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# Evaluation of hardness values in machine part surface hardening process by fuzzy quality control and process capability analysis

## Makine parçası yüzey sertleştirme işleminde sertlik değerlerinin bulanık kalite kontrolü ve süreç yeterlilik analizi ile değerlendirilmesi

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### Abstract

In the manufacture of machine parts, quenching is a method used in all areas of the industry in different environments and conditions. The obtained hardness values must be in the appropriate value range according to the requirements of the function of the relevant machine part. It is common to measure hardness values according to ASTM E10-01 (Standard Test Method for Brinell Hardness of Metallic Materials), which is one of the hardness measurement methods. In the study, Shewart Average ( $\bar{X}$ ) and Range (R) quality control charts and process capability analysis were used to test the compliance of the hardness values obtained as a result of the quenching-tempering process according to the technical drawing of the relevant part in a company manufacturing machine parts. In addition, due to the approximate value of the observation values, the observation values were converted into fuzzy numbers and fuzzy quality control analyzes were performed with the "Fuzzy rules method for TFN case" method, and process adequacy was measured. According to both methods, as a result of the study, it was determined that the process variability was high and therefore the process was not sufficient.

**Keywords:** Heat treatment, Surface hardness, Fuzzy quality control charts, Shewart.

### Öz

Makine parçası imalatında, su verme yoluyla sertleştirme, endüstrinin her alanında farklı ortam ve koşullarda kullanılan bir yöntemdir. Elde edilen sertlik değerlerinin, ilgili makine parçasının işlevinin gerekliliklerine göre uygun değer aralığında olması gerekmektedir. Sertlik değerlerinin, sertlik ölçme yöntemlerinden biri olan ASTM E10-01 (Metalik Malzemelerin Brinell Sertliği için Standart Test Yöntemi)'e göre ölçümü yaygındır. Çalışmada makine parçası imal eden bir işletmede ilgili parçanın Teknik resmine göre yapılan su verme-menevişleme işlemi neticesinde elde edilen sertlik değerlerinin spesifikasyonlara uygunluğunu test etmek için, Shewart Ortalama ( $\bar{X}$ ) ve Aralık (R) kalite kontrol grafikleri ve süreç yeterlilik analizi kullanılmıştır. Ayrıca, gözlem değerlerinin yaklaşık değer içermesi nedeniyle, gözlem değerleri bulanık sayılara çevrilip "Fuzzy rules method for TFN case" yöntemi ile bulanık kalite kontrol analizleri yapılmış ve süreç yeterliliği ölçülmüştür. Her iki yönetime göre, çalışma sonucunda, proses değişkenliğinin fazla olduğu ve bundan dolayı da prosesin yeterli olmadığı tespit edilmiştir.

**Anahtar kelimeler:** Isıl işlem, Yüzey sertliği, Bulanık kalite kontrol grafikleri, Shewart.

## 1 Introduction

Materials used in engineering fall into three basic categories: metals, ceramics, and polymers. Metals are composed of one or more metallic elements (for example, iron, aluminum, copper, titanium, gold, and nickel) and usually relatively small amounts of non-metallic elements (for example, carbon, nitrogen, and oxygen). Atoms in metals and their alloys are arranged very regularly. All metals, many ceramic materials, and certain polymers form crystalline structures under normal solidification conditions. Some properties of crystalline solids depend on the crystal structure of the material, i.e. the spatial arrangement of atoms, ions, or molecules. The mechanical behavior of the metal alloy (strength, hardness, ductility, etc.) is strongly influenced by its microstructure. Many materials' mechanical and other properties depend on their microstructures, which are often produced because of phase transformations. Steel composed of iron, carbon, silicon, manganese, and small amounts of phosphorus and sulfur is called carbon steel. When austenitized iron-carbon alloys are

rapidly cooled to a relatively low temperature (Quenching), a phase called martensite is formed. Martensite is an unstable single-phase structure resulting from the diffusionless transformation of austenite. Martensitic transformation occurs when the quenching rate is fast enough to prevent carbon diffusion [1].

Increasing the strength of machine parts by increasing the hardness of the machine parts by martensitic transformation is a widely used application in the industry. Hardness is called the resistance of the material to inserting or scratching [2].

In this study, the heat treatment applied to the tractor front axle made of AISI 4140 steel, containing 0.38-0.43% carbon, 1%Cr, 0.9%Mn, 0.2%Mo, 0.03%P, 0.2%Si, and 0.02%S, (quenching at 850 °C-tempering process at 400 °C for 4 hours) of an enterprise manufacturing agricultural equipment part in Ankara as a result, the surface hardness values (HB-Brinell Hardness) obtained depending on the martensitic internal structure were taken as the quality variable. The firm measures

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the surface hardness values by the ASTM E10-01 Standard Test Method for Brinell Hardness of Metallic Materials.

In order to determine whether a product is of good quality or not, this product must be among the predetermined standard values in terms of certain features. Statistical process control (SPC) has an important place in quality studies. The beginning of modern quality control has emerged with the use of statistical methods in the industry since the 1920s. In these years, for the first time; Shewhart, Dodge, Roming, Pearson, etc. Scientists have used statistical methods to solve quality problems encountered in industry. It has been observed that these methods provide cost reduction, improve quality and increase productivity in the enterprise. SPC, processes in the production system; means the use of these methods to monitor, control, evaluate, analyze and improve. With the quality management systems, they have established, companies monitor the quality at every stage of the production stage by making periodic measurements and keeping control in this way. SPC techniques allow the system to work within the determined quality control limits by detecting the special causes that occur/may arise in the system. Quality control charts are used to determine and analyze quality control limits. Quality control charts are graphs that show the changes over time of the data obtained from the sample taken at certain time intervals. To detect the changes that may occur in the process over time, to keep the process under control, and to take precautions, abnormal changes that may occur during the process can be observed with quality control charts.

Process capability analysis (*PCA*) measures the immutability of the process. That is, it measures the functional parameters of the product, not the process itself [3]. Process Capability Indices (*PCIs*) such as  $C_p$ ,  $C_{pk}$ ,  $C_{pm}$ ,  $C_{pmk}$  and  $C_a$  have been proposed in the literature for the measurement of *PCA* [4]. The most used indices are  $C_p$  and  $C_{pk}$ . The  $C_p$  index measures the potential capability of the process, while the  $C_{pk}$  index controls the spread of the process and its deviation from the target value [5].

While performing quality control, measurement errors may occur due to the obtained variables from the system or the person measuring the variable value. For this reason, Fuzzy logic, which produces more flexible results, is used to perform quality control of the system and measure process capability. With the Fuzzy logic developed by Zadeh (1965), very good results are obtained in the modeling of random and stochastic systems [6]. After the inception of the notion of fuzzy sets by Zadeh (1965), many authors have applied this approach to very different areas such as statistics, multi-criteria decision analysis, optimization techniques, and quality control [4]. Studies on the use of fuzzy logic in quality control have been carried out in the literature. Some of the studies on the development of Fuzzy Quality control methods in the literature are as follows; Bradshaw (1983) defined fuzzy economic control limits by using the theory of fuzzy sets, instead of traditional quality control charts, in order to make a correct distinction between “good” and “bad” product [7]. Wang and Raz (1990) and Raz and Wang (1990) developed  $\bar{X}$  and  $R$  control charts by converting linguistic data to fuzzy numbers [8], [9]. Kanagawa et al (1993) proposed a different method for linguistic variables than Wang and Ranz. With their proposed method, they developed a fuzzy control chart that controls the process variability and mean [10]. Wang and Chen (1995) proposed a method for SPC by combining the heuristic method

