the necessity of re-meshing and allows predicting the optimal topology of the structure (Chu et al., 1997).

ESO should be applied so that the elements corresponding to the design domain are equally sized. If this requirement is not met, the rejection criterion, which also considers the varying sizes of the elements, should be used (Tanskanen, 2002).

Chu et al. (1997) has demonstrated that the mesh size has little effect on the weight, even though it affects the details of the final design. However, even coarse mesh can provide a rough idea of the shape and topology of the optimal design.

But according to the study of Abolbashari and Keshavarzmanesh (2006), element sizes have a significant effect on the histories of minimum stresses and on the volume reduction histories. Also, the volume reduction is more sensitive to the smaller element sizes. That is, the volume reduction is gained less as the element size becomes larger. It can be concluded that using the fine mesh results in a lighter shape and a lower maximum stress level. Users should make their own decision on getting a lighter shape and therefore paying for more computational time.

If anything certain must be told about the ESO parameters (RR, ER, ERR, element size and even element type, etc.), the most suitable design can be reached only by experimental study and surely the values of these parameters vary for each initial design and also for the final design that is expected. In fact, ESO needs very small computation time in comparison to other optimization methods and therefore many experimental operations with different parameters and environmental conditions can be carried out to find the most suitable designs like done for the examples in this work.

3. DESIGN EXAMPLES

As mentioned above, one of the most attractive features of ESO is the ability of producing truss-like structures. Both of the beam examples presented in this section have been presented by Xie and Steven (1993) and completely same results have been reached in this study and the especially optimum design in the first example obviously resembles Michell trusses.

For all the problems, a concentrated load is applied at the middle of the bottom and the static analysis is carried out by using a mesh of 1250 (50x25) square four-node plane stressed elements. To minimize the weight of the structure and make the structure almost fully stressed, stress based criterion has been used for the evolutionary optimization. To avoid an extra operation of checking the connectivity of elements, changing the elasticity modulus instead of hard-kill method has been taken as the element removing method for this study. In a manner of speaking, although element existence can be chosen, elasticity modulus of each element has been preferred as the design variable and they have been multiplied by a factor of 10-6 for removal as done by Hinton and Sienz (1995).

3. 1. Example 1

First example is a typical optimization problem for ESO. The bottom corners of the beam are assumed as fixed. The evolution starts with the initial rejection ratio RR0=1%, taking the evolution rate ER=0.5% and the element removal ratio is assumed as ERR=1%.

Initial, intermediate and final optimum designs with the stress distributions have been shown in Figure 3. Intermediate designs are presented to show that the whole optimization process should be considered to decide the most optimum design.

The optimization aim RR=25% has been reached at 180 iterations by considering manufacturability, stress distribution throughout the whole structure and displacement constraints. The weight of the structure has been reduced nearly 91% of the initial design. Also the minimum stress has increased 1075 times and the maximum stress by 49%.

3. 2. Example 2

The same beam and loading is used for the second example to examine the effect of changing support on the optimization history and the final design. The right support is replaced from fixed to rolling.

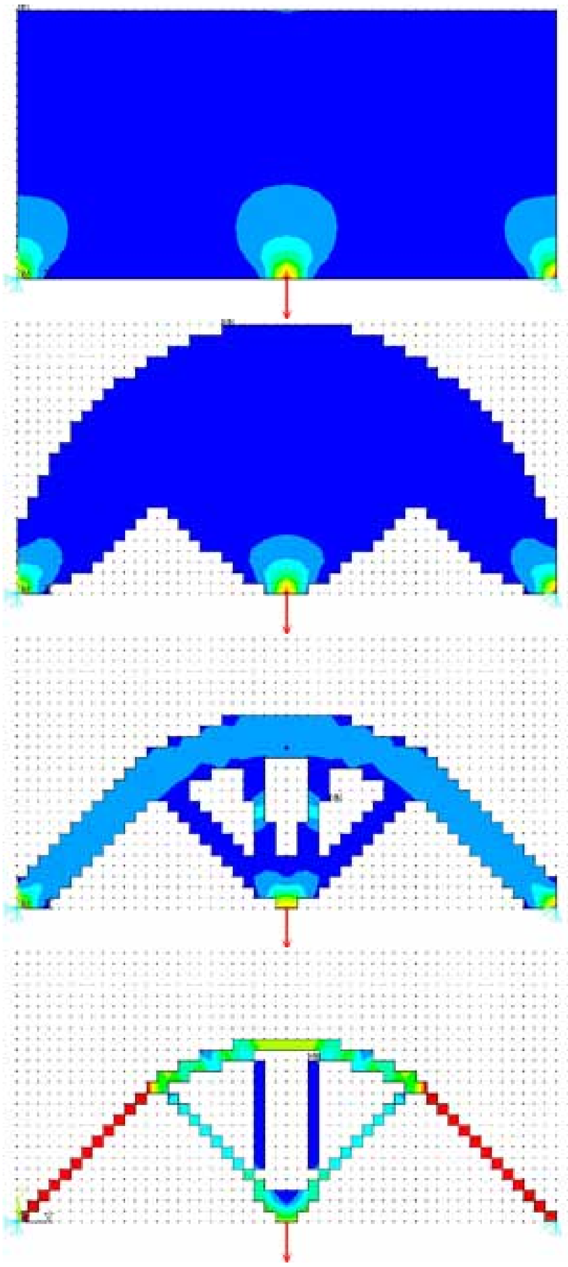

Figure 3. Initial, intermediate and final optimum designs of Example 1.

