Reorder Point and Replenishment Point of Dynamic Inventory Model under Shortages

Yrd. Doç. Dr. İbrahim Zeki AKYURT
İstanbul Üniversitesi İşletme Fakültesi
İşletme Bölümü
zekiakyurt@gmail.com

Abstract

In this study, single item dynamic inventory control model is analyzed. In this model the decision maker counts the inventory periodically, calculates the reorder point and the replenishment point, and decides to replenish the stock according to the inventory position. This calculation is difficult and requires complex mathematical transactions when the demand and lead time are stochastic. For this reason, in this study, the simulation method and genetic algorithms method are used to calculate the reorder point and replenishment point by using total cost function per period. In this function, the ordering cost, the holding cost and the penalty cost are taken into account. The results of these two methods are compared with classic method based on real data where the demand distribution is normal, and the lead time distribution is uniform. Thereafter, three cost calculations and their effects on reorder point and replenishment point are analyzed at two different levels.

Keywords: *Replenishment Point, Reorder Point, Inventory Control, Simulation, Genetic Algorithms, Stochastic Demand.*
JEL Classification Codes: M1, L6, C6.

1. Introduction

Inventory management is one of the key process in operation management. A manufacturer company deals with purchasing raw-materials from various suppliers, holding these goods till they are inserted to the line according to the production plan and at last holding the finished goods for a certain period of time. When the manufacturer’s functions in the chain is considered, the importance of the inventory management should be taken in to account. From the manufacturer’s point of view, inventory can be considered as a raw-material for production, a partially finished item on the line, a finished good to be delivered and even a tool or a spare part for machines for maintenance (William, 2009). The key variable in the problem is the outcome of the various directions of demand and supply for each product. As a result of different names given and different characteristics in each step, it is a must for the decision maker to consider the inventory control separately in each of the production steps and develop policies. The decision maker focuses on to keep the minimum inventory on hand during the developing phase of these policies while dealing with any different inventory type. Consequently, keeping the minimum inventory on hand is the primary goal. Beside this, it is aimed to keep the inventory purchase and holding costs as

low as possible. It is not enough to realize just these two condition to design a good inventory policy. The number of items produced or purchased at the beginning of each period should meet the demand of that period. Briefly, a good inventory management policy should focus on reducing the inventory related costs while redounding the customer service level. Also, low unit costs and high inventory turnover rates are sub-targets.
There are some factors effecting on the decision makers judge. These factors can be collected under four basic groups. These are demand, replenishment, lead time and cost factors. Many inventory control models had been developed on the basis of the fundamental goals of inventory management, using these factors. When diversification is made with a demand oriented view; deterministic and stochastic inventory control models had been came out, according to structure of the demand. While the policies are determined, a mathematical model is structured according to the factors mentioned. By adopting all the costs to the model, an inventory control model with the minimum cost is achieved. The outcome of this model is the answers of “when to replenish” and “how many to purchase/produce”. In order to determine these cost values various mathematical methods can be used.

The (s,S) inventory model put forward by Arrov et al. (1951) is used by many firms and found a wide application area. However, in cases where inventory records cannot be reviewed continuously, the model transforms to periodic review (R,s,S) inventory model. In the literature, both models are called optional replenishment policy or model (Rabta & Aissani, 2005; Yin, Liu & Johnson, 2002; Buchan & Koenigsberg, 1966; Nielsen & Larsen, 2005). The optional replenishment model used in this study is based on periodic review. These models are preferred by many companies as inventory policy in order to avoid from high levels of inventory in low demand periods, and from stock-out in high demand periods. This model reacts very quickly to demand changes. However, the success of the model stems from the quality of data analysis and of determination of reorder point (s) and replenishment point (S). The most problematic part of this model is the difficulty to compute s and S parameters. There are a lot of methods which enables the computation of these two parameters. But these methods are complex and difficult to solve (Silver, Pyke & Peterson, 1998: 336).
The basis of (R, s, S) model depends on Arrow (1951) and Karlin et al. (1958). But the optimization issue for finite periods is studied by Scarf (1960), under specific assumptions. Scraf’s model is studied by Iglehart (1963) for infinite periods under fixed demand and cost structure. Veniott (1966) and Porteus (1971) had researches on this field. Federguen and Zheng determined the calculation of the parameters with average cost per period for infinite periods, in case of discrete demand distribution for each period. Beside this, there are some other heuristic methods. Robert (1962), Sivazlian

(1971), Nador (1975), Wagner (1975) and Schneider (1978) studied on the reorder point in certain service levels. Ehrhardt (1979-1984) Freeland and Porteus (1980), Tijims & Groenevelt (1984), Zheng & Federgruen (1991) developed an algorithm to find these parameters under discrete demand distribution. Bollapragad & Morton (1999), Larson et al. (2001) analyzed such a policy with stochastic demand.