with the fuzzy mathematical programming method [11]. Kahraman et al (1995) proposed quality control charts using fuzzy triangular numbers [12]. Taleb and Limam (2002) used the fuzzy and probability theory-based approaches proposed by Wang and Ranz (1990) for the quality control of a porcelain manufacturing company [13]. Gülbay et al (2004) re-proposed the fuzzy quality control schemes proposed by Wang and Ranz (1990) using  $\alpha$ -cut levels [14]. Aytac (2006) converted linguistic variables from the Stone and Soil Industry into triangular fuzzy numbers and drew Fuzzy Quality control charts [15]. Gülbay et al (2008) developed a “direct fuzzy approach” with an  $\alpha$ -cutoff level [16]. Şentürk and Erginel (2009), in their paper, they created  $\alpha$ -segment fuzzy and control charts and presented an application for fuzzy control charts. They have increased the flexibility of conventional control limits by using fuzzy control charts [17]. Alizdaeh and Ghomi (2011) developed interval and mean diagrams in a fuzzy environment using different methods [18]. Aslangiray (2011), in her studies, used of fuzzy logic in the creation of process control charts and a combination of fuzzy quality control charts demonstrated with practice. In the fuzzy control charts, the “Direct Fuzzy Approach (DBM)” suggested by Gülbay and Kahraman (2006) was used [19]. Kaya and Kahraman (2011) proposed a new method to prevent information loss when converting fuzzy numbers to exact values [4]. Kaya and Kahraman (2011) proposed a new method for both triangular (TFN) and trapezoidal fuzzy numbers (TrFN) to prevent information loss when converting fuzzy numbers to exact values [4]. Alakoç (2012), in his study, the so-called ratio approach, proposed a new approach developed for fuzzy  $c$  quality control charts [20]. Kaya et al (2017), in their paper, proposed control charts for stock price for the BIST-30 index to analyze a fuzzy model mean and variance based on individual and moving ranges. In their paper, they also show that control charts can be designed to monitor and detect the financial sector [21]. Şentürk and Antucheviciene (2017), in their paper, the structure of theoretically spaced type-2 fuzzy  $c$ -control charts was first proposed and applied. It was used in quality control studies of a food company [22]. Tekşen and Anagün (2018) the purpose of the paper is to create an innovation using the ranking methods, which have not been used for control charts inaccessible literature, for the fuzzy control charts with interval type-2 fuzzy sets [23]. Pekin Alakoç and Apaydın (2018), the key features of the approach they propose are: The type of fuzzy control charts is not limited to variables or attributes, and the approach can be easily modified for different processes and fuzzy number types by the decision maker's evaluation or judgment. The proposed approach is designed for the fuzzy  $c$  quality control scheme and is explained with an example table [24]. Tekşen and Anagün (2018), the purpose of this paper is to demonstrate how to create control limits of  $X - R$  control charts for a specified data set of interval type-2 fuzzy sets [25]. Zahir Khan et al (2018) proposed Fuzzy EWMA quality control charts by taking linguistic variables as fuzzy numbers [26]. Santos Mendes et al (2019), in their work, the values of the quality characteristic were converted to fuzzy numbers by adding the uncertainties and converted to representative values for better comparison with conventional control charts. Performance of a control chart measured by mean running length (ARL) and extra quadratic loss (EQL) [27]. Hesamian et al (2019), in their work, almost all the processes required by classical statistical quality control were developed in a completely fuzzy environment with fuzzy observations, fuzzy mean, fuzzy variance, and fuzzy control limits [28]. Razali

et al (2020), in the paper, which is to classify the types of applications of fuzzy control chart (type 1 and type 2 fuzzy control chart) and identify the past and current developments in the fuzzy control chart for the last five years [29]. Rodriguez-Akvarez et al (2021), in paper, the method to convert individual data to fuzzy numbers are based on the sigma level process as a first stage, and then, the fuzzy individual and moving range control charts are introduced using the  $\alpha$ -cut fuzzy midrange approach [30]. Teksen and Anagün (2020), the aim of their article is to obtain a  $c$ -control graph for heuristic fuzzy sets. For this purpose, defuzzification and probability methods were used. In particular, they applied the probability method to the heuristic fuzzy control charts [31].

The company, contacted for the implementation phase of the study, performs quality control in the form of product acceptance/rejection in the quality control evaluations and daily determinations made within the quality control system. The company does not perform statistical quality control such as quality control charts and process adequacy measurement within the quality control system. In the study, Shewart Mean ( $\bar{X}$ ) and Range ( $R$ ) quality control charts and process capability analysis were evaluated to test the compliance of the enterprise's quality control data with specifications. Then, since the observation values and specifications contain approximate values, the observation values were converted into fuzzy numbers and fuzzy quality control analyzes and process adequacy were measured using the "Fuzzy rules method for TFN case" method. In the literature, generally, theoretical studies have been made about Fuzzy quality control charts. There are very few practical studies. In addition, there is no study in the literature on the Fuzzy Quality Control of Hardness Values in the surface hardening process of machine parts. The flow chart of the study can be summarized as follows (Figure 1).

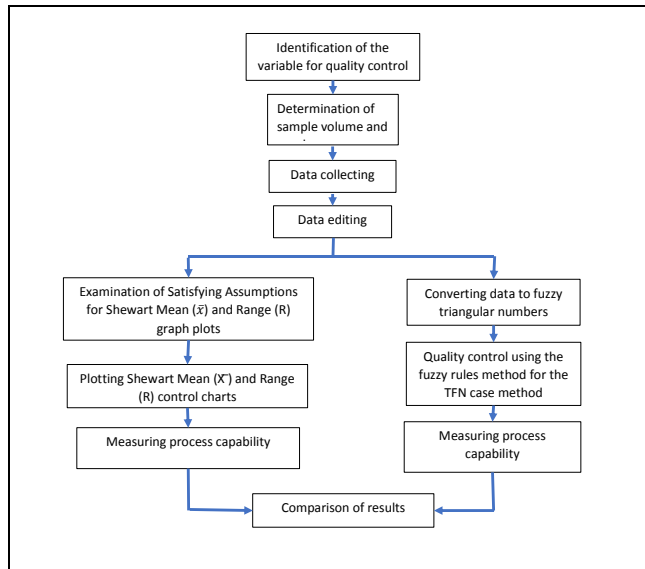


Figure 1. The path followed in the study.

In the next parts of the study, Shewart Quality Control charts and process capability analysis, Fuzzy process capability indices, and control charts for TFN case, application and result are included.

## 2 Shewart control chart

The purpose of control charts is to determine whether the process performance is at an acceptable quality level. Distinguish between controlled and uncontrollable changes due to general and specific causes, Dr. Walter A. Shewart introduced it. For this reason, quality control charts are called Shewart control charts [32], [3]. Values on quality control charts are the values or statistics of the process over time. These values are obtained by regularly measuring or determining the quality characteristic in small samples. Quality control charts mainly consist of Lower Control Limit (LCL), Upper Control Limit (UCL), and Center Line (CL), which were obtained using data from the sample. The average of the data collected over a given time period is indicated by CL. LCL and UCL are calculated based on the sample taken and almost all random variables in the graph fall within this range when the process is under control. [3]. A quality control chart process is defined as "process under control" or "process out of control". The fact that the process is under control means that the production is stable and in the desired standards. The fact that the process is not under control indicates that the process needs improvement in order to meet customer expectations. As a result of the process being out of control, it may be due to a specific reason or an intervention in the process.

Below are the necessary equations to obtain the Mean ( $\bar{X}$ ) and Range ( $R$ ) control charts [33].

