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MULTIPLE CRITERIA DECISION MAKING APPROACH FOR INDUSTRIAL ENGINEER SELECTION USING FUZZY AHP-FUZZY TOPSIS

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ABSTRACT

In this study, a fuzzy multiple criteria decision-making approach is proposed to select an industrial engineer among ten candidates in a manufacturing environment. The industrial engineer selection problem is a special case of the personal selection problem. This problem, which has hierarchical structure of criteria and many decision makers, contains many criteria. The evaluation process of decision makers also includes ambiguous parameters. The fuzzy AHP is used to determine the weights for evaluation criteria. The consistencies of pair-wise comparisons matrices are controlled and a crisp overall performance value is obtained for each candidate based on the Best Non-fuzzy Performance Value. The sensitivity of the candidates' overall performance values is analyzed by taking into account both the weight of decision makers and the weights of basis criteria. The candidates are also evaluated by fuzzy TOPSIS method and the result obtained by fuzzy AHP is compared with the result achieved by fuzzy TOPSIS.

Keywords: Personnel selection, Fuzzy MCDM, Fuzzy AHP, Fuzzy TOPSIS, Sensitivity analysis

1. INTRODUCTION

Personnel selection is the process of selecting individuals with good qualifications to perform a defined job. The increasing competition in global markets encourages organizations to focus on personnel selection process [1]. The personnel selection problem is a strategic issue and has a significant impact on the efficiency of industrial systems. The selecting personnel performances such as capability, knowledge, skill, and other abilities play an important role to reach the goals of business process in organizations. The main objectives of a personnel selection process are to evaluate the differences among candidates and to predict the future performances. The prediction of future performances is a challenging task, as larger samples are needed and other temporal changes may affect employees. This process is generally managed by Human Resources Departments of in the organizations. Human resources management performs many functions such as employing well-qualified labor in business to evaluate performance, compensation management and labor training. Some organizations choose the best candidate by utilizing rigorous and costly selection procedures while others decide to fill positions quickly and inexpensively based only on the information stated on the application forms. Because of the wide area of applications and difficulties existing in finding the best personnel, personnel selection problems are one of the common objects for application of analytical methods.

Industrial engineering is one of the engineering branches dealing with the optimization of complex processes or systems. The industrial engineers are more concerned with increasing productivity of integrated systems including people, money, knowledge, information, equipment, energy and materials. Industrial engineering activities form a bridge between management goals and operational performance. Industrial engineers work in multidisciplinary teams, and are usually interested in the planning, installation, control and improvement of production activities. Therefore, they are preferred by different organizations such as government, manufacturing industry, research and consulting institutions, health care units, banks, insurance companies, nonprofit organizations, etc. Industrial

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engineer selection process is a critical issue for increasing productivity and competition. As a result of increased competition, firms need to manage their processes and resources as efficient and the demand for the industrial engineers rises in economy. According to industrial engineering qualifications, the industrial engineering selection process differs partially from a general personnel selection process.

Since opinions concerned with personal characteristics may vary from one person to another, the evaluation of personal attributes is fuzzy. Personnel selection process is a Multiple Criteria Decision-Making (MCDM) process. According to Lu and Ruan [2], in most real world contexts, an MCDM problem at tactical and strategic levels often involves fuzziness in its criteria (attributes) and decision makers' judgments. This kind of decision problems is called fuzzy multi-criteria decision-making (FMCDM).

In the literature, there are many studies about fuzzy AHP approach for personnel selection. Among the studies in the literature, there are both analytical models and empirical studies. Liang and Wang [3] used the fuzzy set theory introduced by Zadeh [4] and the weighted complete bipartite graph to solve the personnel placement problem. Liang and Wang [5] also developed a fuzzy MCDM algorithm that used both subjective and objective assessments to select employees. Karsak [6] used a fuzzy multiple objective programming approach for personnel selection. His proposed method integrated the decision-makers' linguistic assessments about subjective factors such as excellence in oral communication skills, personality, leadership, and quantitative factors such as aptitude test score within the multiple objective programming framework. Capaldo and Zollo [7] developed the effectiveness of personnel assessment within a large Italian corporation operating in the research sector. Toroslu [8] proposed a variation of the personnel assignment problem with some ordering constraints on the partitions of the bipartite graph. Bali and Gencer [9] used AHP, fuzzy AHP and fuzzy logic algorithm and compared the results. Kaptanoğlu and Özok [10] used a fuzzy AHP based model and used different fuzzy ranking methods one of which was proposed by Liou and Wang [11] and the other one was proposed by Abdel-Kader and Dugdale [12]. Dağdeviren [13] offered an algorithm for personnel selection with fuzzy AHP. The defuzzification of fuzzy weights was done with a different defuzzification operation based on α -cut and optimism index. Golec and Kahya [14] performed through a competency-based fuzzy model for the process of matching an employee with a certain job. Özdağoğlu [15] analyzed the criteria set and their importance for the selection of the manufacturing employee in a firm producing shoe machines. Aydın [16] evaluated the assignment system by FMCDM for Turkish Army Forces. Güngör et al. [17] applied the fuzzy AHP to evaluate the best adequate personnel dealing with the rating of both qualitative and quantitative criteria and compared the result obtained by fuzzy AHP with results produced by Yager's weighted goals method. Huang et al. [18] formulated a bi-objective binary integer-programming model in a fuzzy environment. Çelik et al. [19] proposed fuzzy integrated multi-stages evaluation model under multiple criteria in order to manage the academic personnel selection. Dursun and Karsak [1] developed an FMCDM algorithm using the principles of fusion of fuzzy information, 2-tuple linguistic representation model, and technique for order preference by similarity to ideal solution (TOPSIS) for hiring an industrial engineer. Şen and Çınar [20] integrated a fuzzy AHP method, a max–min approach and a non-parametric statistical test in order to evaluate operator's performance. Ozdaban and Ozkan [21] examined personal evaluation and job evaluation process. Rouyendegh and Erkan [22] examined a fuzzy AHP for selecting the most suitable academic staff while Rouyendegh and Erkan [23] dealt with actual application of academic of staff selection using the Fuzzy ELECTRE method. Over the past decade, several researchers have used different fuzzy MCDMs for the different selection problems [24 - 31].

This study purposes an FMCDM process that supports group decision making to solve the industrial engineer selection problem, which allows us use verbal statements in evaluation, which considers decision makers' priority at the management hierarchy. Recent process for industrial engineer selection is presented in this paper. This process forms the basis of choosing an industrial engineer between an industrial engineering department and an auto components industry to improve the

existing method of industrial engineer selection. In the existing personnel selection process of the Auto Components Industry, managers decide to fill positions quickly and inexpensively based only on the information stated on the application forms. However, this approach causes that the employees leave the job or the job productivity decreases. The managers of auto components industry request to collaborate with the industrial engineering department. One of the reasons of this demand is to have reliable knowledge about the candidates obtained by faculty members of the departments. We focus on the collaboration between the firms and departments for selecting industrial engineers and propose an FMCDM process to improve the existing method of the firm. The proposed process consists of five stages such as preliminary screening, evaluation of candidates' curriculum vitae (CV), industrial engineer selection, sensitivity analysis and comparing the result obtained by fuzzy AHP with results achieved by fuzzy TOPSIS. The knowledge obtained by faculty members of the industrial engineering department about candidates in period of training is included in the process of preliminary screening stage. At this stage, the managers of auto components industry are presented criteria obtained by researching literature. Additional criteria different from the researched literature are determined by asking managers and faculty members. The criteria obtained by researching literature and determined by the managers are illustrated in hierarchical structure of AHP. At the second stage, five candidates are selected by faculty members of the industrial engineering department among ten candidates according to the evaluation of candidates' CV. At the industrial engineer selection stage, the most preferred candidate is determined by integrating fuzzy AHP and FMCDM. The fuzzy AHP is only used to determine the weights of criteria. The candidates are evaluated by using FMCDM. As stated by Dağdeviren et al. [13], the full AHP-fuzzy AHP solution is only practically usable if the number of criteria and alternatives is sufficiently low so that the number of pair-wise comparisons performed by evaluator must remain below a reasonable threshold. Managers joined the selection process are very busy people. They generally don't want to make the large number of pair-wise comparisons. Therefore, to avoid an unreasonably large number of pair-wise comparisons for evaluating candidates, the FMCDM is employed to achieve the final ranking results. The consistency of matrices isn't controlled mostly and sensitivity analysis isn't performed in the related literature. The consistency of the pair-wise comparisons matrices is controlled by considering Consistency Index (CI). The sensitivity of candidates' overall performance value to both decision makers' weights and weights of basis criteria is analyzed. The result obtained by fuzzy AHP is compared with results produced by fuzzy TOPSIS. Although the fuzzy AHP is ranking the candidates, it is not determined whether the first rank is proper person for that job or not. Therefore, in order to support the fuzzy AHP results, the fuzzy TOPSIS is used to evaluate these results by using the positive and negative ideal solutions.

The outline of this paper as follows: In section 2, candidates' evaluation approach consists of fuzzy AHP, FMCDM and fuzzy TOPSIS is defined step by step. An application of the proposed approach is given in section 3. Finally, conclusions are given in section 4.

2. BUILDING EVALUATION APPROACH OF CANDIDATES

Personnel selection is an important matter for organizations. The concept of combining the fuzzy theory and MCDM is referred as FMCDM. This approach is presented to choose the right person for the right job. The expected criteria of candidates and their relative weights for choosing the job are determined by more than one decision makers. Thus, each decision maker can determine if the weights of criteria are important for themselves.

In this study, FMCDM process is defined to collaborate among industrial engineering departments that train candidates and companies that employ industrial engineers. This methodology was applied to senior students of industrial engineering department in Dumlupınar University in order to collaborate with the Auto Component Industry. The advantages of selecting among senior students for the industry are to reduce orientation training cost, to reach truly information of senior students obtained by the faculty members and to utilize that the senior students get oriented in Kütahya. To benefit from

these advantages, only senior students were considered as candidates. The proposed process consists of five stages such as preliminary screening, evaluation of candidates' curriculum vitae (CV), industrial engineer selection, sensitivity analysis and comparing the result obtained by fuzzy AHP with results achieved by fuzzy TOPSIS.

To show the proposed FMCDM approach, a procedure regarding stage-by-stage unification is firstly presented:

Stage 1. Preliminary screening: Form an evaluation and selection hierarchy by considering the organization's strategic business goals. Choose p_1 candidates have the best bachelor scores among last year students of industrial engineering department.

Stage 2. Evaluation of candidates' CV: Choose p_2 candidates by integrating fuzzy AHP and FMCDM among p_1 candidates determined at the stage 1. It is clear that p_2 is less than p_1 . Let m_1 be number of decision makers (faculty members).

Stage 3. Industrial engineer selection: Select an industrial engineer among p_2 candidates determined at stage 2 by integrating fuzzy AHP and FMCDM. Let m_2 be number of decision makers consist of managers of industrial organization and faculty members.

Stage 4. Sensitivity analysis: Analyze changes of decision makers' weights and weights of basis criteria to rank the performance of the candidates selected at stage 3.

Stage 5. Comparison of the results produced by the proposed approach and fuzzy TOPSIS: Apply fuzzy TOPSIS approach to the candidates' selection problem at stage 3. Then, the result obtained by integrating fuzzy AHP and FMCDM is compared with the results produced by fuzzy TOPSIS.

Figure 1 illustrates a flowchart for the proposed approach for the industrial engineering selection. The details of each stage are explained in the following sections.

Conclusion section should state clearly the main conclusions of the research and give a clear explanation of their importance and relevance. Summary illustrations may be included.

2.1. Fuzzy AHP

To solve this MCDM problem, AHP was proposed by Saaty [32]. The conventional AHP may not reflect human cognitive processes truly especially in the situations where problems are not fully defined and/or solving these problems involves uncertain data (so-called 'fuzzy' problems). To make up for this shortcoming, Laarhoven and Pedrycz [33] finally therefore introduced the concept of 'fuzzy theory' to AHP assessments. This so-called 'fuzzy AHP' is able to solve the uncertain 'fuzzy' problems and to rank excluded factors according to their weight ratios.

