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A Note on Spherical Product Surface with L_1 -Pointwise 1-Type Gauss Map

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Abstract

In this study, we deal with the spherical product surface whose Gauss map G satisfies the equality $L_1G = f(G + C)$ where L_1 is the Cheng-Yau operator in Galilean 3-space G_3 . We obtain the necessary and sufficient conditions for spherical product surface to have L_1 -pointwise 1-type Gauss map in G_3 .

Keywords: Spherical product surface, Cheng-Yau operator, Galilean 3-space.

1. Introduction

Finite type immersions are first given by Chen (Chen 1983), (Chen 1984), (Chen 1996). Let M be a submanifold in m -dimensional Euclidean space IE^m . Isometric immersion $x: M \rightarrow IE^m$ is called as a finite type if it can be written as a finite sum of eigenvectors of the Laplacian Δ of M for a constant map x_0 , and non-constant maps

x_1, x_2, \dots, x_k , i.e.,

$$x = x_0 + \sum_{i=1}^k x_i.$$

Here, $\Delta x = \lambda_i x_i$, $\lambda_i \in \mathbb{R}$, $1 \leq i \leq k$. The

submanifold is said to be k -type if the numbers λ_i s are different (Chen 1984).

Then, by Chen and Piccinni, these immersions are generalized to the Gauss map G of M as

$$\Delta G = a(G + C)$$

for a constant vector C and a real number a in (Chen and Piccinni 1987). A submanifold that satisfies the last equality has 1-type Gauss map. After that, in the last equality, a non-constant differentiable function f is taken instead of a . Namely, the last equality turns into

$$\Delta G = f(G + C). \quad (1)$$

A submanifold that satisfies the equation (1) is said to have pointwise 1-type Gauss map. Also, if the vector C is zero, the pointwise 1-type Gauss map is called as the first kind. Otherwise, it is called as the second kind (Chen et al. 2005). Surfaces satisfying the equation (1) are the subject of many studies such as (Arslan et al. 2011), (Arslan et al. 2014), (Arslan and Milousheva 2016), (Choi and Kim 2001), (Kişi and Öztürk 2018), (Kişi and Öztürk 2019a), (Kişi and Öztürk 2019b).

In (Alias and Gürbüz 2006), (Kashani 2009), the notion of finite type submanifolds is generalised by replacing the Laplacian operator with operators L_k ($k = 1, 2, \dots, n-1$) that represent the linear operators of the first variation of the $(k+1)$ -th mean curvature of a submanifold. Here, $L_0 = -\Delta$ and L_1 is the Cheng-Yau operator. Recently, some papers published about the surfaces having L_1 -pointwise 1-type Gauss map in some spaces, such as (Güler and Turgay 2019), (Kim et al 2016), (Kim and Turgay 2013), (Kim and Turgay 2017), (Mohammadpouri 2018), (Qian and Kim 2015), (Yoon et al. 2015).

Embeddings of product spaces defined by Kuiper with a new type in the IE^{m+n+d} as follows:

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Let $g : S^n \rightarrow IE^{n+1}$ and $f : M \rightarrow IE^{m+d}$ be embeddings from n -sphere into IE^{n+1} and from an m -dimensional manifold M into IE^{m+d} , respectively. Then, the new type embedding is

$$X = f \otimes g : M \times S^n \rightarrow IE^{m+n+d}, \quad (2)$$

$$X(u, v) = (f_1(u), f_2(u), \dots, f_{m+d-1}(u), f_{m+d}(u)g(v))$$

(Kuiper 1970).

In (Arslan et al. 2008), (Bulca et al. 2012), authors entitled this type embedding as rotational embedding. They deal with a special case of the rotational embedding which is called as spherical product surfaces in Euclidean spaces. They obtained this surface by writing $m = n = 1$, and $d = 1, 2$ in (2), respectively. Moreover, in Galilean 3-space, spherical product surface is studied in (Ayдын and Öğrenmiş 2016).

2. Preliminaries

Here, some preliminaries about Galilean geometry are given. For more detailed information, the studies (Röschel 1986), (Yaglom 1979) can be examined.