Sezen & Erdoğmuş (2005) determined the inventory policy with the help of simulation method. Köchel & Nielander (2005) offered a simulation based optimization method for multi echelon inventory systems. Ye& You (2016) developed a simulation based optimization method for reducing the cost with a constant service level. Escuín, & Ciprés, (2016), deal with a multi-product dynamic lot-sizing problem under stochastic demand and compare make o order strategy and vendor managed strategy by simulation. DeYong, & Cattani (2015) analyzed the deterministic and stochastic problems as dynamic programs by simulation. The other simulation method is used in inventory management can be seen in the literatüre (White& Censlive, 2015; do Rego & de Mesquita 2015; Agrawal & Sharda, 2012; Kouki & Jouini, 2015; Duan & Liao, 2013; Thiel et al. 2010)

Zhou, et al., (2013) solved a multi-echelon inventory control problem by genetic algorithms. Maiti & Maiti (2006), deal with a multi-item inventory model with two-storage facilities by a fuzzy simulation-based single/multi-objective genetic algorithms. The other genetic algorithms methods used in inventory management problem can be found (Diabat & Deskoores, 2015; García et al., 2015; Nia et al., 2014; Rezaei & Davoodi, 2011; Hwang et al., 2005).

Taleizadeh et al. (2013) assumed that the time between two replenishments is an independent random variable. They demonstrated that the model of this problem is a kind of integer-nonlinear-programming. A hybrid method of fuzzy simulation (FS) and genetic algorithm (GA) were proposed to solve this problem. Genetic algorithms and simulation methods is widely used in inventory management (Lin et al, 2010; Jana et al. 2014).

In this study, real data is taken from a manufacturing company which uses refined steel as raw material. Demand values and lead time are random variables. Where the demand distribution is normal, the lead time is uniform. Additionally, holding cost, penalty cost (stock-out cost) and ordering cost are used. Reorder point and replenishment point are determined using simulation method and genetic algorithms method, and the cost results of these methods are compared. The effect of cost variation on the inventory model is analyzed by genetic algorithms method. Two levels of analysis are developed for these three cost calculations, and an experimental design is formulated accordingly. The new reorder and replenishment points are compared.

2. Dynamic Inventory Model

In single item inventory systems, from the economical aspect of replenishment, order and stock out case, this method can be managed to be favorable in comparison to other models under certain assumptions. Consequently, many studies had been achieved in the last century and many are in progress. In stochastic (R, s, S) inventory policy inventory position is reviewed every R periods. But the decision to order is given according to a certain order point of inventory position determined previously. If the inventory position is equal to or lower than the reorder point (s), an order is given to fill the inventory up to the replenishment point (S). If not, no order is given. At the beginning of the period, on hand inventory is displayed as x_n and transit inventory as Q_0. The sum of these two values is called inventory position. The order quantity (Q) at the beginning of the period is calculated as follows:

$$Q = 0 \qquad x_n + Q_0 > s$$

$$Q = S - x_n - Q_0 \qquad x_n + Q_0 \leq s$$

The total cost is calculated as in following equations:

$$TC = (cQ) + (C_s + k * Q) + ((\frac{x_n + Q}{2}) * (c * c_h)$$

$$or$$

$$TC = (cQ) + (C_s + k * Q) + (\frac{x_n + Q}{2}) * C_h + C_p \qquad D_n > x_n + Q$$

Where the:

c	Unit cost
C_h	*Holding Cost percentage*
C_p	Penalty (stock out) Cost
C_s	Ordering Cost (fixed)
k	Variable ordering cost
TC	Total cost
D_n	Demand of n. period