Control limits for In Shewart's drawing of  $\bar{X} - R$  control charts diagram;

Control limits for  $\bar{X}$  diagram

$$UCL = \bar{\bar{x}} + A_2 \bar{R} \quad (1)$$

$$CL = \bar{\bar{x}} \quad (2)$$

$$LCL = \bar{\bar{x}} - A_2 \bar{R} \quad (3)$$

Control limits for  $R$  diagram;

$$UCL = \bar{R} D_4 \quad (4)$$

$$CL = \bar{R} \quad (5)$$

$$LCL = \bar{R} D_3 \quad (6)$$

$$\mu = \bar{\bar{x}} \quad (7)$$

$$\sigma = \frac{\bar{R}}{d_2} \quad (7)$$

$$UCL = \bar{R} D_4 \quad (8)$$

$\bar{\bar{x}}$  = average of the averages

$\bar{R}$  = range mean

$\mu$  = population average

$\sigma$  = standard deviation

$A_2, D_3, D_4$  and  $d_2$  = control chart constants

These fixed values are  $A_2=0.577$ ;  $D_3=0.000$ ;  $D_4=2.114$  and  $d_2=2.326$  the values that it will take, since the sample size is  $n=5$  in this study.

Process capability analysis usually measures functional parameters on the product, not the process itself. The process capability indices used for process capability analysis, the process's potential ( $C_p$ ) and actual ( $C_{pk}$ ) capability indices are called "relative capability indices or coefficients" and are a measure of the relevance of the system's specification width to

six sigma width [3], [34]. ( $C_p$ ) and ( $C_{pk}$ ) indices are found with the following equations.

$$C_p = \frac{USL - LSL}{6\sigma} \quad (9)$$

$$C_{pk} = \min\{C_{pl}, C_{pu}\} \quad (10)$$

$$C_{pl} = \frac{\mu - LSL}{3\sigma} \quad (11)$$

$$C_{pu} = \frac{USL - \mu}{3\sigma} \quad (12)$$

USL (Upper Specification Limit) and LSL (Lower Specification Limit) represents customer and/or product requirements which are defined as specification limits (SLs). The value of index  $C_p$  gives us an opinion about process' performance. The  $C_p$  should be at least 1.33 and increase to reach 1.67; 2.00; 2.33; 2.67; 3.00; 3.67, ... for enterprises that are in continuous improvement. If it is greater than 1.33 which corresponds to 63 nonconforming parts per million (ppm) for a centered process, we conclude that process performance is satisfactory [3],[4],[34].

In a stable process it is  $C_p = C_{pk}$ . If  $C_p < C_{pk}$ , the process mean is not centered [34]. The interpretation of the  $C_p$  index is given in Table 1 [35].

Table 1. Quality conditions and  $C_p$  values.

Quality condition	$C_p$ values
Super excellent	$2 \leq C_p$
Excellent	$1.67 \leq C_p < 2.00$
Satisfactory	$1.33 \leq C_p < 1.67$
Capable	$1.00 \leq C_p < 1.33$
Inadequate	$0.67 \leq C_p < 1.00$
Poor	$C_p < 0.67$

By narrowing the control limits in the enterprise, it can be ensured that the  $C_p$  value rises above 1. Reducing the control limits is possible by reducing the variability of the quality characteristic of the product. [36].

### 3 Fuzzy process capability indices and control charts for TFN case

Many methods have been proposed in the literature to obtain Fuzzy quality control charts. In this study, the "Fuzzy rules method for TFN case" proposed by Kaya and Kahraman (2011), which provides the opportunity to interpret fuzzy numbers without converting them to classical numbers, was used. The steps of the method are given below. In the equations, TFN numbers are shown as (a, b, c) [4].

After a sample of size  $n$  is measured, the average of this sample ( $\bar{\tilde{x}}$ ) can be calculated as follows:

$$\bar{\tilde{x}} = \left( \frac{\sum_{i=1}^n a_i}{n}, \frac{\sum_{i=1}^n b_i}{n}, \frac{\sum_{i=1}^n c_i}{n} \right) = TFN(o_1, o_2, o_3) \quad (13)$$

Also the range of the sample ( $\tilde{R}$ ) can be calculated as follows:

$$\tilde{R} = [(a_{max} - c_{min}), (b_{max} - b_{min}), (c_{max} - a_{min})] = TFN(g_1, g_2, g_3) \quad (14)$$

After  $m$  samples are checked, fuzzy grand average ( $\bar{\tilde{x}}$ ) and average range of samples ( $\tilde{R}$ )

$$\bar{\tilde{x}} = \left( \frac{\sum_{i=1}^m o_{1i}}{m}, \frac{\sum_{i=1}^m o_{2i}}{m}, \frac{\sum_{i=1}^m o_{3i}}{m} \right) = TFN(\mu_1, \mu_2, \mu_3) \quad (15)$$

$$\tilde{R} = \left( \frac{\sum_{i=1}^m g_{1i}}{m}, \frac{\sum_{i=1}^m g_{2i}}{m}, \frac{\sum_{i=1}^m g_{3i}}{m} \right) = TFN(\bar{r}_1, \bar{r}_2, \bar{r}_3) \quad (16)$$

Then control limits for  $\bar{\tilde{x}} - \tilde{R}$  control charts can be calculated as follows:

For  $\bar{\tilde{x}}$  control chart,

$$U\tilde{C}\bar{L}_{\bar{x}} = \bar{\tilde{x}} + A_2\tilde{R} = (\mu_1 + A_2\bar{r}_1, \mu_2 + A_2\bar{r}_2, \mu_3 + A_2\bar{r}_3) = TFN(UCLx_1, UCLx_2, UCLx_3) \quad (17)$$

$$\tilde{C}\bar{L}_{\bar{x}} = \bar{\tilde{x}} = (\mu_1, \mu_2, \mu_3) = TFN(CLx_1, CLx_2, CLx_3) \quad (18)$$

$$L\tilde{C}\bar{L}_{\bar{x}} = \bar{\tilde{x}} - A_2\tilde{R} = (\mu_1 - A_2\bar{r}_1, \mu_2 - A_2\bar{r}_2, \mu_3 - A_2\bar{r}_3) = TFN(LCLx_1, LCLx_2, LCLx_3) \quad (19)$$

where

$$= \begin{cases} (LCLx_1, LCLx_2, LCLx_3) \\ LCLx_1 = \begin{cases} \mu_1 - A_2\bar{r}_3, & \text{If } (\mu_1 - A_2\bar{r}_3) \geq 0 \\ 0, & \text{If } (\mu_1 - A_2\bar{r}_3) < 0 \end{cases} \\ LCLx_2 = \begin{cases} \mu_2 - A_2\bar{r}_2, & \text{If } (\mu_2 - A_2\bar{r}_2) \geq 0 \\ 0, & \text{If } (\mu_2 - A_2\bar{r}_2) < 0 \end{cases} \\ LCLx_3 = \begin{cases} \mu_3 - A_2\bar{r}_1, & \text{If } (\mu_3 - A_2\bar{r}_1) \geq 0 \\ 0, & \text{If } (\mu_3 - A_2\bar{r}_1) < 0 \end{cases} \end{cases} \quad (20)$$

For  $\tilde{R}$  control chart,

$$U\tilde{C}L_R = \tilde{R}D_4 = (\bar{r}_1D_4, \bar{r}_2D_4, \bar{r}_3D_4) = TFN(UCLr_1, UCLr_2, UCLr_3) \quad (21)$$

$$\tilde{C}L_R = \tilde{R} = (\bar{r}_1, \bar{r}_2, \bar{r}_3) = TFN(CLR_1, CLR_2, CLR_3) \quad (22)$$

$$L\tilde{C}L_R = \tilde{R}D_3 = (\bar{r}_1D_3, \bar{r}_2D_3, \bar{r}_3D_3) = TFN(LCLr_1, LCLr_2, LCLr_3) \quad (23)$$