2.1.1. Fuzzy number

Fuzzy numbers are a fuzzy subset of real numbers, representing the expansion of the idea of the confidence interval. According to the definition of Laarhoven and Pedrycz [33], a triangular fuzzy number (TFN) should possess the following basic features.

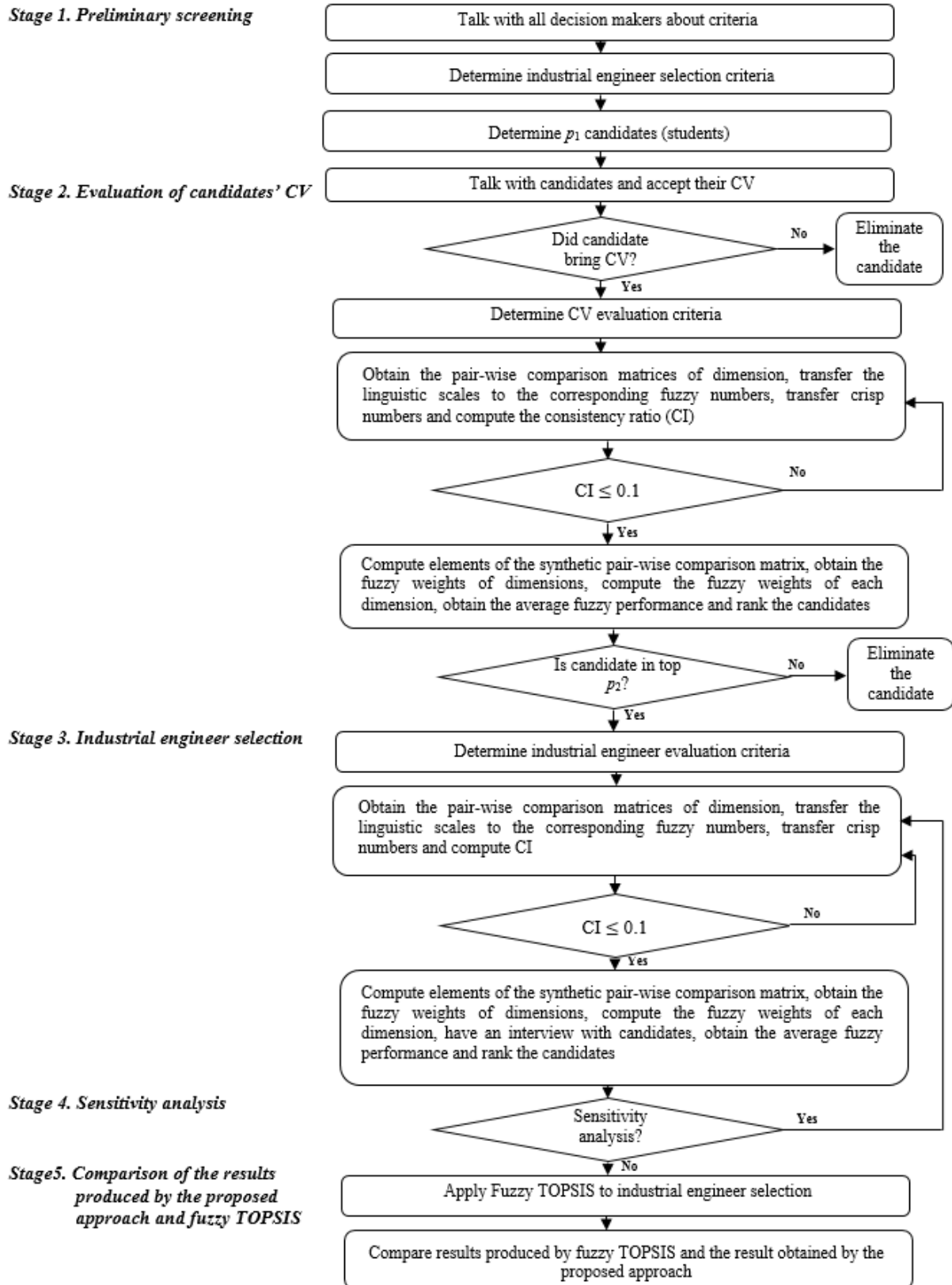


Figure 1. The proposed approach for industrial engineer selection

A fuzzy number \tilde{A} on R to be a TFN if its membership function $\mu_{\tilde{A}}(x): R \rightarrow [0,1]$ is equal to

$$\mu_{\tilde{A}}(x) = \begin{cases} (x - L)/(M - L), & L \leq x < M, L \neq M, \\ 1, & x = M, \\ (U - x)/(U - M), & M < x \leq U, M \neq U, \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$

where L and U stand for the lower and upper bounds of the fuzzy number \tilde{A} , respectively, and M is for the modal value. The TFN can be denoted by $\tilde{A} = (L, M, U)$ and the following is the operational laws of two TFNs $\tilde{A}_1 = (L_1, M_1, U_1)$ and $\tilde{A}_2 = (L_2, M_2, U_2)$, as shown [34]:

$$\begin{aligned} \text{Addition of a fuzzy number } \oplus : \tilde{A}_1 \oplus \tilde{A}_2 &= (L_1, M_1, U_1) \oplus (L_2, M_2, U_2) \\ &= (L_1 + L_2, M_1 + M_2, U_1 + U_2) \end{aligned} \quad (2)$$

$$\begin{aligned} \text{Subtraction of a fuzzy number } \ominus : \tilde{A}_1 \ominus \tilde{A}_2 &= (L_1, M_1, U_1) \ominus (L_2, M_2, U_2) \\ &= (L_1 - U_2, M_1 - M_2, U_1 - L_2) \end{aligned} \quad (3)$$

$$\begin{aligned} \text{Multiplication of a fuzzy number } \otimes : \tilde{A}_1 \otimes \tilde{A}_2 &= (L_1, M_1, U_1) \otimes (L_2, M_2, U_2) \\ &= (L_1 \cdot L_2, M_1 \cdot M_2, U_1 \cdot U_2) \text{ for } L_i, M_i, U_i > 0, \quad i = 1, 2 \end{aligned} \quad (4)$$

$$\begin{aligned} \text{Division of a fuzzy number } \oslash : \tilde{A}_1 \oslash \tilde{A}_2 &= (L_1, M_1, U_1) \oslash (L_2, M_2, U_2) \\ &= (L_1/U_2, M_1/M_2, U_1/L_2) \text{ for } L_i, M_i, U_i > 0, \quad i = 1, 2 \end{aligned} \quad (5)$$

$$\text{Reciprocal of a fuzzy number } \otimes : \tilde{A}_1^{-1} = (L_1, M_1, U_1)^{-1} = (1/U_1, 1/M_1, 1/L_1) \text{ for } L_i, M_i, U_i > 0, \quad i = 1, 2 \quad (6)$$

2.1.2. Linguistic variables

In this paper, the computational technique is based on the following fuzzy numbers as in Table 1. Linguistic variables are primarily used to assess the linguistic ratings given by decision makers for pair-wise comparisons of the importance of criteria in fuzzy AHP. Performance of candidates for each criterion are also used as a way to measure by using linguistic terms as “very good”, “good”, “fair”, “poor” and “very poor”. The procedure for determining the evaluation criteria weights by fuzzy AHP can be summarized as follows:

Step 1. Construct pair-wise comparison matrices among all the elements/criteria in the dimensions of the hierarchy system. Assign linguistic terms to the pair-wise comparisons by asking which is the more important of each two element/criteria.

Table 1. Membership function of linguistic scale

Fuzzy number	Linguistic scales	Scale of fuzzy number
$\tilde{1}$	Equally importance	(1,1,1)
$\tilde{2}$	Intermediate values between $\tilde{1}$ and $\tilde{3}$	(1,2,3)
$\tilde{3}$	Moderate importance	(2,3,4)
$\tilde{4}$	Intermediate values between $\tilde{3}$ and $\tilde{5}$	(3,4,5)
$\tilde{5}$	Essential importance	(4,5,6)
$\tilde{6}$	Intermediate values between $\tilde{5}$ and $\tilde{7}$	(5,6,7)
$\tilde{7}$	Very vital importance	(6,7,8)
$\tilde{8}$	Intermediate values between $\tilde{7}$ and $\tilde{9}$	(7,8,9)
$\tilde{9}$	Extreme vital importance	(9,9,9)

Step 2. To use geometric mean technique to define the fuzzy geometric mean and fuzzy weights of each criterion by Buckley [35] are as follows:

$$\begin{aligned}\tilde{r}_i &= (\tilde{a}_{i1} \otimes \tilde{a}_{i2} \otimes \dots \otimes \tilde{a}_{in})^{1/n}, \\ \tilde{w}_i &= \tilde{r}_i \otimes (\tilde{r}_1 \oplus \dots \oplus \tilde{r}_n)^{-1}\end{aligned}\quad (7)$$

where \tilde{a}_{in} is fuzzy comparison value of criterion i to criterion n , thus, \tilde{r}_i is geometric mean of fuzzy comparison value of criterion i to each criterion, \tilde{w}_i is the fuzzy weight of the i th criterion, can be indicated by a TFN, $\tilde{w}_i = (Lw_i, Mw_i, Uw_i)$, where Lw_i , Mw_i and Uw_i are the lower, middle and upper values of the fuzzy weight of the i th criterion.

2.2. Fuzzy MCDM

The FMCDM can be given as follows [36]:

(1) *Candidates measurement:* Using the measurement of linguistic variables to demonstrate the criteria performance by expressions such as “very good”, “good”, “fair”, “poor”, “very poor” the decision makers are asked for conduct their subjective judgments, and each linguistic variable can be indicated by a TFN within the scale range 0-100.

Take \tilde{E}_{ij}^k to indicate the fuzzy performance value of decision maker k towards candidate i under criterion j , and all of the evaluation criteria will be indicated by $\tilde{E}_{ij}^k = (LE_{ij}^k, ME_{ij}^k, UE_{ij}^k)$. This study uses the notion of average value to integrate the fuzzy judgment values of m decision makers, that is,

$$\tilde{E}_{ij} = (1/m) \otimes (\tilde{E}_{ij}^1 \oplus \tilde{E}_{ij}^2 \oplus \dots \oplus \tilde{E}_{ij}^m) \quad (8)$$

The end-point values LE_{ij} , ME_{ij} and UE_{ij} of the average fuzzy number \tilde{E}_{ij} can be solved by the method by Buckley [35], that is,

$$LE_{ij} = (\sum_{k=1}^m LE_{ij}^k) / m; ME_{ij} = (\sum_{k=1}^m ME_{ij}^k) / m; UE_{ij} = (\sum_{k=1}^m UE_{ij}^k) / m. \quad (9)$$

(2) *Fuzzy synthetic decision:* According to the each criterion weight \tilde{w}_j derived by fuzzy AHP, the criteria weight vector $\tilde{w} = (\tilde{w}_1, \dots, \tilde{w}_j, \dots, \tilde{w}_n)^t$ can be obtained, whereas the fuzzy performance matrix \tilde{E} of each of the candidates can also be obtained from the fuzzy performance value of each candidate under n criteria, that is, $\tilde{E} = (\tilde{E}_{ij})$.

The approximate fuzzy number \tilde{R}_i , of the fuzzy synthetic decision of each candidate can be shown as $\tilde{R}_i = (LR_i, MR_i, UR_i)$, where LR_i , MR_i and UR_i are the lower, middle and upper synthetic performance values of the candidate i , that is,

$$LR_i = \sum_{j=1}^n LE_{ij} \times Lw_j; MR_i = \sum_{j=1}^n ME_{ij} \times Mw_j; UR_i = \sum_{j=1}^n UE_{ij} \times Uw_j; \quad (10)$$

(3) *Ranking the fuzzy number:* In this study, the procedure of defuzzification is to locate the Best Non-fuzzy Performance Value (BNP) which is simple and practical method and there is no need to bring in

the preferences of any decision makers. The BNP value of the fuzzy number \tilde{R}_i can be found by the following equation:

$$BNP_i = [(UR_i - LR_i) + (MR_i - LR_i)]/3 + LR_i, \forall i \quad (11)$$

According to the value of the calculated BNP for each of the candidates, the ranking of the candidates for selecting the most preferred candidate.

