Gazi University Journal of Science
GU J Sci
26(2): 157-163 (2013)

A General Fixed Point Theorem In Complete G - Metric Spaces For Weakly Compatible Pairs

Valeriu POPA[1], Alina Michaela PATRICIU[1,♠]

Vasile Alecsandri University of Bacau Faculty of Sciences Department of Mathematics ROMANIA

Received: 19.11.2012 Accepted:05.06.2013

ABSTRACT

In this paper a general fixed point theorem in complete G - metric space for weakly compatible pairs of mappings is proved, which generalize the results by Theorems 3.2 and 3.3 [18] and obtained another particular results.

Key words: complete G - metric space, fixed point, weakly compatible mappings, implicit relation.

♠Corresponding author, e-mail:alina.patriciu@ub.ro

1. INTRODUCTION

Let (X,d) be a metric space and S, $T:(X,d)\to(X,d)$ be two mappings. In 1994, Pant [13] introduced the notion of pointwise R - weakly commuting mappings. It is proved in [14] that the notion of pointwise R - weakly commutativity is equivalent to commutativity in coincidence points. Jungck [4] defined S and T to be weakly compatible if $Sx=Tx$ implies $STx=TSx$. Thus, S and T are weakly compatible if and only if S and T are pointwise R - weakly commuting.

In [2], [3] Dhage introduced a new class of generalized metric spaces, named D - metric space. Mustafa and Sims [6], [7] proved that most of the claims concerning the fundamental topological structures on D - metric spaces are incorrect and introduced appropriate notion of generalized metric space, named G - metric space. In fact, Mustafa, Sims and other authors studied many fixed point results for self mappings in G - metric spaces under certain conditions [5] – [12], [17].

Quite recently, Srivastava et al. [18] proved two fixed point theorems for weakly compatible mappings in complete G - metric spaces.

In [15] and [16], Popa initiated the study of fixed points for mappings satisfying implicit relations.

The purpose of this paper is to prove a general fixed point theorem in G - metric spaces for weakly compatible pairs of mappings satisfying an implicit relation which generalize the results from Theorems 3.2 and 3.2 [18].

2. PRELIMINARIES

Definition 2.1 [7] *Let X be a nonempty set and $G:X^3\to\mathbf{R}_+$ be a function satisfying the following properties:*

$(G_1): G(x,y,z)=0$ if $x=y=z$,

$(G_2): 0<G(x,x,y)$ for all $x,y\in X$ with $x\neq y$,

$(G_3): G(x,x,y)\le G(x,y,z)$ for all $x,y,z\in X$ with $z\neq y$,

$(G_4): G(x,y,z)=G(y,z,x)=G(z,x,y)=...$ (symmetry in all three variables),

$(G_5): G(x,y,z)\le G(x,a,a)+G(a,y,z)$ for all $x,y,z,a\in X$ (rectangle inequality).

Then the function G is called a G - metric on X and the pair (X,G) is called a G - metric space.

Note that $G(x,y,z)=0$, then $x=y=z$.

Definition 2.2 [7] *Let (X,G) be a G - metric space. A sequence (x_n) in X is said to be*

a) G - convergent if for $\varepsilon>0$, there exists an $x\in X$ and $k\in\mathbf{N}$ such that for all $m,n\ge k$, $G(x,x_n,x_m)<\varepsilon$,

b) G - Cauchy if for each $\varepsilon>0$, there exists $k\in\mathbf{N}$ such that for all $n,m,p\ge k$, $G(x_n,x_m,x_p)<\varepsilon$, that is $G(x_n,x_m,x_p)\to 0$ as $n,m,p\to\infty$.

c) A G - metric space is said to be G - complete if every G - Cauchy sequence is G - convergent.

Lemma 2.1 [7] *Let (X,G) be a G - metric space. Then, the following properties are equivalent:*

1) (x_n) is G - convergent to x;

2) $G(x_n,x_n,x)\to 0$ as $n\to\infty$;

3) $G(x_n,x,x)\to 0$ as $n\to\infty$;

4) $G(x_n,x_m,x)\to 0$ as $m,n\to\infty$.

Lemma 2.2 [7] *If (X,G) is a G - metric space and $(x_n)\in X$, then the following properties are equivalent:*

1) (x_n) is G - Cauchy;

2) For every $\varepsilon>0$, there exists $k\in\mathbf{N}$ such that $G(x_n,x_m,x_m)<\varepsilon$ for all $n,m\ge k$.

