PAPER DETAILS

TITLE: Compact and Matrix Operators on the Space \$\left\vert \overline N_p^{\phi }\right\vert _{k}\$

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PAGES: 124-133

ORIGINAL PDF URL: https://dergipark.org.tr/tr/download/article-file/1584931



Fundamental Journal of Mathematics and Applications

Journal Homepage: www.dergipark.org.tr/en/pub/fujma ISSN: 2645-8845 doi: https://dx.doi.org/10.33401/fujma.882309



Compact and Matrix Operators on the Space $\left|\overline{N}_{p}^{\varphi}\right|_{L}$

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Article Info

Abstract

Keywords: Absolute summability, Compact operator, Hausdorff measure of noncompactness, Matrix transformations, Operator norm, Sequence spaces, Weighted mean 2010 AMS: 40C05, 46B45, 40F05, 46A45 Received: 18 February 2021 Accepted: 4 June 2021 In this paper, determining the operator norm, we give certain characterizations of matrix transformations from the space $\left|\overline{N}_{p}^{\phi}\right|_{k}$, the space of all series summable by the absolute weighted mean summability method, to one of the classical sequence spaces c_{0}, c, l_{∞} . Also, we obtain the necessary and sufficient conditions for each matrix in these classes to be compact and establish a number of estimates or identities for the Hausdorff measures of noncompactness of the matrix operators in these classes.

1. Introduction

Available online: 17 June 2021

The summability theory has an important role in analysis, applied mathematics and engineering sciences, and has been studied by many authors for a long time. One of the main subjects in the summability theory is the theory of sequence spaces that concerns with the generalization of the notions of convergence for sequences and series. The main purpose is to assign a limit value for non-convergent series or sequences by using a transformation which is given by the most general linear mappings of infinite matrices. In this concept, the literature has still enlarged, concerned with characterizing completely all matrices which transform one given sequence space into another and also, many sequence spaces defined as domain of special matrices such as Euler, Nörlund, Hausdorff, Cesàro and weighted mean matrices and related matrix operators have been investigated by several authors (see, [?, ?]). On the other hand, from a different point of view, using the concept of absolute summability, several new spaces of series summable by the absolute summability methods have taken place in the literature (see, for instance, [?]-[?]). In a recent paper, the sequence space $\left|\overline{N}_p^{\phi}\right|_k$ has introduced and studied by Sarıgöl [?, ?], Mohapatra and Sarıgöl [?].

The present paper aims to characterize the infinite matrix classes $(\left|\overline{N}_{p}^{\phi}\right|_{k}, c), (\left|\overline{N}_{p}^{\phi}\right|_{k}, c_{0}), (\left|\overline{N}_{p}^{\phi}\right|_{k}, l_{\infty})$ and to determine the operator norms for $1 \le k < \infty$. Further, the necessary and sufficient conditions for each matrix in these classes to be compact are obtained and certain identities or estimates for the Hausdorff measure of noncompactness are established.

A vector subspace of ω , the space of all sequences of real or complex numbers, is called a *sequence space*. The sequence spaces Φ , l_{∞} , c, c_0 , b_s , c_s and l_k ($k \ge 1$) stand for the sets of all finite, bounded, convergent and null sequences and the sets of all bounded, convergent and *k*-absolutely convergent series, respectively.

Let Λ and Γ be two arbitrary sequence spaces and $R = (r_{nv})$ be an infinite matrix of complex components. The transform sequence $R(\lambda)$ of the sequence $\lambda = (\lambda_v)$ is deduced by the usual matrix product and the components of $R(\lambda)$ are written as

$$R_n(\lambda) = \sum_{\nu=0}^{\infty} r_{n\nu} \lambda_{\nu},$$

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provided that the series converges for all $n \in \mathbb{N}$. If the sequence $R(\lambda)$ exists and $R(\lambda) \in \Gamma$ for $\lambda \in \Lambda$, then, it is said that *R* is a matrix mapping from Λ into Γ . The collection of all such infinite matrices is denoted by (Λ, Γ) . The set

$$\Lambda_R = \{\lambda \in \omega : R(\lambda) \in \Lambda\}$$

is called domain of an infinite matrix *R* in the space Λ . Note that it is also a sequence space. The β -dual of $\Lambda \subset \omega$ is the set

$$\Lambda^{\beta} = \left\{ a : \forall \lambda \in \Lambda, \sum_{\nu=0}^{\infty} a_{\nu} \lambda_{\nu} \text{ converges} \right\}.$$

Let Λ and Γ be Banach spaces. By $\mathscr{B}(\Lambda, \Gamma)$, we mean the set of all bounded (continuous) linear operators *L* from Λ to Γ . $\mathscr{B}(\Lambda, \Gamma)$ is also a Banach space with the operator norm given by

$$\|L\| = \sup_{\lambda \in S_{\Lambda}} \|L(\lambda)\|_{\Gamma}$$

for all $L \in \mathscr{B}(X, Y)$. Here, S_{Λ} represents the unit sphere in Λ , i.e.,

$$S_{\Lambda} = \{\lambda \in \Lambda : \|\lambda\| = 1\}$$

If $a \in \omega$ and $\Lambda \supset \Phi$ is a *BK*-space, a Banach space on which all coordinate functional $p_n(\lambda) = \lambda_n$ are continuous for all *n*, then

$$\|a\|_{\Lambda}^* = \sup_{\lambda \in S_{\Lambda}} \left| \sum_{k=0}^{\infty} a_k \lambda_k \right|$$

provided the expression on the right side is defined and finite which is the case whenever $a \in \Lambda^{\beta}$. If, for each $\lambda \in \Lambda$,

$$\left\|\lambda - \sum_{j=0}^m \lambda_j e^{(j)}\right\| \to 0 \text{ as } m \to \infty$$

then it is said that the BK-space Λ has AK property, and in this case we write $\lambda = \sum_{j=0}^{\infty} \lambda_j e^{(j)}$ where $e^{(j)}$ is a sequence whose only non-zero term is one in *j*th place for $j \in \mathbb{N}$.