The process situations for  $\bar{\tilde{x}}$  control chart ( $Cx_i$ ) and for  $\tilde{R}$  control chart ( $Cr_i$ ) are defined as follows:

$$Cx_i = \begin{cases} 1, & \text{If } (o_{3i} \leq UCLx_1) \wedge (o_{1i} \geq LCLx_3) \\ 0, & \text{If } (o_{1i} > UCLx_3) \vee (o_{3i} < LCLx_1) \\ 1 - \frac{(o_{3i} - UCLx_1)}{(o_{3i} - o_{1i})}, & \text{If } (o_{3i} > UCLx_1) \\ 1 - \frac{(LCLx_3 - o_{1i})}{(o_{3i} - o_{1i})}, & \text{If } (o_{1i} < LCLx_3) \\ \min\left\{1 - \frac{(o_{3i} - UCLx_1)}{(o_{3i} - o_{1i})}, 1 - \frac{(LCLx_3 - o_{1i})}{(o_{3i} - o_{1i})}\right\}, & \text{If } (o_{3i} > UCLx_1) \wedge (o_{1i} < LCLx_3) \end{cases} \quad (24)$$

$$Cr_i = \begin{cases} 1, & \text{If } (g_{3i} \leq UCLr_1) \wedge (g_{1i} \geq LCLr_3) \\ 0, & \text{If } (g_{1i} > UCLr_3) \vee (g_{3i} < LCLr_1) \\ 1 - \frac{(g_{3i} - UCLr_1)}{(g_{3i} - g_{1i})}, & \text{If } (g_{3i} > UCLr_1) \\ 1 - \frac{(LCLr_3 - g_{1i})}{(g_{3i} - g_{1i})}, & \text{If } (g_{1i} < LCLr_3) \\ \min\left\{1 - \frac{(g_{3i} - UCLr_1)}{(g_{3i} - g_{1i})}, 1 - \frac{(LCLr_3 - g_{1i})}{(g_{3i} - g_{1i})}\right\}, & \text{If } (g_{3i} > UCLr_1) \wedge (g_{1i} < LCLr_3) \end{cases} \quad (25)$$

And finally the process control decision is defined as follows:

Process control

$$= \begin{cases} \text{"in control",} & \text{If } (Cx_i = 1) \wedge (Cr_i = 1) \\ \text{"out of control",} & \text{If } (Cx_i = 0) \vee (Cr_i = 0) \\ \text{"rather in control",} & \text{If } (Cx_i \geq \beta) \wedge (Cr_i \geq \beta) \\ \text{"rather out of control",} & \text{If } (Cx_i < \beta) \vee (Cr_i < \beta) \end{cases} \quad (26)$$



$\beta$  is a constant that indicates the extent to which the sample width should remain within the control limit. This ratio may vary between  $0 \leq \beta < 1$  according to the study.

The following equations are used to find "Fuzzy process capability indices for TFN case". Specification limits (SLs) can be defined as follows:

$$U\tilde{S}L = TFN(u_1, u_2, u_3) \quad (27)$$

$$L\tilde{S}L = TFN(l_1, l_2, l_3) \quad (28)$$

Also fuzzy process mean ( $\tilde{\mu}$ ) and standard deviation ( $\tilde{\sigma}$ ) can be calculated as follows:

$$\tilde{\mu}_{isby} = \tilde{\bar{x}} = TFN(\mu_1, \mu_2, \mu_3) \quad (29)$$

$$\tilde{\sigma} = \frac{\tilde{R}}{d_2} = \left( \frac{\bar{r}_1}{d_2}, \frac{\bar{r}_2}{d_2}, \frac{\bar{r}_3}{d_2} \right) = TFN(s_1, s_2, s_3) \quad (30)$$

Fuzzy process capability indices can be calculated as follows:

$$\tilde{C}_p = \frac{U\tilde{S}L - L\tilde{S}L}{6\tilde{\sigma}} = TFN\left(\frac{u_1 - l_3}{6s_3}, \frac{u_2 - l_2}{6s_2}, \frac{u_3 - l_1}{6s_1}\right) \quad (31)$$

$$\tilde{C}_{pu} = \frac{U\tilde{S}L - \tilde{\mu}}{3\tilde{\sigma}} = TFN\left(\frac{u_1 - \mu_3}{3s_3}, \frac{u_2 - \mu_2}{3s_2}, \frac{u_3 - \mu_1}{3s_1}\right) \quad (32)$$

$$\tilde{C}_{pl} = \frac{\tilde{\mu} - L\tilde{S}L}{3\tilde{\sigma}} = TFN\left(\frac{\mu_1 - l_3}{3s_3}, \frac{\mu_2 - l_2}{3s_2}, \frac{\mu_3 - l_1}{3s_1}\right) \quad (33)$$

$$\tilde{C}_{pk} = \min\{\tilde{C}_{pu}, \tilde{C}_{pl}\} \quad (34)$$

#### 4 Application

In this study, heat treatment was applied to the tractor "front wheel axle" manufactured by an enterprise manufacturing machine parts in Ankara. To obtain a completely martensitic structure; After austenitization at 850 °C, it is rapidly quenched to obtain a completely martensitic structure. Then, in order to obtain a fully tempered structure, it was kept at 450 °C for austempering process in a separate tempering furnace for 4 hours until the isothermal transformation was completed, and then cooled to ambient temperature. The surface hardness values (HB) formed due to the martensitic internal structure obtained as a result of the heat treatment were evaluated as a quality variable. In the study, in order to measure the surface hardness values in accordance with the ASTM E10-01 Standard, 5 samples were taken from each heat treatment furnace basket for 11 days, and the surface hardness values of the samples were taken.

During the implementation phase, the constraints and assumptions considered in the study were determined as follows.

- Surface hardness values (HB-Brinell Hardness) control records in the enterprise consist of 11-day data taken in February and March,
- In SPC applications, it is expected that the quantitative variable in the process to be controlled will be measured with which measuring instrument and the measuring method is expected to be reliable and the measuring instrument to be of sufficient sensitivity, it is assumed that these elements are provided in the study,

- All formulas used in SPC applications and control charts, and process capability analysis are valid if the process conforms to the normal distribution [3], [32]. The data in the study fit the normal distribution. In the normality test for the data, the skewness coefficient is -1.32, and the kurtosis coefficient; It was -0.51. Since both values are between -1.96 and +1.96, it can be said that the data are normally distributed,
- The number of samples drawn from the system is very important in terms of determining the reason for the variability in the system. It is recommended that the number of samples be at least 25, each of which is 5 (sample volume) [3],
- In quality control applications, the selection of the quality variable is an important constraint. There are many variables in a production. The selection of the appropriate variable is an important constraint, since preparing a control chart for each variable would be time-consuming and tiring [19]. As a matter of fact, in the study, the quality variable was determined as "surface hardness values (HB)" according to ASTM standards.

To be able to measure process variability well, it is preferred that the sample volume to be drawn from the process is at least 5. The sample volume is the amount of product to be withdrawn from the process at once. On the other hand, the number of samples is the determination of the time when the samples are randomly selected from different parts of the population. The approach taken into consideration for the sample size and sample number is to sample at frequent intervals at the beginning and then to reduce the sample frequency as the development of the process is followed. In the literature, the table below provides guidance to determine the sample size and sample number [3].

