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ORIGINAL ARTICLE

Some Fixed Point Results for Multi Valued Mappings in Ordered G-Metric Spaces

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ABSTRACT

Using the setting of G− metric spaces, some new fixed point theorems for multivalued monotone mappings in ordered G− metric space X are proved, where the partial ordered $\leq$ in X is obtained by a pair of functions (ψ, φ).

Key Words: *Common fixed point, generalized weak contractive condition, lower semicontinous functions, G− metric space.*
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1. INTRODUCTION AND PRELIMINARIES

Many authors studied many fixed and common fixed points in metric and order metric spaces. Dhage introduced the concept of *D*-metric spaces and studied several fixed point results (see [1]-[4]). Mustafa and Sims [5] showed that the structure of *D* metric spaces didn't generate a metric space. They introduced a new concept of generalized metric spaces, called *G*-metric spaces. Since then many authors introduced many fixed and common fixed point results using the concept of *G*-metric spaces (see [5]-[25]).

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In 1976, Caristi [26] defined an order relation in a metric space by using a functional

as follows: Let (*X, d*) be a metric space, $\varphi : X \to R$ be a functional. Define the relation $\leq$ on *X* by

$x \leq y$ iff $d(x,y) \leq \varphi(x) - \varphi(y)$.

Then $\leq$ is a partial order relation on *X* introduced by φ and $(X,\leq)$ is called an ordered metric space introduced by φ. After that many authors discussed the existence of a fixed point and a common fixed point using Caristi type mapping (see [26]-[31]).

Consistent with Mustafa and Sims [6], the following definitions and results will be needed in the sequel.

Definition 1.1. Let *X* be a nonempty set. Suppose that a mapping $G : X \times X \times X \to R^+$ satisfies

(G1) $G(x,y,z)=0$ if $x=y=z$;

(G2) $0<G(x,y,z)$ for all $x,y,z \in X$ with $x \neq y$

(G3) $G(x,x,y) \leq G(x,y,z)$ for all $x,y,z \in X$ with $y \neq z$

(G4) $G(x,y,z)=G(x,z,y)=G(y,z,x)=\ldots$ (symmetry in all three variables); and

(G5) $G(x,y,z) \leq G(x,a,a) + G(a,y,z)$ for all $x,y,z,a \in X$.

Then *G* is called a *G*-metric on X and (*X,G*) is called a *G*-metric space.

Definition 1.2. A sequence $\{x_n\}$ in a G-metric space *X* is:

(i) a *G*-Cauchy sequence if for any $\varepsilon > 0$, there is a natural number $n_0 \in N$ such that for all $n,m,l \geq n_0$, $G(x_n,x_m,x_l) < \varepsilon$,

(ii) a *G*-convergent sequence if for any $\varepsilon > 0$, there is an $x \in X$ and an $n_0 \in N$ such that for all $n,m \geq n_0$, $G(x_n,x_m,x) < \varepsilon$.

A *G*-metric space on *X* is said to be *G*-complete if every *G*-Cauchy sequence in *X* is *G*-convergent in *X*. It is known that $\{x_n\}$ *G*-converges to $x \in X$ if and only if

$G(x_n,x_m,x) \to 0$ as $n,m \to +\infty$.

Proposition 1.3. [6] Let *X* be a *G*-metric space. Then the following are equivalent:

1. The sequence $\{x_n\}$ is G-convergent to *x*.
2. $G(x_n,x_n,x) \to 0$ as $n \to +\infty$.
3. $G(x_n,x,x) \to 0$ as $n \to +\infty$.
4. $G(x_n,x_m,x) \to 0$ as $n,m \to +\infty$.

Proposition 1.4. [6] Let *X* be a *G*-metric space. Then the following are equivalent:

1. The sequence $\{x_n\}$ is G-Cauchy.
2. For every $\varepsilon > 0$, there exists $n_0 \in N$, such that for all $n,m \geq n_0$, $G(x_n,x_m,x_l) < \varepsilon$; that is $G(x_n,x_m,x) \to 0$ as $n,m \to +\infty$.

Definition 1.5. A *G*-metric on *X* is said to be symmetric if $G(x,x,y)=G(x,y,y)$ for all $x,y \in X$.