Throughout the whole paper, assume that $\phi = (\phi_n)$ is a sequence of positive constants and $R = (r_{nv})$ is an infinite matrix of complex numbers for all $n, v \in \mathbb{N}$. Also, k^* is the conjugate of k, that is, $1/k + 1/k^* = 1$ for k > 1 and $1/k^* = 0$ for k = 1. Let $\sum \lambda_n$ be an infinite series with its partial sum s_n . The series $\sum \lambda_v$ is said to be summable $|R, \phi_n|_k$, if (see[?]).

$$\sum_{n=1}^{\infty} \phi_n^{k-1} |\Delta R_n(s)|^k < \infty$$

where $1 \le k < \infty$ and $\Delta R_n(s) = R_n(s) - R_{n-1}(s)$. In the special case, when *R* is a weighted mean matrix, the summability method $|R, \phi_n|_k$ is reduced to $|\overline{N}, p_n, \phi|_k$ [?]. In recent paper, $|\overline{N}_p^{\phi}|_k$ has been generated from the space l_k as a set of all series summable by the absolute weighted mean method by Mohapatra and Sarıgöl [?] and Sarıgöl [?, ?]. The space $|\overline{N}_p^{\phi}|_k$ can be expressed as

$$\left|\overline{N}_{p}^{\phi}\right|_{k} = \left\{\lambda = (\lambda_{\nu}): \sum_{n=1}^{\infty} \phi_{n}^{k-1} \left|\frac{p_{n}}{P_{n}P_{n-1}} \sum_{\nu=1}^{n} P_{\nu-1}\lambda_{\nu}\right|^{k} < \infty\right\},\$$

or equivalently, according to notation of domain, $\left|\overline{N}_{p}^{\phi}\right|_{k} = (l_{k})_{T^{(p)}}$ where the matrix $T^{(p)}$ is given by

$$t_{nv}^{(p)} = \begin{cases} 1, n = 0, v = 0\\ \phi_n^{1/k^*} \frac{p_n P_{v-1}}{P_n P_{n-1}}, \ 1 \le v \le n,\\ 0, \quad v > n, \end{cases}$$

whose inverse $S^{(p)}$ is

$$s_{nv}^{(p)} = \begin{cases} 1, & n = 0, v = 0\\ -\phi_{n-1}^{-1/k^*} \frac{P_{n-2}}{p_{n-1}}, & v = n-1\\ \phi_n^{-1/k^*} \frac{P_n}{p_n}, & v = n\\ 0, & v \neq n-1, n. \end{cases}$$
(1.1)

Besides, it is obvious that the space $\left|\overline{N}_{p}^{\phi}\right|_{k}$ is a *BK*-space with the norm $\left\|\lambda\right\|_{\left|\overline{N}_{p}^{\phi}\right|_{k}} = \left\|T^{(p)}(\lambda)\right\|_{l_{k}}$ and it is also linearly isomorphic to the space l_{k} for $1 \leq k < \infty$ [?].

We recall the following lemmas which are useful in proving our results:

Lemma 1.1. [?] Let U be a triangle. Then,

(*i*) For $\Lambda, \Gamma \subset \omega$, $R \in (\Lambda, \Gamma_U)$ iff $B = UR \in (\Lambda, \Gamma)$.

(ii) If Λ, Γ are BK-spaces and $R \in (\Lambda, \Gamma_U)$, then $||L_R|| = ||L_B||$.

Lemma 1.2. [?] The following statements hold:

1. $R \in (l,c) \Leftrightarrow (i) \lim_{n} r_{nv}$ exists for all $v \ge 0$, (ii) $\sup_{n,v} |r_{nv}| < \infty$ and $R \in (l,l_{\infty}) \Leftrightarrow (ii)$ holds.

- 2. If $1 < k < \infty$, then, $R \in (l_k, c) \Leftrightarrow (i)$ holds, (iii) $\sup_{n} \sum_{\nu=0}^{\infty} |r_{n\nu}|^{k^*} < \infty$ and $R \in (l_k, l_{\infty}) \Leftrightarrow (iii)$ holds. 3. $R \in (l, c_0) \Leftrightarrow (ii)$ holds, (iv) $\lim_{n} r_{n\nu} = 0$ for all $\nu \ge 0$.
- 4. If $1 < k < \infty$, then, $R \in (l_k, c_0) \stackrel{n}{\Leftrightarrow} (iii)$ and (iv) hold.

Lemma 1.3. [?] Let $1 \le k < \infty$. Then, $R \in (l, l_k)$ iff

$$||R||_{(l,l_k)} = \sup_{v} \left\{ \sum_{n=0}^{\infty} |r_{nv}|^k \right\}^{1/k}$$

Lemma 1.4. [?] Let $1 < k < \infty$. Then, $R \in (l_k, l)$ iff

$$\|R\|'_{(l_k,l)} = \left\{\sum_{\nu=0}^{\infty} \left(\sum_{n=0}^{\infty} |r_{n\nu}|\right)^{k^*}\right\}^{1/k^*} < \infty.$$

Since

$$\|R\|_{(l_k,l)} \leq \|R\|_{(l_k,l)} \leq 4 \|R\|_{(l_k,l)},$$

there exists $1 \leq \xi \leq 4$ such that $\|R\|'_{(l_k,l)} = \xi \|R\|_{(l_k,l)}$ where

$$\|R\|_{(l_k,l)} = \sup_{N \in \mathfrak{F}} \left\{ \sum_{\nu=0}^{\infty} \left| \sum_{n \in N}^{\infty} r_{n\nu} \right|^{k^*} \right\}^{1/k}$$

and \mathfrak{F} represents the collection of all finite subsets of \mathbb{N} .