The classical method for solving the reorder point, the replenishment point and the review period is as follows:

$$R = \sqrt{\frac{2C_s}{D * c * C_h}}$$

$$s = D(L+R) + z\sqrt{(L+R)\sigma_D^2 + D^2\sigma_L^2}$$

$$S = s + Q$$

$$Q = \sqrt{\frac{2C_sD}{c * C_h}}$$

Another model which is developed by Ehrhardt (1979) is called *power approximation method*. The model contains the ordering cost, the holding cost and the penalty (backorder) cost. The review period and the lead time are constant. The average cost per period in infinite-horizon is analyzed. This method is an algorithm which helps to find approximate optimum values of inventory model parameters by using mean demand μ and variance σ^2 . The parameters are calculated as follows:

$$Q_p = (1,463)\mu^{0,364}(C_s / C_h)^{0,498}\left[(L+1)\sigma^2\right]^{0.0691}$$

$$s_p = (L+1)\mu + \left[(L+1)\mu\right]^{0,416}(\sigma^2/\mu)^{0,603}\left[0,220/z + 1,142 - 2,866z\right]$$

$$s_o = (L+1) + v\sqrt{(L+1)\sigma^2}$$

$$\int_{-\infty}^{v} \sqrt{2\pi}\,\exp(-x^2/2)\,dx = C_p / C_p + C_h$$

$$\mu_L = (L+1)\mu \;\; and \; \sigma_L = \sigma\sqrt{L+1}$$

$$Q_p = (1{,}463)\mu^{0,364}(\frac{C_s}{C_h})^{0,498}\sigma_L{}^{0,138}$$

$$s_p = \mu_L + \sigma_L{}^{0,832}(\frac{\sigma^2}{\mu})^{0,817}\left[\frac{0{,}220}{z} + 1{,}142 - 2{,}866z\right]$$

$$z = \sqrt{\frac{Q_p}{(1 + C_p / C_h)\sigma_L}}$$

If $Q_p / \mu > 1,5$; $s = s_p$, $S = s_p + Q_p$

If not: $s = \min\{s_p, s_o\}$, $S = \min\{s_p + Q_p, s_o\}$

3. Numerical Example

In this study, real data is analyzed taken from a company which operates in windows & door industry in Turkey. The company produces espagnolettes and supplies them to the companies in the PVC window industry. It is a make to order supplier. The main raw material used to produce espagnolettes is 15.2mm*2.25mm hot-rolled steel (refined steel). It is an A type inventory item. The company counts its inventory position at the beginning of any week in tons and order is given or not based on the reorder point. Therefore, it uses (R,s,S) inventory policy. The refined steel is purchased as steel coils of one ton, counted and stored. Every week it is forwarded to production according to job order. In the production process, the steel wires are straightened before getting cut and covered. The covered steel is assembled and becomes a final product of espagnolette. In this study, the real data concerning the demand for raw material in 52-weeks-period is analyzed using EasyFit 5.6 Professional. The weekly demand distribution is normal with a mean of 99.401, and a standard deviation of 8.8272. The fitting results of demand are as follows:

Table 1: Goodness of Fit – Summary

#	Distribution	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Beta	0,11383	3	0,83336	2	4,8798	2
2	Normal	0,0695	1	0,28604	1	2,172	1
3	Power Function	0,15086	4	2,6105	3	15,789	3
4	Uniform	0,08854	2	8,0964	4	N/A	

Table 2: Descriptive Statistic Report

Statistic	Value
Sample Size	52
Range	38
Mean	99,401
Variance	77,92
Std. Deviation	8,8272
Coef. of Variation	0,0888
Std. Error	1,2241
Skewness	0,10317
Excess Kurtosis	-0,37069

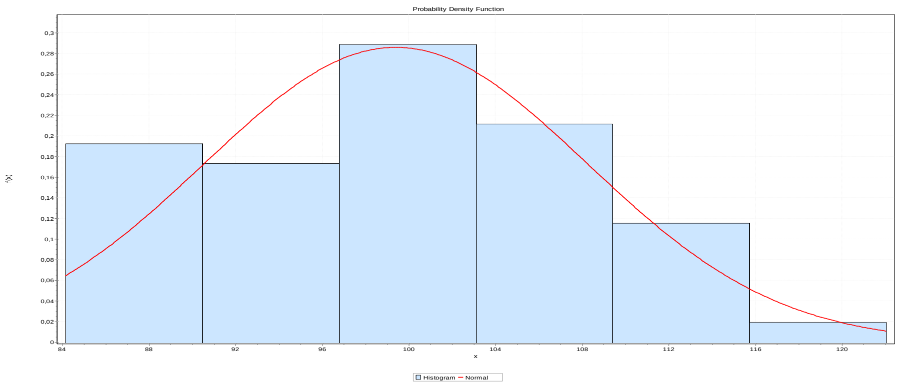

Figure 1: Demand Distribution Graph

Steel is supplied from France because of its high quality. The company imports steel by seaway. If an order is placed, it arrives after one, two, three or four weeks with uniform distribution. The fixed cost for placing an order is €100, and the variable cost for placing a unit is €1. The unit cost per ton is €750, the holding cost for one ton is %15 of unit cost per year. If weekly demand can't be satisfied from on hand inventory, an emergency order is placed by highway. This order arrives instantaneously with a cost of €300 per order. Otherwise the production would stop. So, the total cost function is given as in:

