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On Multiplier of Hyper BCI Algebras

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Abstract

In this paper, we introduce the notion of multiplier of a hyper BCI- algebra, and discuss some properties of hyper BCI-algebras. Also we introduced notion of hyper isotone multiplier.

1. INTRODUCTION

The study of BCK-algebras was started by Y.Imai and K.Iseki [1] in 1966 as a generalization of the concept of set-theoretic difference and propositional calculus.

The hyperstructure theory(called also multialgebras) was introduced by F. Marty [2] in 1934.

Moreover the hyper structure was applied to BCI-algebras and was introduced the concepts of hyper BCI-algebras which is a generalization of BCI-algebras by X.X. Long [3] in 2006.

In this paper, we introduce the notion of multiplier of a hyper BCI- algebra, and discuss some properties of hyper BCI-algebras. Also we introduce the notion of hyper isotone multiplier on hyper BCI-algebras.

2. PRELIMINARIES

Definition 2.1. [3] Let H be a nonempty set and "$\circ$" be a hyper operation on H. Then H is said to be a hyper BCI-algebra, if it contains a constant 0 and the following conditions hold:

(b1) $(x \circ z) \circ (y \circ z) << x \circ y$,

(b2) $(x \circ y) \circ z = (x \circ z) \circ y$,

(b3) $x << x$,

(b4) $x << y$, $y << x \Rightarrow x = y$,

(b5) $0 \circ (0 \circ x) << x$

for all $x, y, z \in H$ where $x << y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$ $A << B$ is defined by for all $a \in A$, there exists $b \in B$ such that $a << b$. In such case $''<<''$ is called the hyper order in H.

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Let $(H,\circ,0)$ be a hyper BCI-algebra. By H^{+} we mean $H^{+}=\{x\in H \mid 0\in 0\circ x\}$

We have $0\in H^{+}$, thus $H^{+}\neq\varnothing$.

Proposition 2.2. [4]Let $(H,\circ,0)$ be a hyper BCI-algebra, the following hold:

(i) $x<<x\circ 0$

(ii) $A<<A$

(iii) $y<<z$ implies $x\circ z<<x\circ y$,

for all $x,y,z\in H$ and for all nonempty subsets A and B of H.

Definition 2.3. [4] Let $(H,\circ,0)$ be a hyper BCI-algebra.Then the set $S_k=\{x\in H: x\circ H<<\{x\}\}$ is called as hyper BCK-part of H .If $H\neq S_k$, then H is said to be a proper hyper BCI-algebra.

A hyper BCI-algebra H is called a

(i) weak proper hyper BCI-algebra if H is proper and $H^{+}=H$. In the other word if 0 is the smallest element of H,

(ii) strong proper hyper BCI-algebra if $H^{+}\neq H$. We note that if $x\notin H^{+}$, then $0\notin 0\circ x$. Thus $0\circ x \not<< \{0\}$

Therefore, $0\circ H \not<< \{0\}$ and $(H,\circ,0)$ is proper.

Definition 2.4. [4] Let I be a nonemptysubset of hyperBCI-algebra H and $0\in I$ .Then I is said to be a

(i)weak hyper BCI-ideal of H if $x\circ y\subseteq I$ and $y\in I$ imply that $x\in I$, for all $x,y\in H$,

(ii)hyper BCI-ideal of H if $x\circ y<<I$ and $y\in I$ imply that $x\in I$, for all $x,y\in H$,

(iii)strong hyper BCI-ideal of H if $x\circ y\approx I$ and $y\in I$ imply that $x\in I$, for all $x,y\in H$, where $x\circ y\approx I$ means $x\circ y\cap I\neq\varnothing$.

Definition 2.5. [5] Let I be a nonempty subset of a hyper BCI-algebra H and $0\in I$ .Then I is called to be hyper subalgebra of H if $x\circ y\subseteq I$ for all $x,y\in I$.

Definition 2.6. [6] Let $(H_1,\circ_1,0_1)$ and $(H_2,\circ_2,0_2)$ be two hyper BCI-algebras and $f: H_1\to H_2$ be a function.Then f is defined a homomorphism if and only if

$f(x\circ_1 y)=f(x)\circ_2 f(y)$, for all $x,y\in H_1$.

If f is one to one $(onto)$ then f is monomorphism $(epimorphism)$ and if f is both one to one and onto, then f is a isomorphism and $(H_1,\circ_1,0_1)$ and $(H_2,\circ_2,0_2)$ are isomorphic.

3. MULTIPLIER OF HYPER BCI-ALGEBRAS

In the following, the notion of multiplier of a hyper BCI-algebra is given.

Definition 3.1. Let $(H,\circ,0)$ be a hyper BCI-algebra. A map $d: H\to H$ is said to be a multiplier if for all $x,y\in H$ $d(x\circ y)=d(x)\circ y.$

Example 3.1. Let $H = \{0, \alpha, \beta\}$ and $(H, \circ, 0)$ be a hyper BCI-algebra with Cayley table as follows

Table 1. *Cayley table*

∘	0	α	β
0	{0}	{0}	{β}
α	{α}	{0,α}	{β}
β	{β}	{β}	{0}

Define a map $d_1 : H \to H$ $d_1(x) = \begin{cases} \beta, & x = 0, \alpha \\ 0, & x = \beta \end{cases}$

Hence it is easily checked that d_1 is a multiplier of hyper BCI-algebra.

Therefore H is strong proper hyper BCI-algebra.

If $I_1 = \{0, \beta\} \subseteq H$ then I_1 is ideal of H.

Example 3.2. Cayley table given in Example 3.1 and $d_2 = I_H$ then d_2 is multiplier of H.