As seen on the initial design at Figure 4, lateral stresses especially at the bottom zone increase by the replacement of the right support. ERR=4% is chosen to produce more suitable designs faster, while keeping RR0 and ER same as in Example 1. The optimization aim RR=25% has been reached at 141 iterations and the weight of the structure has been reduced by about 80% of the initial design. Also the minimum stress has been increased more than 562 times and the maximum stress by only 0.42%.

4. CONCLUSIONS

Considering the engineering aspects, ESO seems to have some attractive features: the ESO method is very simple to program via the FEA packages and requires a relatively small amount of FEA time. A major advantage of ESO is its simple incorporation into any standard FEA code and its versatility in the range of criteria and physics it can handle. Unlike many other FEA based structural optimization techniques, the ESO method does not require re-generating new finite element meshes even when the final structure has departed substantially from the initial design. This is also a great advantage of the ESO method.

Briefly, it can be said that ESO is a standard mathematical and kind of intuitive programming method minimizing a predefined object function (weight, maximum stress, buckling load, i.e.). When the results of the examples in the literature are examined, the attractive features of ESO will be realized. It is obvious that the weight saving up to high values is very important for the acceptability of an optimization method.

Removing the elements that are not effectively used and consequently getting the minimum stress increased up to the higher values, constitute relatively fully stressed designs and also keeping the maximum stress values close to the initial values expands the applicability of the optimum designs. To make a different statement, optimized designs that have the least weight (volume), supports the loads by the most effective topology.

Even though it may be suspicious to reach the absolute best result, ESO gives the designer at

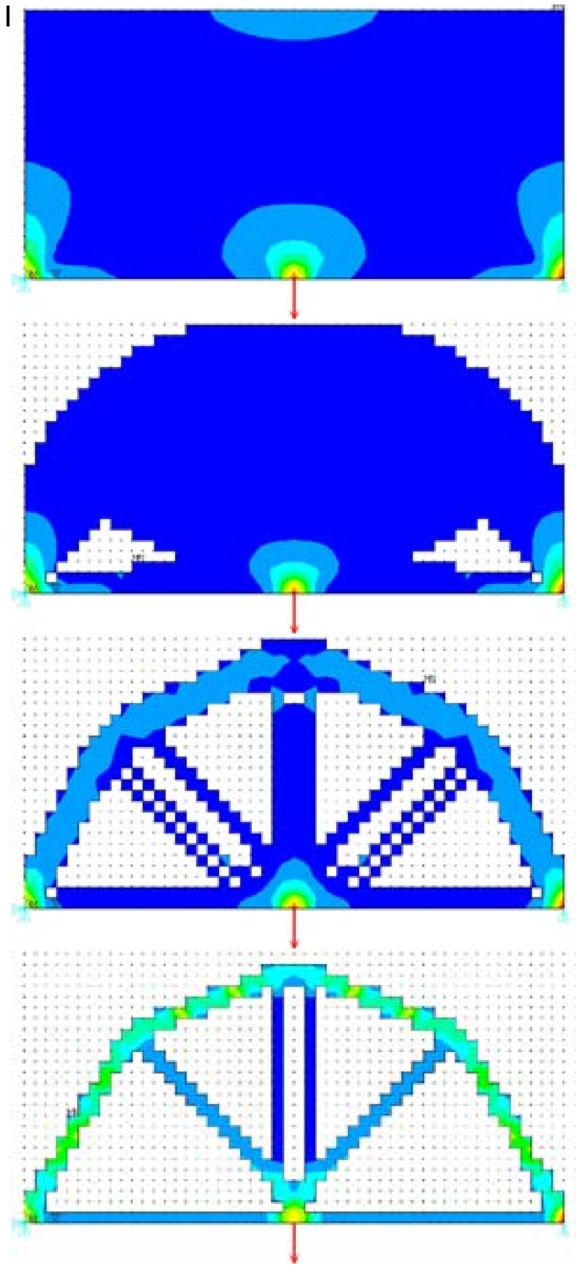

Figure 4. Initial, intermediate and final optimum designs of Example 2.

tures under definite conditions. Most of the optimum designs achieved by ESO can be applied by the manufacturers. ESO method can be used in any industry field and these optimized designs can be used by making some changes due to the necessities. Additionally, unsmooth boundaries of the optimum designs, achieved by ESO may be assumed as a problem by the manufacturers. But there have been some solutions for this subject. Some studies, for instance, by Huang and Xie (2007) and Keane et al. (2002) have offered nodal based approaches to handle the boundary problem.

Finally, it can be concluded that ESO is not just an intuitive method, as it has a very distinct theoretical basis. It is also very simple to employ in engineering design problems. For this reason, ESO has potential to become a tool for design engineers.

Although the potential of modern structural optimization techniques has been realized by aeronautical, automotive, and mechanical industries, they are still viewed by civil engineers as academic exercises. Strut-and-tie model system, as studied by some researchers, is a very important sign for the applicability of this method especially for civil engineering. But those works have been limited to only numerical applications. Experimental studies concerning the material properties and real responses of the structures should be planned.

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