Gazi University Journal of Science
GU J Sci
29(1):143-148 (2016)

On Left Primary and Weakly Left Primary Ideals in Γ-LA- Rings

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Received: 14/05/2014 Revised:29/05/2014 Accepted: 28/10/2014

ABSTRACT

In this paper, we study left ideals, left primary and weakly left primary ideals in Γ-LA-rings. Some characterizations of left primary and weakly left primary ideals are obtained. Moreover, we investigate relationships left primary and weakly left primary ideals in Γ-LA-rings. Finally, we obtain necessary and sufficient conditions of a weakly left primary ideal to be a left primary ideals in Γ-LA- rings.

Keywords: *Γ-LA-ring, left primary ideal, weakly left primary ideal, left ideal.*

1. INTRODUCTION

Abel-Grassmann's groupoid (AG-groupoid) is the generalization of semigroup theory with the wide range of usages in theory of flocks [6]. The fundamentals of this non-associative algebraic structure were the first discovered by Kazim and Naseeruddin [1]. AG-groupoid is a non-associative algebraic structure mid way between a groupoid and a commutative semigroup. It is interesting to note that an AG-groupoid with right identity becomes a commutative monoid [4]. This structure is closely related with a commutative semigroup because if an AG-groupoid contains a right identity, then it becomes a commutative monoid [4]. A left identity in an AG-groupoid is unique. Ideals in AG-groupoids have been discussed by Mushtaq and Yousuf [4, 5].

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In 1981, the notion of Γ -semigroups was introduced by Sen. Let S and Γ be any nonempty sets. If there exists a mapping $S \times\Gamma\times S \to S$ written (a,α,c) by $a\alpha c$, S is called a Γ -semigroup if S satisfies the identity:

$$(a\alpha b)\beta\, c = a\alpha(b\beta c)$$

for all $a,b,c\in S$ and $\alpha,\beta \in\Gamma$. A Γ -AG-groupoids analogous to Γ -semigroups. A groupoid S is called a Γ -AG-groupoid if it satisfies the left invertive law:

$$(a\gamma b)\delta c = (c\gamma b)\delta a$$

for all $a,b,c,d \in S$ and $\gamma ,\delta \in\Gamma$ [2]. This structure is also known as left almost semigroup (LA-semigroup).

S.M. Yusuf in [18] introduces the concept of a left almost ring (LA-ring). That is, a non-empty set R with two binary operations "+" and " · " is called a left almost ring, if $(R,+)$ is a LA-group, $(R,\cdot)$ is a LA-semigroup and distributive laws of "·" over "+" holds. Further in [12] T. Shah and I. Rehman generalize the notions of commutative semigroup rings into LA-semigroup LA-rings. However T. Shah and Fazal ur Rehman in [12] generalize the notion of a LA-ring into an nLA-ring. A near left almost ring (nLA-ring) N is a LAgroup under "+", a LA-semigroup under "·" and left distributive property of "·" over "+" holds.

T. Shah, Fazal ur Rehman and M. Raees asserted that a commutative ring $(R,+,\cdot)$, we can always obtain a LA-ring $(R,\oplus,\cdot)$ by defining, for $a,b,c\in R, a\oplus b = b-a$ and $a\cdot$b is same as in the ring. Furthermore, in this paper we characterize the left primary and weakly left primary ideals in Γ -LA-rings. Moreover, we investigate relationships left primary and weakly left primary ideals in Γ -LA-rings. Finally, we obtain necessary and sufficient conditions of a weakly left primary ideal to be a left primary ideals in Γ -LArings.

2. IDEALS IN Γ -LA-RINGS

The results of the following lemmas seem play an important role to study Γ -LA-ring; these facts will be used so frequently that normally we shall make no reference to this lemma.

Definition 2.1. Let $(R,+)$ and $(\Gamma,+)$ be a two LA-groups, R is called a Γ - left almost ring (Γ -LA-ring) if there exists a mapping $R\times\Gamma\times R\to R$ by $(a,\alpha,b)\mapsto a\alpha b$, for all $a,b\in R$ and $\alpha \in\Gamma$ satisfying the following conditions

1. $a\alpha(b +c) = a\alpha b + a\alpha c$
2. $(a +b)\alpha c = a\alpha c +b\alpha c$
3. $a(\alpha +\beta)b = a\alpha b+ a\beta b$
4. $(a\alpha b)\beta\, c = (c\alpha b)\beta\, a$, for all $a,b,c\in$R and $\alpha,\beta \in\Gamma$.

