

PAPER DETAILS

TITLE: Standardized Likelihood Ratio Test for Homogeneity of Variance Based on Likelihood Ratio Under Normality

AUTHORS: Esra Gökpınar

PAGES: 223-235

ORIGINAL PDF URL: <https://dergipark.org.tr/tr/download/article-file/342785>



Standardized Likelihood Ratio Test for Homogeneity of Variance under Normality

Esra GÖKPINAR^{1,*}

¹ Gazi University, Department of Statistics, Ankara, Turkey.

Article Info

Received: 20/04/2017

Accepted: 12/06/2017

Keywords

Standardized likelihood
ratio test
Homogeneity of
variances
Type I error

Abstract

In this article, standardized likelihood ratio test is proposed for the homogeneity of variances under normality. The proposed method was compared with some of the existing methods via Monte Carlo simulation for various parameter combinations, different group sizes and sample sizes in terms of type I error rate and power of test. According to numeric results, the proposed method performs quite well according to its alternatives.

1. INTRODUCTION

Testing the homogeneity of variances is one of the main assumptions of some common statistical procedures and also it has been broadly used in many scientific application areas. The assumption of homogeneity of variances is required in many experimental design models such as analysis of variance (ANOVA). Testing the homogeneity of variances is also of interest as data from different sources is pooled to yield an improved estimated variance [12]. Besides, it is important to determine uniformity in the quality control of manufacturing processes, in biology, in agricultural production systems and in the development of educational methods [3]. Taguchi [28] demonstrates that reducing variability is an integral part of improving quality. In robust designs, experimentation is used to determine the factor levels so that the product or production processes is insensitive to potential variations in operating, environmental, and market conditions [25]. Besides, in quality control work, testing the homogeneity of variances is often a useful endpoint in analysis [12]. Biologists are interested in differences in the variability of populations for several reasons, for instance, as an indicator of generic diversity and in the investigation of mechanisms of adaptation [3]. Furthermore, testing the homogeneity of variances is used as a prelude to dose response modeling or discriminant analysis [6].

In the literature, there are many methods for testing the homogeneity of variances. Neyman and Pearson [26] suggested a statistic which is the ratio of a weighted geometric mean to a weighted arithmetic means of the mean squares. Bartlett [1] proposed an analogous test in which the sums of squares are weighted with their associated degrees of freedom instead of with the numbers of observations as in Neyman Pearson criterion [20]. Box and Andersen [4] modified the Bartlett test under permutation distribution for homogeneity of variances [18]. Cochran [11] proposed a test statistic in which the variance of only one group is greater than others. Hartley [19] suggested the F-max statistic which is the ratio of the maximum variance to the minimum variance of groups. Levene [22] gave a testing procedure based on ANOVA which uses the absolute difference of observations from their means. Brown and Forsythe [5] modified the Levene test using sample medians instead of sample means. Recently, Bhandary and Dai [2] proposed a test based on Bonferroni type adjustment procedure on the ordered p values to control the family wise type I error rate. Liu and Xu [23] proposed a test, using the generalized p value approach, and compared it with the Bartlett test. For homogeneity of variances, Gökpınar and Gökpınar [16] proposed a test statistic based on computational approach test (CAT) which is a special case of parametric bootstrap. The CAT method based

*Corresponding author, e-mail: eyigit@gazi.edu.tr

on simulation and numerical computations uses the maximum likelihood estimates (MLEs), and does not require the knowledge of any sampling distribution. Some papers regarding the CAT are given as Chang and Pal [8], Chang et al. [7-9], Gökpinar and Gökpinar [13,16, 17], Gökpinar et al. [14], Mutlu et al. [24], etc. The likelihood ratio test (LRT) for homogeneity of variances performs poorly with respect to the type I error rate. Recently, Chang et al. [10] suggested a CAT version based on LRT for homogeneity of variances. Krishnamoorthy and Oral [25] proposed the standardized likelihood ratio test (SLRT) to improve the LRT for testing the equality of means of several log-normal distributions. In this study, by taking an idea from Krishnamoorthy and Oral [21], we proposed the SLRT for homogeneity of variances.

The rest of this study is organized as follows. For the homogeneity of variances, Bartlett test [1], Bhandary and Dai test [2], generalized p value approach given by Liu and Xu [23] and CAT given by Gökpinar and Gökpinar [16] were presented in Section 2. In Section 3, we developed the SLRT for homogeneity of variances. In Section 4, simulation results based on estimated Type I errors and powers of tests were presented. Concluding remarks were summarized in Section 5.

2. TEST STATISTICS

We consider the problem of testing the homogeneity of variances of k populations given random samples $\{X_{ij} : j = 1, \dots, n_i, i = 1, \dots, k\}$ from $N(\mu_i, \sigma_i^2)$ where μ_i and σ_i^2 are the mean and the variance of the i th population, respectively. The sample means and variances are

$$\bar{X}_i = \sum_{j=1}^{n_i} X_{ij} / n_i, \quad \text{and} \quad S_i^2 = \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2 / (n_i - 1), \quad i=1, \dots, k,$$

respectively and the pooled sample mean and the variance are

$$\bar{X} = \sum_{i=1}^k n_i \bar{X}_i / N \quad \text{and} \quad S_p^2 = \sum_{i=1}^k (n_i - 1) S_i^2 / (N - k),$$

respectively, where $N = \sum_{i=1}^k n_i$. The MLEs of the parameters μ_i and σ_i^2 , $i = 1, 2, \dots, k$ are obtained as follows:

$$\hat{\mu}_{i(MLE)} = \sum_{j=1}^{n_i} X_{ij} / n_i = \bar{X}_i \quad \text{and} \quad \hat{\sigma}_{i(MLE)}^2 = \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2 / n_i = S_i^{2*}, \quad i = 1, 2, \dots, k \quad (1)$$

The null hypothesis of interest is

$$H_0 : \sigma_1^2 = \sigma_2^2 = \dots = \sigma_k^2 \quad \text{against} \quad H_1 : \sigma_i^2 \neq \sigma_l^2 \quad \text{for at least one } i \neq l.$$

In the rest of this section, for testing Eq. (2), the Bartlett's test (BT), Bhandary and Dai's test (BDT), generalized p value approach (GPA) and CAT were given briefly.