2.3. Fuzzy TOPSIS

The proposed method by Chen [37] is based on the concept that the chosen candidate should have the shortest distance from the positive-ideal solution (i.e., achieving the minimal gaps in each criterion) and the longest distance from the negative-ideal solution (i.e., achieving the maximal levels in each criterion). TOPSIS defines an index called similarity to the positive-ideal solution and the remoteness from the negative-ideal solution. Then, the method chooses a candidate with the maximum similarity to the positive-ideal solution [38, 39]. It is often difficult for a decision maker to assign a precise performance rating to a candidate for the criteria under consideration. The merit of using a fuzzy approach is to assign the relative importance of criteria using fuzzy numbers instead of precise numbers for suiting the real world in fuzzy environment. We briefly review the rationale of fuzzy theory before the development of fuzzy TOPSIS.

Table 2 presents the linguistic variables and fuzzy ratings defined by Chen [37] for the criteria and Table 3 presents the linguistic variables and fuzzy ratings defined by Chen [37] for the candidates.

Table 2. Linguistic variables for the importance weight of each criterion

Linguistic variables	Scale of fuzzy number
Very low (VL)	(0.0,0.0,0.1)
Low (L)	(0.0,0.1,0.3)
Medium low (ML)	(0.1,0.3,0.5)
Medium (M)	(0.3,0.5,0.7)
Medium high (MH)	(0.5,0.7,0.9)
High (H)	(0.7,0.9,1.0)
Very high (VH)	(0.9,1.0,1.0)

Definition 1. Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between them as

$$d(\tilde{a}, \tilde{b}) = \left(\frac{1}{3} [(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2] \right)^{1/2} \quad (12)$$

Table 3. Linguistic variables for the ratings

Linguistic variables	Scale of fuzzy number	Linguistic variables	Scale of fuzzy number
Very poor (VP)	(0,0,1)	Medium good (MG)	(5,7,9)
Poor (P)	(0,1,3)	Good (G)	(7,9,10)
Medium poor (MP)	(1,3,5)	Very good (VG)	(9,10,10)
Fair (F)	(3,5,7)		

The steps of fuzzy TOPSIS can be given as follows:

Step 1. Determine the weighting of evaluation criteria. This research employs fuzzy AHP to find the fuzzy preference weights.

Step 2. Construct the fuzzy performance/decision matrix and choose the appropriate linguistic variables for the p candidates with respect to n criteria

$$\tilde{D} = \begin{matrix} & B_1 & B_2 & \cdots & B_n \\ \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_p \end{matrix} & \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{p1} & \tilde{x}_{p2} & \cdots & \tilde{x}_{pn} \end{bmatrix} \end{matrix}, \quad i = 1, 2, \dots, p; j = 1, 2, \dots, n$$

$$\tilde{x}_{ij} = \frac{1}{m} [\tilde{x}_{ij}^1 \oplus \dots \oplus \tilde{x}_{ij}^k \oplus \dots \oplus \tilde{x}_{ij}^m], \quad \tilde{w}_j = \frac{1}{m} [\tilde{w}_j^1 \oplus \dots \oplus \tilde{w}_j^k \oplus \dots \oplus \tilde{w}_j^m] \quad (13)$$

where \tilde{x}_{ij}^k is the performance rating of candidate C_i with respect to criterion B_j evaluated by k th decision maker, and $\tilde{x}_{ij}^k = (l_{ij}^k, m_{ij}^k, u_{ij}^k)$.

Step 3. Normalize the fuzzy-decision matrix. The normalized fuzzy-decision matrix denoted by \tilde{R} is shown as following formula:

$$\tilde{R} = [r_{ij}]_{p \times n} \quad i = 1, 2, \dots, p; j = 1, 2, \dots, n \quad (14)$$

Then, the normalization process can be performed by following formula: $\tilde{r}_{ij} = \left(\frac{l_{ij}}{u_j^+}, \frac{m_{ij}}{u_j^+}, \frac{u_{ij}}{u_j^+} \right)$, $u_j^+ = \max_i \{u_{ij} | i = 1, 2, \dots, n\}$ or we can set the best-aspired level u_j^+ and $j = 1, 2, \dots, n$ is equal one; otherwise, the worst is zero.

The normalized \tilde{r}_{ij} is still TFNs. For trapezoidal fuzzy numbers, the normalization process can be conducted in the same way. The weighted fuzzy normalized decision matrix is shown as following matrix \tilde{V} :

$$\tilde{V} = [\tilde{v}_{ij}]_{n \times n} \quad i = 1, 2, \dots, p; j = 1, 2, \dots, n \quad (15)$$

where $\tilde{v}_{ij} = \tilde{r}_{ij} \otimes \tilde{w}_j$.

Step 4. Determine the fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS). According to the weighted normalized fuzzy-decision matrix, we know that the elements \tilde{v}_{ij} are normalized positive TFN and their ranges belong to the closed interval $[0,1]$. Then, we can define the FPIS A^+ (aspiration levels) and FNIS A^- (the worst levels) as following formula:

$$A^+ = (\tilde{v}_1^+, \dots, \tilde{v}_j^+, \dots, \tilde{v}_n^+) \quad (16)$$

$$A^- = (\tilde{v}_1^-, \dots, \tilde{v}_j^-, \dots, \tilde{v}_n^-) \quad (17)$$

where $\tilde{v}_j^+ = (1,1,1) \otimes \tilde{w}_j = (lw_j, mw_j, uw_j)$ ve $\tilde{v}_j^- = (0,0,0)$, $j = 1, 2, \dots, n$.

Step 5. Calculate the distance of each candidate from FPIS and FNIS.

The distances (d_i^+ and d_i^-) of each candidate from A^+ and A^- can be currently calculated by the area compensation method.

$$d_i^+ = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^+), \quad i = 1, 2, \dots, p \quad (18)$$

$$d_i^- = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^-), \quad i = 1, 2, \dots, p \quad (19)$$

Step 6. Obtain the closeness coefficients (relative gaps-degree) and improve candidates for achieving aspiration levels in each criterion.

The CC_i is defined to determine the fuzzy gaps-degree based on fuzzy closeness coefficients for improving candidates; once the d_i^+ and d_i^- of each candidate have been calculated. Calculate similarities to ideal solution. This step solves the similarities to an ideal solution by formula:

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-} = 1 - \frac{d_i^+}{d_i^+ + d_i^-}, \quad i = 1, 2, \dots, p \quad (20)$$

where we define $\frac{d_i^-}{d_i^+ + d_i^-}$ as fuzzy satisfaction degree in i th candidate and $\frac{d_i^+}{d_i^+ + d_i^-}$ as fuzzy gap degree in i th candidate. We can know which and how fuzzy gaps should be improved for achieving aspiration levels and getting the best win-win strategy from among a fuzzy set of feasible candidates.

3. AN APPLICATION OF THE PROPOSED APPROACH FOR INDUSTRIAL ENGINEER SELECTION

In this section, the proposed approach is applied to an Auto Components Industry for selecting an industrial engineer among ten candidates. The proposed approach consists of five stages such as preliminary screening, evaluation of candidates' curriculum vitae (CV), industrial engineer selection, sensitivity analysis and comparing the result obtained by fuzzy AHP with results produced by fuzzy TOPSIS.

3.1. Preliminary Screening

The knowledge obtained by the faculty members of the industrial engineering department about candidates in period of training is included process in the preliminary screening stage. At this stage, the managers of auto components industry are presented the criteria obtained by researching literature. Additional criteria that are different from the researched literature are determined by asking the managers. The criteria obtained by researching literature and determined by managers are illustrated in the hierarchical structure of AHP. Consequently, ten candidates are determined by using their bachelor scores among last year students in the department.

3.2. Evaluation of Candidates' CV

The selected ten candidates at the first stage are held in a meeting and given information about selection process and business. Then, they give their CV to the department. Five students are determined by integrating the fuzzy AHP and FMCDM approaches. A committee of two decision-makers (two faculty members of industrial engineering department) has been formed to conduct the interview and to select the most suitable five candidates. Five selection criteria are considered as below:

- i. Bachelor degree
- ii. Foreign language
- iii. Computer skills
- iv. Projects
- v. Trainings

3.3. Industrial Engineer Selection

The five candidates are evaluated at this stage. A committee of three decision-makers (factory manager, manager of human resources department and chair of industrial engineering department) has been formed to conduct the interview and to select the most suitable candidate. The dimensions and

criteria of the problem are defined by decision makers and hierarchical structure is built at the first stage as given in Figure 2.

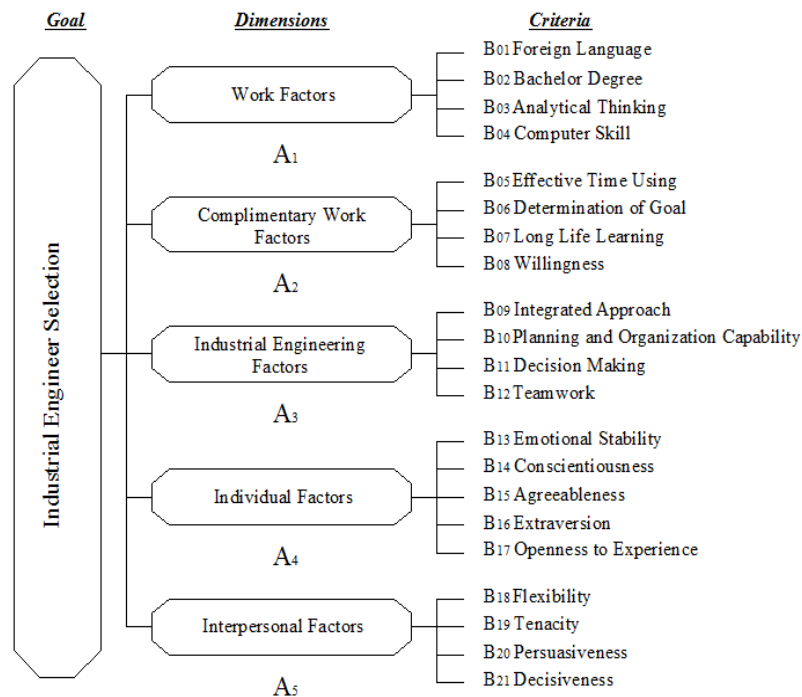


Figure 2. The hierarchical structure of evaluation

After the construction of the hierarchy, the different priority weights of each dimensions, criteria and candidates are calculated by integrating the fuzzy AHP and FMCDM approaches. The comparison of the importance or preference of one dimension, criterion or candidate over another is done with the help of the questionnaire. The method of calculating priority weights of the candidates is discussed below.

Step 1. Firstly dimensions (A1, A2, A3, A4 and A5) are evaluated by three decision makers with linguistic scales and they turned into fuzzy numbers. Factory manager, chair of industrial engineering department and manager of human resources department are indicated by DM1, DM2 and DM3, respectively. The pair-wise comparisons are given in Table 4.

Table 4. The pair-wise comparisons matrices of decision makers for dimensions

a) DM1						b) DM2						c) DM3					
	A1	A2	A3	A4	A5		A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	$\tilde{7}^{-1}$	$\tilde{7}^{-1}$	$\tilde{9}^{-1}$	$\tilde{7}^{-1}$	A1	1	$\tilde{5}$	$\tilde{5}^{-1}$	$\tilde{1}$	$\tilde{3}^{-1}$	A1	1	$\tilde{5}$	$\tilde{3}$	$\tilde{4}^{-1}$	$\tilde{7}$
A2	$\tilde{7}$	1	$\tilde{1}$	$\tilde{8}^{-1}$	$\tilde{1}$	A2	$\tilde{5}^{-1}$	1	$\tilde{9}^{-1}$	$\tilde{5}^{-1}$	$\tilde{7}^{-1}$	A2	$\tilde{5}^{-1}$	1	$\tilde{3}^{-1}$	$\tilde{5}^{-1}$	$\tilde{3}$
A3	$\tilde{7}$	$\tilde{1}$	1	$\tilde{8}^{-1}$	$\tilde{1}$	A3	$\tilde{5}$	$\tilde{9}$	1	$\tilde{5}$	$\tilde{3}$	A3	$\tilde{3}^{-1}$	$\tilde{3}$	1	$\tilde{3}^{-1}$	$\tilde{4}$
A4	$\tilde{9}$	$\tilde{8}$	$\tilde{8}$	1	$\tilde{8}$	A4	$\tilde{1}$	$\tilde{5}$	$\tilde{5}^{-1}$	1	$\tilde{5}^{-1}$	A4	$\tilde{4}$	$\tilde{5}$	$\tilde{3}$	1	$\tilde{7}$
A5	$\tilde{7}$	$\tilde{1}$	$\tilde{1}$	$\tilde{8}^{-1}$	1	A5	$\tilde{3}$	$\tilde{1}$	$\tilde{3}^{-1}$	$\tilde{1}$	1	A5	$\tilde{7}^{-1}$	$\tilde{3}^{-1}$	$\tilde{4}^{-1}$	$\tilde{7}^{-1}$	1

Step 2. According to Kwong and Bai [40], the consistency index, CI , and the consistency ratio, CR , for a comparison matrix can be computed with the use of following equations.

$$CI = \frac{(\lambda_{max} - n)}{(n-1)} \quad (21)$$

$$CR = \left(\frac{CI}{RI(n)} \right) \times 100\% \quad (22)$$

where, λ_{max} is the largest eigenvalue of the comparison matrix, n is the dimension of the matrix, and $RI(n)$ is a random index, that depends on n , as shown in Table 5.