Lemma 2.3 [7] 1 *Let (X,G) be a G - metric space, then the function $G(x,y,z)$ is jointly continuous in all three of its variables.*

Lemma 2.4 [7] *Let (X,G) be a G - metric space. Then $G(x,y,y)\le 2G(y,x,x)$ for all $x,y\in X$.*

Quite recently, the following theorems are proved in [18].

Theorem 2.1 *Let (X,G) be a complete G - metric space and let $S,T:X\to X$ be two mappings which satisfy the following conditions:*

(i) $T(X)\subset S(X)$,

(ii) $T(X)$ or $S(X)$ is G - complete, and

(iii) $G(Tx,Ty,Tz)\le\alpha G(Sx,Sy,Sz)+\beta G(Tx,Sx,Sx)+\gamma G(Ty,Sy,Sy)+\delta G(Tz,Sz,Sz)+\eta G(Tx,Sy,Sy)$,

for all $x,y,z\in X$, where $\alpha,\beta,\gamma,\delta,\eta\ge 0$ and $\alpha+2\beta+2\gamma+2\delta+2\eta<1$.

Then S and T have an unique point of coincidence in X. Moreover, if S and T are weakly compatible, then S and T have an unique common fixed point.

Theorem 2.2 *Let (X,G) be a complete G - metric space and let $S,T:X\to X$ be two mappings which satisfy the following conditions:*

(i) $T(X)\subset S(X)$,

(ii) $T(X)$ or $S(X)$ is G - complete, and

$$\text{(iii) } G(Tx,Ty,Tz)\le \alpha\max\{G(Sx,Sy,Sz),G(Tx,Sx,Sx),G(Ty,Sy,Sy),G(Tz,Sz,Sz),(Tx,Sy,Sy)\},$$

for all $x,y,z\in X$, where $\alpha\in\left(0,\frac{1}{2}\right)$.

Then S and T have an unique point of coincidence in X. Moreover, if S and T are weakly compatible, then S and T have an unique common fixed point in X.

3. IMPLICIT RELATIONS

Definition 3.1 [2] *Let $\mathcal{F}_S$ be the set of all continuous functions $F(t_1,...,t_5):\mathbf{R}_+^5\to\mathbf{R}$ satisfying the following conditions:*

(F_1) F is nonincreasing in variables t_3 and t_4,

(F_2) There exists $h\in[0,1)$ such that for all $u,v\ge 0$, $F(u,v,2v,2u,0)\le 0$ implies $u\le hv$,

(F_3) $F(t,t,0,0,t)>0$, $\forall t>0$.

Example3.1
$F(t_1,...,t_5)=t_1-at_2-bt_3-(c+d)t_4-et_5$, *where* $a,b,c,d,e\ge 0$ *and* $a+2b+2c+2d+e<1$.

(F_1): Obviously.

(F_2): Let $u,v\ge 0$ be and $F(u,v,2v,2u,0)=u-av-2bv-2(c+d)u\le 0$. Then, $u\le hv$ where $0\le h=\frac{a+2b}{1-2(c+d)}$.

(F_3): $F(t,t,0,0,t)=t(1-(a+e))>0,\forall t>0$.

Example 3.2 $F(t_1,...,t_5)=t_1-k\max\{t_2,t_3,t_4,t_5\}$, *where* $k\in\left[0,\frac{1}{2}\right)$.

(F_1): Obviously.

(F_2): Let $u,v\ge 0$ be and $F(u,v,2v,2u,0)=u-k\max\{v,2v,2u\}\le 0$. If $u>v$, then $u(1-2k)\le 0$, a contradiction. Hence $u\le v$ and $u\le hv$, where $0\le h=2k<1$.

(F_3): $F(t,t,0,0,t)=t(1-k)>0,\forall t>0$.

Example3.3
$F(t_1,...,t_5)=t_1^2-t_1(at_2+bt_3+ct_4)-dt_5^2$, *where* $a,b,c,d\ge 0$, $a+2b+2c<1$ *and* $a+d<1$.

(F_1): Obviously.

(F_2): Let $u,v\ge 0$ be and $F(u,v,2v,2u,0)=u^2-u(av+2bv+2cu)\le 0$. If $u>0$, then $u-av-2bv-2cu\le 0$ which implies $u\le hv$, where $0\le h=\frac{a+2b}{1-2c}<1$. If $u=0$ then $u\le hv$.