Lemma 1.5. [?] Let $1 < k < \infty$ and k^* denote the conjugate of k. Then, we have $l_k^\beta = l_{k^*}$ and $l_{\infty}^\beta = c_0^\beta = l$, $l^\beta = l_{\infty}$. Also, if $\Lambda \in \{l_{\infty}, c, c_0, l, l_k\}$ then, we have

$$\|a\|_{\Lambda}^* = \|a\|_{\Lambda^{\beta}}$$

for all $a \in \Lambda^{\beta}$, where $\|.\|_{\Lambda^{\beta}}$ is the natural norm on Λ^{β} .

Lemma 1.6. *[?]* Let $\Lambda \supset \Phi$ be a BK-space and $\Gamma \in \{c, c_0, \ell_\infty\}$. If $R \in (\Lambda, \Gamma)$, then

$$||L_R|| = ||R||_{(\Lambda, l_{\infty})} = \sup_n ||R_n||_{\Lambda}^* < \infty.$$

The Hausdorff measure of noncompactness χ was introduced by Goldenstein, Gohberg and Markus [?]. Using the Hausdorff measure of noncompactness, some compact operators on various sequence spaces are characterized by many authors. For example, Mursaleen and Noman in [?, ?], Malkowsky and Rakocevic in [?] have used the Hausdorff measure of noncompactness method to characterize the class of compact operators on some known spaces, (see also [?, ?, ?, ?], [?]-[?]).

Let (Λ, d) be a metric space and $H, M \subset \Lambda$. If there exists an $h \in H$ such that $d(h, m) < \varepsilon$ for every $m \in M$, then it is said that H is an ε -net of M; if H is finite, then the ε -net H of M is called a finite ε -net of M. Let Q be a bounded subset of the metric space Λ . Then, the Hausdorff measure of noncompactness of Q is defined by

$$\chi(Q) = \inf \{ \varepsilon > 0 : Q \text{ has a finite } \varepsilon - \text{net in } \Lambda \},\$$

and χ is called the Hausdorff measure of noncompactness.

Let Λ, Γ be Banach spaces. A linear operator L from Λ into Γ is called compact if its domain is all of Λ and, for every bounded sequence (λ_n) in Λ , $(L(\lambda_n))$ has a convergent subsequence in Γ . The class of all compact operators in $\mathscr{B}(\Lambda,\Gamma)$ is denoted by $\mathscr{C}(\Lambda,\Gamma).$

The following lemmas give a calculation method for the Hausdorff measure of noncompactness of a bounded subset and the necessary and sufficient conditions a linear operator to be compact.

Lemma 1.7. [?] Let Λ be one of the spaces c_0 or l_k for $1 \le k < \infty$ and Q be a bounded subset of Λ . If $P_r : \Lambda \to \Lambda$ is the operator described by $P_r(\lambda) = (\lambda_0, \lambda_1, ..., \lambda_r, 0, 0, ...)$ for all $\lambda \in \Lambda$, then

$$\chi(Q) = \lim_{r \to \infty} \left(\sup_{\lambda \in Q} \left\| (I - P_r)(\lambda) \right\| \right).$$

Assume that χ_1, χ_2 are two Hausdorff measures on the spaces Λ, Γ and Q is a bounded subset of Λ . The linear operator $L: \Lambda \to \Gamma$ is said to be (χ_1, χ_2) -bounded if L(Q) is a bounded subset of Γ and there exists a positive constant M such that $\chi_2(L(Q)) \leq M\chi_1(Q)$ for every Q. If an operator L is (χ_1, χ_2) -bounded, then the number

$$\|L\|_{(\chi_1,\chi_2)} = \inf \{M > 0 : \chi_2(L(Q)) \le M\chi_1(Q) \text{ for all bounded sets } Q \subset \Lambda \}$$

is called the (χ_1, χ_2) -measure noncompactness of L. In particular, for $\chi_1 = \chi_2 = \chi$, it is written by $\|L\|_{(\chi, \chi)} = \|L\|_{\chi}$.

Lemma 1.8. [?] $L \in \mathscr{B}(\Lambda, \Gamma)$ and S_{Λ} be the unit sphere in X. Then,

$$\|L\|_{\chi} = \chi\left(L(S_{\Lambda})\right)$$

and

L is compact
$$\Leftrightarrow \|L\|_{\gamma} = 0$$
.

Lemma 1.9. [?] Let Λ be a normed sequence space, $U = (u_{nv})$ be an infinite triangle matrix, χ_U and χ denote the Hausdorff measures of noncompactness on M_{Λ_U} and M_{Λ} , the collections of all bounded sets in Λ_U and Λ , respectively. Then, $\chi_U(Q) = \chi_U(Q)$ $\chi(U(Q))$ for all $Q \in M_{\Lambda_U}$.

Lemma 1.10. [?] Let $\Lambda \supset \Phi$ be a BK-space with AK property or $\Lambda = I_{\infty}$. If $R \in (\Lambda, c)$, then we have

$$\lim_{n\to\infty} r_{nk} = \alpha_k \quad \text{exists for all } k,$$
$$\alpha = (\alpha_k) \in \Lambda^{\beta},$$
$$\sup_n \|R_n - \alpha\|_{\Lambda}^* < \infty,$$
$$\lim_{n\to\infty} R_n(\lambda) = \sum_{k=0}^{\infty} \alpha_k \lambda_k \text{ for every } \lambda = (\lambda_k) \in \Lambda.$$

Lemma 1.11. *[?]* Let $X \supset \Phi$ be a BK-space. Then, (a) If $R \in (\Lambda, c_0)$, then

$$\|L_R\|_{\chi} = \lim_{r \to \infty} \left(\sup_{n > r} \|R_n\|_{\Lambda}^* \right).$$

