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PAGES: 677-688

ORIGINAL PDF URL: https://dergipark.org.tr/tr/download/article-file/83521

Gazi University Journal of Science
GU J Sci
25(3):677-688 (2012)

ORIGINAL ARTICLE

The Power of Four Tests in Two Way Models with Interaction and No Replication

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Received: 21.10.2011 Accepted:05.03.2012

ABSTRACT

This article presents tests for the interaction in a two way table with one observation per cell. The conventional linear model theory cannot be used to check for the interaction when there is exactly one observation per cell (no replication) in two way ANOVA models. For this purpose, two different approaches are presented in the literature. The first of these approaches is to assume a specific functional form for the interaction terms. The second approach is not to assume a specific functional form for the interaction terms. In this article, we present four additivity tests by proposed Tusell (1990), Boik (1993a), Piepho (1994), Kharrati-Kopaei and Sadooghi-Alvandi (2007) for second approach. We compared their performance by means of simulation studies with respect to the power.

Key Words: *Two-way ANOVA models, fixed effects, interaction, no replication, power.*

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1. INTRODUCTION

Estimation and testing the effect of the interaction is obtained by the classical ways with two way ANOVA models. If a factor's effect on the response variable is not the same at the other factor's each level, it is said that there is an interaction between the factors. In general, each treatment combination has more than one repetition. However, it is not possible in many application fields. Each treatment combination has only one replication. Estimation and test of the effect of the interaction cannot be obtained by classical ways because zero degrees of freedom is left to the error term in such factorial arrays without replication.

In two way ANOVA model, two factors called factor A and factor B is given as

$$y_{ij} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ij} \quad , \quad i = 1,2,\dots,a \;\; ; \;\; j = 1,2,\dots,b \tag{1}$$

where y_{ij} is the observation value at *i*th level of factor A and *j*th level of factor B. μ is the grand mean, α_i is the main effect of *i*th level of factor A, β_j is the main effect of *j*th level of factor B, γ_{ij} is the interaction effect of *i*th level of factor A and *j*th level of factor B and ϵ_{ij} is error term. ϵ_{ij} are distributed independently and normally with mean zero and variance σ_ε^2. y_{ij} has a normal distribution with mean $E(y_{ij}) = \mu + \alpha_i + \beta_j + \gamma_{ij}$ and variance σ_ε^2.

Factors are assumed to be fixed and errors are assumed to have $\epsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$. The following restrictions are imposed on main effects and interaction effects:

$$\sum_{i=1}^{a} \alpha_i = \sum_{j=1}^{b} \beta_j = \sum_{i=1}^{a} \gamma_{ij} = \sum_{j=1}^{b} \gamma_{ij} = 0$$

These restrictions on parameters and assumptions about the distribution of error will be valid for the rest of the article.

When there is no replication (i.e. $\gamma_{ij} = 0$ for all *i* and *j*), model (1) is given by

$$y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij} \quad , \quad i = 1,2,\dots,a \;\; ; \;\; j = 1,2,\dots,b \tag{2}$$

In model (2) the expected value of y_{ij} is $E(y_{ij}) = \mu + \alpha_i + \beta_j$. This article is to assume that there is no interaction between factor A and factor B. We are interested in testing $H_0: E(y_{ij}) = \mu + \alpha_i + \beta_j \;\; ; \;\forall\, i,j$ versus $H_1: E(y_{ij}) = \mu + \alpha_i + \beta_j + \gamma_{ij} \;\; ; \; some\ i\ and\ j$. Briefly can be expressed in the following form:

$$H_0: \gamma_{ij} = 0 \quad ; \quad \forall\, i,j$$

$$H_1: \gamma_{ij} \neq 0 \quad ; \quad \forall\, i,j$$

Because of the model (1) includes the interaction term γ_{ij} , it is called as "nonadditivity model" and the model (2) does not contain interaction term γ_{ij} it is called as "additivity model".

Several methods have been developed to test the effect of the interaction in the literature. These methods were used two different approaches.

1. The aim of the first approaches is to have a specific functional form for the interaction. In this case, many methods have been developed in the literature. Tukey (1949) was the first to develop "one degree of freedom" procedure for the interaction with the model (1). These procedure which assume that the interactions are of the form $\gamma_{ij} = k\alpha_i\beta_j$, where k is constant and α_i and β_j is row and column effects, respectively. Tukey's nonadditivity test statistics has an F-distribution with [1, (a-1)(b-1)-1] degrees of freedom under the null hypothesis $H_0: k = 0$. Other methods for testing interaction with one observation per cell proposed by Mandel (1961, 1971), Johnson and Graybill (1972), Hegemann and Johnson (1976) and Yochmowitz and Cornell (1978). The main problem with this approach is that if the functional form for the interactions is misspecified, the power of the test is low. It is therefore useful to have a test which does not rely on a specific form for the interactions.

2. The aim of the second approaches is not to have a specific functional form for the interaction. In this case, many methods have been developed in the literature. Some of methods proposed by Tusell (1990), Boik (1993a, 1993b), Piepho (1994), Speed and Speed (1994) and Kharrati-Kopaei and Sadooghi-Alvandi (2007).

In this article, we present four additivity tests for testing interaction which does not rely on a specific form for the interactions. These tests are introduced in Section 2 and illustrated with one example. In section 3, we compared their performance by means of simulation studies with respect to the power and some interpretations were mentioned.