(F_3): $F(t,t,0,0,t)=t^2(1-(a+d))>0,\forall t>0$.

Example 3.4 $F(t_1,...,t_5)=t_1-a\frac{t_2+t_3}{2}-b\frac{t_4+t_5}{2}$, *where* $a,b\ge 0$ *and* $3a+2b<2$.

(F_1): Obviously.

(F_2): Let $u,v\ge 0$ be and $F(u,v,2v,2u,0)=u-a\frac{3v}{2}-bu\le 0$. Hence $u\le hv$, where $0\le h=\frac{3a}{2-2b}<1$.

(F_3): $F(t,t,0,0,t)=t\left(1-\frac{a+b}{2}\right)>0,\forall t>0$.

Example 3.5 $F(t_1,...,t_5)=t_1^2-at_2^2-b\frac{t_3^2+t_4^2}{1+t_5^2}$, *where* $a+8b<1$.

(F_1): Obviously.

(F_2): Let $u,v\ge 0$ be and $F(u,v,2v,2u,0)=u^2-av^2-(4u^2+4v^2)b\le 0$ which implies $u\le hv$, where $0\le h=\sqrt{\frac{a+4b}{1-4b}}$.

(F_3): $F(t,t,0,0,t)=t^2(1-a)>0,\forall t>0$.

Example3.6
$F(t_1,...,t_5)=t_1-at_2-bt_3-c\min\{t_4,t_5\}$, *where* $a,b,c\ge 0$ *and* $a+2b<1$.

(F_1): Obviously.

(F_2): Let $u, v \geq 0$ and $F(u,v,2v,2u,0) = u - av - 2bv \leq 0$ which implies $u \leq hv$, where $0 \leq h = a + 2b < 1$.

(F_3): $F(t,t,0,0,t) = t(1-a) > 0, \forall t > 0$.

Example 3.7 $F(t_1,...,t_5) = t_1 - c\max\{t_2, t_3, \sqrt{t_4 t_5}\}$, *where* $c \in \left(0, \frac{1}{2}\right)$.

(F_1): Obviously.

(F_2): Let $u, v \geq 0$ and $F(u,v,2v,2u,0) = u - 2cv \leq 0$, which implies $u \leq hv$, where $0 \leq h = 2c < 1$.

(F_3): $F(t,t,0,0,t) = t(1-c) > 0, \forall t > 0$.

Example3.8
$$F(t_1,...,t_5) = t_1 - k\max\left\{t_2, t_3, \frac{t_3 + 2t_4}{2}, \frac{t_4 + 2t_5}{2}\right\},$$
where $k \in \left(0, \frac{1}{4}\right)$.

(F_1): Obviously.

(F_2): Let $u, v \geq 0$ and

$$F(u,v,2v,2u,0) = u - k\max\left\{v, 2v, \frac{2v + 4u}{2}, u\right\} = u - k\max\{2v, v + 2u\} \leq 0$$

, which implies $u \leq 2k(u+v)$. Hence $u \leq hv$, where $0 \leq h = \frac{2k}{1-2k} < 1$.

(F_3): $F(t,t,0,0,t) = t(1-k) > 0, \forall t > 0$.

4. MAIN RESULTS

Definition 4.1 *Let* S *and* T *two self mappings of a nonempty set* X. *If* $w = Tx = Sx$ *for some* $x \in X$, *then* x *is called a coincidence point of* S *and* T *and* w *is called a point of coincidence of* T *and* S.

Lemma 4.1 [1] *Let* T *and* S *be weakly compatible self mappings of a nonempty set* X. *If* T *and* S *have an unique point of coincidence* $w = Tx = Sx$, *then* w *is the unique common fixed point of* T *and* S.

Theorem 4.1 *Let* (X, G) *be a* G *- metric space and* T, S *self mappings of* X *such that*

$$F(G(Tx,Ty,Ty), G(Sx,Sy,Sy), G(Tx,Sx,Sx), G(Ty,Sy,Sy), G(Tx,Sy,Sy)) \leq 0 \quad (4.1)$$

for all $x, y \in X$ *and* F *satisfying property* (F_3). *Then* T *and* S *have at most a point of coincidence.*

Proof. Suppose that $u = Tp = Sp$ and $v = Tq = Sq$ are two distinct points of coincidence. Then, by (4.1) we have successively:

$$F(G(Tq,Tp,Tp), G(Sq,Sp,Sp), G(Tq,Sq,Sq), G(Tp,Sp,Sp), G(Tq,Sp,Sp)) \leq 0,$$

$$F(G(Sq,Sp,Sp), G(Sq,Sp,Sp), 0, 0, G(Sq,Sp,Sp)) \leq 0,$$

a contradiction of (F_3) if $G(Sq,Sp,Sp) > 0$. Hence $G(Sq,Sp,Sp) = 0$, so $Sq = Sp$ which implies $u = v$.