2.1. Bartlett's Test

Bartlett [1] test statistic, B , is given by

$$B = \frac{(N - k) \log S_p^2 - \sum_{i=1}^k (n_i - 1) \log S_i^2}{1 + (1/3(k-1)) \left(\sum_{i=1}^k 1/(n_i - 1) - 1/(N - k) \right)}.$$

For a given level α , and an observed value B_h of B , H_0 is rejected if $B_h > \chi_{k-1, \alpha}^2$ where $\chi_{k-1, \alpha}^2$ is the Chi-square distribution with degrees of freedom $k-1$ under the null hypothesis.

2.2. Bhandary and Dai's test

In this section, simplified algorithm of Bhandary and Dai [2] test based on Bonferroni type adjustment procedure is given as follows:

1. Calculate the pooled sample variance on the merged data except i th group as follows:

$$S_{p:i}^2 = ((N-k)S_p^2 - (n_i-1)S_i^2) / (N-n_i-(k-1)) \quad i=1, 2, \dots, k.$$

Determine F-test statistics to be $F_i = \frac{S_i^2}{S_p^2}$ and $F_i' = \frac{S_{p:i}^2}{S_i^2}$ $i=1, 2, \dots, k$.

2. Calculate $P_i = P(X > F_i)$ where $X \sim F_{n_i-1, r_i}$ and $P_i' = P(X' > F_i')$ where $X' \sim F_{r_i, n_i-1}$ and $r_i = \sum_{j \neq i}^k (n_j - 1)$.

3. Sort the P_i and P_i' values and denote them by $P_{(1)} < P_{(2)} < \dots < P_{(2k)}$. Reject H_0 if $P_{(i)} < \frac{i}{2k} \alpha$ for some $i \in \{1, 2, \dots, 2k\}$.

2.3. The generalized p value approach

In this section the algorithm of GPA obtained by Liu and Xu [23] is given as follows:

- 1) For a given data $(x_{i1}, x_{i2}, \dots, x_{in_i})$, $i=1, \dots, k$, calculate observed statistics $(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_k)$ and (s_1^2, \dots, s_k^2) .

- 2) Calculate the generalized pivotal quantity of $A \ln \sigma^2$, $R_{A \ln \sigma^2}$

$$R_{A \ln \sigma^2} = A R_{\ln \sigma^2} = A(R_{\ln \sigma_1^2}, R_{\ln \sigma_2^2}, \dots, R_{\ln \sigma_k^2})',$$

where $R_{\ln \sigma_i^2} = \ln \frac{n_i s_i^2}{U_i}$, $U_i \sim \chi_{n_i-1}^2$, $i=1, \dots, k$ and

$$A = \begin{pmatrix} 1 & 0 & \dots & 0 & -1 \\ 0 & 1 & \dots & 0 & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & -1 \end{pmatrix}_{(k-1) \times k}.$$

- 3) Calculate μ_R and Σ_R as follows:

$$\mu_R = A \left(E(R_{\ln \sigma_1^2} / (\bar{x}, s^2)), E(R_{\ln \sigma_2^2} / (\bar{x}, s^2)), \dots, E(R_{\ln \sigma_k^2} / (\bar{x}, s^2)) \right)' \quad \text{and}$$

$$\Sigma_R = A \text{diag} \left(\text{Var}(R_{\ln \sigma_1^2} / (\bar{x}, s^2)), \text{Var}(R_{\ln \sigma_2^2} / (\bar{x}, s^2)), \dots, \text{Var}(R_{\ln \sigma_k^2} / (\bar{x}, s^2)) \right) A',$$

where

$$E\left(R_{\ln \sigma_i^2} / (\bar{x}, s^2)\right) = \ln(n_i s_i^2) - E(\ln U_i)$$

and

$$\text{Var}\left(R_{\ln \sigma_i^2} / (\bar{x}, s^2)\right) = E\left((\ln U_i)^2\right) - \left(E(\ln U_i)\right)^2.$$

4) Calculate $\|d\|^2 = \mu'_R \Sigma_R^{-1} \mu_R$.

5) Calculate $\|D\|^2 = (R_{A \ln \sigma^2} - \mu_R)' \Sigma_R^{-1} (R_{A \ln \sigma^2} - \mu_R)$.

6) Repeat step 2-5 a total m times and calculate p-value as $p = \#(\|D_j\|^2 > \|d_j\|^2) / m$, $j = 1, \dots, m$. In the case of $p < \alpha$, H_0 is rejected

2.4. The Computational Approach Test Approach

Gökpınar and Gökpınar [16] proposed a test based on the CAT for the homogeneity of variances under normality. The test statistic, η , is given as

$$\eta = \sum_{i=1}^k n_i \left(\log \sigma_i^2 - \log \overline{\sigma^2} \right)^2, \quad (3)$$

where $\overline{\sigma^2} = \sum_{i=1}^k n_i \sigma_i^2 / \sum_{i=1}^k n_i$. The algorithm of this approach is given as follows:

1) The MLE of σ_i^2 is given in Eq. (1), then the test statistic is calculated as

$$\hat{\eta}_{(ML)} = \sum_{i=1}^k n_i \left(\log S_i^{2*} - \log \overline{S^2} \right)^2, \text{ where } \overline{S^2} = \sum_{i=1}^k n_i S_i^{2*} / \sum_{i=1}^k n_i.$$

2. Under H_0 , the restricted MLEs (RMLEs) of parameters are obtained as

$$\hat{\mu}_{i(RMLE)} = \frac{\sum_{j=1}^{n_i} X_{ij}}{n_i} = \bar{X}_i \quad \text{and} \quad \hat{\sigma}_{(RMLE)}^2 = \frac{\sum_{i=1}^k \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2}{\sum_{i=1}^k n_i} = \frac{\sum_{i=1}^k n_i S_i^{*2}}{\sum_{i=1}^k n_i} = \overline{S^2} \quad (4)$$

3. Generate artificial sample $X_{i1}, X_{i2}, \dots, X_{in_i}$, $i = 1, \dots, k$ i.i.d from $N(\hat{\mu}_{i(RMLE)}, \hat{\sigma}_{RMLE}^2)$ a large of number of times(say, m times). For each of these replicated samples, recalculate the values of $\hat{\eta}_{MLE}^{(j)}$ ($j = 1, \dots, m$).

4. Calculate the p -value as $p = \#(\hat{\eta}_{MLE}^{(j)} > \hat{\eta}_{MLE}) / m$. In the case of $p < \alpha$, H_0 is rejected.

3. THE TESTS BASED ON LIKELIHOOD RATIO TEST FOR HOMOGENEITY OF VARIANCES

To obtain LRT statistic, we need to give the likelihood functions under H_1 and H_0 respectively. Under H_1 , log-likelihood function can be written as follows:

$$\ln L_1 = -\sum_{i=1}^k \frac{n_i}{2} \ln(2\pi\sigma_i^2) - \frac{1}{2} \left[\sum_{j=1}^{n_1} \frac{(X_{1j} - \mu_1)^2}{\sigma_1^2} + \dots + \sum_{j=1}^{n_k} \frac{(X_{kj} - \mu_k)^2}{\sigma_k^2} \right].$$

Under H_0 , log-likelihood function can be given as follows:

$$\ln L_0 = -\sum_{i=1}^k \frac{n_i}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \left[\sum_{j=1}^{n_1} (X_{1j} - \mu_1)^2 + \dots + \sum_{j=1}^{n_k} (X_{kj} - \mu_k)^2 \right].$$

The LRT can be given as follows:

$$\Lambda = \sum_{i=1}^k n_i \left(\log S_i^{2*} - \log \overline{S^2} \right). \quad (5)$$

Λ has an approximate chi-square distribution with $k-1$ degrees of freedom under the null hypothesis.

3.1. The CAT Approach Based On The Likelihood Ratio Test

Chang et al. [10] suggested a CAT approach based on likelihood ratio test for homogeneity of variances. The test procedure is given as below:

1. Compute the LRT statistic Λ in Eq. (5)
2. Obtain the RMLEs of parameters μ and σ^2 in Eq. (4).
3. Generate artificial sample $X_{i1}, X_{i2}, \dots, X_{in_i}$, $i=1, \dots, k$ i.i.d from $N(\hat{\mu}_{i(RMLE)}, \hat{\sigma}_{RMLE}^2)$ a large of number of times (say, m times), that is, $X_{ij} \sim N(\hat{\mu}_{i(RMLE)}, \hat{\sigma}_{RMLE}^2)$. For each of these replicated samples, recalculate the values of $\Lambda^{(l)}$ ($l=1, \dots, m$).
4. Calculate the p -value as $p = \#(\Lambda^{(l)} > \Lambda) / m$. In the case of $p < \alpha$, H_0 is rejected.

3.2. The Proposed Test: The Standardized Likelihood Ratio Test

We proposed the standardized likelihood ratio test (SLRT) for testing the equality of variances of several normal distributions. The SLRT is defined as

$$\Lambda_s = \sqrt{2(k-1)} \left(\frac{\Lambda - m(\Lambda)}{SD(\Lambda)} \right) + (k-1), \quad (6)$$

where $m(\Lambda)$ and $SD(\Lambda)$ are the mean and standard deviation of Λ , respectively. Λ_s has an approximate chi-square distribution with $k-1$ degrees of freedom under the null hypothesis. Krishnamoorthy and Oral [21] estimated the expressions of $m(\Lambda)$ ve $SD(\Lambda)$ through simulation because they are difficult to obtain. The proposed SLRT can be computed through the following steps.

1. Calculate the LRT statistic Λ in Eq. (5).
2. Generate artificial sample $X_{i1}, X_{i2}, \dots, X_{in_i}$, $i=1, \dots, k$ i.i.d from $N(\hat{\mu}_{i(RMLE)}, \hat{\sigma}_{RMLE}^2)$ a large of number of times (say, m times), that is, $X_{ij} \sim N(\hat{\mu}_{i(RMLE)}, \hat{\sigma}_{RMLE}^2)$. For each of these replicated samples, recalculate the values of the LRT statistic Λ in Eq. (5).
3. Calculate the mean ($m(\Lambda)$) and standard deviation ($SD(\Lambda)$) of these simulated LRT, and find the SLRT statistic Λ_s in Eq. (6).
4. If $\Lambda_s > \chi_{k-1; 1-\alpha}^2$, then H_0 is rejected.

4. SIMULATION STUDY

In this section, the proposed SLRT was compared with GPA, BDT, BT, CAT test [17] and CATLR by means of simulation based on 5000 Monte Carlo runs. These tests were evaluated in terms of the type I error rate and power of test. The study includes different combination of sample sizes, number of groups and population variances. The samples were generated from the normal distribution. Without loss of generality, μ was taken to be equal to 0. For the specified nominal level of $\alpha = 0.05$, the estimated type I error rates of five tests were presented in from Table 1 to Table 4.