2. ADDITIVITY TESTS

In this article, we present four additivity tests proposed by Tusell's test (LRS), Boik's test (LBI), Piepho's test (T1) and Kharrati-Kopaei ve Sadooghi-Alvandi's test (F^*).

The following notation will be used: Let y_{ij} $(i = 1,2,\dots,a;\ j = 1,2,\dots,b)$ be sample observations. We assume a≥b; the roles of rows and columns are however interchangeable. Under the null hypothesis of no interaction; we consider the additivity model in (2) for fixed factors.

Denote the a×b matrix of residuals in model (2) by R:

$$e_{ij} = y_{ij} - \overline{y_{i.}} - \overline{y_{.j}} + \overline{y_{..}} \quad .$$

$$R = \{e_{ij}\} \quad , i = 1,2,\dots,a \quad ; j = 1,2,\dots,b$$

where $\{\overline{y_{i.}}\}$, $\{\overline{y_{.j}}\}$ and $\{\overline{y_{..}}\}$ are row, column and overall means, respectively. Let p=min (a-1, b-1)=b-1 and q=max (a-1, b-1)=a-1. The rank of the residual matrix is p (with probability 1) rather than p+1 because each row and each column of R sums of zero. Invariant tests depend on the data only through the (eigenvalues of *R'R* / trace(*R'R*)), namely

$$L_1 > L_2 > \cdots . > L_p > 0 \tag{3}$$

where

$$L_k = \frac{l_k}{\sum_{i=1}^{p} l_i}$$

and l_k for $(k = 1,2 \dots, p)$ are the ordered eigen value of *R'R* (Boik, 1993a).

2.1. Tusell's Test

Tusell derived a test for $\gamma_{ij} = 0$. Tusell's procedure consists of employing well-known LR sphericity (LRS) test as a test of additivity. A vector random variable, **Y**, is usually said to have a spherical distribution if its covariance matrix Σ_Y is of the form k**I**, that is, if all variances are equal, where k is any constant and I is the identity matrix. Testing $H_0: \Sigma_Y = \sigma_\varepsilon^2 I_p$ against $H_1: \Sigma_Y \neq \sigma_\varepsilon^2 I_p$ is called "test for sphericity" with p=min (a-1. b-1)=b-1 (Aylin and Kurt, 2006).

The idea underlying the test proposed by Tusell (1990) is to perform successive linear operations on the rows and columns of **Y** so as to annihilate the μ, α_i, β_j parts of y_{ij}. Tusell (1990) suggest that if the interaction is not present, choosing adequately the linear operations we end up with centered and independent vectors with diagonal covariance matrix with equal variances. Tusell's (1990) additivity test rejects $H_0: \Sigma_Y = \sigma^2 I_p$ if

$$\mathrm{LRS} = \frac{|R'R|}{(tr(R'R)/(b-1))^{b-1}} = \frac{l_1 l_2 \dots l_{b-1}}{\left[(b-1)^{-1}\sum_{k=1}^{b-1} l_k\right]^{b-1}} \tag{4}$$

is small. Where tr(.) is the trace function. Critical values for this test statistics are given e.g. in Kresh (1972). Note that these tables are to be used with p=(b-1) and N=a (Tusell, 1990).

2.2. Boik's Test

Boik derived a test for $\gamma_{ij} = 0$ that was designed to maximize power against local alternatives. Boik's test called as the "locally best invariant (LBI) test of additivity". The LBI test is sensitive to nonadditive structures for which the variability among $L_1, L_2, \dots, L_p$ in (3) is not too small. Testing $H_0: E(R) = 0$ against $H_0: E(R) \neq 0$ is rejected if

$$\hat{\epsilon}_{p,q} = \frac{[tr(R'R)]^2}{p\,tr[(R'R)^2]} \tag{5}$$

is small. Here, the subscripts on $\hat{\epsilon}$ give table size parameters and L_k is defined (3). Also, p=min (a-1. b-1) with the restriction of being greater than or equal to 2 and q=max (a-1, b-1). The test statistic in equation (5) is monotonic decreasing function of the coefficient of variation among the eigenvalues of $R'R$. A large coefficient of variation among the sample eigenvalues is evidence against additivity (Boik, 1993a). Coefficients of polynomials for approximating critical values for the LBI test are given in Table 1 (Boik, 1993a).

2.3. Piepho's Test

Milliken&Rasmusson (1977) propose $\sigma_i^2 \ (i = 1,2,\dots,a)$ to determine the variance of observations within each row, where

$$\sigma_i^2 = \frac{\sum_{j=1}^{a}(y_{ij} - \overline{y_{i.}})^2}{a-1}$$

If a test rejects the null hypothesis $H_0: \sigma_1^2 = \sigma_2^2 = \cdots = \sigma_a^2$, one may infer that there is interaction in the data. This may be done by any test for one-way homoscedasticity in two-way ANOVA model. The aim is to test the hypothesis that i th row equal variances. Several such tests are available. Anscombe (1981) and Brindley & Bradley (1985) proposed tests which involve statistics computed from Grubbs's estimates. Shukla (1982) suggested a Bartlett-type test for homogeneity of the sample variances of residuals in the ith row. Shukla's test is based on the statistic:

$$Q \equiv a \ln\left(a^{-1}\sum_{i=1}^{a} W_i\right) - \sum_{i=1}^{a}\ln(W_i) \quad \text{with} \quad W_i = \sum_{j=1}^{b}\left(y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..}\right)^2$$

Under H_o the test statistic

$$T_1 = (a-1)(b-1)Q/d \tag{6}$$

is approximately distributed as χ^2 with (a-1) degrees of freedom. Shukla investigated both first and second moment approximations to the χ^2 distribution; both gave satisfactory results for very small numbers of rows and columns. Piepho (1992a) denotes the first moment approximation by *d*.