Proposition 1.6. Every *G*-metric on *X* will define a metric d_G on *X* by

$d_G(x,y) = G(x,y,y) + G(y,x,x)$ for all $x,y \in X$.

For a symmetric *G*-metric space, one obtains

$d_G(x,y) = 2G(x,y,y)$ for all $x,y \in X$.

However, if *G* is not symmetric, then the following inequality holds:

$\frac{3}{2}G(x,y,y) \leq d_G(x,y) \leq 3G(x,y,y)$ for all $x,y \in X$.

Definition 1.7. The two classes of following mappings are defined as

$\Phi = \{\varphi / \varphi : [0,+\infty) \to [0,+\infty)$ is lower semi continuous, $\varphi(t) > 0$ for all $t > 0$, $\varphi(0) = 0\}$,

$\Psi = \{\psi / \psi : [0,+\infty) \to [0,+\infty)$ is continuous and nondecreasing with $\psi(t) = 0$ if and only if $t = 0\}$.

Using the setting of *G*– metric spaces, some new fixed point theorems for multivalued

monotone mappings in ordered *G*– metric space *X* are proved, where the partial ordered

$\leq$ in *X* is obtained by a pair of functions (ψ, φ).

2. MAIN RESULTS

Throughout this paper, we let $\psi : [0,+\infty) \to [0,+\infty)$ be a function with following properties:

1. ψ is nondecreasing continuous.
2. $\psi^{-1}(\{0\}) = \{0\}$.
3. $\psi(a+b) \leq \psi(a) + \psi(b)$ for all $a, b \in [0,+\infty)$.

Let (*X,G*) be a *G*-metric space, define a relation $\leq$ by using functional $\phi : X \to R$ and ψ

as follows:

$$x \leq y \text{ iff } \psi(G(x,y,y)) \leq \varphi(x) - \varphi(y)$$

for all $x, y \in X$. Then it is an easy matter to prove the following lemma:

Lemma 2.1 $\leq$ is partial order and $(X, \leq)$ is a partial ordered set.

Proof: $\leq$ is reflexive because $\psi(G(x,x,x)) = \varphi(x) - \varphi(x)$ for all $x \in X$.

$\leq$ is antisymmetric because if $x, y \in X$ with $x \leq y$ and $y \leq x$, then

$$\psi(G(x,y,y)) \leq \varphi(x) - \varphi(y)$$

and

$$\psi(G(y,x,x)) \leq \varphi(y) - \varphi(x).$$

Thus

$$\psi(G(x,y,y)) + \psi(G(y,x,x)) = 0.$$

Hence $\psi(G(x,y,y)) = \psi(G(y,x,x)) = 0$. Therefore $G(x,y,y) = 0$ and hence $x = y$.

$\leq$ is transitive because if $x, y, z \in X$ with $x \leq y$ and $y \leq z$, then

$$\psi(G(x,y,y)) \leq \varphi(x) - \varphi(y)$$

and

$$\psi(G(y,z,z)) \leq \varphi(y) - \varphi(z).$$

Thus

$$\psi(G(x,y,y)) + \psi(G(y,z,z)) \leq \varphi(x) - \varphi(z).$$

Using (G5) of the definition *G*-metric space and property (3) of the function ψ, we get

$$\begin{aligned}\psi(G(x,z,z) &\leq \psi(G(x,y,y) + G(y,z,z)) \\ &\leq \psi(G(x,y,y)) + \psi(G(y,z,z)) \\ &\leq \varphi(x) - \varphi(z).\end{aligned}$$

Thus, we have $x \leq z$. ∎

From now on, we let $(X, G, \leq)$ be an ordered *G*-metric space introduced by (ψ, φ).

Let $(X, G, \leq)$ be an ordered *G*-metric space introduced by (ψ, φ). For $x, y \in X$ we define

the ordered interval in *X* as:

$$[x,y] = \{z \in X : x \leq z \leq y\},$$

$$[x,+\infty) = \{z \in X : x \leq z\},$$

$$(-\infty, x] = \{z \in X : z \leq x\}.$$

Let $F : X \to 2^X$ be a multivalued mapping, we say that *F* is upper semi-continuous if whenever $x_n \in X$ and $y_n \in F(x_n)$ with $x_n \to x_0 \in X$ and $y_n \to y_0 \in X$, then $y_0 \in F(x_0)$.