Theorem 4.2 *Let* (X, G) *be a* G *- metric space and let* $T, S : (X,G) \to (X,G)$ *be two mappings such that*

2. *(i)* $T(X) \subset S(X)$,

(ii) $T(X)$ *or* $S(X)$ *is* G *- complete,*

(iii) T *and* S *satisfy the inequality (4.1) for all* $x, y \in X$ *and* $F \in \mathcal{F}_S$.

Then T *and* S *have an unique point of coincidence. Moreover, if* T *and* S *are weakly compatible, then* T *and* S *have an unique common fixed point.*

Proof. Let $x_0 \in X$ be an arbitrary point. Then, there exists $x_1 \in X$ such that $Tx_0 = Sx_1$. In this way we defined a sequence $\{Sx_n\}$ with $Tx_{n-1} = Sx_n$ for $n = 1,2,...$. Then by (4.1) we have successively:

$$F(G(Tx_{n-1},Tx_n,Tx_n), G(Sx_{n-1},Sx_n,Sx_n), G(Tx_{n-1},Sx_{n-1},Sx_{n-1}), G(Tx_n,Sx_n,Sx_n), G(Tx_{n-1},Sx_n,Sx_n)) \leq 0,$$

$$F(G(Sx_n,Sx_{n+1},Sx_{n+1}), G(Sx_{n-1},Sx_n,Sx_n), G(Sx_n,Sx_{n-1},Sx_{n-1}), G(Sx_{n+1},Sx_n,Sx_n), 0) \leq 0.$$

By Lemma 2.4

$G(Sx_{n+1},Sx_n,Sx_n) \leq 2G(Sx_n,Sx_{n+1},Sx_{n+1})$ and

$G(Sx_n, Sx_{n-1}, Sx_{n-1}) \le 2G(Sx_{n-1}, Sx_n, Sx_n)$. By (F_1) we obtain:

$$F(G(Sx_n, Sx_{n+1}, Sx_{n+1}), G(Sx_{n-1}, Sx_n, Sx_n), 2G(Sx_{n-1}, Sx_n, Sx_n), 2G(Sx_n, Sx_{n+1}, Sx_{n+1}), 0) \le 0$$

which implies by (F_2) that

$G(Sx_n, Sx_{n+1}, Sx_{n+1}) \le hG(Sx_{n-1}, Sx_n, Sx_n)$.

By repeated application of the above inequality, we have

$G(Sx_n, Sx_{n+1}, Sx_{n+1}) \le h^n G(Sx_0, Sx_1, Sx_1)$.

Then for $n, m \in \mathbf{N}$, $n < m$, we have by rectangle inequality that

$$\begin{aligned} G(Sx_n, Sx_m, Sx_m) &\le G(Sx_n, Sx_{n+1}, Sx_{n+1}) + G(Sx_{n+1}, Sx_{n+2}, Sx_{n+2}) + \\ &\quad + \ldots + G(Sx_{m-1}, Sx_m, Sx_m) \\ &\le (h^n + h^{n+1} + \ldots + h^{m-1}) G(Sx_0, Sx_1, Sx_1) \\ &\le \frac{h^n}{1-h} G(Sx_0, Sx_1, Sx_1). \end{aligned}$$

Taking limit as $n, m \to \infty$, we get $\lim_{n,m\to\infty} G(Sx_n, Sx_m, Sx_m) = 0$. Hence $\{Sx_n\}$ is a G - Cauchy sequence. Now, since $S(X)$ is G - complete, there exists a point $q \in S(X)$ such that $Sx_n \to q$ as $n \to \infty$. Consequently, we can find a point $p \in X$ such that $Sp = q$.

If $T(X)$ is G - complete, there exists $q \in T(X)$ such that $Sx_n \to q$ as $T(X) \subset S(X)$, we have $q \in Sx$. Then, there exists $p \in X$ such that $Sp = q$.