2.4. Kharrati-Kopaei and Sadooghi-Alvandi's Test

Kharrati-Kopaei and Sadooghi-Alvandi (2007) suggests that partitioning $a \times b$ data table, under restriction $a \geq b$ and $a \geq 4$, according to the rows, into two sub-tables: the first table consisting of the first a_1 rows and the second sub-table consisting of the remaining $a_2 = a_1 - 1$ rows. Let *RSS1* and *RSS2* denote the residual sum of squares for the two sub-tables, respectively. Then $RSS1/\sigma_\varepsilon^2$ and $RSS2/\sigma_\varepsilon^2$ have independent chi-square distributions with $(a_1-1)(b-1)$ and $(a_2-1)(b-1)$ degree of freedom and non centrality parameters

$$\tau_1 = \sum_{i=1}^{a_1}\sum_{j=1}^{b}\frac{\gamma_{1ij}^2}{\sigma^2}$$ and

$$\tau_2 = \sum_{i=1}^{a_2}\sum_{j=1}^{b}\frac{\gamma_{2ij}^2}{\sigma^2}$$

where γ_{1ij}^2 and γ_{2ij}^2 denote the interaction terms for the two sub-tables, respectively. In the absence of interaction, both τ_1 and τ_2 equal zero and the statistic

$$F = \frac{(a_2-1)\,RSS1}{(a_1-1)\,RSS2} \quad (7)$$

has a F distribution with $(a_1-1)(b-1)$ and $(a_2-1)(b-1)$ degrees of freedom.

For calculation of p-values, it is more convenient to slightly modify the F-statistic (7) and reject the hypothesis of no interaction if

$$F^* = \max(F, 1/F) \quad (8)$$

is too large. Since

$$\Pr(F^* \leq x) = \Pr(F \leq x) - \Pr\left(F \leq \frac{1}{x}\right), \quad x \geq 1$$

the p-value for F^* can easily be calculated from the *F*-distribution. Critical values for this test statistic are given in Kharrati-Kopaei ve Sadooghi-Alvandi (2007).

3. NUMERICAL EXAMPLE

In this example, 73 Aeromonas species were isolated from food and environmental samples. Antibiotic suspectibility was tested with the standard disc diffusion method using CLSI (2008). Levels of antibiotic resistance found in food and environmental isolates, respectively, were 84.2%, 5.3% and 10.5% for cefotaxime (CTX). 29.0%, 2.6% and 68.4% for meropenem (MEM). 89.5%, 2.6% and 7.9% for trimethoprim (W), 39.5%. 57.9% and 2.6% for neomycin (N) (Table 1), (Yucel and Erdogan, 2010).

Table 1. The results of antibiotic susceptibility testing of Aeromonas strains isolated from foods.

	10	15	20
Antibiotics	resistant	middle-sensitive	sensitive
Cefotaxime (CTX)	84.2	5.3	10.5
Meropenem (MEM)	29.0	2.6	68.4
Trimethoprim (W)	89.5	2.6	7.9
Neomycin (N)	39.5	57.9	2.6

Analysis of variance table for the results of this data table is shown below (Table 2).

Table 2. Sum of squares for the results of antibiotic susceptibility testing.

source	degree of freedom	sum of squares
antibiotics	3	81.000
susceptibility	2	4158.27
residual	6	7920.2

Also the matrix of residuals is

$$R=\begin{bmatrix} 23.65 & -11.80 & -11.85 \\ -31.55 & -14.50 & 46.05 \\ 28.95 & -14.50 & -14.45 \\ -21.05 & 40.80 & -19.75 \end{bmatrix}$$

The sum of squared of residuals is tr($R'R$)=7920.20 and the non-zero eigenvalues of $R'R/tr(R'R)$ are $\{l_k\}=$(0.5788 0.4212).

Interaction graph for this data table is given below (Figure 1). Here, three different type of antibiotic susceptibility testing of four parallel lines due to the lack of interaction is thought to exist.

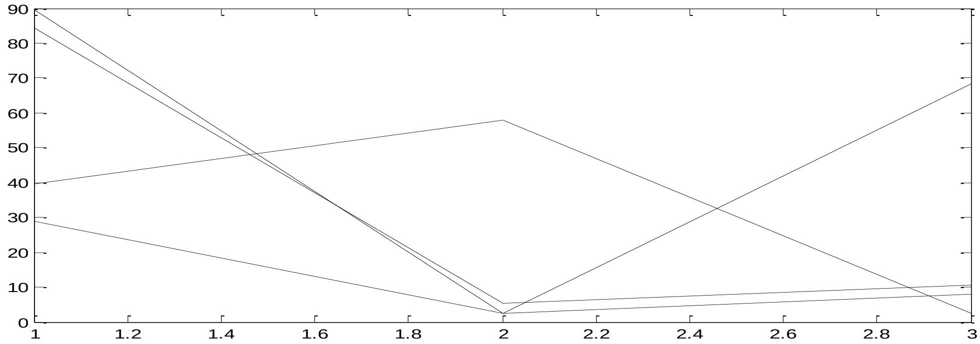

Figure 1. Interaction plot about the results of antibiotic susceptibility testing.

