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THE ANALYTIC HIERARCHY AND ANALYTIC NETWORK PROCESSES

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Abstract

The Analytic Hierarchy Process (AHP) is a method for decision making which includes qualitative factors. In this method, ratio scales are obtained from ordinal scales which are derived from individual judgments for qualitative factors using the pairwise comparison matrix. The Analytic Network Process (ANP) also uses a pairwise comparison matrix to obtain ratio scales. The difference between these two methods appears in modelling the problem and computing the final priorities for the alternatives from ratio scales previously obtained. This paper gives a brief look at the foundation of AHP and ANP. Furthermore, an illustrative example is given for ANP.

Keywords: Decision making, Analytic network process (ANP), Analytic hierarchy process (AHP), Feedback model.

1. Introduction

Decision making is an essential part of almost all human life. Both administrators or others must make real life decisions. Socio-economical, political, cultural and psychological factors should be taken into account in the solution of many decision problems. These factors are generally ignored by many decision making methods since they are expressed by qualitative variables. However, qualitative factors may be just as important, or even more important, than the quantitative factors. For this reason, it is clear that we need a systematic and comprehensive approach to decision making in which both quantitative and qualitative variables can be included in the evaluation.

Since the 1960' s, many studies concerning multi-criteria decision making have been made. In the late 1960' s, Thomas Saaty, one of the pioneers of Operations Research, developed a decision making method in which qualitative variables can also be included in the evaluation. This is the Analytic Hierarchy Process (AHP), which is a general theory of measurement.

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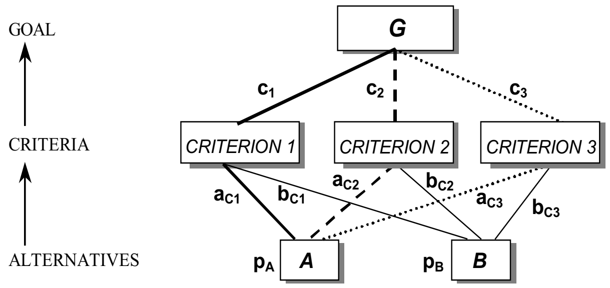
2. The Analytic Hierarchy Process

AHP allows decision makers to model a complex problem in a hierarchical structure (Figure 1). In this method, a simple hierarchical model consists of a goal, criteria and alternatives. In Figure 1, the hierarchical structure shows the relationships of the goal, criteria and alternatives from the top to the bottom. AHP copes with using original data, experience and intuition in the same model in a logical and through way [2]. AHP is composed of several previously existing but unassociated concepts and techniques, such as hierarchical structuring, pairwise comparisons, the eigenvector method for deriving weights and consistency considerations [2]. According to Saaty this method has three phases: decomposition, comparative judgment and synthesizing [7].

Figure 1. A simple hierarchical model



In the decomposition phase, the elements of the decision problem are arranged in the form of a hierarchy. The top element of the hierarchy is the overall goal of the decision making. In the next level, which is known as the cluster, there are general criteria which impact the goal directly. The hierarchy descends from the general to the more particular, until a level of operational sub-criteria is reached, against which the decision alternatives of the lowest level of the hierarchy can be evaluated. The hierarchical structure of the basic AHP allows dependencies among elements to be only between the levels of the hierarchy, and the only possible direction of impact is toward the top of the hierarchy. This eliminates the possibility of including feedback relations in the model. Also the elements of a given level are assumed to be mutually independent [3].

In the comparative judgment phase, elements of one level of a hierarchy are compared pairwise as to the strength of their influence on an element of the next higher level. It is not always possible to find a standard scale for comparing elements with each other. These comparisons may be taken from actual measurements (absolute measurement) or from a fundamental scale that reflects the relative strength of preferences and feelings [8]. Saaty has suggested a scale of 1 to 9 when comparing two elements, with a score of 1 representing indifference between the two elements and 9 representing the overwhelming dominance of that element over the other [5]. Forman has also stated that using a scale of 1 to 9 is logical because the human brain is limited in both its short term memory capacity and its discrimination ability (channel capacity) to about seven to nine things. These comparisons lead to dominance matrices which are called *pairwise comparison matrices*. Ratio scales are derived in the form of principle eigenvectors from these matrices [6]. In this operation, firstly the largest eigenvalue and its related eigenvector are computed. The normalized form of this eigenvector is what we are looking for. The elements of this vector give the local priorities as ratio scales representing the relative importance of the elements on the same level over an element of the next higher level with respect to a common attribute.