We prove that p is a coincidence point for T and S. By (4.1) we have successively:

$$F(G(Tx_{n-1}, Tp, Tp), G(Sx_{n-1}, Sp, Sp), G(Tx_{n-1}, Sx_{n-1}, Sx_{n-1}), G(Tp, Sp, Sp), G(Tx_{n-1}, Sp, Sp)) \le 0,$$

$$F(G(Sx_n, Tp, Tp), G(Sx_{n-1}, Sp, Sp), G(Sx_n, Sx_{n-1}, Sx_{n-1}), G(Tp, Sp, Sp), G(Sx_n, Sp, Sp)) \le 0.$$

Letting n tend to infinity, we obtain

$F(G(Sp, Tp, Tp), 0, 0, G(Tp, Sp, Sp), 0) \le 0$.

By Lemma 2.4, $G(Tp, Sp, Sp) \le 2G(Sp, Tp, Tp)$.

By (F_1) we obtain $F(G(Sp, Tp, Tp), 0, 0, 2G(Sp, Tp, Tp), 0) \le 0$.

By (F_2), $G(Sp, Tp, Tp) = 0$ which implies $w = Tp = Sp$ and p is a coincidence point of T and S. By Theorem 4.1, w is the unique point of coincidence of T and S. Moreover, if T and S are weakly compatible, by Lemma 4.1 w is the unique common fixed point of T and S.

If $S(X)$ is complete, the proof it follows by $T(X) \subset S(X)$.

Corollary 4.1 *Let T and S be self mappings of a G - metric space satisfying the following conditions:*

(i) $T(X) \subset S(X)$,

(ii) $S(X)$ *or* $T(X)$ *is* G *- complete,*

(iii) One of the following inequalities hold for all $x, y \in X$ *(1)*

$G(Tx, Ty, Ty) \le aG(Sx, Sy, Sy) + bG(Tx, Sx, Sx) + (c+d)G(Ty, Sy, Sy) + eG(Tx, Sy, Sy)$, *(3)*

where $a, b, c, d, e \ge 0$ *and* $a + 2b + 2c + 2d + e < 1$. *(2)*

$G(Tx, Ty, Ty) \le k \max\{G(Sx, Sy, Sy), G(Tx, Sx, Sx), G(Ty, Sy, Sy), G(Tx, Sy, Sy)\}$, *where* $k \in \left(0, \frac{1}{2}\right)$. *(3)*

$$G^2(Tx,Ty,Ty) \le G(Tx,Ty,Ty)[aG(Sx,Sy,Sy) + bG(Tx,Sx,Sx) + cG(Ty,Sy,Sy)] + dG^2(Tx,Sy,Sy),$$

where $a,b,c,d \ge 0$, $a + 2b + 2c < 1$ *and* $a + d < 1$. *(4)*

$$G(Tx,Ty,Ty) \le a\frac{G(Sx,Sy,Sy) + G(Tx,Sx,Sx)}{2} + b\frac{G(Ty,Sy,Sy) + G(Tx,Sy,Sy)}{2},$$

where $a,b \ge 0$ *and* $3a + 2b < 2$. *(5)*

$$G^2(Tx,Ty,Ty) \le aG^2(Sx,Sy,Sy) + b\frac{G^2(Tx,Sx,Sx) + G^2(Ty,Sy,Sy)}{1 + G^2(Tx,Sy,Sy)},$$

where $a,b \ge 0$ *and* $a + 8b < 1$. *(6)*

$$G(Tx,Ty,Ty) \le aG(Sx,Sy,Sy) + bG(Tx,Sx,Sx) + c\min\{G(Ty,Sy,Sy), G(Tx,Sy,Sy)\},$$

where $a,b,c \ge 0$ *and* $a + 2b < 1$. *(7)*

$$G(Tx,Ty,Ty) \le c\max\{G(Sx,Sy,Sy), G(Tx,Sx,Sx), [G(Ty,Sy,Sy)\cdot G(Tx,Sy,Sy)]^{1/2}\},$$

where $c \in \left(0, \frac{1}{2}\right)$. *(8)*

$$G(Tx,Ty,Ty) \le k\max\{G(Sx,Sy,Sy), G(Tx,Sx,Sx), \frac{1}{2}[G(Tx,Sx,Sx) + 2G(Ty,Sy,Sy)], \frac{1}{2}[G(Ty,Sy,Sy) + 2G(Tx,Sy,Sy)]\},$$

where $k \in \left(0, \frac{1}{4}\right)$.