Our first result is:

Theorem 2.1 Let $(X, G, \leq)$ be an ordered complete *G*-metric space introduced by (ψ, φ),

where $\varphi : X \to R$ be a function bounded below. Let $F : X \to 2^X$ be a multivalued mapping

and

$$M = \{x \in X : F(x) \cap [x,+\infty) \neq \phi.$$

Suppose that:

i. *F* is upper semi-continuous;

ii. for each $x \in M$, $F(x) \cap M \cap [x,+\infty) \neq \phi\}$;

iii. $M \neq \phi$.

Then there exists a sequence (x_n) with

$$x_{n-1} \leq x_n \in F(x_{n-1}), \quad \forall\, n \in N,$$

and F has a fixed point x^* such that $x_n \to x^*$. Moreover if φ is lower semi-continuous, then $x_n \le x^*$ for all n.

Proof: Since $M \neq \phi$, we choose $x_0 \in M \subseteq X$.. By (ii), we have

$$F(x_0) \cap M \cap [x_0, +\infty) \neq \phi.$$

Thus we choose

$$x_1 \in F(x_0) \cap M \cap [x_0, +\infty).$$

Therefore $x_0 \le x_1$. Again by (ii), we have

$$F(x_1) \cap M \cap [x_1, +\infty) \neq \phi.$$

Thus, we choose

$$x_2 \in F(x_1) \cap M \cap [x_1, +\infty).$$

Hence $x_1 \le x_2$. Continuing in the same process, we construct a sequence (x_n) in X such that

$$x_{n-1} \le x_n \in F(x_{n-1}), \quad \forall\, n \in N.$$

Since $(X, G, \le)$ is an ordered G-metric space introduced by (ψ, φ), we get that

$$\psi(G(x_{n-1}, x_n, x_n)) \le \varphi(x_{n-1}) - \varphi(x_n).$$

Since ψ is a nonnegative function, we get that

$$\varphi(x_{n-1}) - \varphi(x_n) \ge 0 \quad \forall\, n \in N.$$

Thus

$$\varphi(x_{n-1}) \ge \varphi(x_n) \quad \forall\, n \in N.$$

Since φ is a function which is bounded below, we have $(\varphi(x_n))$ is a decreasing sequence which is bounded below. By completeness property of **R**, we have

$$\lim_{n \to +\infty} \varphi(x_n) = \inf\{x_n : n \in N\}.$$

For $m > n$, we have $x_n \le x_m$. Thus, we get

$$\psi(G(x_n, x_m, x_m)) \le \varphi(x_n) - \varphi(x_m).$$

Let $n, m \to +\infty$., then

$$\lim_{n,m \to +\infty} \psi(G(x_n, x_m, x_m)) \le \lim_{n \to +\infty} \varphi(x_n) - \lim_{m \to +\infty} \varphi(x_m).$$

Thus

$$\lim_{n,m \to +\infty} \psi(G(x_n, x_m, x_m)) = 0.$$

Using the continuity of ψ and the fact that $\psi^{-1}(\{0\}) = \{0\}$, we get that

$$\lim_{n,m \to +\infty} G(x_n, x_m, x_m) = 0.$$

Hence (x_n) is a Cauchy sequence in X. Since X is G-complete, then there is $x^* \in X$ such that (x_n) is G-convergent to x^*. Since $x_{n-1} \in X$, $x_n \in F(x_{n-1})$, $x_{n-1} \to x^*$ and $x_n \to x^*$ by definition of upper semi-continuous of F, we have $x^* \in F(x^*)$. Now, suppose that φ is lower semi-continuous, then for each $n \in N$. we have

$$\begin{aligned}
\psi(G(x_n, x^*, x^*) &= \lim_{m \to +\infty} \psi(G(x_n, x_m, x_m) \\
&\le \limsup_{m \to +\infty} \varphi(x_n) - \varphi(x_m) \\
&= \varphi(x_n) - \liminf_{m \to +\infty} \varphi(x_m) \\
&\le \varphi(x_n) - \varphi(x^*).
\end{aligned}$$