The next phase is to synthesize the priorities. A simple example can be given to explain how the resulting priorities of the alternatives are established. Figure 1 gives a simple hierarchical model which evaluates two alternatives A and B, with respect to CRITERION1, CRITERION2 and CRITERION3 of the overall goal G. In the same figure it is seen that, with respect to G, the weight of CRITERION1 is c_1, the weight of CRITERION2 is c_2 and that of CRITERION3 is c_3. The priorities of alternatives A and B with respect to CRITERION1 are a_{C1} and b_{C1}, respectively. In the same way, with respect to CRITERION2 and CRITERION3 the priorities of these alternatives are a_{C2}, b_{C2}, a_{C3} and b_{C3}, respectively. The resulting priorities of the alternatives are denoted by p_A and p_B. These weights and priority values are calculated from four different pairwise comparison matrices. In the first matrix, the criteria are compared with respect to G and the weights c_1, c_2 and c_3 are calculated. In the second matrix the alternatives A and B are compared with respect to CRITERION1, and a_{C1}, b_{C1} are calculated. With the third and fourth matrices the priorities a_{C2}, b_{C2}, a_{C3} and b_{C3} are calculated for the alternatives A and B with respect to CRITERION2 and CRITERION3, respectively [1].

It is shown in Figure 1 that the priority of alternative A has been obtained from three different contributions which are represented by the solid, dashed and dotted lines. The priority of the alternative A is computed by summing these contributions. Consequently, the resulting priorities of alternatives A and B are obtained from the following matrix product:

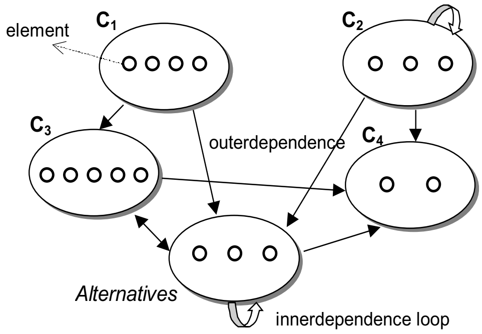
$$\begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} a_{C1} & b_{C1} \\ a_{C2} & b_{C2} \\ a_{C3} & b_{C3} \end{bmatrix} = \begin{bmatrix} p_A & p_B \end{bmatrix}$$

As it is seen in this equation AHP has a linear structure in synthesizing priorities but the ANP has a nonlinear structure [9].

3. The Analytic Network Process

Many decision problems cannot be structured hierarchically when the interaction of higher level elements with lower level elements and their dependency should be taken into account. The ANP provides a solution for problems which cannot be structured hierarchically. Not only does the importance of the criteria determine the importance of the alternatives, as in a hierarchy, the importance of the alternatives themselves determine the importance of the criteria [8]. Therefore, a good many problems can be modelled using a diagram called a "network," as presented in Figure 2.

Figure 2. Feedback Network

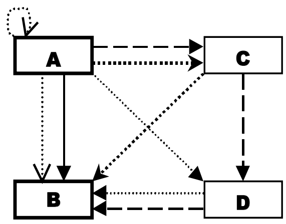


Network models do not have to show a hierarchical structure, which means they do not have to be linear from the top to the bottom. In fact the ANP uses a network for which it is not necessary to specify levels, as in a hierarchy [9]. Therefore the term *level* in AHP is replaced by the term *cluster* in ANP. The network model has cycles connecting its clusters of elements and loops that connect a cluster to itself. This kind of model are called systems-with-feedback [4]. In practice, many decision problems involve feedback. We need a tool to manage complexity resulting from feedback [8]. ANP is a tool which can meet this necessity by enabling a systematic and comprehensive approach.