Lemma 2.2. If R is a Γ -LA-ring with left identity, then $a\gamma b = a\beta b$, for all $a,b\in R$ and $\gamma ,\beta \in\Gamma$.

Proof. Let R is a Γ -LA-ring and e be the left identity of $a,b\in R$ and let $\gamma ,\beta \in\Gamma$ therefore we have

$$\begin{aligned} a\gamma\, b &= a\gamma\, (e\beta b) \\ &= e\gamma\, (a\beta b) \\ &= a\beta\, b. \end{aligned}$$

Hence $a\gamma\, b = a\beta b$.

Lemma 2.3. Let R be a Γ -LA-ring with left identity e. Then $R\Gamma R = R$ and $R = e\Gamma R = R\Gamma e$.

Proof. Let R be a Γ -LA-ring with left identity e and let $r\in R$ then $r= e\alpha r\in R\Gamma R$, for all $\alpha \in\Gamma$, so that $R \subseteq R\Gamma R$. Since R is a Γ -LA-ring, we have $R\Gamma R \subseteq R$. Thus $R\Gamma R = R$. Now as e is a left identity in $R, e\alpha a = a$, for all $a\in R$ and $\alpha \in\Gamma$. Then $R = e\Gamma R$. Since $(a\alpha b)\beta\, c = (c\alpha b)\beta\, a$, for all $a,b,c\in R$ and $\alpha,\beta \in\Gamma$, we have $(R\Gamma R)\Gamma e = (e\Gamma R)\Gamma R$. Now,

$$R\Gamma e = (R\Gamma R)\Gamma e = (e\Gamma R)\Gamma R = R\Gamma R = R.$$

Hence $R = e\Gamma R = R\Gamma e$.

Definition 2.4. A nonempty subset I of a Γ -LA-ring R is a subring of R if under the binary operations in R, form a Γ -LA-ring.

Definition 2.5. A subring I of R is called a left (right) ideal of R if $R\Gamma I \subseteq I$ ($I\Gamma R \subseteq I$) and is called ideal if it is left as well as right ideal.

Lemma 2.6. If R is a Γ -LA-ring with left identity, then every right ideal is a left ideal.

Proof. Let R be a Γ -LA-ring with left identity and let A be a right ideal of R. Then for $a\in A$, $r\in R$ and $\alpha \in\Gamma$, consider

$$\begin{aligned} r\alpha a &= (e\beta\, r)\alpha a \\ &= (a\beta\, r)\alpha e \\ &\in (A\Gamma R)\Gamma R \\ &\subseteq A\Gamma R \\ &\subseteq A, \end{aligned}$$

where e is a left identity and $\beta \in\Gamma$, that is $r\alpha a\in A$. Therefore A is left ideal of R.

Lemma 2.7. If I is a left ideal of a Γ -LA-ring R with left identity, and if for any $a\in R$, $\gamma \in\Gamma$, then $a\gamma\, I$ is a left ideal of R.

Proof. Let I be a left ideal of R, consider

$$\begin{aligned} s\gamma\, (a\gamma\, i) &= (e\gamma\, s)\gamma\, (a\gamma\, i) \\ &= (e\gamma\, a)\gamma\, (s\gamma\, i) \\ &= a\gamma\, (s\gamma\, i)\in a\gamma\, I \end{aligned}$$

and $(a\gamma\, i) +(a\gamma\, j) = a\gamma\, (i + j)\in a\gamma\, I$. Hence $a\gamma\, I$ is a left ideal of R.

Lemma 2.8. Let R be a Γ -LA-ring with left identity, and $a\in R$, $\gamma \in\Gamma$. Then $R\gamma a$ is a left ideal of R.

Proof. Let R be a Γ -LA-ring with left identity, and $a\in R,\gamma \in\Gamma$. Then

$$\begin{aligned} R\gamma (R\gamma a) &= (R\gamma R)\gamma (R\gamma a) \\ &= (a\gamma R)\gamma (R\gamma R) \\ &= (a\gamma R)\gamma R \\ &= (R\gamma R)\gamma a \\ &= R\gamma a \end{aligned}$$

and $(r\gamma a) + (s\gamma a) = (r + s)\gamma a \in R\gamma a$. Hence $R\gamma a$ is a left ideal of R.