Table 5. Consistency index, RI , of random matrices [41]

n	3	4	5	6	7	8	9
$RI(n)$	0.58	0.9	1.12	1.24	1.32	1.41	1.45

If the calculated CR of a comparison matrix is less than 10%, the consistency of the pair-wise judgment can be thought of as being acceptable. Otherwise the judgments expressed by the decision makers are considered to be inconsistent, and the decision maker has to repeat the pair-wise comparison matrix. A triangular fuzzy number, denoted as $M = (l, m, u)$, can be defuzzified to a crisp number as follows:

$$M_{crisp} = \frac{(4m+l+u)}{6} \quad (23)$$

Taking the comparison matrix DM1 as an example, the corresponding crisp matrix can be obtained as shown in Table 6:

Table 6. The pair-wise comparison matrix converted to crisp number for DM1

	A1	A2	A3	A4	A5
A1	1	0.15	0.15	0.12	0.15
A2	7	1	1	0.17	1
A3	7	1	1	0.17	1
A4	9	6	6	1	6
A5	7	1	1	0.17	1

The largest eigenvalue of matrix for DM1, λ_{max} , is calculated to be 5.3841. The dimension of the matrix, n , is five and the random index, $RI(5)$, is 1.12 by reference to Table 5. Therefore, the consistency index and the consistency ratio of the matrix is calculated as follows:

$$CI = (\lambda_{max} - n)/(n - 1) = (5.3841 - 5)/(5 - 1) = 0.0960,$$

$$CR = (CI / RI(5)) \times 100\% = (0.0960/1.12) = 0.0857 \leq 0.1.$$

After calculating the consistency ratios of all the other comparison matrices, it was found that they are all less than 10%. Therefore, the consistency of the judgment in all the comparison matrices is acceptable.

Step 3. Geometric mean method suggested by Buckley [35] is used to obtain the synthetic pair-wise comparison matrix and the comparison is given in Table 7.

Step 4. The calculations of fuzzy geometric means (\tilde{r}_i) can be given as follows:

$$\begin{aligned} \tilde{r}_1 &= (\tilde{a}_{11} \otimes \tilde{a}_{12} \otimes \tilde{a}_{13} \otimes \tilde{a}_{14} \otimes \tilde{a}_{15})^{1/5} \\ &= ((1 \times 1.277 \times \dots \times 0.580)^{1/5}, (1 \times 1.554 \times \dots \times 0.709)^{1/5}, (1 \times 1.82 \times \dots \\ &\quad \times 0.8794)^{1/5}) = (0.597, \quad 0.688, \quad 0.79) \end{aligned}$$

Table 7. Synthetic pair-wise comparison matrix for dimensions

	A1	A2	A3	A4	A5
A1	1.000	(1.277,1.554,1.829)	(0.354,0.448,0.554)	(0.289,0.311,0.344)	(0.580,0.709,0.879)
A2	(0.558,0.654,0.794)	1.000	(0.311,0.344,0.392)	(0.163,0.190,0.232)	(0.638,0.766,0.879)
A3	(1.817,2.283,2.885)	(2.621,3.000,3.302)	1.000	(0.531,0.661,0.844)	(1.817,2.290,2.715)
A4	(3.000,3.302,3.557)	(4.309,5.313,6.316)	(1.194,1.533,1.913)	1.000	(1.958,2.426,3.037)
A5	(1.160,1.466,1.759)	(1.145,1.335,1.588)	(0.369,0.440,0.554)	(0.339,0.425,0.514)	1.000

Likewise, $\tilde{r}_2 = (0.448, 0.505, 0.576)$, $\tilde{r}_3 = (1.357, 1.596, 1.852)$,
 $\tilde{r}_4 = (1.977, 2.306, 2.649)$ and $\tilde{r}_5 = (0.698, 0.818, 0.955)$.

For the weight (\tilde{w}_i) of each dimension, they can be done as follows:

$$\begin{aligned}\tilde{w}_1 &= \tilde{r}_1 \otimes (\tilde{r}_1 \oplus \tilde{r}_2 \oplus \tilde{r}_3 \oplus \tilde{r}_4 \oplus \tilde{r}_5)^{-1} \\ &= (0.597, 0.688, 0.79) \\ &\quad \otimes (1/(0.79 + \dots + 0.955), 1/(0.688 + \dots + 0.818), 1/(0.597 + \dots + 0.698)) \\ &= (0.088, 0.117, 0.156).\end{aligned}$$

Likewise, $\tilde{w}_2 = (0.066, 0.086, 0.114)$, $\tilde{w}_3 = (0.199, 0.271, 0.365)$,
 $\tilde{w}_4 = (0.29, 0.391, 0.522)$ and $\tilde{w}_5 = (0.103, 0.139, 0.189)$.

Table 8. Weights of dimensions and criteria

Dimensions and criteria	Local weights			Overall weights			BNP
Work factors	(0.088,	0.117,	0.156)				0.121
Foreign language	(0.316,	0.439,	0.600)	(0.028,	0.052,	0.094)	0.014
Bachelor degree	(0.094,	0.131,	0.183)	(0.009,	0.016,	0.029)	0.005
Analytical thinking	(0.251,	0.353,	0.501)	(0.023,	0.042,	0.079)	0.011
Computer skill	(0.057,	0.079,	0.114)	(0.006,	0.010,	0.018)	0.003
Complimentary work factors	(0.066,	0.086,	0.114)				0.089
Effective time using	(0.064,	0.083,	0.107)	(0.005,	0.008,	0.013)	0.003
Determination of goal	(0.148,	0.195,	0.259)	(0.010,	0.017,	0.030)	0.006
Long life learning	(0.308,	0.410,	0.536)	(0.021,	0.036,	0.062)	0.012
Willingness	(0.238,	0.315,	0.420)	(0.016,	0.028,	0.048)	0.009
Industrial Engineering factors	(0.199,	0.271,	0.365)				0.279
Integrated approach	(0.141,	0.198,	0.276)	(0.029,	0.054,	0.101)	0.013
Planning and organization capability	(0.065,	0.088,	0.126)	(0.013,	0.024,	0.046)	0.006
Decision making	(0.267,	0.358,	0.478)	(0.054,	0.098,	0.175)	0.028
Teamwork	(0.267,	0.358,	0.478)	(0.054,	0.098,	0.175)	0.028
Individual factors	(0.290,	0.391,	0.522)				0.401
Emotional stability	(0.038,	0.051,	0.072)	(0.012,	0.020,	0.038)	0.006
Conscientiousness	(0.354,	0.489,	0.664)	(0.103,	0.192,	0.347)	0.051
Agreeableness	(0.161,	0.218,	0.298)	(0.047,	0.086,	0.156)	0.024
Extraversion	(0.098,	0.138,	0.195)	(0.029,	0.054,	0.102)	0.013
Openness to experience	(0.075,	0.107,	0.156)	(0.022,	0.042,	0.082)	0.009
Interpersonal factors	(0.103,	0.139,	0.189)				0.144
Flexibility	(0.109,	0.161,	0.241)	(0.012,	0.023,	0.046)	0.004
Tenacity	(0.177,	0.276,	0.412)	(0.019,	0.039,	0.078)	0.006
Persuasiveness	(0.126,	0.182,	0.270)	(0.013,	0.026,	0.052)	0.004
Decisiveness	(0.268,	0.383,	0.555)	(0.028,	0.054,	0.105)	0.011

Step 5. Use the Eq. (11) to compute the BNP value of the fuzzy weights of each dimension. To take the BNP value of the weight of A1 (work factors) as an example, the calculation process is as follows:

$$BNP_{w1} = \frac{[(U_{w1} - L_{w1}) + (M_{w1} - L_{w1})]}{3} + L_{w1} = \left[\frac{[(0.156 - 0.088) + (0.117 - 0.088)]}{3} \right] + 0.088 = 0.121.$$

Then, the weights for the remaining dimensions and criteria can be found as shown in Table 8. According to the fuzzy AHP results, it is clear that the first two important dimensions for industrial engineer selection are individual factors (0.401) and industrial engineering factors (0.279). Moreover, the less important dimension is complimentary work factors (0.089).

Step 6. Each decision makers evaluated the candidates under the defined criteria based on the expressions given in Table 9 and decision makers' expressions are given in Table 9 as DM1, DM2 and DM3, respectively.

Table 9. Range for the linguistic variables of decision makers

Decision makers	very poor	poor	fair	good	very good
1	(0,0,15)	(20,30,40)	(35,45,65)	(60,70,80)	(75,90,100)
2	(0,0,20)	(15,25,45)	(40,50,60)	(60,75,90)	(90,100,100)
3	(0,0,15)	(15,25,35)	(35,45,60)	(70,80,90)	(80,100,100)

For the candidate C1 as an example, the average fuzzy performance value of criterion - B01 (foreign language) from decision makers' judgment is obtained as follows:

$$\begin{aligned} & \begin{matrix} E^1 & E^2 & E^3 \\ \text{[very good]} & \text{good} & \text{good} \end{matrix} = [\begin{matrix} E^1 & E^2 & E^3 \\ (75,90,100) & (60,75,90) & (70,80,90) \end{matrix}] \\ & \tilde{E}_{11} = \left(\left(\sum_{k=1}^3 LE_{11}^k \right) / 3, \left(\sum_{k=1}^3 ME_{11}^k \right) / 3, \left(\sum_{k=1}^3 UE_{11}^k \right) / 3 \right) = (68, 82, 93) \end{aligned}$$

The remainder elements of fuzzy performance values of each criterion of decision makers for each candidate can be obtained by the similar way and they are shown in Table 10.

Table 10. Synthetic performance values of candidates

Criteria	C1	C2	C3	C4	C5
Foreign language	(68,82,93)	(55,67,82)	(72,88,97)	(67,82,90)	(57,67,77)
Bachelor degree	(78,90,97)	(67,82,90)	(67,82,90)	(82,97,100)	(57,67,77)
Analytical thinking	(63,75,87)	(72,88,97)	(57,67,77)	(53,68,77)	(72,88,97)
Computer skill	(52,63,77)	(63,75,87)	(47,60,67)	(63,75,87)	(67,82,90)
Effective time using	(53,65,78)	(82,97,100)	(67,82,90)	(73,83,90)	(82,97,100)
Determination of goal	(67,82,90)	(72,88,97)	(68,82,93)	(60,73,80)	(65,80,87)
Long life learning	(67,82,90)	(72,88,97)	(77,90,93)	(67,82,90)	(67,82,90)
Willingness	(65,80,87)	(72,88,97)	(82,97,100)	(77,90,93)	(72,88,93)
Integrated approach	(57,67,77)	(77,90,93)	(63,75,87)	(77,90,93)	(67,82,90)
Planning and organization capability	(48,58,72)	(77,90,93)	(63,75,87)	(67,82,90)	(60,73,80)
Decision making	(60,73,80)	(72,88,97)	(68,82,93)	(60,73,80)	(67,82,90)
Teamwork	(57,67,87)	(67,82,90)	(72,88,97)	(67,82,90)	(67,82,90)
Emotional stability	(57,67,77)	(73,83,90)	(67,82,90)	(73,83,90)	(68,82,93)
Conscientiousness	(67,82,90)	(63,75,87)	(67,82,90)	(72,88,97)	(67,82,90)
Agreeableness	(63,75,87)	(67,82,90)	(67,82,90)	(72,88,97)	(67,82,90)
Extraversion	(57,67,77)	(78,90,97)	(77,90,93)	(78,90,97)	(67,82,90)
Openness to experience	(67,82,90)	(72,88,97)	(65,80,87)	(67,82,90)	(67,82,90)

Table 10. (Continues) Synthetic performance values of candidates

Criteria	C1	C2	C3	C4	C5
Flexibility	(45,55,67)	(77,90,93)	(63,75,87)	(63,75,87)	(63,75,87)
Tenacity	(60,73,80)	(67,82,90)	(68,82,93)	(67,82,90)	(72,88,97)
Persuasiveness	(57,67,77)	(72,88,97)	(57,67,77)	(60,73,80)	(57,67,77)
Decisiveness	(57,67,77)	(72,88,97)	(57,67,77)	(60,73,80)	(67,82,90)

After calculations of synthetic performance values, fuzzy numbers have to be turned into nonfuzzy forms. BNP values are also used in this phase and the results are given in Table 11. Ranking of the candidates is determined based on BNP values and ratios are calculated. These values are also given in Table 11.