The scalar product and the cross product of the two vectors $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$ in G_3 are defined as

$$\langle a, b \rangle = \begin{cases} a_1 b_1, & \text{if } a_1 \neq 0 \text{ or } b_1 \neq 0 \\ a_2 b_2 + a_3 b_3 & \text{if } a_1 = 0 \text{ and } b_1 = 0, \end{cases} \quad (3)$$

and

$$a \times b = \begin{vmatrix} 0 & e_2 & e_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}, \quad (4)$$

respectively. Here, $e_2 = (0, 1, 0)$ and $e_3 = (0, 0, 1)$ are orthonormal unit vectors. The length of the vector $a = (a_1, a_2, a_3)$ is given as follows:

$$\|a\| = \begin{cases} |a_1|, & \text{if } a_1 \neq 0 \\ \sqrt{a_2^2 + a_3^2}, & \text{if } a_1 = 0 \end{cases} \quad (5)$$

(Pavkovic and Kameranovic 1987).

An admissible unit speed curve $\alpha : I \subset \mathbb{R} \rightarrow G_3$ is given with the parametrization

$$\alpha(u) = (u, y(u), z(u)).$$

For an isometric immersion $X : M \rightarrow \tilde{M}$ from a hypersurface M from an $(n+1)$ -dimensional Riemannian manifold \tilde{M} , and for the Levi-Civita connections $\tilde{\nabla}$ of \tilde{M} and ∇ of M , the Gauss formula is given by

$$\tilde{\nabla}_X Y = \nabla_X Y + \langle S(X), Y \rangle,$$

where $X, Y \in \chi(M)$ and S is the shape operator of M . It is known that the eigenvalues k_1, k_2, \dots, k_n of S are called as the principal curvatures of M . For a smooth function f on M , linear operators $L_k S$ are defined as

$$L_k(f) = \text{div}(P_k(\nabla f))$$

where ∇ is the gradient, div is the divergence operator and

$$P_k = \sum_{i=0}^k (-1)^i s_{k-i} S^i$$

is the Newton k -th transformation, $s_k = \binom{n}{k} H_k$ is the k -th mean curvature (Cheng and Yau 1977). Thus, for $k = 0$, $P_0 = I_n$ (I_n is the identity matrix), and for $k = 1$, $P_1 = \text{tr}(S)I_n - S$.

Now, let M be a surface, e_1, e_2 be the principal directions correspond to the curvatures k_1, k_2 of M . From (2), for a smooth function f , the Cheng-Yau operator $L_1 f$ can be given as

$$L_1 f = \text{div}(P_1(\nabla f)) = e_1[k_2]e_1 f + e_2[k_1]e_2 f + k_2(e_1 e_1 - \nabla_{e_2} e_2) f + k_1(e_2 e_2 - \nabla_{e_1} e_1) f.$$

Hence, the Cheng-Yau operator L_1 can be given by

$$L_1 = e_1[k_2]\tilde{\nabla}_{e_1} + e_2[k_1]\tilde{\nabla}_{e_2} + k_2(\tilde{\nabla}_{e_1}\tilde{\nabla}_{e_1} - \tilde{\nabla}_{\nabla_{e_2} e_2}) + k_1(\tilde{\nabla}_{e_2}\tilde{\nabla}_{e_2} - \tilde{\nabla}_{\nabla_{e_1} e_1})$$

(Kim and Turgay 2013).

Let the surface M parametrized with

$$X(u_1, u_2) = (x(u_1, u_2), y(u_1, u_2), z(u_1, u_2))$$

in G_3 . To represent the partial derivatives, we use

$$x_{,i} = \frac{\partial x}{\partial u_i} \quad \text{and} \quad x_{,ij} = \frac{\partial^2 x}{\partial u_i \partial u_j}, \quad 1 \leq i, j \leq 2.$$

If $x_{,i} \neq 0$ for some $i = 1, 2$, then the surface is admissible (i.e. having not any Euclidean tangent planes). The first fundamental form I of the surface M is defined as

$$I = (g_1 d_{u_1} + g_2 d_{u_2})^2 + \varepsilon(h_{11} d_{u_1}^2 + 2h_{12} d_{u_1} d_{u_2} + h_{22} d_{u_2}^2),$$

where $g_i = x_{,i}$, $h_{ij} = y_{,i} y_{,j} + z_{,i} z_{,j}$; $i, j = 1, 2$ and

$$\varepsilon = \begin{cases} 0, & \text{if } d_{u_1} : d_{u_2} \text{ is non-isotropic,} \\ 1, & \text{if } d_{u_1} : d_{u_2} \text{ is isotropic.} \end{cases}$$