Lemma 2.9. If I is an ideal of a Γ-LA-ring R with left identity, and if for any $a \in R, \gamma \in \Gamma$, then $a^2\gamma I$ is an ideal of R.

Proof. By Lemma 2.7, we have $a^2\gamma I$ is a left ideal of R. Now consider

$$\begin{aligned} (a^2\gamma r)\gamma s &= ((a\gamma a)\gamma r)\gamma s \\ &= ((r\gamma a)\gamma a)\gamma s \\ &= [e\gamma ((r\gamma a)\gamma a)]\gamma s \\ &= [s\gamma ((r\gamma a)\gamma a)]\gamma e \\ &= [(r\gamma a)\gamma (s\gamma a)]\gamma e \\ &= [((s\gamma a)\gamma a)\gamma r]\gamma e \\ &= [((a\gamma a)\gamma s)\gamma r]\gamma e \\ &= [(r\gamma s)\gamma (a\gamma a)]\gamma e \\ &= [e\gamma (a\gamma a)]\gamma (r\gamma s) \\ &= (a\gamma a)\gamma (r\gamma s) \\ &= a^2\gamma (r\gamma s) \in a^2\gamma I. \end{aligned}$$

Hence $a^2\gamma I$ is an ideal of R.

Lemma 2.10. Let R be a Γ-LA-ring with left identity, and $a \in R$, $\gamma \in \Gamma$. Then $R\gamma a^2$ is an ideal of R.

Proof. Let R be a Γ-LA-ring with left identity, and $a \in R$, $\gamma \in \Gamma$. Now consider

$$\begin{aligned} R\gamma a^2 &= (R\Gamma R)\gamma a^2 \\ &= a^2\gamma (R\Gamma R) \\ &= a^2\gamma R \end{aligned}$$

By Lemma 2.9, we have $R\gamma a^2$ is an ideal of R.

Lemma 2.11. Let R be a Γ-LA-ring with left identity, and let A,B be left ideals of R. Then $(A:\Gamma: B)$ is a left ideal in R, where $(A:\Gamma: B) = \{r \in R: B\Gamma r \subseteq A\}$.

Proof. Suppose that R is a Γ-LA-ring. Let $s \in R$ and let $a,b \in (A:\Gamma: B)$. Then $B\Gamma a \subseteq A$ and $B\Gamma b \subseteq A$ so that

$$\begin{aligned} B\Gamma(a + b) &= (B\Gamma a) + (B\Gamma b) \\ &\subseteq A + A \\ &= A \end{aligned}$$

and

$$\begin{aligned} B\Gamma(s\gamma a) &= s\Gamma(B\gamma a) \\ &= s\Gamma A \\ &= A. \end{aligned}$$

Therefore $a + b \in (A:\Gamma: B)$ and $s\gamma a \in (A:\Gamma:B)$ so that $R\Gamma(A:\Gamma: B) \subseteq (A:\Gamma: B)$. Hence $(A:\Gamma: B)$ is a left ideal in R.

Corollary 2.12. Let R be a Γ-LA-ring with left identity, and let A be left ideals of R Then $(A:\gamma :b)$ is a left ideal in R, where $(A:\gamma :b) = \{r \in R: b\gamma r \in A\}$.

Proof. This follows from Lemma 2.11.

Remark.1. Let R be a Γ-LA-ring and let A be a left ideal of R. It is easy to verify that $A \subseteq (A:\gamma : r)$.

2. Let R be a Γ-LA-ring with left identity e, and let A be a proper left (right) ideal of R. By Corollary 2.12 , we have $e \notin (A:\gamma : r)$, where $r \in R - A$.

3. Let R be a Γ-LA-ring and let A,B,C be left ideals of R. It is easy to verify that $(A:\Gamma:C) \subseteq (A:\Gamma: B)$,where $B \subseteq C$.