The calculated values for each test statistic and critical values for this data table are given in the table below (Table 3).

Table 3. Calculated test statistics and critical values

Test statistics	Calculated values	Critical values (CV)		
		$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
Boik's Test (LBI)	*LBI*=0.9758	0.5025	0.5128	0.5263
Tusell's Test (LRS)	*LRS*=0.9752	0.0100	0.0500	0.1000
Piepho's Test	T1= 1.0368 (p-value=0.2076)	11.300	7.8147	6.2514
Kharrati-Alvandi's Testi	F*=1.5165 (p-value=0.7948)	0.0033	0.0170	0.0340

According to the above four different test statistics, since the calculated the test statistics values are LBI> CV, LRS> CV, T1 <CV, F* (p-value) > CV at 0.01, 0.05 and 0.10 levels of significance, the hypothesis of no interaction cannot be rejected at that significance level. Thus we may conclude that the data are non additive and the model is additivity model.

4. SIMULATION RESULTS

We considered four additivity tests for testing interaction which does not rely on a specific form for the interactions in two way tables with one replication per cell. These tests proposed by Tusell(1990), Boik(1993a), Piepho(1994) and Kharrati-Alvandi(2007). Piepho (1994) proposed three test statistics. We chose his third test, T1, for our comparisons.

In the conducted simulation study, two-way tables were produced by considering data matrix with model given equation (1). Because choosing μ, α_i and β_j values in this model equation is a constant for the tests, these values were taken as zero. Random error term ε_{ij} was chosen within the range of zero mean and σ_ε^2 variance values increasing with 1 increment at each step from 0 to 20. The interaction term γ_{ij} was produced from the zero mean σ_γ^2 variance normal distribution with rank-r (r=1.2.…p). Rank-r corresponds to the number of smallest linear independent rows or columns. The rest lines or columns depend linearly on each other. This indicates existence of the interaction. σ_ε^2 and σ_γ^2 were chosen at same values. For σ_ε^2's each value, observation values in 10000 two-way tables were produced by adding the errors to the interaction values. For each test statistic, it was determined how many times H_0 null hypothesis indicating there was no interaction was rejected to predict power of each test. Superposition tests were checked at the significance level of $\alpha = 0.05$.

All handled data matrixes were examined separately according to their rank-r structures. For example, the graphics related to each test statistics for the 5×4, 6×4 and 7×4 data matrices are given below, respectively (Figure 2, Figure 3, Figure 4).

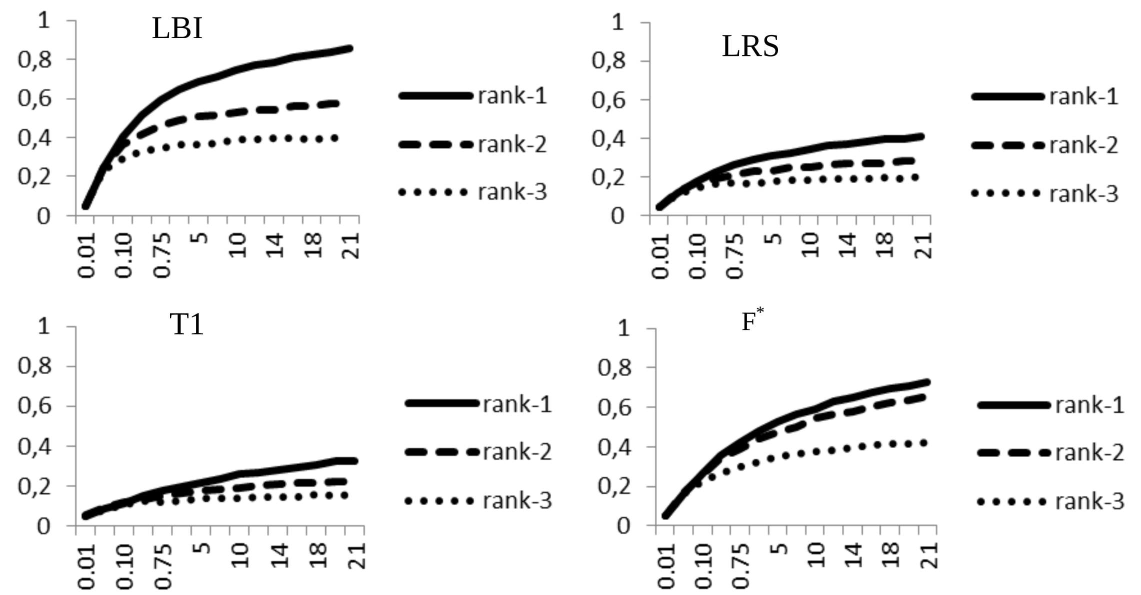

Figure 2. Powers of the LRS, LBI, T1 and F^* tests: $\alpha = 0.05, a = 5, b = 4$.