Let a function W is given by

$$W = \sqrt{(x_{,1} z_{,2} - x_{,2} z_{,1})^2 + (x_{,2} y_{,1} - x_{,1} y_{,2})^2} \quad (6)$$

Then, the unit normal vector field is given as

$$G = \frac{1}{W} (0, -x_{,1} z_{,2} + x_{,2} z_{,1}, x_{,1} y_{,2} - x_{,2} y_{,1}). \quad (7)$$

Similarly, the second fundamental form II of the surface M is defined as

$$II = L_{11} d_{u_1}^2 + 2L_{12} d_{u_1} d_{u_2} + L_{22} d_{u_2}^2,$$

where

$$L_{ij} = \frac{1}{g_1} \langle g_1(0, y_{,ij}, z_{,ij}) - g_{i,j}(0, y_{,1}, z_{,1}), N \rangle, \quad g_1 \neq 0$$

or

$$L_{ij} = \frac{1}{g_2} \langle g_2(0, y_{,ij}, z_{,ij}) - g_{i,j}(0, y_{,2}, z_{,2}), N \rangle, \quad g_2 \neq 0.$$

The Gaussian and the mean curvatures of M are defined as

$$K = \frac{L_{11}L_{22} - L_{12}^2}{W^2} \quad \text{and} \quad H = \frac{g_2^2 L_{11} - 2g_1 g_2 L_{12} + g_1^2 L_{22}}{2W^2}.$$

(8)

A surface is called as flat (resp. minimal) if its Gaussian (resp. mean) curvatures vanish (Aydm et al. 2019), (Röschel 1986).

Lemma 2.1 (Kim and Turgay 2013) Let M be an oriented surface in IE^3 and K and H be the Gaussian and the mean curvatures of M , respectively. Then the Gauss map G of M satisfies

$$L_1 G = -\nabla K - 2HKG. \quad (9)$$

Definition 2.2 (Kim and Turgay 2013) Let M be an oriented surface in IE_3 . Then, M is said to have L_1 -harmonic Gauss map if its Gauss map satisfies $L_1 G = 0$.

Definition 2.3 (Kim and Turgay 2013) Let M be an oriented surface in IE^3 . Then, M is said to have L_1 -pointwise 1-type Gauss map if its Gauss map satisfies

$$L_1 G = f(G + C) \quad (10)$$

for a smooth function f and a constant vector C .

Moreover, if the vector C is zero, the pointwise L_1 -type Gauss map is called as the first kind. Otherwise, it is called as the second kind.

3. Spherical Product Surface with L_1 Pointwise 1-Type Gauss Map in G_3

For the smooth functions p and q , let $\alpha(u) = (u, p(u))$ and $\beta(v) = (v, q(v))$ be the unit speed plane curves in Galilean 2-space G_2 . Then the spherical product patch is given as

$$X = \alpha \otimes \beta : G_2 \rightarrow G_3, \quad (11)$$

$$X(u, v) = (u, p(u)v, p(u)q(v)).$$

The surface M given with (11) is called as spherical product surface in G_3 .

From the parametrization of the surface,

$$g_1 = u_{,1} = 1, \quad g_2 = u_{,2} = 0. \quad (12)$$

An orthonormal frame $\{e_1, e_2, G\}$ of M is given by

$$e_1 = \frac{X_u}{\|X_u\|} = (1, p'(u)v, p'(u)q(v)), \quad \|X_u\| = 1 \quad (13)$$

$$e_2 = \frac{X_v}{\|X_v\|} = \frac{(0, 1, q'(v))}{\sqrt{1 + (q'(v))^2}}, \quad \|X_v\| = p\sqrt{1 + (q'(v))^2}$$

and

$$G = \frac{(0, -q', 1)}{\sqrt{1 + (q')^2}}, \quad (14)$$

where $W = p\sqrt{1+(q')^2}$. The coefficients of the second fundamental form are obtained as

$$L_{11} = \frac{p''A}{\sqrt{1+(q')^2}}, \quad L_{12} = 0, \quad L_{22} = \frac{pq''}{\sqrt{1+(q')^2}}, \quad (15)$$

where $A = A(v) = -vq'(v) + q(v)$. From, (12) and (15), curvature functions of M are obtained as

$$K = \frac{p''q''A}{p(1+(q')^2)^2}, \quad H = \frac{q''}{2p(1+(q')^2)^{\frac{3}{2}}} \quad (16)$$

(Aydm and Öğrenmiş 2016).