Table 1. The estimated type I error rates for $\alpha = 0.05$ and $k=3$

(n_1, n_2, n_3)	BT	GPA	BDT	CAT	CATLR	SLRT
3,5,7	0.050	0.050	0.042	0.049	0.050	0.052
3,6,9	0.046	0.047	0.040	0.049	0.047	0.051
3,8,13	0.055	0.058	0.045	0.054	0.058	0.055
10,15,20	0.048	0.044	0.044	0.045	0.047	0.052
20,25,30	0.050	0.051	0.047	0.051	0.051	0.052
10,20,30	0.053	0.054	0.050	0.053	0.055	0.046
15,25,35	0.046	0.046	0.044	0.044	0.046	0.054
20,30,40	0.047	0.047	0.043	0.049	0.050	0.046
3,3,3	0.046	0.050	0.048	0.049	0.049	0.047
5,5,5	0.051	0.053	0.048	0.053	0.052	0.050
7,7,7	0.048	0.050	0.046	0.051	0.050	0.047
9,9,9	0.047	0.049	0.043	0.049	0.048	0.055
15,15,15	0.048	0.047	0.045	0.045	0.047	0.051
20,20,20	0.050	0.051	0.045	0.050	0.050	0.050
25,25,25	0.048	0.047	0.045	0.048	0.048	0.046
30,30,30	0.047	0.046	0.043	0.046	0.047	0.051

Table 2. The estimated type I error rates for $\alpha = 0.05$ and $k=5$

$(n_1, n_2, n_3, n_4, n_5)$	BT	GPA	BDT	CAT	CATLR	SLRT
3,3,5,7,7	0.050	0.052	0.052	0.051	0.053	0.053
3,3,6,9,9	0.047	0.048	0.049	0.048	0.049	0.048
3, 3, 8,13,13	0.052	0.051	0.051	0.048	0.049	0.053
10,10,15,20,20	0.051	0.051	0.049	0.054	0.051	0.055
20,20,25,30,30	0.050	0.051	0.047	0.049	0.050	0.048
10,10, 20,30,30	0.047	0.045	0.048	0.045	0.046	0.050
15, 15, 25,35,35	0.050	0.052	0.053	0.051	0.050	0.047
20, 20, 30,40,40	0.048	0.051	0.054	0.050	0.049	0.054
3,3,3,3,3	0.045	0.051	0.051	0.051	0.050	0.047
5,5,5,5,5	0.045	0.044	0.048	0.043	0.045	0.049
7,7,7,7,7	0.049	0.049	0.046	0.048	0.049	0.051
9,9,9,9,9	0.051	0.049	0.052	0.052	0.051	0.047
15,15,15,15,15	0.056	0.054	0.052	0.054	0.055	0.048
20,20,20,20,20	0.051	0.051	0.046	0.051	0.051	0.051
25,25,25,25,25	0.049	0.046	0.046	0.045	0.049	0.051
30,30,30,30,30	0.050	0.052	0.051	0.053	0.050	0.050

Table 3. The estimated type I error rates for $\alpha = 0.05$ and $k=7$

$(n_1, n_2, n_3, n_4, n_5, n_6, n_7)$	BT	GPA	BDT	CAT	CATLR	SLRT
3, 3, 5, 5, 5, 7, 7	0.050	0.052	0.051	0.052	0.053	0.051
3, 3, 6, 6, 6, 9, 9	0.049	0.050	0.055	0.052	0.052	0.049
3, 3, 8, 8, 8, 13,13	0.050	0.050	0.047	0.053	0.051	0.047

10,10,15, 15, 15, 20,20	0.050	0.050	0.048	0.049	0.049	0.048
20,20,25,25,25, 30,30	0.053	0.055	0.053	0.053	0.053	0.054
10,10, 20, 20, 20, 30,30	0.057	0.056	0.051	0.053	0.055	0.051
15, 15, 25, 25, 25, 35, 35	0.046	0.045	0.052	0.049	0.046	0.049
20, 20, 30, 30, 30, 40,40	0.050	0.048	0.049	0.048	0.050	0.054
3, 3, 3, 3, 3, 3, 3	0.042	0.050	0.051	0.046	0.047	0.051
5, 5, 5, 5, 5, 5, 5	0.045	0.051	0.049	0.050	0.047	0.053
7, 7, 7, 7, 7, 7, 7	0.052	0.054	0.052	0.055	0.053	0.048
9, 9, 9, 9, 9, 9, 9	0.045	0.049	0.046	0.048	0.044	0.050
15,15,15,15,15,15,15	0.049	0.049	0.046	0.049	0.049	0.053
20,20,20,20,20,20,20	0.043	0.041	0.040	0.041	0.043	0.051
25,25,25,25,25,25,25	0.054	0.053	0.049	0.052	0.054	0.050
30,30,30,30,30,30,30	0.050	0.050	0.048	0.051	0.050	0.054