Although ANP and AHP are similar in the comparative judgement phase, there are differences in the synthesizing phase. In the ANP, ratio scale priority vectors derived from pairwise comparison matrices are not synthesized linearly as in AHP. Saaty has an improved "supermatrix" technique to synthesize ratio scales. Each ratio scale is appropriately introduced as a column in a matrix to represent the impact of elements in a cluster on an element in another cluster (outerdependence) or on elements of the cluster itself (innerdependence). In that case, the supermatrix is composed of several submatrices, each of whose columns is a principal eigenvector that represents the impact of all elements in a cluster on each of the elements in another (or the same) cluster. There is no requirement that every element of a cluster has an influence on an element in another cluster. In such a case, these elements are given a zero value for their contribution. The supermatrix, which is composed of ratio scale priority vectors derived from pairwise comparison matrices and the zero vectors, must be stochastic to obtain meaningful limiting results. The supermatrix has clusters. Each block of column vectors are weighted by the priority of the corresponding cluster, with their elements displayed vertically on the left side of the matrix and horizontally at the top of the matrix. To ensure that this matrix is stochastic we need to compare clusters themselves that are on the left with respect to their impact on each cluster at the top. The resulting priorities of the clusters are then used to weight column vector clusters on the left with respect to the corresponding cluster on the top. Thus, the supermatrix is column stochastic [8].

It is especially necessary to be careful when synthesizing ratio scale priority vectors derived from pairwise comparisons matrices in systems-with-feedback. Elements do not only interact directly in systems-with-feedback. There may be many interactions between elements indirectly. As an example, four elements (A, B, C, D) and various impacts between them are shown in Figure 3.

Figure 3. Different sources of impact of element A on element B



The total impact of element A on element B has many components. The direct impact (or first order impact) of element A on element B is represented with a solid line in Figure 3. All the first order impacts can be obtained directly from the supermatrix. There are also indirect impacts of element A on element B over a third element. For instance, there is an impact of element A on element B over element C. In Figure 3 this second order impact is represented with a dotted line. The contribution of this impact to the total impact of element A on element B can be obtained by multiplying the impact of element A on element C by the impact of element C on element B. Another second order impact of element A on element B is over element D. This second order impact is also represented with a dotted line, using a different arrow head. The last second order impact of element A on element B is over the element A itself. This second order impact is also represented with a dotted line, again with a different arrow head. All the second order impact totals can be obtained from the square of the supermatrix. As can be seen in Figure 3, there is also a third order impact of element A on element B, represented with a dashed line. The contribution of this impact on the total impact of element A on element B can be obtained by multiplying the impact of element A on element C by the impact of element C on element D and by the impact of element D on element B. All the third order impacts totals can be obtained from the third power of the supermatrix, and so on.

Thus, we need to compute the limiting power of a supermatrix which is column stochastic. This concept is parallel to the Markov chain process [5]. The limiting power of a supermatrix has an equilibrium distribution, as in the Markov chain process. Alternatives in the model can be ordered by using limiting priorities obtained from the equilibrium distribution of the supermatrix.

4. An Illustrative Example

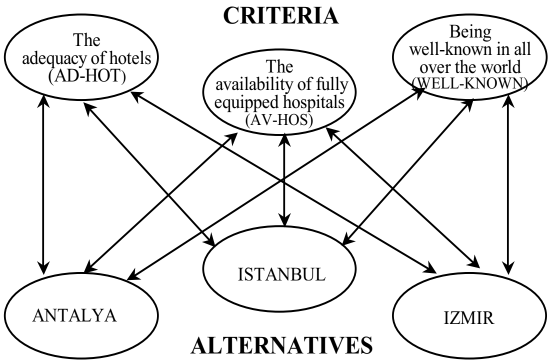
The Formula One World Championship is firmly established as one of the most prestigious, popular, exciting and technically sophisticated competitions in the international sporting calendar. This championship is held every year over an eight month period, generally on the basis of one Grand Prix every two weeks [10]. Europe is the traditional home of Grand Prix racing. The majority of races in the Formula One World Championship have always been held in Europe. Eleven of the seventeen races are in Europe; Italy and Germany have two races each at present, South America has just one, while Africa and the Middle East are not represented. There is now a rising demand for more races to be held outside Europe. China, India, Dubai and Egypt have been mentioned as possible venues while Turkey is another candidate [11]. Bernie Ecclestone, Formula One Administration (FOA) chief executive, said in August 2002 in Istanbul that Turkey had a 99,9 percent chance of hosting a Grand Prix [12].