Table 11. BNP values, rank and ratios of candidates

Candidates	DM1		DM2		DM3		Compromised	
	BNP_i	Ranking	BNP_i	Ranking	BNP_i	Ranking	BNP_i	Ranking
C1	78.969	5	73.722	5	108.452	5	88.375	5
C2	83.292	2	107.672	1	111.104	4	98.495	1
C3	82.730	3	95.897	3	112.304	3	95.966	3
C4	86.997	1	99.585	2	115.630	1	96.816	2
C5	80.310	4	94.252	4	115.137	2	95.434	4

It can be seen from Table 11 that the candidate C2 is the most preferred candidate considering the three decision makers' weights. However, the candidate C4 is the best candidate by the weight of DM1 and DM3 (Factory manager and manager of human resources department, respectively) clearly different from DM2 (chair of industrial engineering department). This difference is related to the weights of dimensions according to DMs. Although the most important dimensions of DM1 and DM3 is the individual factors, the industrial engineering factors are most important dimension for DM2.

3.4. Sensitivity Analysis

Because the pair-wise comparison matrices are based on decision makers' judgments, they can vary from one decision maker to another. Therefore the weights of criteria and the importance of the decision makers can change in selection process. Sensitivity analysis allows us analyze effects of these changes on the relative weights of the criteria and decision makers. Sensitivity analysis is performed for different levels of weights for decision makers and dimensions.

The weights of decision makers are accepted as equal in the application. w_1 , w_2 and w_3 respectively stand for the weights of DM1, DM2 and DM3. Firstly, sensitivity analysis is performed in the different levels of weights for decision makers. Thus, ranking the candidates is given in Figure 3 for seven different weight vectors. The rank of the candidate C1 isn't affected by the seven different weight vectors of decision makers.

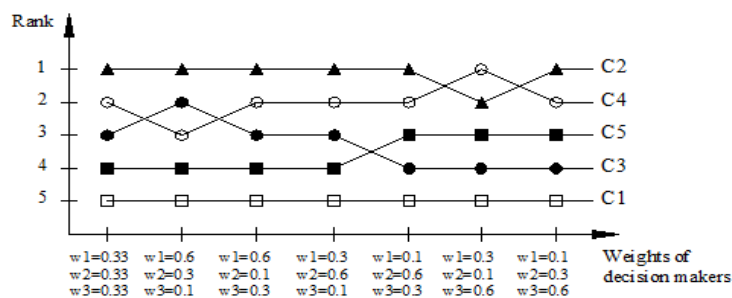


Figure 3. Sensitivity analysis for the different levels of weights

The rank of the candidate C2 changes only at sixth weight vector, that is if w_1 , w_2 and w_3 are 0.3, 0.1 and 0.6, respectively, the rank of candidate C2 is second rank. Also, the rank of the candidate C2 is rarely affected by the changes of weight vectors of decision makers.

Secondly, sensitivity analysis is performed for the dimensions. If only one dimension is attended to select industrial engineer and the others are neglected, Figure 4 shows how to be influenced ranking the candidates by only one dimension. The ranking candidates indicated by “General” shows the ranks of candidates in the application. For example, if only the first dimension is attended to select and the others are neglected, the A1 column in Figure 4 shows how the ranking candidates change. The candidate C1 is in the first rank and the candidate C5 is in the last rank. The other columns in Figure 4 can be explained in a similar way. The best candidate for each dimensions changes.

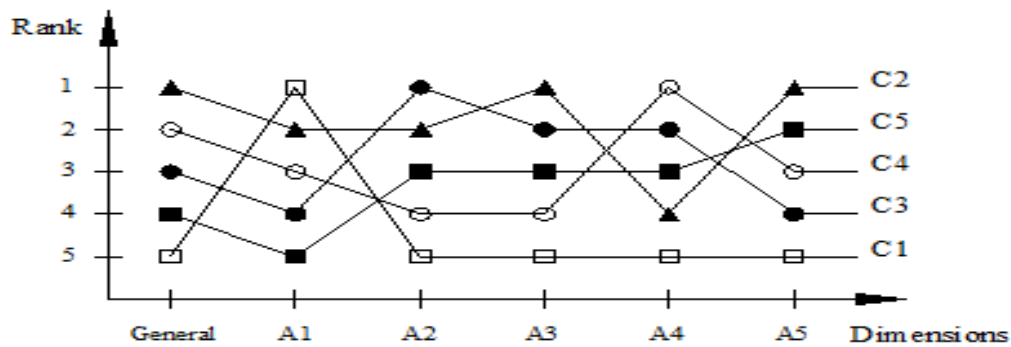


Figure 4. Sensitivity analysis for only one dimension

3.5. Comparison of the Results Produced by the Proposed Approach and Fuzzy TOPSIS

At the fuzzy TOPSIS approach, three decision makers evaluate dimensions, criteria and candidates attended to select industrial engineer by using the linguistic variables.

The committee provided linguistic assessments for the twenty-one criteria using rating scales given in Table 2 and to the three candidates using rating scales of Table 3. Tables 12 and 13 present the linguistic assessments for the criteria and the candidates.

Table 12. Linguistic assessments for dimensions and criteria

Dimensions and criteria	DM1	DM2	DM3
Work factors	L	M	H
Foreign language	H	H	MH
Bachelor degree	ML	H	L
Analytical thinking	MH	MH	VH
Computer skill	L	L	ML
Complimentary work factors	MH	L	ML
Effective time using	ML	H	L
Determination of goal	M	H	M
Long life learning	M	VH	H
Willingness	VH	L	H
Industrial Engineering factors	MH	VH	MH
Integrated approach	M	VH	L
Planning and organization capability	ML	M	ML
Decision making	H	MH	H
Teamwork	H	MH	H

Table 12. (Cont.) Linguistic assessments for dimensions and criteria

Dimensions and criteria	DM1	DM2	DM3
Individual factors	VH	M	VH
Emotional stability	VL	ML	L
Conscientiousness	H	H	VH
Agreeableness	H	MH	M
Extraversion	MH	MH	M
Openness to experience	L	ML	MH
Interpersonal factors	MH	H	L
Flexibility	VH	ML	L
Tenacity	MH	H	ML
Persuasiveness	ML	MH	MH
Decisiveness	MH	MH	VH

Then, the aggregated fuzzy weights (\tilde{w}_{ij}) for each criterion are calculated using Eq. (13). For example, for criterion B01 (Foreign language), the aggregated fuzzy weight is given by $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$ where

$$\begin{aligned}
 & \begin{matrix} \text{DM1} & \text{DM2} & \text{DM3} \\ [(0,0.1,0.3) & (0.2,0.5,0.7) & (0.4,0.6,0.9) \end{matrix}] = [\quad (0.19, \quad 0.39, \quad 0.64)] \\
 & \tilde{w}_{11} = \left(\left(\sum_{k=1}^3 Lw_{11} \right) / 3, \left(\sum_{k=1}^3 Mw_{11} \right) / 3, \left(\sum_{k=1}^3 Uw_{11} \right) / 3 \right) = (0.19, \quad 0.39, \quad 0.64)
 \end{aligned}$$

Table 13. Linguistic assessments for the five candidates

Criteria	DM1					DM2					DM3				
	A1	A2	A3	A4	A5	A1	A2	A3	A4	A5	A1	A2	A3	A4	A5
Foreign language	VG	P	VG	G	G	G	G	G	G	F	G	G	VG	VG	G
Bachelor degree	VG	G	G	VG	G	VG	G	G	VG	F	G	VG	VG	VG	G
Analytical thinking	G	VG	G	P	VG	G	G	F	G	G	G	VG	G	VG	VG
Computer skill	G	G	P	G	G	G	G	F	G	G	F	G	VG	G	VG
Effective time using	VG	VG	G	G	VG	P	VG	G	VG	VG	G	VG	VG	G	VG
Determination of goal	G	VG	VG	G	VG	G	G	G	F	F	VG	VG	G	VG	VG
Long life learning	G	VG	G	G	G	G	G	VG	G	G	VG	VG	VG	VG	VG
Willingness	VG	VG	VG	G	VG	F	G	VG	VG	G	VG	VG	VG	VG	VG
Integrated approach	G	G	G	G	G	F	VG	G	VG	G	G	VG	G	VG	VG
Planning and organization capability	G	G	G	G	G	P	VG	G	G	F	G	VG	G	VG	VG
Decision making	G	VG	VG	G	G	F	G	G	F	G	VG	VG	G	VG	VG
Teamwork	G	G	VG	G	G	F	G	G	G	G	G	VG	VG	VG	VG
Emotional stability	G	G	G	G	VG	F	VG	G	VG	G	G	G	VG	G	G
Conscientiousness	G	G	G	VG	G	G	G	G	G	F	VG	G	VG	VG	VG
Agreeableness	G	G	G	VG	G	G	G	G	G	G	G	VG	VG	VG	VG
Extraversion	G	VG	G	VG	G	F	VG	VG	VG	G	G	G	VG	G	VG
Openness to experience	G	VG	VG	G	G	G	G	F	G	G	VG	VG	VG	VG	VG
Flexibility	G	G	G	G	G	F	VG	G	G	G	F	VG	G	G	G
Tenacity	G	G	VG	G	VG	F	G	G	G	G	VG	VG	G	VG	VG
Persuasiveness	G	VG	G	G	G	F	G	F	F	F	G	VG	G	VG	G
Decisiveness	G	VG	G	G	G	F	G	F	F	G	G	VG	G	VG	VG

Likewise, we compute the aggregate weights for the remaining 20 criteria. The aggregate weights of the 21 criteria are presented in Table 14.