3. LEFT PRIMARY AND WEAKLY LEFT PRIMARY IDEAL IN Γ -LA-RINGS

We start with the following theorem that gives a relation between left primary and weakly left primary ideal in Γ-LA-ring. Our starting points is the following definition:

Definition 3.1. A left ideal P is called left primary if $A\Gamma B \subseteq P$ implies that $(((A\Gamma A)\ \Gamma)\ldots A\ \Gamma A) = A^n \subseteq P$ or $B \subseteq P$ for some positive integer n, where A, B is a left ideals of R.

Definition 3.2. A left ideal P is called weakly left primary if $0 \neq A\Gamma B \subseteq P$ implies that $(((A\Gamma A)\ \Gamma)\ldots A\ \Gamma A) = A^n \subseteq P$ or $B \subseteq P$ for some positive integer n, where A, B is a left ideals of R.

Remark. It is easy to see that every left primary ideal is weakly left primary.

Lemma 3.3. If R is a Γ-LA-ring with left identity, then a left ideal P of R is left primary if and only if $a\gamma b \in P$ implies that $a^n \in P$ or $b \in P$ for some positive integer n, where $\gamma \in \Gamma$ and $a,b \in R$.

Proof. Let P be a left ideal of Γ-LA-ring R with left identity. Now suppose that $a\gamma b \in P$. Then by Definition of left ideal, we get

$$\begin{aligned} (R\gamma a)\beta (R\alpha b) &= (R\gamma R)\beta(a\alpha b) \\ &= R\beta(a\alpha b) \\ &\subseteq R\beta P \\ &\subseteq P. \end{aligned}$$

Then $a = (e\gamma\, a)^n \in (R\gamma\, a)^n \subseteq P$ or $b = e\alpha b \in R\alpha b \subseteq P$, for some positive integer n. Conversely, the proof is easy.

Corollary 3.4. If R is a Γ -LA-ring with left identity, then a left ideal P of R is weakly left primary if and only if $0 \neq a\gamma b \in P$ implies that $a^n \in P$ or $b \in P$ for some positive integer n, where $\gamma \in \Gamma$ and $a,b \in R$.

Proof. This follows from Lemma 3.3.

Let R be a Γ -LA-ring and A be a subset of R. We write

$$\sqrt{A} = \{a \in R: a^k \in A, \text{ for some positive integer } k\}.$$

Theorem 3.5. Let R be a Γ -LA-ring with left identity, and let P be an ideal of R. If P is a weakly left primary ideal that is not let primary. Then $\sqrt{P} = \sqrt{0}$.

Proof. Let R be a Γ -LA-ring with left identity. First, we prove that $P^2 = 0$. Suppose that $P^2 \neq 0$ we show that P is weakly left primary. Let $a\gamma b \in P$, where $a,b \in R$, $\gamma \in \Gamma$. If $a\gamma\, b \neq 0$, then either

$$a \in \sqrt{P} \text{ or } b \in P$$

since P is weakly left primary ideal. So suppose that $a\gamma b = 0$. If $P\gamma b \neq 0$, then there is an element p' of P such that $p'\gamma b \neq 0$, so that

$$0 \neq p'\gamma\, b = (\,p' + a)\gamma b \in P$$

and hence P weakly left primary ideal gives either $p' + a \in \sqrt{P}$ or $b \in P$. As $p' + a \in \sqrt{P}$ and $p' \in P \subseteq \sqrt{P}$ we have either $a \in \sqrt{P}$ or $b \in P$. So we can assume that $P\gamma b = 0$. Similarly, we can assume that $P\gamma a = 0$. Since $P^2 \neq 0$, there exist $c, d \in P$ such that $c\gamma d \neq 0$. Then

$$0 \neq (a + c)\gamma\, (b + d\,) \in P,$$

so either $a + c \in \sqrt{P}$ or $b + d \in P$, and hence either $a \in \sqrt{P}$ or $b \in P$. Thus P is left primary ideal. Clearly, $\sqrt{0} \subseteq \sqrt{P}$. As $P^2 = 0$, we get $\sqrt{P} \subseteq \sqrt{0}$, hence $\sqrt{P} = \sqrt{0}$, P as required.

Corollary 3.6. Let R be a Γ -LA-ring with left identity, and let P an ideal of R. If $\sqrt{P} \neq \sqrt{0}$, then P is left primary if and only if P is weakly left primary.

Proof. This follows from Theorem 3.5.