As an example, ANP is applied to the problem of selecting a city in case a leg of Formula One car races be held in Turkey in the future. There are three Turkish cities which are believed to be the most competitive in hosting the race: the Mediterranean resort of Antalya, the commercial capital Istanbul and the Aegean city of Izmir. This example is only presented for illustrative purposes. A detailed study may also be done jointly with the real decision maker, Ecclestone. In this study, a simple feedback model is structured from the statements at Bernie Ecclestone's press conferences during his visit to examine locations where the Turkish leg of the Formula One car race could be held.

Ecclestone has said that **adequacy of hotels** (AD-HOT) is the main standard required from candidate cities to hold a Formula One car race [13]. In another press conference he has expressed that **the availability of fully equipped hospitals** (AV-HOS) is another criteria [14]. In one of his meetings in Istanbul he had also said that the city they are going to select **should be well-known all over the world** (WELL-KNOWN) [15]. From these statements a simple feedback network model may be structured for selecting a city where the Formula One car races would be held in Turkey, as shown in Figure 4.

The flow of influence in a feedback network model is specified by links. A link from one element, such as an criterion, to other elements, such as alternatives, specifies that influence can flow from the former to the latter. As seen in Figure 4, not only does the importance of the criteria determine the importance of the alternatives as in a hierarchy, but also the importance of the alternatives themselves determines the importance of the criteria.

Figure 4. Overall Goal: Selecting the most appropriate city to hold Formula One car races.



Each criterion in this example has a link to the three alternatives to indicate the flow of influence from the criterion to the alternatives. Pairwise comparisons are made to determine the relative influence which the criterion has on the relative preferences of the alternatives.

Similarly, each alternative in this example has a link to the three criteria to indicate the flow of influence from the alternative to the criteria. Pairwise comparisons are made to determine the relative influence which the alternative has on the relative importance of the criteria.

Once the feedback network model is set up, pairwise comparisons will be made to indicate the relative amount of influence that flows from one element to each of the other elements.

Three alternatives are compared with respect to the criteria Ad-Hot, Av-Hos and Well-Known in Table 1 to Table 3, respectively. To compare cities with respect to Ad-Hot, the number of beds in the hotels of each city are used in Table 1. For example, to compare Antalya and Istanbul with respect to Ad-Hot, the number of beds Antalya has (400.000) is divided by the number of beds Istanbul has (75.000). In Table 2, to compare cities with respect to Av-Hos, the number of hospitals in these cities are used [16]. In Table 3, to compare cities with respect to Well-Known, the number of hits that are found from searching web pages with the Google Search Engine for cities' names (excluding Turkish web pages) are used [17]. For example, to compare Antalya and Istanbul with respect to Well-Known the number of the hits for Antalya (334.000) is divided by the number of the hits for Istanbul (1.955.000). These comparisons are taken from actual measurements.

Table 1. Pairwise comparisons of the three cities with respect to Ad-Hot

Ad-Hot	Antalya	Istanbul	Izmir	Vector Weights
Antalya	1	400/75	400/150	0,63983
Istanbul	75/400	1	75/150	0,12006
Izmir	150/400	150/75	1	0,24011

Table2. Pairwise comparisons of the three cities with respect to AV-HOS

AV-HOS	ANTALYA	ISTANBUL	IZMIR	Vector Weights
ANTALYA	1	26/192	26/49	0,09731
ISTANBUL	192/26	1	192/49	0,71931
IZMIR	49/26	49/192	1	0,18338

Table 3. Pairwise comparisons of the three cities with respect to WELL-KNOWN

WELL-KNOWN	ANTALYA	ISTANBUL	IZMIR	Vector Weights
ANTALYA	1	334/1955	334/338	0,12711
ISTANBUL	1955/334	1	1955/338	0,74426
IZMIR	338/334	338/1955	1	0,12864

From Table 4 through Table 6, three criteria are compared with respect to the alternatives ANTALYA, ISTANBUL and IZMIR, respectively. In Table 4, for example with respect to ANTALYA, which has more hotels than required, a reasonable judgement is that AV-HOS is five times more important than AD-HOT. However, with respect to ISTANBUL, which has a more critical number of beds, AD-HOT is two times more important than AV-HOS. The comparisons in Table 4 through Table 6 have been taken from the relative strength of preferences and feelings as in Table 4.