$$TC = 750Q + 100 + Q + \frac{x_n + Q}{2} * 2$$

$$or$$

$$TC = 750Q + 100 + Q + \frac{x_n + Q}{2} * 2 + 300 \qquad D_n > x_n + Q$$

Unit cost is omitted since there is no discount in the model.

The first method is genetic algorithms to calculate the reorder point and replenishment point. Genetic algorithms (GAs) is a population based optimization technique developed by Holland (Reeves, 1995; Goldberg, 1989). Chromosome is a vector that represents the solution variables. This representation can not only be done with binary coding but with real values as well. With randomly selected values from the pre-determined value range an initial solution is generated. This group of solutions produced with the number of chromosomes (population size) is called initial population. Than evaluates the quality of each solution candidates according to the problem-specific fitness function. Fitness function in our model is considered as minimization of the total cost including holding cost, penalty cost and ordering cost. New solution candidates are created by selecting relatively fit members of the population and recombining them through various operators:

selection, crossover, mutation (Allen & Karjalainen, 1999). The purpose of parent selection in GAs is to offer additional reproductive chances to those population members that are the fittest. One commonly used technique, the roulette-wheel-selection, is used for this proposed GAs. The crossover is the operator for solution space search. It is procedure of creating new chromosomes. There are various versions of crossover. In this paper, one-point crossover is used. Mutation plays decidedly secondary role in the operation of GAs. "In artificial genetic systems the mutation operator protects against such an irrecoverable loss" (Goldberg, 1989). In this study real-value coding is used. The steps of GAs are as follows:

Step 1. Generation of initial population
Step 2. Evaluation of each individual
Step 3. Selection
Step 4. Crossover
Step 5. Mutation
Step 6. If stopping criteria is not met return to Step 2
Step 7. Select the best individual as a final solution (Kiremitci & Akyurt; 2012).

GAs parameters used in calculation inventory model is given Table 3. GAs was run for 100 times and the averages of the results are represented in Table 4. The codes were generated MATLAB 8.5 (R2015a) and the algorithm was run with an Intel(R) Core(TM) i73517U CPU @ 1.90 GHz, 4.0 GB Ram configured PC. In Table 4, it can be seen that better results are reached when mutation rate is 0.2, and population size is 100. It is observed that 1000 iterations are enough for the solution. The reorder point is approximately as 9 and the replenishment point is 109. Average inventory cost is €327 per week.

Table 3: GAs Parameters for the Inventory Model

GAs Parameter	Values
Iteration	5000
Population size	20, 50, 100
Crossover rate	% 100
Mutation Rate	%1, % 10, %20

Table 4: GAs Results for Different Population Sizes (ps)

	Reorder Point	Replenishment Point	Average TC(week)
GAs (ps:20)	14.23	113,46	339
GAs (ps:50)	10,57	110,49	332
GAs (ps:100)	8.87	109,13	327

The other method for determining reorder point and replenishment point is simulation method which is applied using Microsoft Excel spreadsheet by taking the mentioned cost function into account. As seen in the Table 5, a 10x10 trial matrix is formulated. The model searches the reorder point and the replenishment point which result in minimum cost for predetermined values. In each iteration, 52 weeks are simulated according to demand and lead time structure. This model runs for 1000 iterations. In Table 5, the repetition number of the minimum total cost is shown. The reorder point is calculated as 10, and the replenishment point is 110 and it is repeated for 188 times. However, 1000 iterations take 17 minutes with the same PC, and it takes longer compared to the genetic algorithms method. In Table 6, average weekly inventory cost can be seen for different values of reorder point and replenishment point. It’s calculated through a 52-weeks simulation.