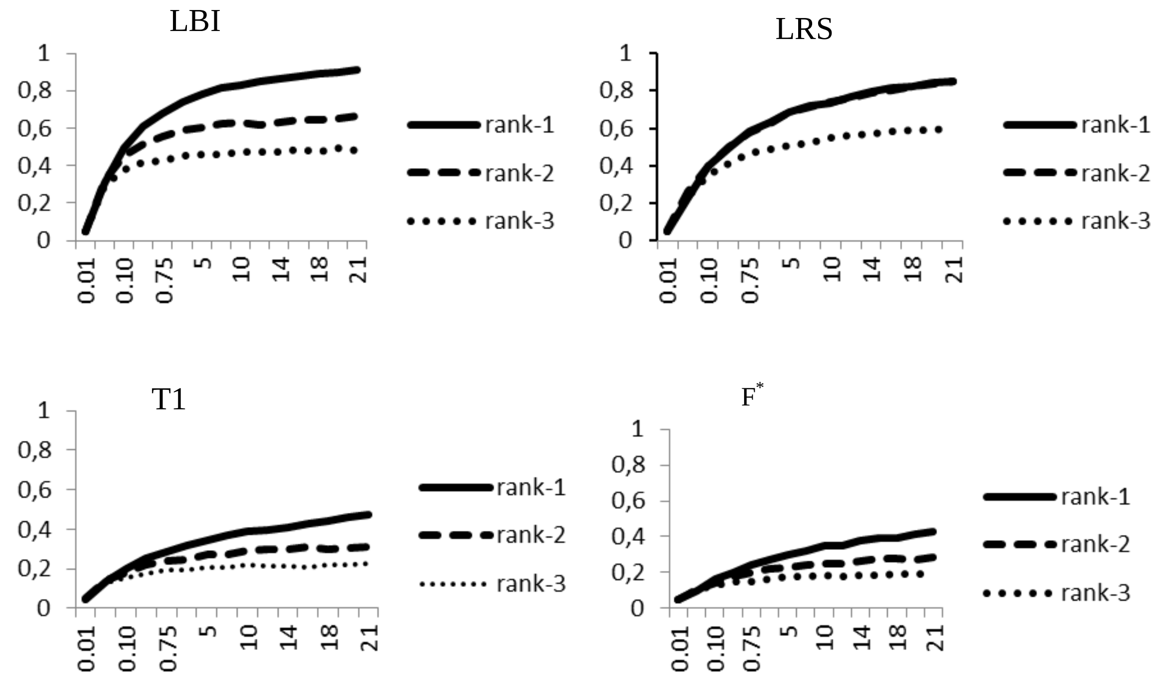

Figure 3. Powers of the LRS, LBI, T1 and F^* tests: $\alpha = 0.05, a = 6, b = 4$.

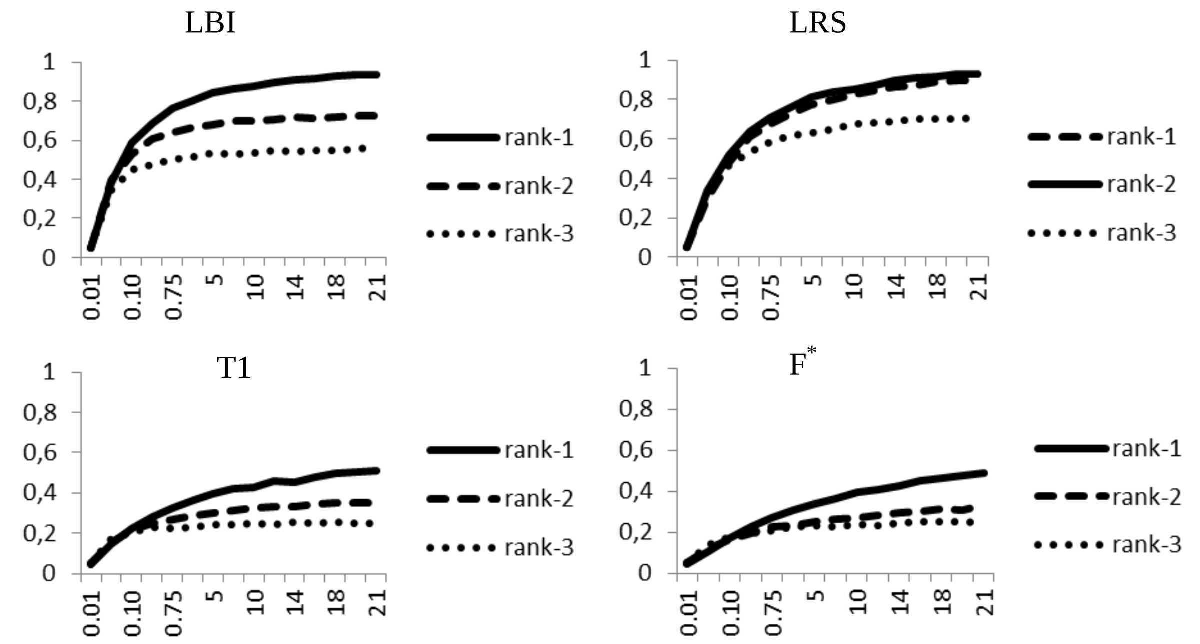

Figure 4. Powers of the LRS. LBI. T1 and F^* tests: $\alpha = 0.05, a = 7, b = 4$.

According to Figure 2, Figure 3 and Figure 4, it was observed that the power curve coincides in LRS test statistics in rank 1 and rank 2 situations. If rank of the interaction matrix was 2, power increased according to the rank 1 under restriction of a≥6, b≥4. If rank of the interaction matrix was 1, power of LBI, T1 and F^* test statistics increased according to the rank 2 under restriction of a≥4, b≥3.