Therefore $x_n \le x^*$ for all $n \in N$. ∎

Corollary 2.1 Let $(X, G, \le)$ be an ordered complete G-metric space introduced by (ψ, φ),

where $\varphi : X \to R$ be a function bounded below. Let $F : X \to 2^X$ be a multivalued mapping

Suppose that:

i. F is upper semi-continuous;

ii. F satisfies the monotonic condition: For each $x, y \in X$ with $x \le y$ and any $u \in F(x)$, there exists $v \in F(y)$ such that $u \le v$.

iii. There exists $x_0 \in X$ such that $F(x_0) \cap [x_0, +\infty) \neq \phi$.

Then there exists a sequence (x_n) in X with

$$x_{n-1} \le x_n \in F(x_{n-1}), \quad \forall\, n \in N,$$

and F has a fixed point x^* such that $x_n \to x^*$. Moreover if φ is lower semi-continuous,

then $x_n \le x^*$ for all n.

Proof : Let

$$M = \{x \in X : F(x) \cap [x, +\infty) \neq \phi\}.$$

By (iii) we conclude that $M \neq \phi.$ For $x \in M,$ take $y \in F(x)$ and $x \leq y$. Since F satisfies

the monotonic condition, there exist $z \in F(y)$ such that $y \leq z.$. Thus $y \in M$, and

$F(x) \cap M \cap [x, +\infty) \neq \phi.$ Thus we get the result from Theorem 2.1. ■

Corollary 2.2 Let $(X, G, \leq)$ be an ordered complete G-metric space introduced by (ψ, φ),

where $\varphi : X \to R$ be a function bounded below. Let $f : X \to X$ be a map.

Suppose that:

i. f is continuous.

ii. f is monotone increasing.

iii. There exists $x_0 \in X$ such that $x_0 \leq f(x_0)$.

Then there exists a sequence (x_n) in X with

$$x_{n-1} \leq x_n \in f(x_{n-1}), \quad \forall\, n \in N,$$

and f has a fixed point x^* such that $x_n \to x^*$. Moreover if φ is lower semi-continuous,

then $x_n \leq x^*$ for all n.

Proof : Define $F : X \to 2^X$ by $F(x) = \{f(x)\}$ for all $x \in X.$ Then F and X satisfy all the

hypotheses of Theorem 2.1. Thus the result follows from Theorem 2.1. ■

Theorem 2.2 Let $(X, G, \leq)$ be an ordered complete G-metric space introduced by (ψ, φ),

where $\varphi : X \to R$ be a function bounded above. Let $F : X \to 2^X$ be a multivalued mapping

and

$$M = \{x \in X : F(x) \cap (-\infty, x] \neq \phi\}.$$

Suppose that:

i. F is upper semi-continuous;

ii. for each $x \in M$, $F(x) \cap M \cap (-\infty, x] \neq \phi\}$;

iii. $M \neq \phi.$

Then there exists a sequence (x_n) with

$$x_{n-1} \geq x_n \in F(x_{n-1}), \quad \forall\, n \in N,$$

and F has a fixed point x^* such that $x_n \to x^*$. Moreover if φ is lower semi-continuous,

then $x_n \geq x^*$ for all n.

Proof: Since $M \neq \phi,$ we choose

$$x_1 \in F(x_0) \cap M \cap (-\infty, x_0].$$

Therefore $x_0 \geq x_1$. Again by (ii), we choose

$$x_2 \in F(x_1) \cap M \cap (-\infty, x_1].$$

Hence $x_1 \geq x_2$. Continuing in the same process, we construct a sequence (x_n) in X such

that

$$x_{n-1} \geq x_n \in F(x_{n-1}), \quad \forall\, n \in N.$$

Since $(X, G, \leq)$ is an ordered G-metric space introduced by (ψ, φ), we get that

$$\psi(G(x_n, x_{n-1}, x_{n-1})) \leq \varphi(x_n) - \varphi(x_{n-1}).$$

Since ψ is a nonnegative function, we get that

$$\varphi(x_n) - \varphi(x_{n-1}) \geq 0 \quad \forall\, n \in N.$$

Thus

$$\varphi(x_n) \geq \varphi(x_{n-1}) \quad \forall\, n \in N.$$

Since φ be a function which is bounded above, we have $(\varphi(x_n))$ is an increasing sequence