Table 2. Number of samples by production amount

Production amount per shift	Number of parts to be inspected per shift
1-65	5
66-110	10
111-180	15
181-300	25
301-500	30
501-800	35
801-1300	40

The study stated that the production amount per shift with the interviewed enterprise was 280. If 280 pieces are produced in a shift, the number of pieces to be checked in a day is 25 according to Table 2. If 5 pieces are checked in each sampling period, 25/5=5 samples should be taken. The sample size and number drawn were determined accordingly. Sample values obtained for quality control are given in Table 3.

Considering the data in Table 3, the values were loaded into the Minitab 18 package program and the following graphics were obtained (Figure 2).

When the mean control chart was examined, it was observed that the samples numbered (5), (6), (25), (28), (29), (32), (34), and (36) were outside the control limit. When the interval control chart was examined, it was observed that the samples (16) and (23) were out of control. When the capability analysis of the process was made, Figure 3 and process proficiency indices were obtained.

Table 3. Surface hardness values.

Samp No	Time	$X_1$ surface hardness (HB) Appr	$X_2$ surface hardness (HB) Appr	$X_3$ surface hardness(HB) Appr	$X_4$ hardness Appr	surface (HB)	$X_5$ surface hardness (HB) Appr
1	08:00	444	435	429		437	427
2	10:00	444	435	429		437	427
3	12:00	444	442	435		435	432
4	14:00	444	442	435		435	432
5	16:00	404	401	406		423	426
6	08:00	404	401	406		423	426
7	10:00	438	444	420		398	448
8	12:00	438	444	420		398	448
9	14:00	438	433	432		426	451
10	16:00	438	433	432		426	451
11	08:00	438	433	438		451	432
12	10:00	420	426	433		451	432
13	12:00	441	464	426		436	425
14	14:00	412	404	401		435	437
15	16:00	441	444	432		398	448
16	08:00	467	464	467		401	442
17	10:00	444	438	432		438	432
18	12:00	451	448	444		420	395
19	14:00	441	464	426		451	440
20	16:00	412	404	401		420	451
21	08:00	451	448	444		435	440
22	10:00	441	444	432		451	430
23	12:00	467	464	467		398	401
24	14:00	444	438	432		420	432
25	16:00	450	467	451		438	450
26	08:00	451	438	429		451	440
27	10:00	451	429	441		449	440
28	12:00	444	457	467		440	462
29	14:00	454	467	451		450	465
30	16:00	451	429	441		450	438
31	08:00	451	438	429		450	440
32	10:00	444	457	467		440	453
33	12:00	420	438	415		440	420
34	14:00	404	398	415		404	420
35	16:00	420	438	415		415	440
36	08:00	404	398	415		406	401
37	10:00	412	438	432		410	440
38	12:00	412	438	432		410	440
39	14:00	441	423	444		440	425
40	16:00	429	438	432		444	420
41	08:00	423	429	438		398	420
42	10:00	429	432	406		430	435
43	12:00	423	444	409		443	410
44	14:00	432	432	423		435	420
45	16:00	444	426	417		430	426
46	08:00	429	423	440		435	442
47	10:00	426	438	426		420	398
48	12:00	429	432	444		440	430
49	14:00	429	435	448		402	440
50	16:00	444	438	441		398	440
51	08:00	448	432	426		444	397
52	10:00	435	455	441		440	420
53	12:00	429	417	434		420	435
54	14:00	441	423	444		410	421
55	16:00	426	406	422		444	401

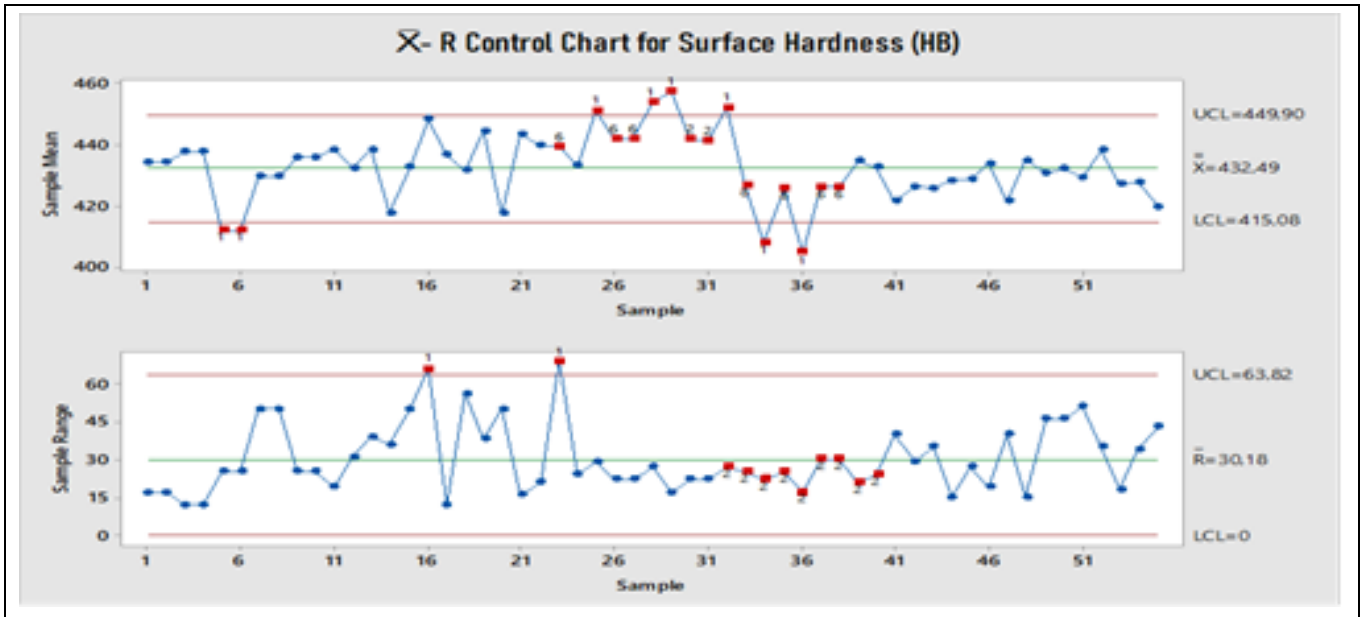


Figure 2. Shewart,  $\bar{X}$  - R control chart for surface hardness.