Table 14. Aggregate fuzzy weights for criteria

Dimensions and criteria	Local weights			Overall weights			Aggregated fuzzy weights
	DM1	DM2	DM3	DM1	DM2	DM3	
A1	(0.0,0.1,0.3)	(0.3,0.5,0.7)	(0.7,0.9,1.0)				
B01	(0.7,0.9,1.0)	(0.7,0.9,1.0)	(0.5,0.7,0.9)	(0.0,0.1,0.3)	(0.2,0.5,0.7)	(0.4,0.6,0.9)	(0.19,0.39,0.64)
B02	(0.1,0.3,0.5)	(0.7,0.9,1.0)	(0.0,0.1,0.3)	(0.0,0.0,0.2)	(0.2,0.5,0.7)	(0.0,0.1,0.3)	(0.07,0.19,0.39)
B03	(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.9,1.0,1.0)	(0.0,0.1,0.3)	(0.2,0.4,0.6)	(0.6,0.9,1.0)	(0.26,0.44,0.64)
B04	(0.0,0.1,0.3)	(0.0,0.1,0.3)	(0.1,0.3,0.5)	(0.0,0.0,0.1)	(0.0,0.1,0.2)	(0.1,0.3,0.5)	(0.03,0.11,0.27)
A2	(0.5,0.7,0.9)	(0.0,0.1,0.3)	(0.1,0.3,0.5)				
B05	(0.1,0.3,0.5)	(0.7,0.9,1.0)	(0.0,0.1,0.3)	(0.1,0.2,0.5)	(0.0,0.1,0.3)	(0.0,0.0,0.2)	(0.02,0.11,0.30)
B06	(0.3,0.5,0.7)	(0.7,0.9,1.0)	(0.3,0.5,0.7)	(0.2,0.4,0.6)	(0.0,0.1,0.3)	(0.0,0.2,0.4)	(0.06,0.20,0.43)
B07	(0.3,0.5,0.7)	(0.9,1.0,1.0)	(0.7,0.9,1.0)	(0.2,0.4,0.6)	(0.0,0.1,0.3)	(0.1,0.3,0.5)	(0.08,0.24,0.48)
B08	(0.9,1.0,1.0)	(0.0,0.1,0.3)	(0.7,0.9,1.0)	(0.5,0.7,0.9)	(0.0,0.0,0.1)	(0.1,0.3,0.5)	(0.18,0.33,0.50)
A3	(0.5,0.7,0.9)	(0.9,1.0,1.0)	(0.5,0.7,0.9)				
B09	(0.3,0.5,0.7)	(0.9,1.0,1.0)	(0.0,0.1,0.3)	(0.2,0.4,0.6)	(0.8,1.0,1.0)	(0.0,0.1,0.3)	(0.32,0.48,0.64)
B10	(0.1,0.3,0.5)	(0.3,0.5,0.7)	(0.1,0.3,0.5)	(0.1,0.2,0.5)	(0.3,0.5,0.7)	(0.1,0.2,0.5)	(0.13,0.31,0.54)
B11	(0.7,0.9,1.0)	(0.5,0.7,0.9)	(0.7,0.9,1.0)	(0.4,0.6,0.9)	(0.5,0.7,0.9)	(0.4,0.6,0.9)	(0.39,0.66,0.90)
B12	(0.7,0.9,1.0)	(0.5,0.7,0.9)	(0.7,0.9,1.0)	(0.4,0.6,0.9)	(0.5,0.7,0.9)	(0.4,0.6,0.9)	(0.39,0.66,0.90)
A4	(0.9,1.0,1.0)	(0.3,0.5,0.7)	(0.9,1.0,1.0)				
B13	(0.0,0.0,0.1)	(0.1,0.3,0.5)	(0.0,0.1,0.3)	(0.0,0.0,0.1)	(0.0,0.2,0.4)	(0.0,0.1,0.3)	(0.01,0.09,0.25)
B14	(0.7,0.9,1.0)	(0.7,0.9,1.0)	(0.9,1.0,1.0)	(0.6,0.9,1.0)	(0.2,0.5,0.7)	(0.8,1.0,1.0)	(0.55,0.79,0.90)
B15	(0.7,0.9,1.0)	(0.5,0.7,0.9)	(0.3,0.5,0.7)	(0.6,0.9,1.0)	(0.2,0.4,0.6)	(0.3,0.5,0.7)	(0.35,0.59,0.78)
B16	(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.3,0.5,0.7)	(0.5,0.7,0.9)	(0.2,0.4,0.6)	(0.3,0.5,0.7)	(0.29,0.52,0.75)
B17	(0.0,0.1,0.3)	(0.1,0.3,0.5)	(0.5,0.7,0.9)	(0.0,0.1,0.3)	(0.0,0.2,0.4)	(0.5,0.7,0.9)	(0.16,0.32,0.52)
A5	(0.5,0.7,0.9)	(0.7,0.9,1.0)	(0.0,0.1,0.3)				
B18	(0.9,1.0,1.0)	(0.1,0.3,0.5)	(0.0,0.1,0.3)	(0.5,0.7,0.9)	(0.1,0.3,0.5)	(0.0,0.0,0.1)	(0.18,0.33,0.50)
B19	(0.5,0.7,0.9)	(0.7,0.9,1.0)	(0.1,0.3,0.5)	(0.3,0.5,0.8)	(0.5,0.8,1.0)	(0.0,0.0,0.2)	(0.25,0.45,0.66)
B20	(0.1,0.3,0.5)	(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.1,0.2,0.5)	(0.4,0.6,0.9)	(0.0,0.1,0.3)	(0.14,0.31,0.54)
B21	(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.9,1.0,1.0)	(0.3,0.5,0.8)	(0.4,0.6,0.9)	(0.0,0.1,0.3)	(0.20,0.41,0.67)

Then, the aggregate fuzzy weights of the candidates are computed using Eq. (12). For example, the aggregate rating for candidate C1 for criterion B01 given by the three decision makers is computed as follows:

$$\begin{matrix} \text{DM1} & \text{DM2} & \text{DM3} \\ [(9,10,10) & (7,9,10) & (7,9,10) \end{matrix}] = [(7.67, 9.34, 10)]$$

$$\tilde{w}_{11} = \left(\left(\sum_{k=1}^3 Lw_{11} \right) / 3, \left(\sum_{k=1}^3 Mw_{11} \right) / 3, \left(\sum_{k=1}^3 Uw_{11} \right) / 3 \right) = (7.67, 9.34, 10)$$

Likewise, the aggregate ratings for the remaining four candidates (C2, C3, C4, and C5) with respect to the 21 criteria are computed. The aggregate fuzzy decision matrix for the candidates is presented in Table 15.

In the next step, we perform normalization of the fuzzy decision matrix of candidates using Eq. (14). For example, the normalized rating for the candidate C1 for criterion B01 (Foreign language) is given by

$$u_j^+ = \max_i (7.67, 9.34, 10)$$

$$\tilde{r}_{11} = \left(\frac{7.67}{10.00}, \frac{9.34}{10.00}, \frac{10.00}{10.00} \right) = (0.77, 0.94, 1.00)$$

Table 15. Aggregate fuzzy weights for candidates

B*	C*	Decision makers			Aggregated ratings	B*	C*	Decision makers			Aggregated ratings
		DM1	DM2	DM3				DM1	DM2	DM3	
B01	C1	(9,10,10)	(7,9,10)	(7,9,10)	(7.67,9.34,10)	B12	C1	(7,9,10)	(3,5,7)	(7,9,10)	(5.67,7.67,9)
	C2	(3,5,7)	(7,9,10)	(7,9,10)	(5.67,7.67,9)		C2	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
	C3	(9,10,10)	(7,9,10)	(9,10,10)	(8.34,9.67,10)		C3	(9,10,10)	(7,9,10)	(9,10,10)	(8.34,9.67,10)
	C4	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)		C4	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
	C5	(7,9,10)	(3,5,7)	(7,9,10)	(5.67,7.67,9)		C5	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
B02	C1	(9,10,10)	(9,10,10)	(7,9,10)	(8.34,9.67,10)	B13	C1	(7,9,10)	(3,5,7)	(7,9,10)	(5.67,7.67,9)
	C2	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)		C2	(7,9,10)	(9,10,10)	(7,9,10)	(7.67,9.34,10)
	C3	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)		C3	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
	C4	(9,10,10)	(9,10,10)	(9,10,10)	(9,10,10)		C4	(7,9,10)	(9,10,10)	(7,9,10)	(7.67,9.34,10)
	C5	(7,9,10)	(3,5,7)	(7,9,10)	(5.67,7.67,9)		C5	(9,10,10)	(7,9,10)	(7,9,10)	(7.67,9.34,10)
B03	C1	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)	B14	C1	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
	C2	(9,10,10)	(7,9,10)	(9,10,10)	(8.34,9.67,10)		C2	(7,9,10)	(7,9,10)	(9,10,10)	(7,9,10)
	C3	(7,9,10)	(3,5,7)	(7,9,10)	(5.67,7.67,9)		C3	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
	C4	(0,1,3)	(7,9,10)	(9,10,10)	(5.34,6.67,7.67)		C4	(9,10,10)	(7,9,10)	(9,10,10)	(8.34,9.67,10)
	C5	(9,10,10)	(7,9,10)	(9,10,10)	(8.34,9.67,10)		C5	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
B04	C1	(7,9,10)	(7,9,10)	(3,5,7)	(5.67,7.67,9)	B15	C1	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)
	C2	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)		C2	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
	C3	(0,1,3)	(3,5,7)	(9,10,10)	(4.5,34,6.67)		C3	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
	C4	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)		C4	(9,10,10)	(7,9,10)	(9,10,10)	(8.34,9.67,10)
	C5	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)		C5	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
B05	C1	(9,10,10)	(0,1,3)	(7,9,10)	(5.34,6.67,7.67)	B16	C1	(7,9,10)	(3,5,7)	(7,9,10)	(5.67,7.67,9)
	C2	(9,10,10)	(9,10,10)	(9,10,10)	(9,10,10)		C2	(9,10,10)	(9,10,10)	(7,9,10)	(8.34,9.67,10)
	C3	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)		C3	(7,9,10)	(9,10,10)	(9,10,10)	(8.34,9.67,10)
	C4	(7,9,10)	(9,10,10)	(7,9,10)	(7.67,9.34,10)		C4	(9,10,10)	(9,10,10)	(7,9,10)	(8.34,9.67,10)
	C5	(9,10,10)	(9,10,10)	(9,10,10)	(7.67,9.34,10)		C5	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
B06	C1	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)	B17	C1	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
	C2	(9,10,10)	(7,9,10)	(9,10,10)	(8.34,9.67,10)		C2	(9,10,10)	(7,9,10)	(9,10,10)	(8.34,9.67,10)
	C3	(9,10,10)	(7,9,10)	(7,9,10)	(7.67,9.34,10)		C3	(9,10,10)	(3,5,7)	(9,10,10)	(7,8.34,9)
	C4	(7,9,10)	(3,5,7)	(9,10,10)	(6.34,8,9)		C4	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
	C5	(9,10,10)	(3,5,7)	(9,10,10)	(6.34,8,9)		C5	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
B07	C1	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)	B18	C1	(7,9,10)	(3,5,7)	(3,5,7)	(4.34,6.34,8)
	C2	(9,10,10)	(7,9,10)	(9,10,10)	(8.34,9.67,10)		C2	(7,9,10)	(9,10,10)	(9,10,10)	(8.34,9.67,10)
	C3	(7,9,10)	(9,10,10)	(9,10,10)	(8.34,9.67,10)		C3	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)
	C4	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)		C4	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)
	C5	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)		C5	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)
B08	C1	(9,10,10)	(3,5,7)	(9,10,10)	(7,8.34,9)	B19	C1	(7,9,10)	(3,5,7)	(9,10,10)	(6.34,8,9)
	C2	(9,10,10)	(7,9,10)	(9,10,10)	(8.34,9.67,10)		C2	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
	C3	(9,10,10)	(9,10,10)	(9,10,10)	(9,10,10)		C3	(9,10,10)	(7,9,10)	(7,9,10)	(7.67,9.34,10)
	C4	(7,9,10)	(9,10,10)	(9,10,10)	(8.34,9.67,10)		C4	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
	C5	(9,10,10)	(7,9,10)	(9,10,10)	(8.34,9.67,10)		C5	(9,10,10)	(7,9,10)	(9,10,10)	(8.34,9.67,10)
B09	C1	(7,9,10)	(3,5,7)	(7,9,10)	(5.67,7.67,9)	B20	C1	(7,9,10)	(3,5,7)	(7,9,10)	(5.67,7.67,9)
	C2	(7,9,10)	(9,10,10)	(9,10,10)	(8.34,9.67,10)		C2	(9,10,10)	(7,9,10)	(9,10,10)	(8.34,9.67,10)
	C3	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)		C3	(7,9,10)	(3,5,7)	(7,9,10)	(5.67,7.67,9)
	C4	(7,9,10)	(9,10,10)	(9,10,10)	(8.34,9.67,10)		C4	(7,9,10)	(3,5,7)	(9,10,10)	(6.34,8,9)
	C5	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)		C5	(7,9,10)	(3,5,7)	(7,9,10)	(5.67,7.67,9)
B10	C1	(7,9,10)	(0,1,3)	(7,9,10)	(4.67,6.34,7.67)	B21	C1	(7,9,10)	(3,5,7)	(7,9,10)	(5.67,7.67,9)
	C2	(7,9,10)	(9,10,10)	(9,10,10)	(8.34,9.67,10)		C2	(9,10,10)	(7,9,10)	(9,10,10)	(8.34,9.67,10)
	C3	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)		C3	(7,9,10)	(3,5,7)	(7,9,10)	(5.67,7.67,9)
	C4	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)		C4	(7,9,10)	(3,5,7)	(9,10,10)	(6.34,8,9)
	C5	(7,9,10)	(3,5,7)	(9,10,10)	(6.34,8,9)		C5	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)
B11	C1	(7,9,10)	(3,5,7)	(9,10,10)	(6.34,8,9)	B*: Criteria C*: Candidates					
	C2	(9,10,10)	(7,9,10)	(9,10,10)	(8.34,9.67,10)						
	C3	(9,10,10)	(7,9,10)	(7,9,10)	(7.67,9.34,10)						
	C4	(7,9,10)	(3,5,7)	(9,10,10)	(6.34,8,9)						
	C5	(7,9,10)	(7,9,10)	(9,10,10)	(7.67,9.34,10)						

We compute the normalized values of the candidates for the remaining criteria by similar way. The normalized fuzzy decision matrix for the five candidates is presented in Table 16.