which is bounded above. By completeness property of **R**, we have

$$\lim_{n \to +\infty} \varphi(x_n) = \sup\{x_n : n \in N\}.$$

For $m > n$, we have $x_n \geq x_m$. Thus, we get

$$\psi(G(x_m, x_n, x_n)) \leq \varphi(x_m) - \varphi(x_n).$$

Let $n, m \to +\infty.$, then

$$\lim_{n,m \to +\infty} \psi(G(x_m, x_n, x_n)) \leq \lim_{m \to +\infty} \varphi(x_m) - \lim_{n \to +\infty} \varphi(x_n).$$

Thus

$$\lim_{n,m \to +\infty} \psi(G(x_m, x_n, x_n)) = 0.$$

Using the continuity of ψ and the fact that $\psi^{-1}(\{0\}) = \{0\}$, we get that

$$\lim_{n,m \to +\infty} G(x_m, x_n, x_n) = 0.$$

Hence (x_n) is a Cauchy sequence in X. Since X is G-complete, then there is $x^* \in X$ such

that (x_n) is G-convergent to x^*. Since $x_{n-1} \in X$, $x_n \in F(x_{n-1})$, $x_{n-1} \to x^*$ and $x_n \to x^*$

by definition of upper semi-continuous of F, we have $x^* \in F(x^*)$. Now, suppose that φ is

lower semi-continuous, then for each $n \in N$, we have

$$\begin{aligned}\psi(G(x^*, x_n, x_n) &= \lim_{m \to +\infty} \psi(G(x_m, x_n, x_n) \\ &\leq \limsup_{m \to +\infty} \varphi(x_m) - \varphi(x_n) \\ &\leq \varphi(x^*) - \varphi(x_n).\end{aligned}$$

Therefore $x_n \geq x^*$ for all $n \in N$. ∎

Corollary 2.3 Let $(X, G, \leq)$ be an ordered complete G-metric space introduced by (ψ, φ),

where $\varphi : X \to R$ be a function bounded above. Let $F : X \to 2^X$ be a multivalued mapping

Suppose that:

i. F is upper semi-continuous;

ii. F satisfies the monotonic condition: For each $x, y \in X$ with $x \geq y$ and any $u \in F(x)$,

there exists $v \in F(y)$ such that $u \geq v$.

iii. There exists $x_0 \in X$ such that $F(x_0) \cap (-\infty, x_0] \neq \phi$.

Then there exists a sequence (x_n) in X with

$$x_{n-1} \geq x_n \in F(x_{n-1}), \quad \forall\, n \in N,$$

and F has a fixed point x^* such that $x_n \to x^*$. Moreover if φ is lower semi-continuous,

then $x_n \geq x^*$ for all n.

Proof : Let

$$M = \{x \in X : F(x) \cap (-\infty, x] \neq \phi\}.$$

By (iii) we conclude that $M \neq \phi$. For $x \in M$, take $y \in F(x)$ and $x \geq y$. Since F satisfies the monotonic condition, there exist $z \in F(y)$ such that $y \geq z$. Thus $y \in M$, and

$F(x) \cap M \cap (-\infty, x] \neq \phi$. Thus we get the result from Theorem 2.2. ∎

Corollary 2.4 Let $(X, G, \leq)$ be an ordered complete G-metric space introduced by (ψ, φ),

where $\varphi : X \to R$ be a function bounded above. Let $f : X \to X$ be a map.

Suppose that:

i. f is continuous.

ii. f is monotone increasing.

iii. There exists $x_0 \in X$ such that $x_0 \geq f(x_0)$.

Then there exists a sequence (x_n) in X with

$$x_{n-1} \geq x_n \in f(x_{n-1}), \quad \forall\, n \in N,$$

and f has a fixed point x^* such that $x_n \to x^*$. Moreover if φ is lower semi-continuous,

then $x_n \geq x^*$ for all n.

Proof : Define $F : X \to 2^X$ by $F(x) = \{f(x)\}$ for all $x \in X$. Then F and X satisfy all the

hypotheses of Theorem 2.2. Thus the result follows from Theorem 2.2. ∎

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