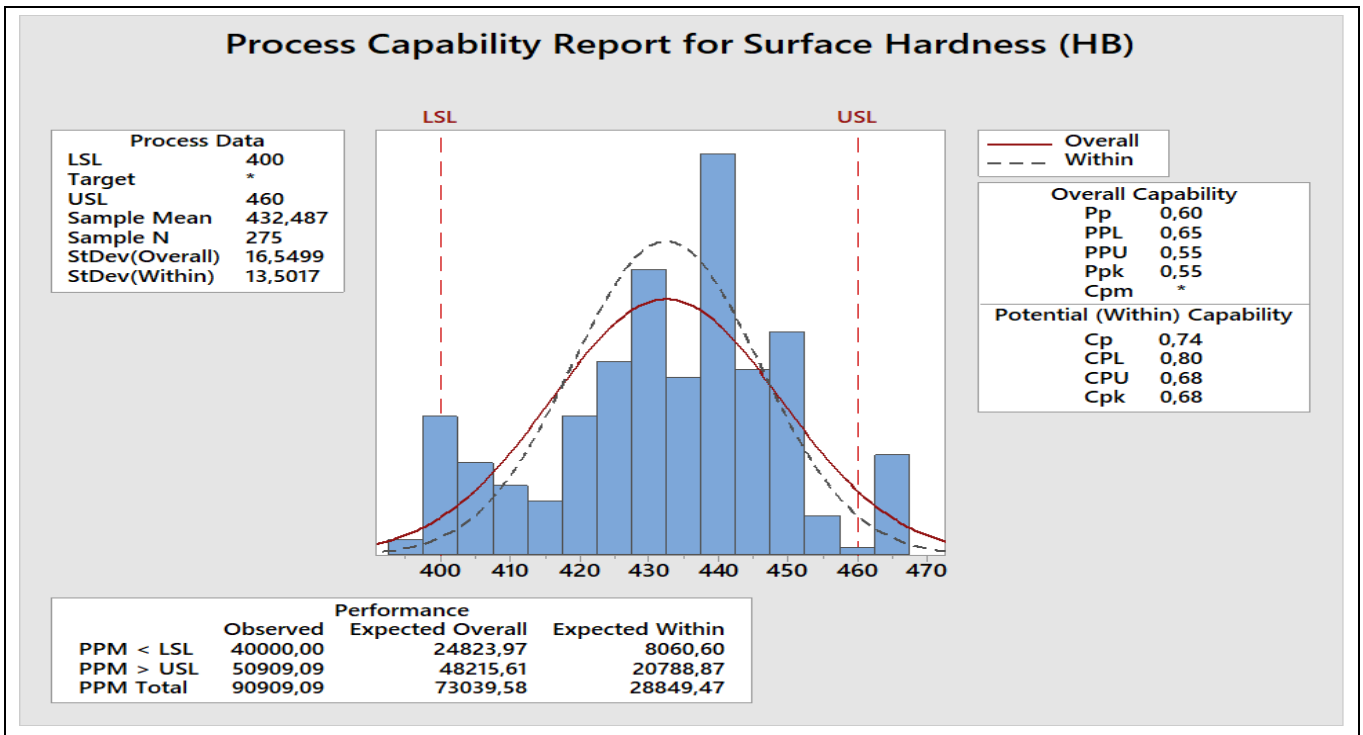


Figure 3. Process Capability Report for Surface Hardness (HB)

Since  $C_p$  value is  $0.67 \leq C_p < 1.00$ , the process is "Inadequate".  $C_{pk}$  value is also below 1. The fact that both values are below 1 indicates that the process is not sufficient. For process competence, the process control limits must be drawn within the process specification limits. It is method necessary to reduce the variability in production. The probability of the process producing a defective product can be found with the help of standard normal variables. As a result of the calculation made, the probability of producing defective products of the enterprise was found to be 1.95%. "Fuzzy rules method for TFN case" has been applied to obtain more precise results. For this,

the data containing approximate values taken from the process were converted into triangular fuzzy numbers, considering a deviation of 0.005 HB. The form of the data converted to fuzzy numbers is given in Table 4.

With the calculations made in Excel, the equations (13), (14), (15) and (16) were used and  $\bar{\tilde{x}} = (432.482, 432.487, 432.492)$  and  $\bar{\tilde{R}} = (30.172, 30.182, 30.192)$  was found. Then, the values for  $\bar{\tilde{x}}$  control chart were found as follows using the equations (17), (18), (19) and (20).



$$U\tilde{C}L_{\bar{x}} = (449.891, 449.902, 449.913)$$

$$\tilde{C}L_{\bar{x}} = (432.482, 432.487, 432.492)$$

$$L\tilde{C}L_{\bar{x}} = (415.062, 415.072, 415.083)$$

For the values of  $\bar{R}$  control chart, the following values were obtained by using the equations (21), (22) and (23).

$$U\tilde{C}L_R = (63.783, 63.804, 63.826)$$

$$\tilde{C}L_R = (30.172, 30.182, 30.192)$$

$$L\tilde{C}L_R = (0.000, 0.000, 0.000)$$

Using the equations (24), (25) and (26) the values related to the quality control of the system are shown in Table 5.

Table 4. Surface hardness values (HB) as TFNs.