(b) If Λ has AK property or $\Lambda = l_{\infty}$ and $R \in (\Lambda, c)$, then

$$\frac{1}{2}\lim_{r\to\infty}\left(\sup_{n>r}\|R_n-\alpha\|^*_{\Lambda}\right)\leq\|L_R\|_{\chi}\leq\lim_{r\to\infty}\left(\sup_{n>r}\|R_n-\alpha\|^*_{\Lambda}\right)$$

where $\alpha = (\alpha_k)$ defined by $\alpha_k = \lim_{k \to \infty} r_{nk}$, for all $n \in \mathbb{N}$. (c) If $R \in (\Lambda, l_{\infty})$, then

$$0 \le \left\|L_R\right\|_{\chi} \le \lim_{r \to \infty} \left(\sup_{n > r} \left\|R_n\right\|_{\Lambda}^*\right)$$

2. Matrix and compact operators on space $\left| \overline{N}_{p}^{\phi} \right|_{L}$

In this section, by computing the operator norms we characterize infinite matrix classes $\left(\left|\overline{N}_{p}^{\phi}\right|_{k}, c\right), \left(\left|\overline{N}_{p}^{\phi}\right|_{k}, c_{0}\right), \left(\left|\overline{N}_{p}^{\phi}\right|_{k}, l_{\infty}\right)$ and also compact matrix classes $\mathscr{C}\left(\left|\overline{N}_{p}^{\phi}\right|_{k}, c\right), \mathscr{C}\left(\left|\overline{N}_{p}^{\phi}\right|_{k}, c_{0}\right), \mathscr{C}\left(\left|\overline{N}_{p}^{\phi}\right|_{k}, l_{\infty}\right)$. Moreover, we establish some identities or estimates for the Hausdorff measure of noncompactness.

For simplicity of notation, in what follows, we use

$$\sigma_{nv} = \Delta r_{nv} \frac{P_v}{p_v} + r_{n,v+1},$$

where $\Delta r_{nv} = r_{nv} - r_{n,v+1}$.

Lemma 2.1. Let $1 < k < \infty$. Then, (i) If $a = (a_v) \in \left\{ \left| \overline{N}_p^{\phi} \right|_k \right\}^{\beta}$, then, $\tilde{a}^{(k)} = (\tilde{a}_v^{(k)}) \in l_{k^*}$ for all $\lambda \in \left| \overline{N}_p^{\phi} \right|_k$ (ii) If $a = (a_v) \in \left\{ \left| \overline{N}_p^{\phi} \right| \right\}^{\beta}$, then, $\tilde{a}^{(1)} = (\tilde{a}_v^{(1)}) \in l_{\infty}$ for all $\lambda \in \left| \overline{N}_p^{\phi} \right|$ and the equality

$$\sum_{\nu=0}^{\infty} a_{\nu} \lambda_{\nu} = \sum_{\nu=0}^{\infty} \widetilde{a}_{\nu}^{(k)} y_{\nu}$$
(2.1)

holds, where $y = T^{(p)}(\lambda)$ and

$$\widetilde{a}_{v}^{(k)} = \phi_{v}^{-1/k^{*}} \left(\Delta a_{v} \frac{P_{v}}{p_{v}} + a_{v+1} \right) \text{ for } v > 0, a_{0} = \widetilde{a}_{0}^{(k)}.$$

Proof. (*i*) Let $a = (a_v) \in \left\{ \left| \overline{N}_p^{\phi} \right|_k \right\}^{\beta}$. By (??), the equation (??) is immediately obtained. Also, it follows from Lemma ?? that $\tilde{a}^{(k)} \in l_{k^*}$ whenever $a \in \left\{ \left| \overline{N}_p^{\phi} \right|_k \right\}^{\beta}$, which completes the proof. The proof of (*ii*) is left to reader.

Lemma 2.2. Let $1 < k < \infty$. Then, we have $\|a\|_{|\overline{N}_p^{\phi}|_k}^* = \|\tilde{a}^{(k)}\|_{l_{k^*}}$ for all $a \in \left\{ \left|\overline{N}_p^{\phi}\right|_k \right\}^{\beta}$ and $\|a\|_{|\overline{N}_p^{\phi}|}^* = \|\tilde{a}^{(1)}\|_{\infty}$ for all $a \in \left\{ \left|\overline{N}_p^{\phi}\right|_k \right\}^{\beta}$.

Proof. Take $a \in \left\{ \left| \overline{N}_{p}^{\phi} \right|_{k} \right\}^{\beta}$. It is obvious from Lemma **??** that $\tilde{a}^{(k)} \in I_{k^{*}}$. Also, it follows from Lemma **??** and Lemma **??** that

$$\left\|a\right\|_{\left|\overline{N}_{p}^{\phi}\right|_{k}}^{*}=\sup_{\lambda\in S_{\left|\overline{N}_{p}^{\phi}\right|_{k}}}\left|\sum_{\nu=0}^{\infty}a_{\nu}\lambda_{\nu}\right|=\sup_{y\in S_{l_{k}}}\left|\sum_{\nu=0}^{\infty}\tilde{a}_{\nu}^{(k)}y_{\nu}\right|=\left\|\tilde{a}^{(k)}\right\|_{l_{k}}^{*}=\left\|\tilde{a}^{(k)}\right\|_{l_{k}}^{*}$$

For $a \in \left\{ \left| \overline{N}_{p}^{\phi} \right| \right\}^{\beta}$, the proof is similar, so it is left to reader.

Theorem 2.3. Let $1 \le k < \infty$, Λ be arbitrary sequence space. Further, let $B = (b_{nj})$ be a matrix satisfying

$$b_{nj} = \phi_n^{1/k^*} \frac{p_n}{P_n P_{n-1}} \sum_{\nu=1}^n P_{\nu-1} r_{\nu j}.$$
(2.2)

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Then, $R \in \left(\Lambda, \left|\overline{N}_{p}^{\phi}\right|_{k}\right)$ iff $B \in (\Lambda, l_{k})$.