In the following figure, for LRS, LBI, T1 and F^* experimental power functions were obtained at the significance level of $\alpha = 0.05$ for a =7 and b = 4,5,6,7 having the rank=1 structure of the interaction (Figure 5).

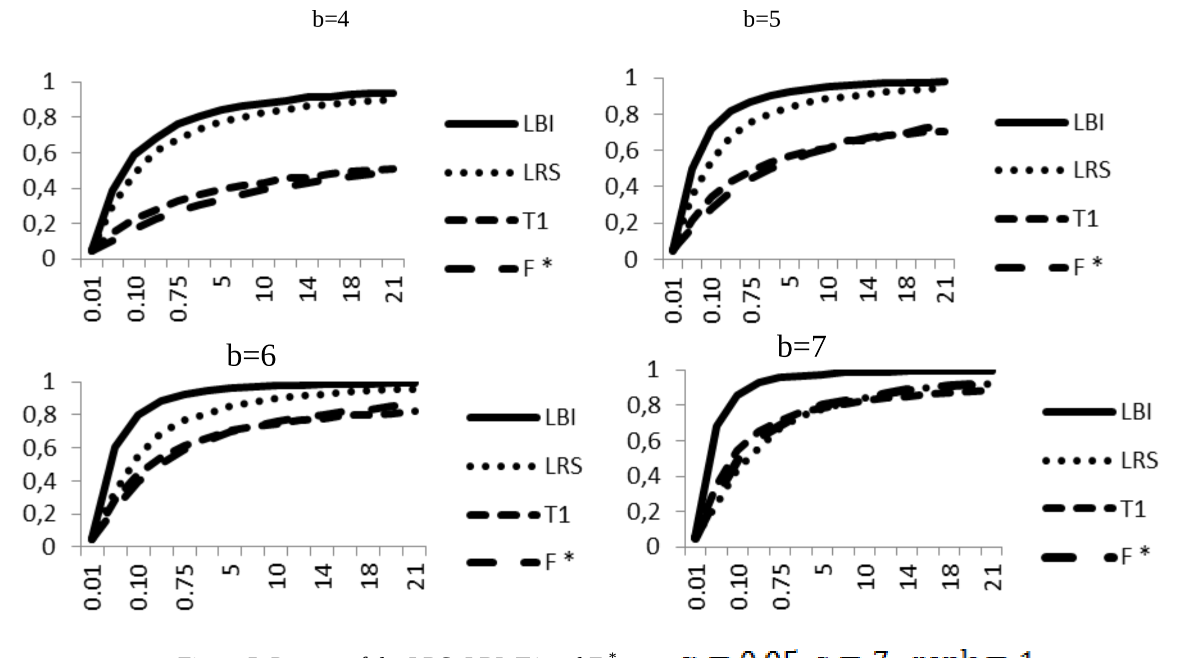

Figure 5. Powers of the LRS. LBI. T1 and F^* tests: $\alpha = 0.05, a = 7, \text{ rank} = 1$.

Table 4 shows power values for all possible situations of LRS, LBI, T1 and F^* test statistics (for σ_ϵ^2=20). Also, number of lines was taken as a≥4 and b=3 was taken as constant. These data matrixes were examined according to their rank-1 and rank-2 structures. As a result, it was seen that if rank of the interaction matrix is 1, LRS and

LBI test statistics were almost equal with regard to power. It was also seen that they yielded better results compared with T1 and F* test statistics with regard to power (Table 4).

Number of lines was taken as a≥4 and b=4 was taken as constant. These data matrixes were examined according to their rank-1, rank-2, and rank-3 structures. As a result, it was seen that if rank of the interaction matrix is 1, LBI test statistic produced better results compared with LRS test statistic with regard to power (Table 4). When the number of columns is constant (b=4), power of all test statistics increases as the number of rows increases. Furthermore, power of T1 statistic produced the better result compared with F* in all data matrixes. Number of rows was taken as a≥4 and b=5 was taken as constant. These data matrixes were examined according to their rank-1, rank-2, rank-3 and rank-4 structures. As a result, it was seen that if rank of the interaction matrix is 1, LBI test statistic produced the better result compared with LRS test statistics. Furthermore, power of F* test statistic is almost same with that of T1 test statistic.

Table 4. Powers of the LRS. LBI. T1 and F* tests for $\sigma_{\varepsilon}^{2} = 20$.