Table 16. Normalized fuzzy decision matrix for candidates

Criteria	Normalized ratings				
	C1	C2	C3	C4	C5
A1					
B01	(0.77,0.94,1.00)	(0.57,0.77,0.90)	(0.84,0.97,1.00)	(0.77,0.94,1.00)	(0.57,0.77,0.90)
B02	(0.84,0.97,1.00)	(0.77,0.94,1.00)	(0.77,0.94,1.00)	(0.90,1.00,1.00)	(0.57,0.77,0.90)
B03	(0.70,0.90,1.00)	(0.84,0.97,1.00)	(0.57,0.77,0.90)	(0.54,0.67,0.77)	(0.84,0.97,1.00)
B04	(0.57,0.77,0.90)	(0.70,0.90,1.00)	(0.40,0.54,0.67)	(0.70,0.90,1.00)	(0.77,0.94,1.00)
A2					
B05	(0.54,0.67,0.77)	(0.90,1.00,1.00)	(0.77,0.94,1.00)	(0.77,0.94,1.00)	(0.77,0.94,1.00)
B06	(0.77,0.94,1.00)	(0.84,0.97,1.00)	(0.77,0.94,1.00)	(0.64,0.80,0.90)	(0.64,0.80,0.90)
B07	(0.77,0.94,1.00)	(0.84,0.97,1.00)	(0.84,0.97,1.00)	(0.77,0.94,1.00)	(0.77,0.94,1.00)
B08	(0.70,0.84,0.90)	(0.84,0.97,1.00)	(0.90,1.00,1.00)	(0.84,0.97,1.00)	(0.84,0.97,1.00)
A3					
B09	(0.57,0.77,0.90)	(0.84,0.97,1.00)	(0.70,0.90,1.00)	(0.84,0.97,1.00)	(0.77,0.94,1.00)
B10	(0.47,0.64,0.77)	(0.84,0.97,1.00)	(0.70,0.90,1.00)	(0.77,0.94,1.00)	(0.64,0.80,0.90)
B11	(0.64,0.80,0.90)	(0.84,0.97,1.00)	(0.77,0.94,1.00)	(0.64,0.80,0.90)	(0.77,0.94,1.00)
B12	(0.57,0.77,0.90)	(0.77,0.94,1.00)	(0.84,0.97,1.00)	(0.77,0.94,1.00)	(0.77,0.94,1.00)
A4					
B13	(0.57,0.77,0.90)	(0.77,0.94,1.00)	(0.77,0.94,1.00)	(0.77,0.94,1.00)	(0.77,0.94,1.00)
B14	(0.77,0.94,1.00)	(0.70,0.90,1.00)	(0.77,0.94,1.00)	(0.84,0.97,1.00)	(0.77,0.94,1.00)
B15	(0.70,0.90,1.00)	(0.77,0.94,1.00)	(0.77,0.94,1.00)	(0.84,0.97,1.00)	(0.77,0.94,1.00)
B16	(0.57,0.77,0.90)	(0.84,0.97,1.00)	(0.84,0.97,1.00)	(0.84,0.97,1.00)	(0.77,0.94,1.00)
B17	(0.77,0.94,1.00)	(0.84,0.97,1.00)	(0.70,0.84,0.90)	(0.77,0.94,1.00)	(0.77,0.94,1.00)
A5					
B18	(0.44,0.64,0.80)	(0.84,0.97,1.00)	(0.70,0.90,1.00)	(0.70,0.90,1.00)	(0.70,0.90,1.00)
B19	(0.64,0.80,0.90)	(0.77,0.94,1.00)	(0.77,0.94,1.00)	(0.77,0.94,1.00)	(0.84,0.97,1.00)
B20	(0.57,0.77,0.90)	(0.84,0.97,1.00)	(0.57,0.77,0.90)	(0.64,0.80,0.90)	(0.57,0.77,0.90)
B21	(0.57,0.77,0.90)	(0.84,0.97,1.00)	(0.57,0.77,0.90)	(0.64,0.80,0.90)	(0.77,0.94,1.00)

Table 17. Weighted normalized candidates

Criteria	Candidates				
	C1	C2	C3	C4	C5
A1					
B01	(0.15,0.37,0.64)	(0.11,0.30,0.58)	(0.16,0.38,0.64)	(0.15,0.37,0.64)	(0.11,0.30,0.58)
B02	(0.06,0.19,0.39)	(0.06,0.18,0.39)	(0.06,0.18,0.39)	(0.07,0.18,0.39)	(0.04,0.15,0.36)
B03	(0.19,0.40,0.64)	(0.22,0.43,0.64)	(0.15,0.34,0.58)	(0.14,0.30,0.50)	(0.22,0.43,0.64)
B04	(0.02,0.09,0.25)	(0.03,0.10,0.27)	(0.02,0.06,0.18)	(0.03,0.10,0.27)	(0.03,0.11,0.27)
A2					
B05	(0.02,0.08,0.23)	(0.02,0.11,0.30)	(0.02,0.11,0.30)	(0.02,0.11,0.30)	(0.02,0.11,0.30)
B06	(0.05,0.19,0.43)	(0.05,0.20,0.43)	(0.05,0.19,0.43)	(0.04,0.16,0.39)	(0.04,0.16,0.39)
B07	(0.07,0.23,0.48)	(0.07,0.24,0.48)	(0.07,0.24,0.48)	(0.07,0.23,0.48)	(0.07,0.23,0.48)
B08	(0.13,0.28,0.45)	(0.15,0.32,0.50)	(0.17,0.33,0.50)	(0.15,0.32,0.50)	(0.15,0.32,0.50)
A3					
B09	(0.19,0.37,0.58)	(0.27,0.47,0.64)	(0.23,0.44,0.64)	(0.27,0.47,0.64)	(0.25,0.45,0.64)
B10	(0.07,0.20,0.42)	(0.11,0.30,0.54)	(0.10,0.28,0.54)	(0.10,0.29,0.54)	(0.09,0.25,0.49)
B11	(0.25,0.53,0.81)	(0.33,0.64,0.90)	(0.30,0.62,0.90)	(0.25,0.53,0.81)	(0.30,0.62,0.90)
B12	(0.23,0.51,0.81)	(0.30,0.62,0.90)	(0.33,0.64,0.90)	(0.30,0.62,0.90)	(0.30,0.62,0.90)
A4					
B13	(0.01,0.07,0.23)	(0.01,0.09,0.25)	(0.01,0.09,0.25)	(0.01,0.09,0.25)	(0.01,0.09,0.25)
B14	(0.43,0.74,0.90)	(0.39,0.72,0.90)	(0.43,0.74,0.90)	(0.46,0.77,0.90)	(0.43,0.74,0.90)
B15	(0.25,0.54,0.78)	(0.27,0.56,0.78)	(0.27,0.56,0.78)	(0.30,0.58,0.78)	(0.27,0.56,0.78)
B16	(0.17,0.40,0.68)	(0.25,0.51,0.75)	(0.25,0.51,0.75)	(0.25,0.51,0.75)	(0.23,0.49,0.75)
B17	(0.13,0.30,0.52)	(0.14,0.31,0.52)	(0.12,0.27,0.47)	(0.13,0.30,0.52)	(0.13,0.30,0.52)
A5					
B18	(0.08,0.21,0.40)	(0.15,0.32,0.50)	(0.13,0.30,0.50)	(0.13,0.30,0.50)	(0.13,0.30,0.50)
B19	(0.16,0.36,0.60)	(0.20,0.42,0.66)	(0.20,0.42,0.66)	(0.20,0.42,0.66)	(0.20,0.42,0.66)
B20	(0.08,0.24,0.49)	(0.12,0.30,0.54)	(0.08,0.24,0.49)	(0.09,0.25,0.49)	(0.08,0.24,0.49)
B21	(0.12,0.32,0.61)	(0.17,0.40,0.67)	(0.12,0.32,0.61)	(0.13,0.33,0.61)	(0.16,0.39,0.67)

Then, the fuzzy weighted decision matrix for the five candidates is constructed using Eq. (15). The \tilde{r}_{ij} values from Table 16 and \tilde{w}_j values from Table 14 are used to compute the fuzzy weighted decision matrix for the candidates. For example, for the candidate C1, the fuzzy weight for criterion B01 (Foreign language) is calculated as follows:

$$\tilde{r}_{11} \otimes \tilde{w}_{11} = (0.77, 0.94, 1.00) \otimes (0.19, 0.39, 0.64) = (0.15, 0.37, 0.64)$$

Similarly, we compute the fuzzy weighted decision matrix. The results are shown in Table 17.

Then, we can define the fuzzy positive-ideal solution and the fuzzy negative-ideal solution for 21 criteria as A^+ and A^- .

$$A^+ = [(1,1,1), (1,1,1)]$$

$$A^- = [(0,0,0), (0,0,0)]$$

Then, we compute the distance $d_v(\cdot)$ for each candidate from the fuzzy positive ideal matrix (A^+) and fuzzy negative ideal matrix (A^-) using Eq. (12). For example, for the candidate C1 and criterion B01, the distances $d_v(C_1, A^+)$ and $d_v(C_1, A^-)$ are computed as follows:

$$d(C_1, A^+) = \sqrt{\frac{1}{3}[(1 - 0.15)^2 + (1 - 0.37)^2 + (1 - 0.64)^2]} = 0.645,$$

$$d(C_1, A^-) = \sqrt{\frac{1}{3}[(0 - 0.15)^2 + (0 - 0.37)^2 + (0 - 0.64)^2]} = 0.436.$$

Likewise, we compute the distances for the remaining criteria for the four candidates. The results are shown in Table 18.

Table 18. Distances $d_v(C_i, A^+)$ and $d_v(C_i, A^-)$ for candidates

Criteria	$d_v(C_1, A^+)$	$d_v(C_2, A^+)$	$d_v(C_3, A^+)$	$d_v(C_4, A^+)$	$d_v(C_5, A^+)$	$d_v(C_1, A^-)$	$d_v(C_2, A^-)$	$d_v(C_3, A^-)$	$d_v(C_4, A^-)$	$d_v(C_5, A^-)$
B01	0.645	0.697	0.638	0.645	0.967	0.436	0.382	0.440	0.436	0.382
B02	0.798	0.802	0.802	0.794	0.827	0.253	0.250	0.250	0.254	0.226
B03	0.618	0.595	0.667	0.702	0.595	0.449	0.463	0.398	0.346	0.463
B04	0.885	0.873	0.916	0.873	0.865	0.154	0.167	0.110	0.167	0.169
B05	0.894	0.865	0.865	0.865	0.865	0.141	0.185	0.185	0.185	0.185
B06	0.792	0.789	0.792	0.816	0.816	0.273	0.275	0.273	0.244	0.244
B07	0.759	0.756	0.756	0.759	0.759	0.310	0.312	0.321	0.310	0.310
B08	0.725	0.692	0.680	0.692	0.692	0.315	0.354	0.360	0.354	0.354
B09	0.640	0.561	0.588	0.561	0.576	0.412	0.484	0.468	0.484	0.474
B10	0.783	0.706	0.716	0.713	0.742	0.272	0.362	0.356	0.359	0.322
B11	0.523	0.443	0.463	0.523	0.463	0.577	0.665	0.654	0.577	0.654
B12	0.538	0.463	0.443	0.463	0.443	0.568	0.654	0.665	0.654	0.654
B13	0.901	0.889	0.889	0.889	0.889	0.139	0.154	0.154	0.154	0.154
B14	0.366	0.392	0.366	0.344	0.366	0.717	0.702	0.717	0.734	0.717
B15	0.524	0.508	0.508	0.488	0.508	0.566	0.576	0.576	0.587	0.576
B16	0.619	0.537	0.537	0.537	0.552	0.466	0.543	0.543	0.543	0.534
B17	0.702	0.694	0.728	0.702	0.702	0.355	0.359	0.321	0.355	0.355
B18	0.781	0.692	0.706	0.706	0.706	0.265	0.354	0.345	0.345	0.345
B19	0.652	0.603	0.603	0.603	0.593	0.414	0.466	0.466	0.466	0.474
B20	0.749	0.701	0.749	0.742	0.749	0.318	0.363	0.318	0.322	0.318
B21	0.680	0.621	0.680	0.673	0.629	0.404	0.461	0.404	0.407	0.457

Then, we compute the distances d_i^+ and d_i^- using Eqs. (18)–(19). Using the distances d_i^+ and d_i^- , we compute the closeness coefficient CC_i for the five candidates using Eq. (20). For example, for the candidate C1, the closeness coefficient is given by

$$CC_1 = \frac{7.805}{14.578+7.805} = 0.349$$

CC_i is computed by similar way for the other four candidates. The final results are shown in Table 19.