Proof. Let $\lambda \in \Lambda$. Then, it follows from (??) that

$$\sum_{j=0}^{\infty} b_{nj} \lambda_j = \phi_n^{1/k^*} \frac{p_n}{P_n P_{n-1}} \sum_{\nu=1}^n P_{\nu-1} \sum_{j=0}^{\infty} \lambda_j r_{\nu j},$$

which implies that $B_n(\lambda) = T_n^{(p)}(R(\lambda))$. This gives that $R_n(\lambda) \in \left|\overline{N}_p^{\phi}\right|_k$ for all $\lambda \in \Lambda$ iff $B(\lambda) \in l_k$ for all $\lambda \in \Lambda$. So, the proof of the theorem is completed.

Let us define the matrix $\tilde{R}^{(k)} = \left(\tilde{r}_{nv}^{(k)}\right)$ with $\tilde{r}_{nv}^{(k)} = \frac{1}{\phi_v^{1/k^*}}\sigma_{nv}$ for v > 0, $\tilde{r}_{n0}^{(k)} = r_{n0}$. It is clear that the matrices R and $\tilde{R}^{(k)}$ are connected by (??).

Theorem 2.4. (*i*) Let $1 < k < \infty$ and $\Lambda \in \{c_0, c, l_{\infty}\}$. Then,

$$R \in \left(\left|\overline{N}_{p}^{\phi}\right|_{k}, \Lambda\right) \Rightarrow \|L_{R}\| = \sup_{n} \left\|\widetilde{R}_{n}^{(k)}\right\|_{l_{k^{*}}} = \sup_{n} \left(\sum_{\nu=0}^{\infty} \left|\widetilde{r}_{n\nu}^{(k)}\right|^{k^{*}}\right)^{1/\kappa}$$
$$R \in \left(\left|\overline{N}_{p}^{\phi}\right|, \Lambda\right) \Rightarrow \|L_{R}\| = \sup_{n} \left\|\widetilde{R}_{n}^{(1)}\right\|_{l_{\infty}} = \sup_{n,\nu} \left|\widetilde{r}_{n\nu}^{(1)}\right|.$$

(ii) Let $1 < k < \infty$. Then, there exists $1 \le \xi \le 4$ such that

$$R \in \left(\left|\overline{N}_{p}^{\phi}\right|_{k}, l\right) \Rightarrow \left\|L_{R}\right\| = \frac{1}{\xi} \left\|\tilde{R}^{(k)}\right\|_{(l_{k}, l)}^{\prime} = \frac{1}{\xi} \left\{\sum_{\nu=0}^{\infty} \left(\sum_{n=0}^{\infty} \left|\tilde{r}_{n\nu}^{(k)}\right|\right)^{k^{*}}\right\}^{1/k^{*}},$$
$$R \in \left(\left|\overline{N}_{p}^{\phi}\right|, l_{k}\right) \Rightarrow \left\|L_{R}\right\| = \left\|\tilde{R}^{(1)}\right\|_{(l, l_{k})} = \sup_{\nu} \left\{\sum_{n=0}^{\infty} \left|\tilde{r}_{n\nu}^{(1)}\right|^{k}\right\}^{\frac{1}{k}},$$
$$R \in \left(\left|\overline{N}_{p}^{\phi}\right|, l\right) \Rightarrow \left\|L_{R}\right\| = \left\|\tilde{R}_{n}^{(1)}\right\|_{(l, l)} = \sup_{\nu} \sum_{n=0}^{\infty} \left|\tilde{r}_{n\nu}^{(1)}\right|.$$

Proof. The proof of the theorem is obtained from Lemma ??, Lemma ??, Lemma ??, and Lemma ??.

Theorem 2.5. Let $1 < k < \infty$. Then, a) $R \in \left(\left| \overline{N}_{p}^{\phi} \right|_{k}, c_{0} \right)$ iff

$$\lim_{n \to \infty} \tilde{r}_{nv}^{(k)} = 0 \text{ for all } v \tag{2.3}$$

$$\sup_{n} \sum_{\nu=0}^{\infty} \left| \tilde{r}_{n\nu}^{(k)} \right|^{k^*} < \infty$$

$$(2.4)$$

$$\sup_{m} \left\{ \sum_{\nu=1}^{m-1} \frac{1}{\phi_{\nu}} \left| \sigma_{n\nu} \right|^{k^{*}} + \frac{1}{\phi_{m}} \left| r_{nm} \frac{P_{m}}{p_{m}} \right|^{k^{*}} \right\} < \infty$$
(2.5)

hold. b) $R \in \left(\left|\overline{N}_{p}^{\phi}\right|_{k}, c\right)$ iff (??), (??) and

$$\lim_{n\to\infty}\tilde{r}_{nv}^{(k)} \text{ exists for all } v$$

hold. c) $R \in \left(\left| \overline{N}_{p}^{\phi} \right|_{k}, l_{\infty} \right)$ iff (??), (??) hold.

Proof. Prove only the part (*a*) since the proofs of the other parts can be made the same way. $R \in \left(\left|\overline{N}_{p}^{\phi}\right|_{k}, c_{0}\right)$ if and only if $(r_{nv})_{v=0}^{\infty} \in \left\{\left|\overline{N}_{p}^{\phi}\right|_{k}\right\}^{\beta}$ and $R(\lambda) \in c_{0}$ for every $\lambda \in \left|\overline{N}_{p}^{\phi}\right|_{k}$. It is seen immediately from Theorem 2.1 in [?] $(r_{nv})_{v=0}^{\infty} \in \left\{\left|\overline{N}_{p}^{\phi}\right|_{k}\right\}^{\beta}$ if and only if (??) holds. Also, if any matrix $R \in (l_{k}, c_{0})$, then the series $\sum_{v} r_{nv} \lambda_{v}$ converges uniformly in *n* and so

$$\lim_{n} \sum_{\nu} r_{n\nu} \lambda_{\nu} = \sum_{\nu} \lim_{n} r_{n\nu} \lambda_{\nu}.$$
(2.6)