Lemma 3.7. Let R be a Γ -LA-ring with left identity, and let P be a proper ideal of R. If P is a weakly left primary ideal of R, then

$$(P{:}\Gamma{:}\ R\Gamma a) = P \cup (0{:}\Gamma{:}\ R\Gamma a),$$

where $a \in R - \sqrt{P}$.

Proof. Let R be a Γ -LA-ring with left identity, and let P be a weakly left primary ideal of R. Clearly,

$$P \cup (0{:}\Gamma{:}\ R\Gamma a) \subseteq (P{:}\Gamma{:}\ R\Gamma a).$$

For the other inclusion, suppose that $m \in (P{:}\Gamma{:}\ R\Gamma a)$, so that

$$\begin{aligned}(R\Gamma a)\Gamma(R\Gamma m) &= (m\Gamma R)\Gamma(a\Gamma R)\\ &= (m\Gamma a)\Gamma(R\Gamma R)\\ &= (m\Gamma a)\Gamma R\\ &= (R\Gamma a)\Gamma m\\ &\subseteq P.\end{aligned}$$

If $0 \neq (R\Gamma a)\Gamma m$, then $m = e\gamma\, m \in R\Gamma m \subseteq P$ since P is weakly left primary. If $0 = (R\Gamma a)\Gamma m$, then $m \in (0{:}\Gamma{:}\ R\Gamma a)$ so we have the equality.

Corollary 3.8. Let R be a Γ -LA-ring with left identity, and let P be a proper ideal of R. If P is a weakly left primary ideal of R, then

$$(P{:}\Gamma{:}\ a) = P \cup (0{:}\Gamma{:}\ a),$$

where $a \in R - \sqrt{P}$.

Proof. This follows from Lemma 3.7.

Corollary 3.9. Let R be a Γ -LA-ring with left identity, and let P be a proper ideal of R. If $(P{:}\Gamma{:}\ R\Gamma a) = P \cup (0{:}\Gamma{:}\ R\Gamma a)$, then

$$(P{:}\Gamma{:}\ R\Gamma a) = P \text{ or } (P{:}\Gamma{:}\ R\Gamma a) = (0{:}\Gamma{:}\ R\Gamma a),$$

where $a \in R - \sqrt{P}$.

Proof. This follows from Lemma 3.7.

Theorem 3.10. Let R be a Γ - LA-ring with left identity, and let P be a proper ideal of R. If $(P{:}\Gamma{:}\ n) = P$ or $(P{:}\Gamma{:}\ n) = (0{:}\Gamma{:}\ n)$, then P is a weakly left primary ideal of R, where $n \in R - \sqrt{P}$.

Proof. Let R be a Γ -LA-ring with left identity, and let P be a proper ideal of R. Suppose that Let $0 \neq m\gamma n \in P$, where $m \in R - \sqrt{P}$, $\gamma \in \Gamma$. Then

$$m \in (P{:}\Gamma{:}\ n) = P \cup (0{:}\Gamma{:}\ n)$$

by Corollary 3.9 hence $m \in P$ since $m\gamma n \neq 0$, as required.

Lemma 3.11. Let $R = R_1 \times R_2$, where each R_i is a Γ -LA-ring with left identity. Then the following hold:

(i) If A is a left ideal of R_1, then $\sqrt{A \times R_2} = \sqrt{A} \times R_2$.

(ii) If A is a left ideal of R_2, then $\sqrt{R_1 \times A} = R_1 \times \sqrt{A}.$

Proof. The proof is straightforward.

Theorem 3.12. Let $R = R_1 \times R_2$, where each R_i is a Γ-LA-ring with left identity. If P is a weakly left primary (left primary) ideal of R_1, then $P \times R_2$ is a weakly left primary (left primary) ideal of *R*.

Proof. Suppose that $R = R_1 \times R_2$, where each R_i is a Γ-LA-ring with left identity and *P* is a weakly left primary ideal of R_1. Let

$$0 \neq (a, b)\gamma(c, d) = (a\gamma\, c, b\gamma\, d) \in P \times R,$$

where $(a,b),(c,d) \in R$, $\gamma \in \Gamma$ so either $a \in \sqrt{P}$ or $c \in P$ since *P* is weakly left primary. It follows that either

$$(a, b) \in \sqrt{P} \times R = \sqrt{P \times R_2} \text{ or } (c, d) \in P \times R.$$

By Definition of weakly left primary ideal, we have $P \times R_2$ is a weakly left primary ideal of *R*.