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$
1	(443.995,444,444.005)	(434.995,435,435.005)	(428.995,429,429.005)	(436.995,437,437.005)	(426.995,427,427.005)
2	(443.995,444,444.005)	(434.995,435,435.005)	(428.995,429,429.005)	(436.995,437,437.005)	(426.995,427,427.005)
3	(443.995,444,444.005)	(441.995,442,442.005)	(434.995,435,435.005)	(434.995,435,435.005)	(431.995,432,432.005)
4	(443.995,444,444.005)	(441.995,442,442.005)	(434.995,435,435.005)	(434.995,435,435.005)	(431.995,432,432.005)
5	(403.995,404,404.005)	(400.995,401,401.005)	(405.995,406,406.005)	(422.995,423,423.005)	(425.995,426,426.005)
6	(403.995,404,404.005)	(400.995,401,401.005)	(405.995,406,406.005)	(422.995,423,423.005)	(425.995,426,426.005)
7	(437.995,438,438.005)	(443.995,444,444.005)	(419.995,420,420.005)	(397.995,398,398.005)	(447.995,448,448.005)
8	(437.995,438,438.005)	(443.995,444,444.005)	(419.995,420,420.005)	(397.995,398,398.005)	(447.995,448,448.005)
9	(437.995,438,438.005)	(432.995,433,433.005)	(431.995,432,432.005)	(425.995,426,426.005)	(450.995,451,451.005)
10	(437.995,438,438.005)	(432.995,433,433.005)	(431.995,432,432.005)	(425.995,426,426.005)	(450.995,451,451.005)
11	(437.995,438,438.005)	(432.995,433,433.005)	(437.995,438,438.005)	(450.995,451,451.005)	(431.995,432,432.005)
12	(419.995,420,420.005)	(425.995,456,426.005)	(432.995,433,433.005)	(450.995,451,451.005)	(431.995,432,432.005)
13	(440.995,441,441.005)	(463.995,464,464.005)	(425.995,426,426.005)	(435.995,436,436.005)	(424.995,425,425.005)
14	(411.995,412,412.005)	(403.995,404,404.005)	(400.995,401,401.005)	(434.995,435,435.005)	(436.995,437,437.005)
15	(440.995,441,441.005)	(443.995,444,444.005)	(431.995,432,432.005)	(397.995,398,398.005)	(447.995,448,448.005)
16	(466.995,467,467.005)	(463.995,464,464.005)	(466.995,467,467.005)	(400.995,401,401.005)	(441.995,442,442.005)
17	(443.995,444,444.005)	(437.995,438,438.005)	(431.995,432,432.005)	(437.995,438,438.005)	(431.995,432,432.005)
18	(450.995,451,451.005)	(447.995,448,448.005)	(443.995,444,444.005)	(419.995,420,420.005)	(394.995,395,395.005)
19	(440.995,441,441.005)	(463.995,464,464.005)	(425.995,426,426.005)	(450.995,451,451.005)	(439.995,440,440.005)
20	(411.995,412,412.005)	(403.995,404,404.005)	(400.995,401,401.005)	(419.995,420,420.005)	(450.995,451,451.005)
21	(450.995,451,451.005)	(447.995,448,448.005)	(443.995,444,444.005)	(434.995,435,435.005)	(439.995,440,440.005)
22	(440.995,441,441.005)	(443.995,444,444.005)	(431.995,432,432.005)	(450.995,451,451.005)	(429.995,430,430.005)
23	(466.995,467,467.005)	(463.995,464,464.005)	(466.995,467,467.005)	(397.995,398,398.005)	(400.995,401,401.005)
24	(443.995,444,444.005)	(437.995,438,438.005)	(431.995,432,432.005)	(419.995,420,420.005)	(431.995,432,432.005)
25	(449.995,450,450.005)	(466.995,467,467.005)	(450.995,451,451.005)	(437.995,438,438.005)	(449.995,450,450.005)
26	(450.995,451,451.005)	(437.995,438,438.005)	(428.995,429,429.005)	(450.995,451,451.005)	(439.995,440,440.005)
27	(450.995,451,451.005)	(428.995,429,429.005)	(440.995,441,441.005)	(448.995,449,449.005)	(439.995,440,440.005)
28	(443.995,444,444.005)	(456.995,457,457.005)	(466.995,467,467.005)	(439.995,440,440.005)	(461.995,462,462.005)
29	(453.995,454,454.005)	(466.995,467,467.005)	(450.995,451,451.005)	(449.995,450,450.005)	(464.995,465,465.005)
30	(450.995,451,451.005)	(428.995,429,429.005)	(440.995,441,441.005)	(449.995,450,450.005)	(437.995,438,438.005)
31	(450.995,451,451.005)	(437.995,438,438.005)	(428.995,429,429.005)	(449.995,450,450.005)	(439.995,440,440.005)
32	(443.995,444,444.005)	(456.995,457,457.005)	(466.995,467,467.005)	(439.995,440,440.005)	(452.995,453,453.005)
33	(419.995,420,420.005)	(437.995,438,438.005)	(414.995,415,415.005)	(439.995,440,440.005)	(419.995,420,420.005)
34	(403.995,404,404.005)	(397.995,398,398.005)	(414.995,415,415.005)	(403.995,404,404.005)	(419.995,420,420.005)
35	(419.995,420,420.005)	(437.995,438,438.005)	(414.995,415,415.005)	(414.995,415,415.005)	(439.995,440,440.005)
36	(403.995,404,404.005)	(397.995,398,398.005)	(414.995,415,415.005)	(405.995,406,406.005)	(400.995,401,401.005)
37	(411.995,412,412.005)	(437.995,438,438.005)	(431.995,432,432.005)	(409.995,410,410.005)	(439.995,440,440.005)
38	(411.995,412,412.005)	(437.995,438,438.005)	(431.995,432,432.005)	(409.995,410,410.005)	(439.995,440,440.005)
39	(440.995,441,441.005)	(422.995,423,423.005)	(443.995,444,444.005)	(439.995,440,440.005)	(424.995,425,425.005)
40	(428.995,429,429.005)	(437.995,438,438.005)	(431.995,432,432.005)	(443.995,444,444.005)	(419.995,420,420.005)
41	(422.995,423,423.005)	(428.995,429,429.005)	(437.995,438,438.005)	(397.995,398,398.005)	(419.995,420,420.005)
42	(428.995,429,429.005)	(431.995,432,432.005)	(405.995,406,406.005)	(429.995,430,430.005)	(434.995,435,435.005)
43	(422.995,423,423.005)	(443.995,444,444.005)	(408.995,409,409.005)	(442.995,443,443.005)	(409.995,410,410.005)
44	(431.995,432,432.005)	(431.995,432,432.005)	(422.995,423,423.005)	(434.995,435,435.005)	(419.995,420,420.005)
45	(443.995,444,444.005)	(425.995,426,426.005)	(416.995,417,417.005)	(429.995,430,430.005)	(425.995,426,426.005)
46	(428.995,429,429.005)	(422.995,423,423.005)	(439.995,440,440.005)	(434.995,435,435.005)	(441.995,442,442.005)
47	(425.995,426,426.005)	(437.995,438,438.005)	(425.995,426,426.005)	(419.995,420,420.005)	(397.995,398,398.005)
48	(428.995,429,429.005)	(431.995,432,432.005)	(443.995,444,444.005)	(439.995,440,440.005)	(429.995,430,430.005)
49	(428.995,429,429.005)	(434.995,435,435.005)	(447.995,448,448.005)	(401.995,402,402.005)	(439.995,440,440.005)
50	(443.995,444,444.005)	(437.995,438,438.005)	(440.995,441,441.005)	(397.995,398,398.005)	(439.995,440,440.005)
51	(447.995,448,448.005)	(431.995,432,432.005)	(425.995,426,426.005)	(443.995,444,444.005)	(396.995,397,397.005)
52	(434.995,435,435.005)	(454.995,455,455.005)	(440.995,441,441.005)	(439.995,440,440.005)	(419.995,420,420.005)
53	(428.995,429,429.005)	(416.995,417,417.005)	(433.995,434,434.005)	(419.995,420,420.005)	(434.995,435,435.005)
54	(440.995,441,441.005)	(422.995,423,423.005)	(443.995,444,444.005)	(409.995,410,410.005)	(420.995,421,421.005)
55	(425.995,426,426.005)	(405.995,406,406.005)	(421.995,422,422.005)	(443.995,444,444.005)	(400.995,401,401.005)

Table 5. Control results with fuzzy mean and range of variation values.

Sample	Time	$\bar{\tilde{x}}$	Fuzzy rules method for $\bar{\tilde{x}}$ chart	$\bar{\tilde{R}}$	Fuzzy rules method for $\bar{\tilde{R}}$ chart
1.Day	1 08:00	(434.395,434.400,434.405)		(16.99,17.000,17.010)	
	2 10:00	(434.395,434.400,434.405)		(16.99,17.000,17.010)	
	3 12:00	(437.595,437.600,437.605)		(11.99,12.000,12.010)	
	4 14:00	(437.595,437.600,437.605)		(11.99,12.000,12.010)	
	5 16:00	(411.995,412.000,412.005)	rather in control	(24.99,25.000,25.010)	
2.Day	6 08:00	(411.995,412.000,412.005)	rather in control	(24.99,25.000,25.010)	
	7 10:00	(429.595,429.600,429.605)		(49.99,50.000,50.010)	
	8 12:00	(429.595,429.600,429.605)		(49.99,50.000,50.010)	
	9 14:00	(435.995,436.000,436.005)		(24.99,25.000,25.010)	
	10 16:00	(435.995,436.000,436.005)		(24.99,25.000,25.010)	
3.Day	11 08:00	(434.395,434.400,434.405)		(18.99,19.000,19.010)	
	12 10:00	(432.395,432.400,432.405)		(30.99,31.000,31.010)	
	13 12:00	(434.395,434.400,434.405)		(38.99,39.000,39.010)	
	14 14:00	(417.795,417.800,417.805)		(35.99,36.000,36.010)	
	15 16:00	(432.595,432.600,432.605)		(49.99,50.000,50.010)	
4.Day	16 08:00	(448.195,448.200,448.205)		(65.99,66.000,66.010)	rather in control
	17 10:00	(436.795,436.800,436.805)		(11.99,12.000,12.010)	
	18 12:00	(431.595,431.600,431.605)		(55.99,56.000,56.010)	
	19 14:00	(444.395,444.400,444.405)		(37.99,38.000,38.010)	
	20 16:00	(417.595,417.600,417.605)		(49.99,50.000,50.010)	
5.Day	21 08:00	(443.595,443.600,443.605)		(15.99,16.000,19.010)	
	22 10:00	(439.595,439.600,439.605)		(20.99,21.000,21.010)	
	23 12:00	(439.395,439.400,439.405)		(68.99,69.000,69.010)	rather in control
	24 14:00	(433.195,433.200,433.205)		(23.99,24.000,24.010)	
	25 16:00	(451.195,451.200,451.205)	rather in control	(28.99,29.000,29.010)	
6.Day	26 08:00	(441.795,441.800,441.805)		(21.99,22.000,22.010)	
	27 10:00	(441.995,442.000,442.005)		(21.99,22.000,22.010)	rather in control
	28 12:00	(453.995,454.000,454.005)	rather in control	(26.99,27.000,27.010)	
	29 14:00	(457.395,457.400,457.405)	rather in control	(16.99,17.000,17.010)	
	30 16:00	(441.795,441.800,441.805)		(21.99,22.000,22.010)	
7.Day	31 08:00	(441.595,441.600,441.605)		(21.99,22.000,22.010)	
	32 10:00	(452.195,452.200,452.205)	rather in control	(26.99,27.000,27.010)	
	33 12:00	(426.595,426.600,426.605)		(24.99,25.000,25.010)	
	34 14:00	(408.195,408.200,408.205)	rather in control	(21.99,22.000,22.010)	
	35 16:00	(425.595,425.600,425.605)		(24.99,25.000,25.010)	
8.Day	36 08:00	(404.795,404.800,404.805)	rather in control	(16.99,17.000,17.010)	
	37 10:00	(426.395,426.400,426.405)		(29.99,30.000,30.010)	
	38 12:00	(426.395,426.400,426.405)		(29.99,30.000,30.010)	
	39 14:00	(434.595,434.600,434.605)		(20.99,21.000,21.010)	
	40 16:00	(432.595,432.600,432.605)		(23.99,24.000,24.010)	
9.Day	41 08:00	(421.595,421.600,421.605)		(39.99,40.000,40.010)	
	42 10:00	(426.395,426.400,426.405)		(28.99,29.000,29.010)	
	43 12:00	(425.795,425.800,425.805)		(34.99,35.000,35.010)	
	44 14:00	(428.395,428.400,428.405)		(14.99,15.000,15.010)	
	45 16:00	(428.595,428.600,428.605)		(26.99,27.000,27.010)	
10.Day	46 08:00	(433.795,433.800,433.805)		(18.99,19.000,19.010)	
	47 10:00	(421.595,421.600,421.605)		(39.99,40.000,40.010)	
	48 12:00	(434.995,435.000,435.005)		(14.99,15.000,15.010)	
	49 14:00	(430.795,430.800,430.805)		(45.99,46.000,46.010)	
	50 16:00	(432.195,432.200,432.205)		(45.99,46.000,46.010)	
11.Day	51 08:00	(429.395,429.400,429.405)		(50.99,51.000,51.010)	
	52 10:00	(438.195,438.200,438.205)		(34.99,35.000,35.010)	
	53 12:00	(426.995,427.000,427.005)		(17.99,18.000,18.010)	
	54 14:00	(427.795,427.800,427.805)		(33.99,34.000,34.010)	
	55 16:00	(419.795,419.800,419.805)		(42.99,43.000,43.010)	