On the other hand,

$$\lim_{m}\sum_{\nu=0}^{m}r_{n\nu}\lambda_{\nu}=\lim_{m}\sum_{\nu=0}^{m}b_{m\nu}^{(n)}y_{\nu}$$

where $B^{(n)} = (b_{mv}^{(n)})$ is defined by

$$b_{mv}^{(n)} = \begin{cases} r_{n0}, & v = 0, \\ \frac{P_v}{\phi_v^{1/\mu_v^*} p_v} \left(r_{nv} - \frac{P_{v-1}}{P_v} r_{n,v+1} \right), 1 \le v < m-1 \\ \frac{P_m r_{mm}}{\phi_m^{1/\mu_m^*} p_m}, & v = m, m \ge 1 \\ 0, & v > m. \end{cases}$$

So, it follows from (??)

$$R_n(\lambda) = \lim_{m} \sum_{\nu=0}^m r_{n\nu} \lambda_{\nu} = \lim_{m} \sum_{\nu=0}^m b_{m\nu}^{(n)} y_{\nu} = \sum_{\nu=0}^\infty \tilde{r}_{n\nu}^{(k)} y_{\nu} = \tilde{R}_n^{(k)}(y).$$

It is clear that $R(\lambda) \in c_0$ for every $\lambda \in \left|\overline{N}_p^{\phi}\right|_k$ equals to $\tilde{R}^{(k)}(y) \in c_0$ for every $y \in l_k$ since $\left|\overline{N}_p^{\phi}\right|_k \cong l_k$. This means that $\tilde{R}^{(k)} \in (l_k, c_0)$. Applying Lemma ?? to the matrix $\tilde{R}^{(k)}$ the conditions (??) and (??) are obtained which completes the proof of the part (a).

Theorem 2.6. *The following statements hold: a)* $R \in (\left|\overline{N}_{p}^{\phi}\right|, c_{0})$ *iff*

$$\lim_{n \to \infty} \tilde{r}_{nv}^{(1)} = 0 \text{ for all } v \tag{2.7}$$

$$\sup_{n,v} \left| \tilde{r}_{nv}^{(1)} \right| < \infty \tag{2.8}$$

$$\sup_{\nu} \left\{ \left| \sigma_{n\nu} \right| + \left| r_{n\nu} \frac{P_{\nu}}{p_{\nu}} \right| \right\} < \infty, \text{ for all } n$$
(2.9)

hold.
b)
$$R \in \left(\left| \overline{N}_{p}^{\phi} \right|, c \right)$$
 iff (??), (??) and

$$\lim_{n o\infty} ilde{r}_{nv}^{(1)}$$
 exists for all v

hold.
c)
$$R \in \left(\left|\overline{N}_{p}^{\phi}\right|, l_{\infty}\right)$$
 iff (??), (??) hold.

Proof. (*b*) Let $R \in (\left|\overline{N}_{p}^{\phi}\right|, c)$. $R \in (\left|\overline{N}_{p}^{\phi}\right|, c)$ if and only if $(r_{nv})_{v=0}^{\infty} \in \left\{\left|\overline{N}_{p}^{\phi}\right|\right\}^{\beta}$ and $R(\lambda) \in c$ for every $\lambda \in \left|\overline{N}_{p}^{\phi}\right|$. It follows from Theorem 2.1 in [?], $(r_{nv})_{v=0}^{\infty} \in \left\{\left|\overline{N}_{p}^{\phi}\right|\right\}^{\beta}$ if and only if (??) holds. Further, for any matrix $R \in (l, c)$, the series $\sum_{v} r_{nv} \lambda_{v}$ converges uniformly in *n* and so

$$\lim_{n} \sum_{\nu} r_{n\nu} \lambda_{\nu} = \sum_{\nu} \lim_{n} r_{n\nu} \lambda_{\nu}.$$
 (2.10)

Also,

$$\lim_{m}\sum_{\nu=0}^{m}r_{n\nu}\lambda_{\nu}=\lim_{m}\sum_{\nu=0}^{m}d_{m\nu}^{(n)}y_{\nu}$$

where the matrix $D^{(n)} = (d_{mv}^{(n)})$ is given by

$$d_{mv}^{(n)} = \begin{cases} \frac{P_v}{p_v} \left(r_{nv} - \frac{P_{v-1}}{P_v} r_{n,v+1} \right), 0 \le v < m-1 \\ \frac{P_m}{p_m} r_{nm}, \quad v = m, m \ge 1 \\ 0, \quad v > m. \end{cases}$$

So, it is deduced from (??)

$$R_n(\lambda) = \lim_{m} \sum_{\nu=0}^m r_{n\nu} \lambda_{\nu} = \lim_{m} \sum_{\nu=0}^m d_{m\nu}^{(n)} y_{\nu} = \sum_{\nu=0}^\infty \tilde{r}_{n\nu}^{(1)} y_{\nu} = \tilde{R}_n^{(1)}(y).$$

It is obvious that $R(\lambda) \in c$ for every $\lambda \in \left| \overline{N}_p^{\phi} \right|$ if and only if $\tilde{R}^{(1)}(\lambda) \in c$ for every $y \in l$, i.e., $\tilde{R}^{(1)} \in (l,c)$. Applying Lemma **??** to the matrix $\tilde{R}^{(1)}$ the conditions (**??**), (**??**) are obtained. This completes the proof of the part (*b*). The other parts can be proved by the similar way with Lemma **??**.