Table 4. The estimated type I error rates for $\alpha = 0.05$ and $k=9$

$(n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9)$	BT	GPA	BDT	CAT	CATLR	SLRT
3, 3, 3, 5, 5, 5, 7, 7, 7	0.047	0.050	0.047	0.051	0.049	0.050
3, 3, 3, 6, 6, 6, 9, 9, 9	0.053	0.054	0.054	0.055	0.053	0.046
3, 3, 3, 8, 8, 8, 13, 13, 13	0.050	0.048	0.049	0.045	0.051	0.053
10,10,10, 15, 15, 15, 20, 20, 20	0.049	0.052	0.050	0.054	0.050	0.049
20,20,20,25,25,25, 30, 30, 30	0.048	0.047	0.049	0.046	0.048	0.051
10,10, 10, 20, 20, 20, 30, 30, 30	0.048	0.047	0.054	0.048	0.048	0.053
15, 15, 15, 25, 25, 25, 35, 35, 35	0.051	0.049	0.050	0.048	0.051	0.048
20, 20, 20, 30, 30, 30, 40, 40, 40	0.047	0.047	0.048	0.048	0.047	0.051
3, 3, 3, 3, 3, 3, 3, 3, 3	0.040	0.046	0.047	0.048	0.045	0.054
5, 5, 5, 5, 5, 5, 5, 5, 5	0.055	0.055	0.051	0.054	0.055	0.054
7, 7, 7, 7, 7, 7, 7, 7, 7	0.047	0.052	0.051	0.050	0.048	0.048
9, 9, 9, 9, 9, 9, 9, 9, 9	0.052	0.052	0.050	0.052	0.051	0.052
15,15,15,15,15,15,15,15,15	0.052	0.051	0.044	0.052	0.052	0.049
20,20,20,20,20,20,20,20,20	0.050	0.050	0.049	0.050	0.049	0.055
25,25,25,25,25,25,25,25,25	0.049	0.050	0.048	0.050	0.050	0.049
30,30,30,30,30,30,30,30,30	0.048	0.048	0.051	0.048	0.050	0.048

From the numerical results in Table 1-Table 4, it appears that the estimated type I error rates of six tests are close to the nominal level for all cases.

For specified nominal level of $\alpha = 0.05$, Table 5-Table 8 present the estimated powers of the six tests.

Table 5. The estimated powers of the tests for $k=3$

$(\sigma_1^2, \sigma_2^2, \sigma_3^2)$	(n_1, n_2, n_3)	BT	GPA	BDT	CAT	CATLR	SLRT
(0.25, 0.5, 1)	3,5,7	0.129	0.126	0.131	0.156	0.163	0.157
	3,6,9	0.149	0.150	0.146	0.179	0.192	0.167
	3,8,13	0.177	0.169	0.177	0.185	0.213	0.213
	10,15,20	0.539	0.530	0.523	0.585	0.569	0.572
	20,25,30	0.847	0.848	0.835	0.862	0.857	0.848
	10,20,30	0.634	0.633	0.608	0.697	0.676	0.666
	15,25,35	0.788	0.792	0.774	0.830	0.812	0.822
	20,30,40	0.887	0.886	0.875	0.908	0.899	0.896
	3,3,3	0.083	0.078	0.085	0.081	0.091	0.094
	5,5,5	0.164	0.149	0.161	0.155	0.166	0.169
	7,7,7	0.269	0.253	0.259	0.260	0.270	0.243
	9,9,9	0.356	0.345	0.340	0.349	0.356	0.350
	15,15,15	0.595	0.587	0.578	0.588	0.594	0.601

	20,20,20	0.753	0.749	0.738	0.750	0.750	0.764
	25,25,25	0.849	0.849	0.836	0.848	0.849	0.854
	30,30,30	0.921	0.920	0.913	0.921	0.921	0.914
(0.25,0.75, 2.25)	3,5,7	0.270	0.252	0.263	0.316	0.332	0.332
	3,6,9	0.315	0.294	0.296	0.357	0.378	0.382
	3,8,13	0.404	0.380	0.373	0.401	0.465	0.461
	10,15,20	0.927	0.926	0.914	0.943	0.937	0.938
	20,25,30	0.997	0.998	0.996	0.998	0.998	0.999
	10,20,30	0.972	0.973	0.963	0.982	0.980	0.977
	15,25,35	0.995	0.995	0.993	0.997	0.996	0.997
	20,30,40	0.999	0.999	0.999	0.999	0.999	0.999
	3,3,3	0.149	0.132	0.162	0.143	0.162	0.157
	5,5,5	0.372	0.342	0.369	0.355	0.375	0.382
	7,7,7	0.599	0.581	0.587	0.591	0.601	0.601
	9,9,9	0.738	0.729	0.725	0.733	0.737	0.744
	15,15,15	0.950	0.949	0.943	0.949	0.949	0.954
	20,20,20	0.990	0.989	0.988	0.989	0.989	0.989
	25,25,25	0.998	0.998	0.997	0.998	0.998	0.997
	30,30,30	0.999	0.999	0.999	0.999	0.999	1.000