Theorem 3.4 (Aydm and Öğrenmiş 2016)

Let M be a spherical product surface given with the parametrization (11) in G_3 . Then, M is minimal if and only if the curve β is a line and the surface is an open part of an isotropic plane.

Theorem 3.5 (Aydm and Öğrenmiş 2016)

Let M be a spherical product surface given with the parametrization (11) in G_3 . Then, M is flat if and only if it is either an isotropic plane or the generating curve α is a line.

By (13) and (16), we write the gradient of the Gaussian curvature

$$\nabla K = \frac{q''AB}{p^2(1+(q')^2)^2} e_1 + \frac{p''(1+(q')^2)(q''A+q''A')-4p''(q'')^2q'A}{p(1+(q')^2)^3} e_2, \quad (17)$$

where $B = B(u) = p'''(u)p(u) - p''(u)p'(u)$.

Thus, from (9), (16) and (17), we obtain the Cheng-Yau operator of the Gauss map as

$$L_1G = \frac{-q''AB}{p^2(1+(q')^2)^2} e_1 \quad (18)$$

$$+ \frac{-p''(1+(q')^2)(q'''A+q''A')+4p''(q'')^2q'A}{p(1+(q')^2)^3} e_2$$

$$+ \frac{-p''(q'')^2A}{p^2(1+(q')^2)^{\frac{7}{2}}} G.$$

We assume that the spherical product surface M has L_1 -pointwise 1-type Gauss map of the first kind, i.e. $L_1G = fG$ for a smooth function f . Then, from (13), (14) and (18),

$$\frac{-1}{p^2(1+(q')^2)^4} \left\{ \begin{array}{l} q''(1+(q')^2)^2 AB(1,p'v,p'q) \\ pp''(1+(q')^2)^{\frac{3}{2}}(q''A+q''A')-4pp''(q'')^2q'A \\ +p''(q'')^2A(0,-q',1). \end{array} \right\} (0,1,q)$$

$$= \frac{f}{\sqrt{1+(q')^2}} (0,-q',1) \quad (19)$$

From (19), $q''AB = 0$. Here, $q' \neq 0$, $q'' \neq 0$, $A \neq 0$, and $p' \neq 0$, $p'' \neq 0$, otherwise f would be zero. Then, $p'' = pc^2$ and so $p(u) = c_1e^{cu} + c_2e^{-cu}$. By the second and the third components of (21), we have

$$pp''(1+(q')^2)^{\frac{3}{2}}(q''A+q''A')-4pp''(q'')^2q'A-p''(q'')^2A = fp^2q'(1+(q')^2)^{\frac{7}{2}} \quad (20)$$

and

$$pp''q'(1+(q')^2)^{\frac{3}{2}}(q''A+q''A')-4pp''(q'')^2q'A+p''(q'')^2A = -fp^2(1+(q')^2)^{\frac{7}{2}}. \quad (21)$$

Multiplying (20) with $-q'$, and then adding the obtained equation by (21), we get

$$p''(q'')^2A = -fp^2(1+(q')^2)^{\frac{7}{2}},$$

where $p'' = pc^2$. Hence, we have

$$f = \frac{-p''(q'')^2A}{p^2(1+(q')^2)^{\frac{7}{2}}}. \quad (22)$$

Then, multiplying (21) with q' , and then adding the obtained equation by (20), we get

$$(1+(q')^2)^{\frac{3}{2}}(q'''(-q'v+q)-(q'')^2v)-4(q'')^2q'(-q'v+q)=0 \quad (23)$$

Then, we give the following theorems:

Theorem 3.6 Let M be a spherical product surface given with the parametrization (11) in G_3 .

M has L_1 -pointwise 1-type Gauss map of the first kind if and only if $p(u) = c_1e^{cu} + c_2e^{-cu}$ and $q(v)$ satisfies the differential equation (23).

Theorem 3.7 Let M be a spherical product surface given with the parametrization (11) in G_3 .

M has L_1 -pointwise 1-type Gauss map of the first

kind if and only if $K = \frac{c^2 q'' A}{(1+(q')^2)^2}$ which is a

function of v and $f = \frac{-Kq''}{p(1+(q')^2)^{\frac{3}{2}}}$.