Take the matrix $L = (l_{nj})$ defined by

$$l_{nj} = \begin{cases} 1, \ 0 \le j \le n \\ 0, \quad j > n. \end{cases}$$

Then, since $b_s = \{l_{\infty}\}_L$ and $c_s = \{c\}_L$, the matrix classes $\left(\left|\bar{N}_p^{\phi}\right|_k, c_s\right)$ and $\left(\left|\bar{N}_p^{\phi}\right|_k, b_s\right)$ can be characterized as follows with Lemma **??**:

Corollary 2.7. Let $1 < k < \infty$. $R \in \left(\left|\overline{N}_{p}^{\phi}\right|_{k}, c_{s}\right)$ iff

$$\lim_{n\to\infty}\tilde{r}(n,v) exists for all v$$

$$\sup_{n} \sum_{\nu=0}^{\infty} \left| \tilde{r}(n,\nu) \right|^{k^*} < \infty \tag{2.11}$$

$$\sup_{m} \left\{ \sum_{\nu=1}^{m-1} \left| \tilde{r}(n,\nu) \right|^{k^{*}} + \frac{1}{\phi_{m}} \left| r(n,m) \frac{P_{m}}{p_{m}} \right|^{k^{*}} \right\} < \infty,$$
(2.12)

 $R \in \left(\left|\overline{N}_{p}^{\phi}\right|_{k}, b_{s}\right)$ if and only if (??) and (??) hold where $r(n, v) = \sum_{j=0}^{n} r_{jv}$, R(n, v) and $\tilde{R}(n, v)$ are connected by (??).

Theorem 2.8. Suppose that $1 < k < \infty$. Then, a) $R \in (\left|\overline{N}_{p}^{\phi}\right|_{k}, c_{0})$ iff

$$\left\|L_{R}\right\|_{\chi} = \limsup_{n \to \infty} \left(\sum_{\nu=0}^{\infty} \left|\tilde{r}_{n\nu}^{(k)}\right|^{k^{*}}\right)^{1/k}$$

and $R \in \mathscr{C}(\left|\bar{N}_{p}^{\phi}\right|_{k}, c_{0})$ iff $\limsup_{n \to \infty} \sum_{\nu=0}^{\infty} \left|\tilde{r}_{n\nu}^{(k)}\right|^{k^{*}} = 0.$ b) $R \in (\left|\overline{N}_{p}^{\phi}\right|_{k}, c)$ iff

$$\frac{1}{2} \limsup_{n \to \infty} \left(\sum_{\nu=0}^{\infty} \left| \tilde{r}_{n\nu}^{(k)} - \alpha_{\nu} \right|^{k^*} \right)^{1/k^*} \le \left\| L_R \right\|_{\chi} \le \limsup_{n \to \infty} \left(\sum_{\nu=0}^{\infty} \left| \tilde{r}_{n\nu}^{(k)} - \alpha_{\nu} \right|^{k^*} \right)^{1/k^*}$$

and $R \in \mathscr{C}(\left|\overline{N}_{p}^{\phi}\right|_{k}, c_{0})$ iff $\limsup_{n \to \infty} \sum_{\nu=0}^{\infty} \left|\tilde{r}_{n\nu}^{(k)} - \alpha_{\nu}\right|^{k^{*}} = 0$ where $\alpha_{\nu} = \lim_{n \to \infty} \tilde{r}_{n\nu}^{(k)}$. $c) R \in \left(\left|\overline{N}_{p}^{\phi}\right|_{k}, l_{\infty}\right)$ iff $0 \leq \|L_{R}\|_{\chi} \leq \limsup_{n \to \infty} \left(\sum_{\nu=0}^{\infty} \left|\tilde{r}_{n\nu}^{(k)}\right|^{k^{*}}\right)^{1/k^{*}}$

also, if $\limsup_{n\to\infty}\sum_{\nu=0}^{\infty}\left|\tilde{r}_{n\nu}^{(k)}\right|^{k^*}=0$, then $R\in\mathscr{C}(\left|\overline{N}_p^{\phi}\right|_k,l_{\infty})$.

Proof. To avoid repetition, only the proof of b is made and the proofs of (a) and (c) are left to the reader. (b) Let $R \in \left(\left| \overline{N}_{p}^{\phi} \right|_{k}, c \right)$. To compute the Hausdorff measure of noncompactness of L_{R} , take the unit sphere $S_{\left| \overline{N}_{p}^{\phi} \right|_{k}}$ in the space $\left| \overline{N}_{p}^{\phi} \right|_{k}$. It is written from Lemma **??** that

$$\|L_R\|_{\chi} = \chi(RS_{\left|\overline{N}_p^{\phi}\right|_k}).$$

On the other hand, since $\left|\overline{N}_{p}^{\phi}\right|_{k} \cong l_{k}, R \in \left(\left|\overline{N}_{p}^{\phi}\right|_{k}, c\right)$ if and only if $\tilde{R}^{(k)} \in (l_{k}, c)$, and so

$$\|L_R\|_{\chi} = \chi(RS_{\left|\overline{N}_p^{\phi}\right|_k}) = \chi(\tilde{R}^{(k)}T^{(p)}S_{\left|\overline{N}_p^{\phi}\right|_k}) = \|L_{\tilde{R}^{(k)}}\|_{\chi}$$

which implies, by Lemma ??,

$$\frac{1}{2}\lim_{r\to\infty}\left(\sup_{n\geq r}\left\|\tilde{R}_{n}^{(k)}-\alpha\right\|_{l_{k}}^{*}\right)\leq\left\|L_{R}\right\|_{\chi}\leq\lim_{r\to\infty}\left(\sup_{n\geq r}\left\|\tilde{R}_{n}^{(k)}-\alpha\right\|_{l_{k}}^{*}\right),$$
(2.13)

where $\alpha_v = \lim_{n \to \infty} \tilde{r}_{nv}^{(k)}$, for all $v \ge 0$.

By Lemma ??, $\|\tilde{R}_n^{(k)} - \alpha\|_{l_k}^* = \|\tilde{R}_n^{(k)} - \alpha\|_{l_{k^*}}$. The last equality completes the first part of the proof of (b) with (??). Moreover, the compactness of L_R is immediately deduced from Lemma ??. So, the proof of (b) is completed.