Table 4. Pairwise comparisons of the three criteria with respect to ANTALYA

ANTALYA	AD-HOT	AV-HOS	WELL-KNOWN	Vector Weights
AD-HOT	1	1/5	1/3	0,10948
AV-HOS	5	1	2	0,58144
WELL-KNOWN	3	1/2	1	0,30908

Table 5. Pairwise comparisons of the three criteria with respect to ISTANBUL

ISTANBUL	AD-HOT	AV-HOS	WELL-KNOWN	Vector Weights
AD-HOT	1	2	4	0,58415
AV-HOS	1/2	1	1	0,23183
WELL-KNOWN	1/4	1	1	0,18402

Table 6. Pairwise comparisons of the three criteria with respect to IZMIR

IZMIR	AD-HOT	AV-HOS	WELL-KNOWN	Vector Weights
AD-HOT	1	1/3	1/2	0,16316
AV-HOS	3	1	2	0,53983
WELL-KNOWN	2	1/2	1	0,29700

Then the following supermatrix is constructed by using the vector weights for the alternatives with respect to each criteria and the vector weights of the criteria with respect to each alternative and it is used to assess the results of feedback network model.

	Ad-Hot	Av-Hos	Well-Known	Antalya	Istanbul	Izmir
Ad-Hot	0,00000	0,00000	0,00000	0,10948	0,58415	0,16316
Av-Hos	0,00000	0,00000	0,00000	0,58144	0,23182	0,53983
Well-Known	0,00000	0,00000	0,00000	0,30908	0,18402	0,29700
Antalya	0,63983	0,09731	0,12711	0,00000	0,00000	0,00000
Istanbul	0,12006	0,71931	0,74426	0,00000	0,00000	0,00000
Izmir	0,24011	0,18338	0,12864	0,00000	0,00000	0,00000

The final priorities for both the objectives and alternatives are obtained by multiplying this matrix by itself numerous times until the columns stabilize and become identical in each block. The limiting power of the supermatrix is reached at the 33rd stage as follows:

	Ad-Hot	Av-Hos	Well-Known	Antalya	Istanbul	Izmir
ad-hot	0,00000	0,00000	0,00000	0,36126	0,36126	0,36126
av-hos	0,00000	0,00000	0,00000	0,39560	0,39560	0,39560
well-known	0,00000	0,00000	0,00000	0,24314	0,24314	0,24314
antalya	0,30055	0,30055	0,30055	0,00000	0,00000	0,00000
istanbul	0,50889	0,50889	0,50889	0,00000	0,00000	0,00000
izmir	0,19057	0,19057	0,19057	0,00000	0,00000	0,00000

In the last three rows of this final matrix, the limiting priorities of the three alternative cities are seen. The priorities are in the region of 0,30 0,51 and 0,19 for Antalya, Istanbul and Izmir respectively. As a result of this illustrative example, Istanbul can be said to be a more appropriate city in which to hold Formula One car races than are Antalya and Izmir.

5. Conclusions

The Analytic Hierarchy Process (AHP) is a method for decision making which involves qualitative factors. In this method, ratio scales are obtained from ordinal scales which are derived from individual judgements for qualitative factors using the pairwise comparison matrix. The Analytic Network Process (ANP) also uses a pairwise comparison matrix to obtain ratio scales. The difference between the two methods appears in modelling the problem and computing the final priorities for the alternatives from ratio scales previously obtained. AHP models a decision making problem using a unidirectional hierarchical relationship among decision elements. However the ANP allows for more complex interrelationships among the decision elements. This paper has discussed the foundation of the Analytic Hierarchy Process and of its generalized form, the Analytic Network Process, together with an illustrative example for ANP aimed at selecting an appropriate city where Formula One car races could be held in Turkey. However, it is important to stress again that the aim of this example is marly to illustrate the ANP. There may be other important criteria which we have not taken into account in this example, and including these criteria in the model could change the result entirely. Furthermore, it is not certain that Turkey will hold a leg of Formula One races. In this study, it is useful to take into consideration these points.

Finally, this paper provides an explanatory evaluation of an analytical approach for decision making through a modelling technique that has not been fully explored by researchers or practitioners in various fields.

Added in print: A decision has been taken to hold a leg of the Formula One races in Istanbul, and at the time of going to press work is in progress on preparing the track.

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