Example 3.3. Cayley table given in Example 3.1and define a map $d_3 : H \to H$

$$d_3(x) = \begin{cases} 0, & x = 0, \alpha \\ b, & x = \beta \end{cases}$$

Hence it is easily checked that d_3 is a multiplier of hyper BCI-algebra.

If $I_2 = \{0, \alpha\} \subseteq H$ then I_2 is ideal of H. And also d_3 is an invariant map: $d_3(I_2) \subseteq I_2$.

Proposition 3.2. Let $(H, \circ, 0)$ be a hyper BCI-algebra and d be a multiplier of H .Then it satisfies $d(x \circ d(x)) << 0$ for all $x \in H$.

Proof. Using $(b3)$;

$$d(x \circ d(x)) << 0$$

$$0 \in d(x) \circ d(x)$$

$$0 \in d(x) \circ d(x) \circ 0$$

Definition 3.3. Let $(H, \circ, 0)$ be a hyper BCI-algebra and a map $d : H \to H$ is called to be a regular if $d(0) = 0$.

Example 3.4. d_3 given in Example 3.3 is multiplier of hyper BCI-algebra and regular. That is $d_3(0) = 0$.

Proposition 3.4. Let $(H, \circ, 0)$ be a hyper BCI-algebra and a map $d : H \to H$ is a regular multplier of H. Then the following hold for all $x, y \in H$:

(i) $d(x) << x$,

(ii) $d(x \circ y) << d(x) \circ d(y)$.

Proof.

(i) $0 = d(0) \in d(x \circ x) = d(x) \circ x$, for all $x \in X$. We get $d(x) << x$.

(ii) Let $y \in X$. Using (i) and $Prop.2.2.(iii)$, we have $d(x \circ y) = d(x) \circ y$.

Therefore we get $d(x) \circ y << d(x) \circ d(y)$.

Hence we have $d(x \circ y) << d(x) \circ d(y)$.

Example 3.5. d_3 given in Example 3.3. is multiplier of hyper BCI-algebra. And also it is a homomorphism.

Definition 3.5. Let $(H, \circ, 0)$ be a hyper BCI-algebra and a map $d : H \to H$, if $x << y$ then $d(x) << d(y)$ for all $x, y \in H$, d is said to be hyper isotone.

Example 3.6. d_3 given in Ex. 3.3 is hyper isotone.

Proposition 3.6. Let $(H, \circ, 0)$ be a hyper BCI-algebra and d be a regular multiplier of H. If $d : H \to H$ is an endomorphism, then d is hyper isotone.

Proof. Let $x, y \in X$ and $x << y$

Therefore we get $0 \in x \circ y$. d be a regular multiplier of H and $d : H \to H$ is an endomorphism so we have $d(0) \in d(x \circ y) \ll d(x) \circ d(y)$. Hence we find $d(x) << d(y)$.

Definition 3.7. Let $(H, \circ, 0)$ be a hyper BCI-algebra and d_1, d_2 be two maps. Then a map

$d_1 \bullet d_2 : H \to H$ is defined by $(d_1 \bullet d_2)(x) = d_1(d_2(x))$ for all $x \in H$.

Proposition 3.8. Let $(H, \circ, 0)$ be a hyper BCI-algebra and be d_1, d_2 two maps. $d_1, d_2 : H \to H$ are multipliers of H .Then $d_1 \bullet d_2$ is a multiplier of H.

Proof. Let $x, y \in H$, we get,

$$(d_1 \bullet d_2)(x \circ y) = d_1(d_2(x \circ y))$$

$$= d_1(d_2(x) \circ y)$$

$$= (d_1 \bullet d_2)(x) \circ y$$

And so $d_1 \bullet d_2$ is a multiplier of H.

Definition 3.9. Let $(H, \circ, 0)$ be a hyper BCI-algebra and d be a multiplier of H. A set $Fix_d(H)$ is defined by $Fix_d(H) := \{x \in H \mid d(x) = x\}$.

Proposition 3.10. Let $(H, \circ, 0)$ be a hyper BCI-algebra and d be a regular multiplier of H. If $x \in Fix_d(H)$ and $y \in H$ imply $(d \bullet d)(x \circ y) = (x \circ y)$.

Proof. Let $x, y \in H$, we have,

$$\begin{aligned}(d \bullet d)(x \circ y) &= d\big(d(x \circ y)\big) \\ &= d\big(d(x) \circ y)\big) \\ &= d(d(x)) \circ y \\ &= d(x) \circ y \\ &= x \circ y\end{aligned}$$

Proposition 3.11. Let $(H, \circ, 0)$ be a hyper BCI-algebra and d be a multiplier of H. Then $Fix_d(H)$ is a hyper subalgebra of H.

Proof. Let $x, y \in Fix_d(H)$. We have $d(x \circ y) = d(x) \circ y = x \circ y$.

Hence we find $Fix_d(H)$ is a hyper subalgebra of H.

Proposition 3.12. Let $(H, \circ, 0)$ be a hyper BCI-algebra and d be a multiplier of H. If $x \in H$ and $y \in Fix_d(H)$ then $x \wedge y \in Fix_d(H)$.

Proof. Let $y \in Fix_d(H)$, we get,

$$\begin{aligned}d(x \wedge y)) &= d(y \circ (y \circ x)) \\ &= d(y) \circ (y \circ x) \\ &= y \circ (y \circ x) \\ &= x \wedge y\end{aligned}$$

Proposition 3.13. Let $(H, \circ, 0)$ be a hyper BCI-algebra and d be a multiplier of H. If $x \in H$ and $y \in Fix_d(H)$ then $d(x \circ y) = d(x) \circ d(y)$.

Proof. Let $y \in Fix_d(H)$ and $x \in H$

$$\begin{aligned}d(x \circ y)) &= d(x) \circ y \\ &= d(x) \circ d(y).\end{aligned}$$

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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