Table 6. The estimated powers of the tests for $k=5$

$(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2)$	$(n_1, n_2, n_3, n_4, n_5)$	BT	GPA	BDT	CAT	CATLR	SLRT
(0.25,0.25,0.5,1,1)	3,3,5,7,7	0.160	0.140	0.141	0.200	0.217	0.218
	3,3,6,9,9	0.182	0.158	0.143	0.219	0.249	0.262
	3, 3, 8,13,13	0.208	0.185	0.152	0.233	0.280	0.284
	10,10,15,20,20	0.753	0.738	0.584	0.802	0.792	0.792
	20,20,25,30,30	0.971	0.968	0.905	0.974	0.973	0.976
	10,10, 20,30,30	0.827	0.816	0.631	0.880	0.867	0.865
	15, 15, 25,35,35	0.945	0.944	0.850	0.964	0.957	0.956
	20, 20, 30,40,40	0.986	0.986	0.944	0.991	0.988	0.989
	3,3,3,3,3	0.112	0.085	0.119	0.097	0.125	0.119
	5,5,5,5,5	0.243	0.187	0.203	0.205	0.245	0.240
	7,7,7,7,7	0.391	0.329	0.306	0.347	0.391	0.408
	9,9,9,9,9	0.526	0.472	0.406	0.481	0.527	0.532
	15,15,15,15,15	0.817	0.798	0.693	0.800	0.817	0.820
	20,20,20,20,20	0.935	0.930	0.852	0.929	0.936	0.936
	25,25,25,25,25	0.981	0.979	0.934	0.980	0.980	0.982
	30,30,30,30,30	0.994	0.993	0.975	0.994	0.994	0.994
(0.25,0.25,0.75,2.25,2.25)	3,3,5,7,7	0.382	0.315	0.243	0.430	0.491	0.486
	3,3,6,9,9	0.441	0.371	0.258	0.485	0.552	0.542
	3, 3, 8,13,13	0.518	0.440	0.288	0.535	0.629	0.639
	10,10,15,20,20	0.997	0.996	0.965	0.998	0.998	0.998
	20,20,25,30,30	0.999	0.999	0.999	1.000	0.999	1.000
	10,10, 20,30,30	0.998	0.997	0.982	1.000	0.999	1.000
	15, 15, 25,35,35	1.000	1.000	0.999	1.000	1.000	1.000
	20, 20, 30,40,40	1.000	1.000	1.000	1.000	1.000	1.000
	3,3,3,3,3	0.233	0.153	0.218	0.187	0.249	0.257
	5,5,5,5,5	0.587	0.488	0.440	0.528	0.590	0.595
	7,7,7,7,7	0.816	0.769	0.664	0.784	0.816	0.819
	9,9,9,9,9	0.932	0.913	0.825	0.918	0.932	0.936
	15,15,15,15,15	0.997	0.997	0.987	0.997	0.997	0.999
	20,20,20,20,20	0.999	0.999	0.998	0.999	0.999	1.000
	25,25,25,25,25	1.000	1.000	1.000	1.000	1.000	1.000
	30,30,30,30,30	1.000	1.000	1.000	1.000	1.000	1.000

Table 7. The estimated powers of the tests for $k=7$

$(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2, \sigma_6^2, \sigma_7^2)$	$(n_1, n_2, n_3, n_4, n_5, n_6, n_7)$	BT	GPA	BDT
(0.25, 0.25, 0.5, 0.5, 0.5, 1, 1)	3, 3, 5, 5, 5, 7, 7	0.168	0.137	0.158
	3, 3, 6, 6, 6, 9, 9	0.192	0.160	0.172
	3, 3, 8, 8, 8, 13, 13	0.256	0.208	0.202
	10, 10, 15, 15, 15, 20, 20	0.714	0.672	0.549
	20, 20, 25, 25, 25, 30, 30	0.959	0.955	0.883
	10, 10, 20, 20, 20, 30, 30	0.828	0.802	0.648
	15, 15, 25, 25, 25, 35, 35	0.936	0.927	0.821
	20, 20, 30, 30, 30, 40, 40	0.975	0.973	0.918
	3, 3, 3, 3, 3, 3, 3	0.105	0.075	0.115
	5, 5, 5, 5, 5, 5, 5	0.203	0.142	0.187
	7, 7, 7, 7, 7, 7, 7	0.336	0.265	0.275
	9, 9, 9, 9, 9, 9, 9	0.468	0.407	0.366
	15, 15, 15, 15, 15, 15, 15	0.777	0.741	0.632
	20, 20, 20, 20, 20, 20, 20	0.902	0.887	0.786
	25, 25, 25, 25, 25, 25, 25	0.965	0.961	0.900
	30, 30, 30, 30, 30, 30, 30	0.983	0.982	0.951
(0.25, 0.25, 0.75, 0.75, 0.75, 2.25, 2.25)	3, 3, 5, 5, 5, 7, 7	0.403	0.285	0.287
	3, 3, 6, 6, 6, 9, 9	0.497	0.373	0.329
	3, 3, 8, 8, 8, 13, 13	0.608	0.492	0.409
	10, 10, 15, 15, 15, 20, 20	0.993	0.991	0.950
	20, 20, 25, 25, 25, 30, 30	0.999	0.999	0.999
	10, 10, 20, 20, 20, 30, 30	0.997	0.997	0.978
	15, 15, 25, 25, 25, 35, 35	1.000	1.000	0.998
	20, 20, 30, 30, 30, 40, 40	1.000	1.000	1.000
	3, 3, 3, 3, 3, 3, 3	0.215	0.115	0.217
	5, 5, 5, 5, 5, 5, 5	0.529	0.400	0.410
	7, 7, 7, 7, 7, 7, 7	0.772	0.692	0.618
	9, 9, 9, 9, 9, 9, 9	0.906	0.875	0.782
	15, 15, 15, 15, 15, 15, 15	0.996	0.996	0.979
	20, 20, 20, 20, 20, 20, 20	0.999	0.999	0.997
	25, 25, 25, 25, 25, 25, 25	1.000	1.000	0.999
	30, 30, 30, 30, 30, 30, 30	1.000	1.000	1.000