			TUSELL'S LRS TEST		BOIK'S LBI TEST		PIEPHO'S T1 TEST		KHARRATI-ALVANDI'S F* TEST	
a	b	p	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.10$
4	3	1	0.4021	0.5617	0.4098	0.5672	0.1548	0.2652	0.1252	0.2187
		2	0.2555	0.3975	0.2501	0.3866	0.1139	0.1998	0.0967	0.1763
4	4	1	0.4157	0.5860	0.7197	0.8105	0.3632	0.5038	0.2792	0.3974
		2	0.2954	0.4833	0.4375	0.5636	0.2429	0.3637	0.1927	0.2961
		3	0.1863	0.3234	0.2937	0.4100	0.1621	0.2730	0.1351	0.2282
5	3	1	0.6078	0.7184	0.5959	0.7135	0.1889	0.3049	0.1408	0.2573
		2	0.3930	0.5213	0.3836	0.5052	0.1453	0.2411	0.0987	0.1930
5	4	1	0.7290	0.8231	0.8560	0.8954	0.4103	0.5177	0.3269	0.4804
		2	0.6581	0.7964	0.5729	0.6708	0.2853	0.3968	0.2245	0.3553
		3	0.4247	0.5780	0.4002	0.5141	0.2038	0.3083	0.1576	0.2684
5	5	1	0.6778	0.8040	0.9321	0.9467	0.5758	0.6742	0.5370	0.6780
		2	0.6543	0.8212	0.6858	0.7852	0.4056	0.5146	0.3783	0.5205
		3	0.4535	0.6513	0.5148	0.6235	0.2995	0.4143	0.2565	0.3842
		4	0.2826	0.4513	0.4019	0.5167	0.2242	0.3294	0.1847	0.2978
6	3	1	0.7159	0.7979	0.7150	0.7954	0.2173	0.3317	0.1790	0.2951
		2	0.4902	0.5972	0.4875	0.5948	0.1582	0.2532	0.1251	0.2198
6	4	1	0.8494	0.9024	0.9095	0.9340	0.4714	0.5931	0.4306	0.5758
		2	0.8527	0.9271	0.6635	0.7451	0.3142	0.4345	0.2818	0.4180
		3	0.6006	0.7187	0.4822	0.5876	0.2264	0.3350	0.1900	0.3017
6	5	1	0.8823	0.9266	0.9639	0.9762	0.6463	0.7404	0.6530	0.7718
		2	0.9357	0.9721	0.7892	0.8672	0.4864	0.5790	0.4551	0.6030
		3	0.8259	0.9169	0.6234	0.7201	0.3315	0.4497	0.3713	0.4558
		4	0.5816	0.7270	0.4922	0.6067	0.2628	0.3715	0.2364	0.3581
6	6	1	0.8389	0.9074	0.9831	0.9890	0.7619	0.8323	0.8008	0.8811
		2	0.9044	0.9644	0.9001	0.9677	0.4149	0.3147	0.6195	0.7482
		3	0.8142	0.9316	0.7356	0.8334	0.4312	0.5449	0.4322	0.5705
		4	0.5948	0.7786	0.6006	0.7148	0.3395	0.4500	0.3150	0.4490
		5	0.3859	0.5804	0.5013	0.6221	0.2791	0.3881	0.2418	0.3647

7	3	1	0.7797	0.8429	0.7800	0.8421	0.2462	0.3581	0.2080	0.3346
		2	0.5584	0.6493	0.5588	0.6504	0.1770	0.2752	0.1417	0.2439
7	4	1	0.8980	0.9280	0.9349	0.9517	0.5106	0.6269	0.4862	0.6357
		2	0.9307	0.9693	0.7243	0.8053	0.3517	0.4729	0.3293	0.4784
		3	0.7092	0.7997	0.5643	0.6536	0.2444	0.3565	0.2124	0.3379
7	5	1	0.9453	0.9661	0.9780	0.9885	0.7038	0.7885	0.7328	0.8320
		2	0.9850	0.9949	0.8713	0.9415	0.5199	0.6324	0.5316	0.6694
		3	0.9423	0.9759	0.7060	0.8065	0.3644	0.4800	0.3571	0.4898
		4	0.7613	0.8491	0.5749	0.6907	0.2808	0.4009	0.2545	0.3788
7	6	1	0.9582	0.9750	0.9915	0.9941	0.8225	0.8777	0.8618	0.9201
		2	0.9909	0.9956	0.9872	0.9987	0.6451	0.7395	0.7080	0.8226
		3	0.9876	0.9968	0.8428	0.9197	0.4879	0.5981	0.4977	0.6347
		4	0.9178	0.9678	0.7008	0.8190	0.3756	0.4946	0.3692	0.5077
		5	0.7193	0.8319	0.5862	0.7222	0.3112	0.4224	0.2868	0.4170
7	7	1	0.9242	0.9593	0.9953	0.9966	0.8844	0.9239	0.9233	0.9592
		2	0.9793	0.9936	0.9990	0.9998	0.7384	0.8132	0.8247	0.9051
		3	0.9754	0.9940	0.9299	0.9656	0.5785	0.6798	0.6245	0.7502
		4	0.9082	0.9722	0.8158	0.8975	0.4602	0.5756	0.4726	0.6155
		5	0.7173	0.8710	0.7044	0.8078	0.3771	0.4886	0.3678	0.5130
		6	0.5176	0.7002	0.6203	0.7290	0.3148	0.4318	0.2982	0.4429

Powers of the test statistics for each situation were studied according to tables in different sizes, different variance ratios and different significance levels.

In the previous simulation study, the interaction term was chosen depending on a certain rank structure. Another simulation study was conducted to overlook power of the four tests if the interaction term was taken randomly not depending on a certain rank structure. When the structure of the interaction was randomly taken from normal distribution with zero mean and σ_γ^2 variance not depending on a certain rank structure, it was produced for 100 situations. Observation values or data matrix in the 10000 two-way tables for each situation were produced by adding the random errors obtained from the normal distribution with zero mean and σ_ε^2 variance to the interaction term. Error variance was taken as 1 and variance of the interaction term was taken as 5 depending on the error variance (Kharrati-Kopaei and Sadooghi-Alvandi. 2007). Experimental power functions of all test statistics were obtained. For example, for the data matrix of 7×5, powers of the four tests are given in Figure 6 at the significance level of $\alpha = 0.05$ according to 100 situations.