According to the fuzzy rules method for  $\bar{\tilde{x}}$  chart, as in Shewart's quality control charts (5), (6), (25), (28), (29), (32), (34) and (36) was out of control in the samples. According to the fuzzy rules method for  $\bar{\tilde{R}}$  chart, samples (16), (23), and (27) were out of control. It was observed that Shewart was out of control at one point more than the interval (R) control charts (sample 27).

According to equation (27) and (28) for process capability analysis, the specification limits of the enterprise  $\bar{U\tilde{S}L}$  and  $\bar{A\tilde{S}L}$  are determined as follows.

$$\bar{U\tilde{S}L} = (459.995, 460, 460.005)$$

$$\bar{A\tilde{S}L} = (399.995, 400, 400.005)$$

Then,  $\tilde{\mu}$  and  $\tilde{\sigma}$  values were found according to the equations (29) and (30).

$$\tilde{\mu} = \frac{\tilde{\bar{x}}}{d_2} = (432.482, 432.487, 432.492)$$

$$\tilde{\sigma} = \frac{\tilde{\bar{R}}}{d_2} = (12.972, 12.976, 12.980)$$

With the help of equations (31), (32), (33) and (34), the process adequacy indices took the following values.

$$\tilde{C}_p = (0.770, 0.771, 0.771)$$

$$\tilde{C}_{pu} = (0.706, 0.707, 0.707)$$

$$\tilde{C}_{pl} = (0.834, 0.835, 0.835)$$

$$\tilde{C}_{pk} = (0.706, 0.707, 0.707)$$

Since  $\tilde{C}_p$  and  $\tilde{C}_{pk}$  values are less than 1.33, the process is expressed as "inadequate".

## 5 Conclusions

With the quenching process, it is aimed to increase the strength of the material and the surface friction wear resistance. As a result of quenching and tempering processes applied to AISI-4140 steel material, the target hardness values are reached. For businesses to reach the quality level they want, they need to measure and evaluate the quality of their products. Statistical Quality Control tools provide great convenience in measuring the quality of enterprises. The aim of quality control is to take the processes under control and to eliminate the causes of the processes that go out of control. Control charts and process capability analyzes are frequently used to identify specific causes of variability. In the study, to measure the surface hardness values by the ASTM E10-01 Standard, 5 samples were taken from each heat treatment furnace basket for 11 days, and the surface hardness values of the samples were taken. The obtained data were analyzed with Shewart control charts and Fuzzy quality control. According to Shewart's mean ( $\bar{\tilde{x}}$ ) quality control method, samples (5), (6), (25), (28), (29), (32), (34) and (36) were observed to be outside the control limit. According to the interval (R) quality control chart, it was observed that samples (16) and (23) were out of control. To obtain more flexible results, the data were converted into Fuzzy triangular numbers and the data were re-evaluated with the fuzzy quality control method. Fuzzy mean ( $\bar{\tilde{x}}$ ) also according to the quality control method, (5), (6), (25), (28), (29), (32), (34) and (36) and Fuzzy range. According to ( $\bar{\tilde{R}}$ ) quality control analysis, out-of-control conditions were observed in samples (16), (23) and (27). According to both methods, the process efficiency was low. When we look at the times when there were out-of-control situations, it was seen that there was no concentration at the

same time of the day, and the out-of-control situations increased on some days. It can be expected that the time to stabilize the temperatures of the quenching and tempering furnaces and the variation in the temperature of the refrigerant oil used in the quenching during the day may be effective. But it has been observed that the process does not go out of control during the day, the temperatures of the furnaces and the temperature of the quenching oil in the related heat treatment unit are kept very well under control with the help of automation heating and cooling units. Although the furnace temperature and quenching oil temperature can be controlled very well; To ensure that the edges and middle parts of the basket can be heated at the same degree, it is of great importance that the materials placed in the furnace basket are correctly arranged to ensure that the heat is evenly distributed throughout the furnace. Looking at the fuzzy quality control result table, there was no significant variability between the first hours of the day when the furnace was started and the end of the working hours. The fact that the variability is seen on different days is related to the personnel operating, in other words, there are workmanship errors caused by reasons such as carelessness and lack of training. It is considered that the process will become more efficient if the personnel are trained and competent. In addition, the company contacted for the implementation phase of the study performs quality control in the form of product acceptance/rejection, in its quality control evaluations and daily determinations within the quality control system. However, it does not apply statistical quality control such as quality control charts and process adequacy measurement within the quality control system. To carry out functional and effective quality control, it is necessary to keep quality records and to make comparisons by drawing control charts considering these data. The results of the proposed fuzzy control charts and their reflections can be evaluated with customer complaints and feedback. The business should record such information. It will be possible to make a sound evaluation by keeping such records.

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## 7 Author contribution statements

In the study carried out, Ahmet Bilal ŞENGÜL focused on forming the idea, obtaining the data, evaluating the data and the results, literature review studies; Ümran ŞENGÜL contributed to the determination of the analysis method of the data, the analysis, the literature review and the evaluation of the results.

## 8 Ethics committee approval and conflict of interest statement

There is no need to obtain permission from the ethics committee for the article prepared.

There is no conflict of interest with any person/institution in the article prepared.

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