Table 8. The estimated powers of the tests for $k=9$

$(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2, \sigma_6^2, \sigma_7^2, \sigma_8^2, \sigma_9^2)$	$(n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9)$	BT	GPA	P
(0.25, 0.25, 0.25, 0.5, 0.5, 0.5, 1, 1, 1)	3, 3, 3, 5, 5, 5, 7, 7, 7	0.209	0.154	0
	3, 3, 3, 6, 6, 6, 9, 9, 9	0.240	0.176	0
	3, 3, 3, 8, 8, 8, 13, 13, 13	0.300	0.234	0
	10, 10, 10, 15, 15, 15, 20, 20, 20	0.854	0.823	0
	20, 20, 20, 25, 25, 25, 30, 30, 30	0.992	0.990	0
	10, 10, 10, 20, 20, 20, 30, 30, 30	0.923	0.906	0
	15, 15, 15, 25, 25, 25, 35, 35, 35	0.986	0.983	0
	20, 20, 20, 30, 30, 30, 40, 40, 40	0.998	0.998	0
	3, 3, 3, 3, 3, 3, 3, 3, 3	0.122	0.082	0
	5, 5, 5, 5, 5, 5, 5, 5, 5	0.287	0.191	0
	7, 7, 7, 7, 7, 7, 7, 7, 7	0.451	0.357	0
	9, 9, 9, 9, 9, 9, 9, 9, 9	0.603	0.523	0
	15, 15, 15, 15, 15, 15, 15, 15, 15	0.904	0.884	0
	20, 20, 20, 20, 20, 20, 20, 20, 20	0.973	0.969	0
	25, 25, 25, 25, 25, 25, 25, 25, 25	0.996	0.995	0
	30, 30, 30, 30, 30, 30, 30, 30, 30	0.999	0.999	0
(0.25, 0.25, 0.25, 0.75, 0.75, 0.75, 2.25, 2.25, 2.25)	3, 3, 3, 5, 5, 5, 7, 7, 7	0.521	0.368	0
	3, 3, 3, 6, 6, 6, 9, 9, 9	0.604	0.458	0
	3, 3, 3, 8, 8, 8, 13, 13, 13	0.733	0.593	0
	10, 10, 10, 15, 15, 15, 20, 20, 20	0.999	0.999	0
	20, 20, 20, 25, 25, 25, 30, 30, 30	1.000	1.000	1
	10, 10, 10, 20, 20, 20, 30, 30, 30	1.000	1.000	0
	15, 15, 15, 25, 25, 25, 35, 35, 35	1.000	1.000	1
	20, 20, 20, 30, 30, 30, 40, 40, 40	1.000	1.000	1
	3, 3, 3, 3, 3, 3, 3, 3, 3	0.286	0.146	0
	5, 5, 5, 5, 5, 5, 5, 5, 5	0.694	0.538	0
	7, 7, 7, 7, 7, 7, 7, 7, 7	0.909	0.853	0
	9, 9, 9, 9, 9, 9, 9, 9, 9	0.972	0.959	0
	15, 15, 15, 15, 15, 15, 15, 15, 15	1.000	1.000	0
	20, 20, 20, 20, 20, 20, 20, 20, 20	1.000	1.000	0
	25, 25, 25, 25, 25, 25, 25, 25, 25	1.000	1.000	1
	30, 30, 30, 30, 30, 30, 30, 30, 30	1.000	1.000	1

As seen from Table 5, when the number of groups is $k=3$ and the sample sizes are small and different, the power of the CATLR is higher than the other tests. For example, when $(n_1, n_2, n_3) = (3, 5, 7)$ and $(\sigma_1^2, \sigma_2^2, \sigma_3^2) = (0.25, 0.5, 1)$, powers of tests were obtained as $BT=0.129$, $GPA=0.126$, $BDT=0.131$, $CAT=0.156$, $CATLR=0.163$ and $SLRT=0.157$. As the sample sizes are getting larger, the powers of the CATLR and SLRT approach each other. For example, while $(n_1, n_2, n_3) = (3, 8, 13)$ and $(\sigma_1^2, \sigma_2^2, \sigma_3^2) = (0.25, 0.5, 1)$, the powers of the tests can be given as $BT=0.177$, $GPA=0.169$, $BDT=0.178$, $CAT=0.186$, $CATLR=0.213$ and $SLRT=0.213$. However, when all sample sizes are getting larger, the CAT is slightly more powerful than CATLR and SLRT. When sample sizes are equal, the powers of the CATLR and SLRT are higher than other tests, even the power of the SLRT is slightly higher than CATLR. However, when the equal sample sizes are getting larger, the powers of the CATLR, SLRT and BT approach each other.

When differences between population variances are getting higher and the sample sizes are small, the powers of the CATLR and SLRT are getting higher than the other tests, especially in the case of different sample sizes. For example, while $(n_1, n_2, n_3) = (3, 8, 13)$ and $(\sigma_1^2, \sigma_2^2, \sigma_3^2) = (0.25, 0.75, 2.25)$, powers of tests can be given as $BT=0.405$, $GPA=0.380$, $BDT=0.374$, $CAT=0.401$, $CATLR=0.465$, $SLRT=0.461$. When the equal sample sizes are getting larger, the powers of the CATLR, SLRT and BT approach each other.

When the number of groups is $k=5$, for the small and different sample sizes, the powers of the CATLR and SLRT are close to each other and are higher than the other tests. However, when all sample sizes are getting larger, the CAT is slightly more powerful than the CATLR and SLRT. For the equal sample sizes, the powers of the CATLR and SLRT are close to each other and are higher than other tests, however, for the large and equal sample sizes, the powers of the CATLR, SLRT and BT approach each other. When the differences between population variances are getting higher, the powers of the CATLR and SLRT are getting higher than the other tests, especially in the case of different sample sizes.

As the number of group is getting larger, that is, while $k=7$ and the sample sizes are different, the CATLR and SLRT are more powerful than the other tests. For the equal sample sizes the powers of the CATLR and SLRT are better than the others. Besides, as the equal sample sizes increase, the CATLR, SLRT and BT approach each other. For $k=9$, the results are similar to the ones obtained for the number of group, $k=7$.

All in all, the CATLR and SLRT are better than the others, especially in the cases of small sample sizes. In addition to this, the powers of other tests are close to the CATLR and SLRT as the sample size increases.