Now, we consider that the spherical product surface M has L_1 -pointwise 1-type Gauss map of the second kind, i.e. $L_1 G = f(G+C)$ for a smooth function f and a nonzero constant vector C . Differentiating (13) and (14) with respect to e_1 and e_2 , we obtain the derivatives

$$\begin{aligned}\tilde{\nabla}_{e_1} e_1 &= \frac{p''(qq' + v)}{(1+(q')^2)^{\frac{1}{2}}} e_2 + \frac{p'' A}{(1+(q')^2)^{\frac{1}{2}}} G, \\ \tilde{\nabla}_{e_1} e_2 &= 0, \\ \tilde{\nabla}_{e_1} G &= 0, \\ \tilde{\nabla}_{e_2} e_1 &= \frac{p'}{p} e_2, \\ \tilde{\nabla}_{e_2} e_2 &= \frac{q''}{p(1+(q')^2)^{\frac{3}{2}}} G, \\ \tilde{\nabla}_{e_2} G &= \frac{-q''}{p(1+(q')^2)^{\frac{3}{2}}} e_2.\end{aligned}\quad (24)$$

From the equation (10) and (18), we can write the vector C as

$$C = \frac{-q'' AB}{fp^2(1+(q')^2)^2} e_1$$

$$\begin{aligned}& + \frac{-p''(1+(q')^2)(q'''A + q''A') + 4p''(q'')^2 q' A}{fp(1+(q')^2)^3} e_2 \\ & + \left(\frac{-p''(q'')^2 A}{fp^2(1+(q')^2)^{\frac{7}{2}}} - 1 \right) G.\end{aligned}\quad (25)$$

Since C is a nonzero constant vector, $\tilde{\nabla}_{e_1} C = 0$ and $\tilde{\nabla}_{e_2} C = 0$. From (24) and (25), we get

$$\begin{aligned}0 &= \tilde{\nabla}_{e_1} C = e_1 \left[\frac{-q'' AB}{fp^2(1+(q')^2)^2} \right] e_1 \\ & + \left(e_1 \left[\frac{D}{fp(1+(q')^2)^3} \right] - \frac{p'' q'' (qq' + v) AB}{fp^2(1+(q')^2)^{\frac{5}{2}}} \right) e_2 \\ & + \left(e_1 \left[\frac{-p''(q'')^2 A}{fp^2(1+(q')^2)^{\frac{7}{2}}} - 1 \right] - \frac{p'' q'' A^2 B}{fp^2(1+(q')^2)^{\frac{5}{2}}} \right) G,\end{aligned}\quad (26)$$

and

$$\begin{aligned}0 &= \tilde{\nabla}_{e_2} C = e_2 \left[\frac{-q'' AB}{fp^2(1+(q')^2)^2} \right] e_1 \\ & + \left(e_2 \left[\frac{D}{fp(1+(q')^2)^3} \right] + \frac{q''}{p(1+(q')^2)^{\frac{3}{2}}} \left(\frac{p''(q'')^2 A}{fp^2(1+(q')^2)^{\frac{7}{2}}} + 1 \right) - \frac{-q'' p' AB}{fp^3(1+(q')^2)^2} \right) e_2 \\ & + \left(e_2 \left[\frac{-p''(q'')^2 A}{fp^2(1+(q')^2)^{\frac{7}{2}}} - 1 \right] + \frac{q'' D}{p(1+(q')^2)^{\frac{3}{2}}} \right) G,\end{aligned}\quad (27)$$

where $D = D(u, v) = -p''(1+(q')^2)(q'''A + q''A') + 4p''(q'')^2 q' A$.

Since

$$e_1 \left[\frac{-q'' AB}{fp^2(1+(q')^2)^2} \right] = e_2 \left[\frac{-q'' AB}{fp^2(1+(q')^2)^2} \right] = 0,$$

we get

$$\frac{-q'' AB}{fp^2(1+(q')^2)^2} = c, \quad c \text{ const.}$$

Thus, we have

$$f = \frac{-q'' AB}{cp^2(1+(q')^2)^2}. \quad (28)$$

Theorem 3.8 Let M be a spherical product surface given with the parametrization (11) in G_3 . M has L_1 -pointwise 1-type Gauss map of the second kind if and only if f is a constant function given with (28).

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