Table 5: Simulation Results

		Replenishment Point (S)									
		110	**115**	**120**	**125**	**130**	**135**	**140**	**145**	**150**	**155**
Reorder Point (s)	**10**	188	140	24	4	6	2	0	0	0	0
	15	148	80	18	2	4	0	0	0	0	0
	20	80	52	12	4	4	4	0	0	0	0
	25	32	14	0	4	2	0	0	0	0	0
	30	58	6	2	0	0	0	0	0	0	0
	35	26	8	0	0	0	0	0	0	0	0
	40	28	6	0	0	0	0	0	0	0	0
	45	20	4	0	0	0	0	0	0	0	0
	50	4	0	0	0	0	0	0	0	0	0
	55	12	2	0	0	0	0	0	0	0	0
	Total	596	312	56	14	16	6	0	0	0	0

Table 6: Average Weekly Costs for Different Values of Inventory Parameters

Reorder Point (s)	Replenishment Point (S)								
	110	115	120	125	130	135	140	145	150
10	332,9	334,4	340,1	343,1	345,0	346,6	349,7	352,5	356,4
15	334,6	335,6	340,3	343,2	345,7	347,6	350,3	353,2	356,3
20	336,1	337,3	342,4	343,8	346,9	348,4	350,8	353,2	357,3
25	337,7	338,3	344,3	345,7	347,7	348,7	350,5	353,5	356,9
30	337,5	340,5	345,8	347,8	350,2	349,8	351,8	354,0	357,7
35	339,5	341,7	347,7	350,6	352,8	352,9	353,8	355,5	359,1
40	340,8	341,4	348,5	351,3	353,5	354,2	355,5	357,0	358,9
45	341,2	343,2	349,5	352,4	354,9	355,7	357,4	358,9	360,1
50	341,1	344,0	349,6	352,9	355,2	357,7	359,6	361,5	363,1
55	342,2	344,3	351,0	353,2	356,0	358,0	361,0	362,5	364,9

Table 7 compares results of classic, simulation and genetic algorithms methods. When the classic method is used average cost increases dramatically because of that the reorder point in the formulation is high. Results of two other methods are close, the result generated by genetic algorithm is faster, and has a lower cost. The simulation formulates results according presupposed intervals, and therefore cannot generate solution values directly.

Table 7: Comparison of Results

	Reorder Point	Replenishment Point	Average Cost(weekly)
Classic Method	420	527	785
GAs (ps:100)	8.87	109,13	327
Simulation*	10	110	333

Two-level genetic algorithms' results determined for each cost are given in Table 8. As seen in the table, when only holding cost increases, model parameters stay stable. Similarly, when only ordering cost increases, parameters stay again stable. When both holding cost and ordering cost increase, parameters stay again stable, but only total cost increases slightly. As penalty cost increases, the reorder point increases dramatically, and the model converges to the base stock model (R,S).

Table 8: Experimental Results for Different Costs (GAs)

Experiment Number	Holding Cost	Penalty Cost	Ordering Cost	Reorder Point s	Replenishment Point S	Av. Weekly Total Cost €
1	2	300	100	9	109	327
2	2	300	250	7	110	393
3	2	750	100	65	163	581
4	2	750	250	67	165	639
5	5	300	100	8	104	422
6	5	300	250	7	108	736
7	5	750	100	51	110	691
8	5	750	250	8	113	763

4. Results and Suggestions

In this study, optional replenishment model is analyzed as an inventory model. Although this policy results in optimum values, in practice, it is hard to determine parameters of reorder point (s) and replenishment point (S). In order to determine these two points, the demand structure is analyzed and a cost function is formulated. This function is compared with GAs codes developed in MATLAB with the simulation developed in Microsoft Excel. Weekly average costs are calculated for different reorder and replenishment points. In the chosen case, the demand for raw material is normal, and its lead time shows uniform distribution. When the results are evaluated, it is seen that GAs constitutes a better alternative in terms of both time and cost. However, the cost difference stems from increases of 10 units in the simulation model. When the classic method is used average cost increases dramatically because of that the reorder point in the formulation is high.
In addition, in order to analyze the influence of cost change on reorder point and replenishment point, two levels are determined for holding cost, penalty cost and ordering cost. The model is operated at these levels again by GAs and the results reached are examined. The examination shows that the main cost effecting the model is the penalty cost. As the penalty cost increases, the reorder point also increases. Therefore, the policy becomes the base stock policy.

With the help of some modifications in the structure of parameters of GAs, the solution time can be shortened. Determining R as a variable, the search for a solution can be continued in a wider area. Moreover, the same model can be used for various demand distributions and lead time distributions.

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