We have the following theorems by following the above lines:

Theorem 2.9. (a) If
$$R \in \left(\left|\overline{N}_{p}^{\phi}\right|, c_{0}\right)$$
. Then
 $\left\|L_{R}\right\|_{\chi} = \limsup_{n \to \infty} \left\|\widetilde{R}_{n}^{(k)}\right\|_{l_{\infty}} = \limsup_{n \to \infty} \sup_{v} \left|\widetilde{r}_{nv}^{(1)}\right|_{v}$

and
$$R \in \mathscr{C}\left(\left|\overline{N}_{p}^{\phi}\right|, c_{0}\right)$$
 iff $\limsup_{n \to \infty} \sup_{v} \left|\tilde{r}_{nv}^{(1)}\right| = 0.$
(b) If $R \in \left(\left|\overline{N}_{p}^{\phi}\right|, c\right)$, then
 $\frac{1}{2}\limsup_{n \to \infty} \sup_{v} \left|\tilde{r}_{nv}^{(1)} - \alpha_{v}\right| \leq \left\|L_{R}\right\|_{\chi} \leq \limsup_{n \to \infty} \sup_{v} \left|\tilde{r}_{nv}^{(1)} - \alpha_{v}\right|$
and $R \in \mathscr{C}\left(\left|\overline{N}_{p}^{\phi}\right|, c\right)$ iff $\limsup_{n \to \infty} \sup_{v} \left|\tilde{r}_{nv}^{(1)} - \alpha_{v}\right| = 0$ where $\alpha_{v} = \lim_{n \to \infty} \tilde{r}_{nv}^{(1)}$, for all $v \in \mathbb{N}$.
(c) If $R \in \left(\left|\overline{N}_{p}^{\phi}\right|, l_{\infty}\right)$, then
 $0 \leq \left\|L_{R}\right\|_{\chi} \leq \limsup_{n \to \infty} \sup_{v} \left|\tilde{r}_{nv}^{(1)}\right|$
and $R \in \mathscr{C}\left(\left|\overline{N}_{p}^{\phi}\right|, c_{0}\right)$ if $\limsup_{n \to \infty} \sup_{v} \left|\tilde{r}_{nv}^{(1)}\right| = 0$.

Theorem 2.10. (a) If $R \in \left(\left| \overline{N}_p^{\phi} \right|, l_k \right)$, $1 \le k < \infty$, then

$$\left\|L_{\mathcal{R}}\right\|_{\chi} = \lim_{j \to \infty} \left\{ \sup_{\nu} \left(\sum_{n=j+1}^{\infty} \left| \tilde{r}_{n\nu}^{(1)} \right|^{k} \right)^{1/k} \right\},$$

and *R* is a compact operator iff $\lim_{j\to\infty} \sup_{v} \sum_{n=j+1}^{\infty} \left| \tilde{r}_{nv}^{(1)} \right|^{k} = 0.$ (b) If $R \in \left(\left| \overline{N}_{p}^{\phi} \right|_{k}, l \right)$, $1 < k < \infty$, then there exists $1 \le \xi \le 4$ such that

$$\|L_{R}\|_{\chi} = \frac{1}{\xi} \lim_{j \to \infty} \left\{ \sum_{\nu=0}^{\infty} \left(\sum_{n=j+1}^{\infty} \left| \tilde{r}_{n\nu}^{(k)} \right| \right)^{k^{*}} \right\}^{1/k^{*}},$$

and *R* is compact a compact operator iff $\lim_{j\to\infty}\sum_{\nu=0}^{\infty}\left(\sum_{n=j+1}^{\infty}\left|\tilde{r}_{n\nu}^{(k)}\right|\right)^{k^{*}}=0.$

Proof. (a) Let $S_{|\overline{N}_p^{\phi}|}$ be a unit sphere in the space $|\overline{N}_p^{\phi}|$ and $R = \tilde{R}^{(1)} \circ T^{(p)}$. Since $\lambda \in S_{|\overline{N}_p^{\phi}|}$, $y = T^{(p)}(\lambda) \in S_l$. So, by Lemma ??, Lemma ??, it is written that

$$\begin{split} \|R\|_{\chi} &= \chi \left(RS_{\left|\overline{N_{p}^{\phi}}\right|} \right) = \chi \left(\tilde{R}^{(1)} \circ T^{(p)}S_{\left|\overline{N_{p}^{\phi}}\right|_{k}} \right) \\ &= \lim_{j \to \infty} \left(\sup_{y \in T^{(p)}S_{\left|\overline{N_{p}^{\phi}}\right|}} \left\| (I - P_{j})(\tilde{R}^{(1)}(y)) \right\| \right) \\ &= \lim_{j \to \infty} \sup_{v} \left\{ \sum_{n=j+1}^{\infty} \left| \tilde{r}_{nv}^{(1)} \right|^{k} \right\}^{1/k} \end{split}$$

which completes the proof of the first part with Lemma ??. The proof of (b) is similar, so it is omitted.

3. Conclusion

The approach of constructing a lot of new sequence spaces by means of the matrix domain of some particular limitation methods have recently been employed by several authors in many research papers. Also, with a different point of view, using the concept of absolute summability method new sequence spaces have taken into the literature. For instance, in recent paper, $\left|\overline{N}_{p}^{\phi}\right|_{k}$ has been generated from the space l_{k} as a set of all series summable by the absolute weighted mean method by Mohapatra and Sarıgöl [?] and Sarıgöl [?, ?]. In the present study, as a continuation of these papers, certain compact and matrix operatos from this space to one of the classical sequence spaces c, l_{∞}, c_{0} are characterized and their norms and Hausdorff measures of noncompactness are determined. So, it has been brought a different perspective and studying field.

Acknowledgements

The authors would like to express their sincere thanks to the editor and the anonymous reviewers for their helpful comments and suggestions.

Funding

There is no funding for this work.

Availability of data and materials

Not applicable.

Competing interests

The authors declare that they have no competing interests.

Author's contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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