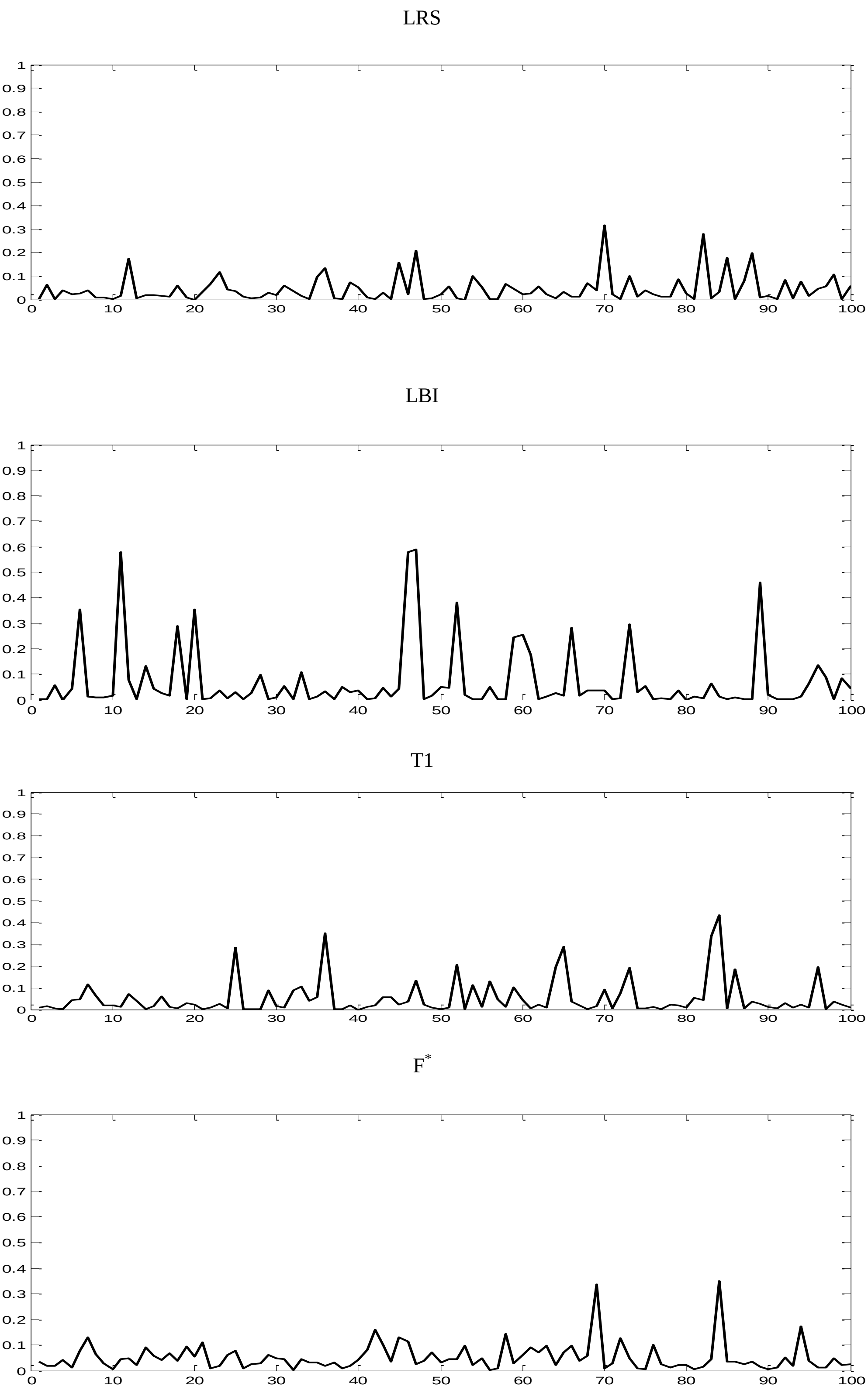

Figure 6. Powers of the LRS. LBI. T1 and F^* tests in 100 cases.

It may be said that the test statistics have power in almost same direction when interactions are randomly produced (Figure 6). However. according to median power of the four tests, it may be said that F^* and LBI test statistics are better in median power (Table 5).

Table 5 . Descriptive statistics in terms of power of all tests.

	mean	variance	median
LRS	0.0524	0.0031	0.0338
LBI	0.0694	0.0169	0.0161
T1	0.0526	0.0068	0.0192
F^*	0.0521	0.0031	0.0377

5. RESULTS

In this article, four different tests were designed to test the interaction in two-way ANOVA models having an observation in each cell. It was observed that if rank structure of interaction was 1, as well as number of lines was $a \geq 4$ and $b \geq 3$, LBI and LRS tests were generally close to each other and yielded better performance compared with T1 and F^* test statistics. Furthermore, all tests in general produced the better result with respect to power according to their rank structures for the interaction matrixes with rank-1. However, rank-2 structures produced the better result with respect to power in comparison with the rank-1 structures LRS test statistics in case of $a \geq 6$ and $b \geq 4$. In case of $a \geq 7$ and $b \geq 5$, T1 and F^* test statistics yielded better results with respect to power compared with other data matrix structures and almost approached to LBI and LRS test statistics with respect to power. It may also be said that all test statistics' performances increase as numbers of lines and columns increase.

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