5. CONCLUSION

In this study, we proposed a standardize likelihood ratio test (SLRT) for the homogeneity of variances. We have compared this approach with existing approaches. Simulation study indicates that regardless of the number of group and sample sizes, the powers of the CATLR and SLRT are close to each other and are higher than the other tests. When the differences between population variances are getting higher, in case of large equal sample sizes, the powers of the CATLR, SLRT and BT are close to each other and are higher than the other tests.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors

REFERENCES

- [1] Bartlett, M.S., "Properties of sufficiency and statistical tests", Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, 160(901):268-282, (1937).

- [2] Bhandary, M., and Dai, H., "An alternative test for the equality of variances for several populations when the underlying distributions are normal", *Communications in Statistics-Simulation and Computation*, 38(1):109-117, (2009).
- [3] Boos, D.D. and Brownie, C., "Comparing variances and other measures of dispersion", *Statistical Science*, 19(4):571-578, (2004).
- [4] Box, G.E. and Andersen, S.L., "Permutation theory in the derivation of robust criteria and the study of departures from assumption", *Journal of the Royal Statistical Society. Series B (Methodological)*, 17(1):1-34, (1955).
- [5] Brown, M.B. and Forsythe, A.B., "Robust tests for the equality of variances", *Journal of the American Statistical Association*, 69(346):364-367, (1974).
- [6] Cahoy, D.O., "A bootstrap test for equality of variances", *Computational Statistics & Data Analysis*, 54(10):2306-2316, (2010).
- [7] Chang, C.H., Lin, J.J. and Pal, N. "Testing the equality of several gamma means: a parametric bootstrap method with applications", *Computational Statistics*, 26(1), 55-76, (2011).
- [8] Chang, C.H. and Pal, N., "A revisit to the Behrens-Fisher problem: comparison of five test methods", *Communications in Statistics—Simulation and Computation*, 37(6), 1064-1085, (2008).
- [9] Chang, C.H., Pal, N. and Lin, J.J., "A note on comparing several poisson means", *Communications in Statistics-Simulation and Computation*, 39(8):1605-1627, (2010).
- [10] Chang, C.H., Pal, N. and Lin, J-J., "A Revisit to Test the Equality of Variances of Several Populations", *Communications in Statistics-Simulation and Computation* (in press), 2016.
- [11] Cochran, W.G., "Testing a linear relation among variances", *Biometrics*, 7(1):17-32, (1951).
- [12] Conover, W. J., Johnson, M. E. and Johnson, M.M., "A comparative study of tests for homogeneity of variances, with applications to the outer continental shelf bidding data", *Technometrics*, 23(4), 351-361, (1981).
- [13] Gökpınar, E. and Gökpınar, F. "A test based on computational approach for equality of means under unequal variance assumption", *Hacettepe Journal of Mathematics and Statistics*, 41(4):605-613, (2012).
- [14] Gökpınar, E., Polat, E., Gökpınar, F. and Günay S., "A new computational approach for testing equality of inverse Gaussian means under heterogeneity", *Hacettepe Journal of Mathematics and Statistics*, 42(5):581-590, (2013).
- [15] Gökpınar, F. and Gökpınar, E., "A Computational approach for testing of coefficients of variation in k normal population", *Hacettepe Journal of Mathematics and Statistics*, 44(5):1197-1213, (2015a).
- [16] Gökpınar, E. and Gökpınar, F., "Testing equality of variances for several normal populations", *Communications in Statistics-Simulation and Computation* (DOI: 10.1080/03610918.2014.955110) (2015b).
- [17] Gökpınar, F. and Gökpınar, E., "Testing the equality of several log-normal means based on a computational approach." *Communications in Statistics-Simulation and Computation* 46(3): 1998-2010, (2017).
- [18] Hall, I.J., "Some comparisons of tests for equality of variances", *Journal of Statistical Computation and Simulation*, 1(2):133-194, (1972).

- [19] Hartley, H.O., "The maximum F-ratio as a short-cut test for heterogeneity of variance", *Biometrika*, 37(3/4):308-312, (1950).
- [20] Jen, T. and Gupta, A.K., "On testing homogeneity of variances for Gaussian models", *Journal of Statistical Computation and Simulation*, 27(2):155-173, (1987).
- [21] Krishnamoorthy, K. and Oral, E., "Standardized likelihood ratio test for comparing several log-normal means and confidence interval for the common mean," *Statistical methods in medical research*, DOI:10.1177/0962280215615160, (2015).
- [22] Levene, H., "Robust tests for equality of variances", In *Contributions to probability and statistics: Essays in honor of Harold Hotelling*, 2:278-292, (1960).
- [23] Liu, X. and Xu, X. "A new generalized p-value approach for testing the homogeneity of variances", *Statistics & probability letters*, 80(19):1486-1491, (2010).
- [24] Mutlu, H.T., Gökpınar, F., Gökpınar, E. and Gül, H.H., "A New Computational Approach Test for One-Way ANOVA under Heteroscedasticity" ,*Communications in Statistics - Theory and Methods* , DOI: 10.1080/03610926.2016.117708, ISSN: 0361-0926, (2016).
- [25] Nair, V.N. and Pregibon, D., "Analyzing dispersion effects from replicated factorial experiments", *Technometrics*, 30(3):247-257, (1988)..
- [26] Neyman, J. and Pearson, E.S., On the problem of k samples. *Bull. Ac. Polonaise Sci. Letters, Serie A3*: 460-481, (1931).
- [27] Pal, N., Lim, W.K. and Ling, C.H., "A computational approach to statistical inferences", *Journal of Applied Probability and Statistics*, 2(1):13-35, (2007).
- [28] Taguchi, G., *Introduction to quality engineering: designing quality into products and processes*, Asian Productivity Organization, Tokyo, (1986).