If S *and* T *are weakly compatible, then* S *and* T *have an unique common fixed point.*

Proof. The proof follows by Theorem 4.2 and Examples 3.1 – 3.8.

Remark 4.1 *Because in Theorem 2.1 and* $a + 2b + 2c + 2d + 2e < 1$, *for* $y = z$ *we obtain*

$$G(Tx,Ty,Ty) \le aG(Sx,Sy,Sy) + bG(Tx,Sx,Sx) + (c+d)G(Ty,Sy,Sy) + eG(Tx,Sy,Sy)$$

and $a + 2b + 2c + 2d + e < 1$, *Theorem 2.1 follows from Corollary 4.1 (iii) (1).*

Remark 4.2 *Because in Theorem 2.2 for* $y = z$ *we obtain*

$$G(Tx,Ty,Ty) \le k\max\{G(Sx,Sy,Sy), G(Tx,Sx,Sx), G(Ty,Sy,Sy), G(Tx,Sy,Sy)\},$$

and Theorem 2.2 follows from Corollary 4.1 (iii) (2).

REFERENCES

[1] Abbas, M. and Rhoades, B.E., "Common fixed point results for noncommuting mappings without continuity in generalized metric spaces", ***Appl. Math. and Computation***, 215: 262 – 269, (2009).

[2] Dhage, B.C., "Generalized metric spaces and mappings with fixed points", ***Bull. Calcutta Math. Soc***., 84: 329 – 336, (1992).

[3] Dhage, B.C., "Generalized metric spaces and topological structures I", ***Anal. St. Univ. Al. I. Cuza, Iasi Ser. Mat.***, 46(1): 3 – 24, (2000).

[4] Jungck, G., "Common fixed points for noncontinuous, nonself maps on nonnumeric spaces", ***Far East J. Math. Sci.***, 4(2): 195 – 215, (1996).

[5] Manro, S., Bahtia, S.S. and Kumar, S., "Expansive mappings theorems in G - metric spaces", ***Intern. J. Contemp. Math. Sci.***, 5(51): 2529 – 2535, (2010).

[6] Mustafa, Z. and Sims, B., "Some remarks concerning D - metric spaces", ***Intern. Conf. Fixed Point Theory and Applications***, Yokohama, 189 – 198, (2004).

[7] Mustafa, Z. and Sims, B., "A new approach to generalized metric spaces", ***J. Nonlinear Convex Analysis***, 7: 289 – 297, (2006).

[8] Mustafa, Z., Obiedat, H. and Awadeh, F., "Some fixed point theorems for mappings on G - complete metric spaces", ***Fixed Point Theory and Applications***, Article ID 189870, 12, (2008).

[9] Mustafa, Z., Shatanawi, W. and Bataineh, M., "Fixed point theorem on uncomplete G - metric spaces", ***J. Math. Statistics***, 4(4): 196 – 201, (2008).

[10] Mustafa, Z. and Sims, B., "Fixed point theorems for contractive mappings in complete G - metric spaces", ***Fixed Point Theory and Applications***, Article ID 917175, 10, (2009).

[11] Mustafa, Z. and Obiedat, H., "A fixed point theorem of Reich in G - metric spaces", ***Cubo A. Math. J.***, 12: 83 – 93, (2010).

[12] Obiedat, H. and Mustafa, Z., "Fixed point results on a nonsymmetric G - metric space", ***Jordan S. Math. Statistics***, 3(2): 65 – 79, (2010).

[13] Pant, R. P., "Common fixed point for noncommuting mappings", ***J. Math. And Appl.***, 188: 436 – 440, (1994).

[14] Pant, R. P., "Common fixed point for four mappings", ***Bull. Calcutta Math. Soc***., 9: 281 – 286, (1998).

[15] Popa, V., "Fixed point theorems for implicit contractive mappings", ***Stud. Cerc. St. Ser. Mat., Univ. Bacău***, 7: 129 – 133, (1997).

[16] Popa, V., "Some fixed point theorems for compatible mappings satisfying an implicit relation", ***Demonstratio Math.***, 32(1): 157 – 163, (1999).

[17] Shatanawi, W., "Fixed point theory for contractive mappings satisfying ϕ - maps in G - metric spaces", ***Fixed Point Theory and Applications***, Article ID 181650, 9, (2010).

[18] Srivastava, R., Agrawal, S., Bhardwaj, R., Yadava, R. N., "Fixed point theorems in complete G - metric spaces", ***South Asian J. Math.***, 2(2): 167 – 174, (2012).