Corollary 3.13. Let $R = R_1 \times R_2$, where each R_i is a Γ-LA-ring with left identity. If *P* is a weakly left primary (left primary) ideal of R_2, then $R_1 \times P$ is a weakly left primary (left primary) ideal of *R*.

Proof. This follows from Theorem 3.12.

Corollary 3.14. Let $R = \prod_{i=1}^{n} R_i$, where each R_i is a Γ-LA-ring with left identity. If P is a weakly left primary (left primary) ideal of R_j, then

$$R_1 \times R_2 \times \ldots \times P_{j-1} \times P_j \times R_{j+1} \times \ldots \times R_n$$

is a weakly left primary (left primary) ideal of R.

Proof. This follows from Theorem 3.12 and Corollary 3.13.

Theorem 3.15. Let $R = R_1 \times R_2$, where each R_i is a Γ-LA-ring with left identity. If P is a weakly left primary ideal of *R*, then either $P = 0$ or *P* is left primary.

Proof. Let $R = R_1 \times R_2$, where each R_i is a Γ-LA-ring with identity and let $P = P_1 \times R_2$ be a weakly left primary ideal of *R*. We can assume that $P \neq 0$. So there is an element (a, b) of *P* with $(a,b) \neq (0,0)$. Then

$$(0,0) \neq (a,e)\gamma\,(e, b) \in P,$$

where $\gamma \in \Gamma$, gives either

$$(a, e) \in \sqrt{P} = \sqrt{P_1 \times R_2} = \sqrt{P_1} \times R_2 \text{ or } (e, b) \in P$$

If $(e, b) \in P$, then $P = R_1 \times P_2$. We will show that P_2 is left primary hence *P* is weakly left primary by Corollary 3.13. Let $c\gamma d \in P_2$, where $c,d \in R_2$. Then

$$(0, 0) \neq (e, c)\gamma\,(e, d) = (e, c\gamma) \in P,$$

so either $(e, c) \in \sqrt{P} = \sqrt{R_1 \times P_2} = R_1 \times \sqrt{P_2}$ or $(e, d) \in P$ and hence either $c \in \sqrt{P_2}$ or $d \in P_2$. By a similar argument, $P = R_1 \times P_2$ is left primary.

Proposition 3.16. Let $A \subseteq P$ be proper ideals of a Γ-LA-ring *R*. Then the following hold:

(i) If *P* is weakly left primary (left primary), then P/A is weakly left primary (left primary).

(ii) If *A* and P/A are weakly left primary (left primary), then P is weakly left primary (left primary).

Proof. (i) Let $0 \neq (a + A)\gamma\,(b + A) = a\gamma\, b + A \in P/A$, where $a,b \in R$, $\gamma \in \Gamma$ so $a\gamma b \in P$. If $a\gamma\, b = 0 \in A$, then

$$(a + A)\gamma\,(b + A) = 0,$$

a contradiction. So if *P* is weakly left primary, then either $a \in \sqrt{P}$ or $b \in P$, hence either $a + A \in P/A$ or $b + A \in P/A$, as required.

(ii) Let $0 \neq a\gamma\, b \in P$, where $a,b \in R$, so $(a + A)\gamma\,(b + A) \in P/A$. For $a\gamma b \in A$, if A is weakly left primary, then either

$$a \in A \subseteq \sqrt{P} \text{ or } b \in A \subseteq P.$$

So we may assume that $a\gamma\, b \notin A$. Then either $a + A \in \sqrt{P/P}$ or $b + A \in P/A$. It follows that either $a \in \sqrt{P}$ or $b \in P$ as needed.

Theorem 3.17. Let *P* and *Q* be weakly left primary ideals of a Γ-LA –ring *R* that are not left primary. Then $P + Q$ is a weakly left primary ideal of *R*.

Proof. Since $(P + Q)/Q \approx Q/(P \cap Q)$, we get that $(P+Q)/Q$ is weakly left primary by Proposition 3.16 (i). Now the assertion follows from Proposition 3.16 (ii).

Acknowledgement

The authors are very grateful to the anonymous referee for stimulating comments and improving presentation of the paper

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