Table 19. d_i^+ , d_i^- and closeness coefficients (CC_i) of five candidates

	C1	C2	C3	C4	C5	Ranking order
d_i^+	14.578	13.879	14.093	14.09	14.06	
d_i^-	7.805	8.534	8.315	8.283	8.368	
$d_i^+ + d_i^-$	22.383	22.413	22.408	22.373	22.428	C2 > C5 > C3 > C4 > C1
CC_i	0.349	0.381	0.372	0.371	0.374	

By comparing the CC_i values of the five candidates, we find that the ranking order of the candidates as C2 > C5 > C3 > C4 > C1. Therefore, the candidate C2 is selected as the most preferred candidate, again by using the fuzzy TOPSIS.

4. CONCLUSION

In this study, we applied the fuzzy AHP approach to determine weights of criteria, the FMCDM approach to rank the candidates and fuzzy TOPSIS to compare the results obtained by integrating the fuzzy AHP and FMCDM approach for solving the industrial engineer selection problem. In the proposed approach, the fuzzy AHP approach is used to evaluate only dimensions and criteria. Candidates are evaluated by using the FMCDM approach. If all of the candidates are evaluated by fuzzy AHP instead of FMCDM, the total number of pair-wise comparison matrices considerably increase due to the increasing number of candidates, dimensions and criteria. For example, in the application, there are five dimensions, twenty one criteria and five candidates. There are six pair-wise comparison matrices in the proposed approach. On the other hand, 105 pair-wise comparison matrices are required for evaluating the candidates by using the full-fuzzy AHP approach. It leads to time-consuming process for each decision maker and may cause inconsistent matrices. So the FMCDM approach is preferred to evaluate the candidates.

The proposed approach is a simple and practical method to support group decision making in multi-criteria, uncertain and vague environment. It is important to define accurate linguistic terms and ranges for obtaining the most preferred candidate. The dimensions and criteria for the proposed approach and the fuzzy TOPSIS approach in the application are determined by three decision makers, but more decision makers can be easily included in the industrial engineer selection process.

In the literature, there are only a few studies about fuzzy AHP approach for personnel selection. In these studies, the consistency of matrices isn't controlled mostly and the sensitivity analysis isn't performed. In this study, the consistency of all matrices is controlled, if a matrix isn't consistent, decision makers are interviewed about the reasons of inconsistency and the pair-wise comparison matrices are obtained again by eliminating inconsistency. The candidate C2 is selected as the most preferred candidate by using the proposed approach and the fuzzy TOPSIS. The ranking order of the candidates in the integrated approach is different from the fuzzy TOPSIS. The results show that although the most important dimension of DM1 and DM3 is the individual factors, the industrial engineering factors are most important dimension for DM2. If the chair of industrial engineering departments hadn't joint the process, the industrial engineering factors would have been neglected by

managers. Furthermore, the sensitivity analysis is performed to determine the influence of criteria weights and the weights of decision makers. These analyses are useful to obtain final decision for DMs. M.S. Excel 2007 is employed for this process, but software can be developed for integrating the fuzzy AHP and FMCDM approach. Furthermore, the proposed approach can be improved by using fuzzy analytic network process, as some of the criteria can be affected each other. This study can be extended by using evaluation of classmates of the candidates in the department, because the knowledge derived from classmates about the candidates is important to predict the future performances.

REFERENCES

- [1] Dursun M, Karsak EE. A fuzzy MCDM approach for personnel selection. *Expert Systems with applications* 2010; 37 (6): 4324-4330.
- [2] Lu J, Ruan D. *Multi-objective group decision making: methods, software and applications with fuzzy set techniques*: Imperial College Press; 2007.
- [3] Liang G-S, Wang M-JJ. Personnel placement in a fuzzy environment. *Computers & operations research* 1992; 19 (2): 107-121.
- [4] Zadeh LA. Fuzzy sets. *Information and control* 1965; 8(3): 338-53.
- [5] Liang G-S, Wang M-JJ. Personnel selection using fuzzy MCDM algorithm. *European journal of operational research* 1994; 78 (1): 22-33.
- [6] Karsak EE. A fuzzy multiple objective programming approach for personnel selection. *Systems, Man, and Cybernetics, 2000 IEEE International Conference on*: IEEE; 2000. 2007-2012.
- [7] Capaldo G, Zollo G. Applying fuzzy logic to personnel assessment: a case study. *Omega* 2001; 29(6): 585-597.
- [8] Toroslu IH. Personnel assignment problem with hierarchical ordering constraints. *Computers & industrial engineering* 2003; 45 (3): 493-510.
- [9] Bali Ö, Gencer C. AHP, Bulanık AHP Ve Bulanık Mantık'la Kara Harp Okuluna Öğretim Elemanı Seçimi. *Savunma Bilimleri Dergisi* 2005; 4 (1): 24-43.
- [10] Kaptanoğlu D, Özok AF. Akademik performans değerlendirmesi için bir bulanık model. *itüdergisi/d mühendislik* 2010; 5 (1): 193-204.
- [11] Liou T-S, Wang M-JJ. Ranking fuzzy numbers with integral value. *Fuzzy sets and systems* 1992; 50 (3): 247-255.
- [12] Abdel-Kader MG, Dugdale D. Evaluating investments in advanced manufacturing technology: A fuzzy set theory approach. *The British Accounting Review* 2001; 33 (4): 455-489.
- [13] Dağdeviren M. Bulanık analitik hiyerarşi prosesi ile personel seçimi ve bir uygulama. *Gazi Üniversitesi Mühendislik-Mimarlık Fakültesi Dergisi* 2007; 22 (4): 791-799.
- [14] Golec A, Kahya E. A fuzzy model for competency-based employee evaluation and selection. *Computers & Industrial Engineering* 2007; 52 (1): 143-161.

- [15] Özdağoğlu A. Analysis of selection criteria for manufacturing employees using fuzzy-AHP. Dokuz Eylül Üniversitesi İşletme Fakültesi Dergisi 2008; 9 (1): 141-160.
- [16] Aydın Ö. FMCDM for personnel assignment in Turkish Armed Forces. Asia-Pacific Journal of Operational Research 2008; 25 (01): 75-87.
- [17] Güngör Z, Serhadlıoğlu G, Kesen SE. A fuzzy AHP approach to personnel selection problem. Applied Soft Computing 2009; 9 (2): 641-646.
- [18] Huang DK, Chiu HN, Yeh RH, Chang JH. A fuzzy multi-criteria decision making approach for solving a bi-objective personnel assignment problem. Computers & Industrial Engineering 2009; 56 (1): 1-10.
- [19] Celik M, Kandakoglu A, Er ID. Structuring fuzzy integrated multi-stages evaluation model on academic personnel recruitment in MET institutions. Expert Systems with Applications 2009; 36 (3): 6918-6927.
- [20] Şen CG, Çınar G. Evaluation and pre-allocation of operators with multiple skills: A combined fuzzy AHP and max–min approach. Expert Systems with Applications 2010; 37(3): 2043-2053.
- [21] Ozdaban I, Ozkan C. A fuzzy method on determining of job and personnel evaluation results, and matching them with suggested model. International Journal of Industrial Engineering: Theory, Applications and Practice 2010; 17 (4): 334-340.
- [22] Rouyendegh BD, Erkart T. Selection of academic staff using the fuzzy analytic hierarchy process (FAHP): a pilot study. Technical Gazette 2012; 19 (4): 923-929.
- [23] Rouyendegh BD, Erkan TE. An application of the fuzzy electre method for academic staff selection. Human Factors and Ergonomics in Manufacturing & Service Industries 2013; 23(2): 107-115.
- [24] Karsak EE, Dursun M. An integrated fuzzy MCDM approach for supplier evaluation and selection. Computers & Industrial Engineering 2015; 82: 82-93.
- [25] Dursun M, Karsak EE. A QFD-based fuzzy MCDM approach for supplier selection. Applied Mathematical Modelling 2013; 37 (8): 5864-5875.
- [26] Büyüközkan G, Karabulut Y, Arsenyan J. RFID service provider selection: an integrated fuzzy MCDM approach. Measurement 2017; 112: 88-98.
- [27] Erol İ, Sencer S, Özmen A, Searcy C. Fuzzy MCDM framework for locating a nuclear power plant in Turkey. Energy Policy 2014; 67: 186-197.
- [28] Önüt S, Kara SS, Işık E. Long term supplier selection using a combined fuzzy MCDM approach: a case study for a telecommunication company. Expert systems with applications 2009; 36(2): 3887-3895.
- [29] Bongo MF, Ocampo LA. A hybrid fuzzy MCDM approach for mitigating airport congestion: a case in Ninoy Aquino International Airport. Journal of Air Transport Management 2017; 63: 1-16.

- [30] Van de Kaa G, Rezaei J, Kamp L, de Winter A. Photovoltaic technology selection: a fuzzy MCDM approach. *Renewable and Sustainable Energy Reviews* 2014; 32: 662-670.
- [31] Quang NH, Vincent FY, Lin AC, Dat LQ, Chou S-Y. Parting curve selection and evaluation using an extension of fuzzy MCDM approach. *Applied Soft Computing* 2013; 13 (4): 1952-1959.
- [32] Saaty TL. *The analytic hierarchy process*. New York: McGraw-Hill; 1980.
- [33] Van Laarhoven P, Pedrycz W. A fuzzy extension of Saaty's priority theory. *Fuzzy sets and Systems* 1983; 11 (1-3): 229-241.
- [34] Chen S-JJ, Hwang CL. *Fuzzy Multiple Attribute Decision Making: Methods and Applications*. NJ, USA: Springer-Verlag New York, Inc.; 1992.
- [35] Buckley JJ. Ranking alternatives using fuzzy numbers. *Fuzzy sets and systems* 1985; 15 (1): 21-31.
- [36] Hsieh T-Y, Lu S-T, Tzeng G-H. Fuzzy MCDM approach for planning and design tenders selection in public office buildings. *International journal of project management* 2004; 22 (7): 573-584.
- [37] Chen C-T. Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy sets and systems* 2000; 114 (1): 1-9.
- [38] Hwang C-L, Yoon K. *Multiple Attribute Decision Making: Methods and Applications A State-of-the-art survey*. New York: Springer Science & Business Media; 1981.
- [39] Wang T-C, Chang T-H. Application of TOPSIS in evaluating initial training aircraft under a fuzzy environment. *Expert Systems with Applications* 2007; 33 (4): 870-880.
- [40] Kwong C-K, Bai H. Determining the importance weights for the customer requirements in QFD using a fuzzy AHP with an extent analysis approach. *Iie Transactions* 2003; 35 (7): 619-626.
- [41] Golden BL, Harker PT, Wasil EE. *The analytic hierarchy process: Applications and studies*. Berlin: Springer-Verlag; 1989.