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# FOUR TOPICS AND AN EXAMPLE $^\dagger$

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Dedicated to the memory of Professor Doğan Çoker

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## Abstract

Four aspects of the theory of topological spaces are presented, and an important example which illustrates all four of these aspects is described.

**Keywords:** Anti-property, Countable subset, Compact, Finite, Bitopological spaces, Change of topology.

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## 1. Introduction

Throughout this paper  $(X, \tau)$  will denote an arbitrary topological space. No (separation) properties are assumed unless explicitly stated. This paper surveys some of the work of the author and his co-authors over more than two decades. It is an expanded version of his lecture at the Coker memorial meeting.

### 2. Spaces in which the only compact subspaces are finite

Such spaces were introduced under the names *pseudo finite* spaces by Wilansky [30] and cf spaces by Levine [14]. In 1976 Hutton and Reilly [11] attempted to obtain a local characterization of such spaces, with the following definition.

**2.1. Definition.** A topological space  $(X, \tau)$  is called *icn* (infinite complement neighbourhood) if for each point p in X and for each infinite subset A of X there is an open set G containing p and such that A - G is infinite.

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They showed that the cf property implied icn, and that for first countable spaces the converse was true. The general converse was left as an open question. Reilly and Vamanamurthy [24] used  $\beta \mathbb{N}$  to show that icn does not imply cf in general. They also proved that the icn property is equivalent to scf, where X is scf if every sequentially compact subspace of X is finite. Furthermore, they showed that cf is equivalent to  $icn^*$ , where  $icn^*$  is defined by changing the condition "A - G is finite" in Definition 1 to "A - G is non-compact". The class of Lc spaces - those in which every Lindelof subspace is countable – was considered by Reilly and Vamanamurthy [24].

Bankston [1] developed a general theory which had the cf spaces as a particular example. Consider a topological property  $\mathcal{P}$ . An operation "anti" is defined on classes of spaces as follows. A space X is called *anti-P* if the only subspaces of X having property  $\mathcal{P}$  are those whose cardinalities require them to have property  $\mathcal{P}$ . For example, anti-compact  $\equiv cf$ , anti-connected  $\equiv$  totally disconnected, and anti-Lindelof  $\equiv Lc$ .

Reilly and Vamanamurthy [25] continued the general study, especially for covering and separation properties. They showed that the  $\operatorname{anti}(\cdot)$  operation does not discriminate well between classes of spaces defined by different separation properties. In fact, the  $\operatorname{anti}(\cdot)$  operation distinguishes only the  $T_0$  spaces from spaces with any higher separation property. It maps the class of  $T_0$  spaces onto the class of indiscrete spaces, and the class of  $T_i$  spaces ( $i \geq 1$ ) onto the class of spaces with totally ordered topologies. Here  $T_3$ means regular and  $T_1, T_4$  means normal and  $T_1$  and so on.

If  $\mathcal{K}$  is a topological class the *spectrum* of  $\mathcal{K}$ , denoted  $\operatorname{spec}(\mathcal{K})$ , is the class of cardinal numbers  $\kappa$  such that any topology on a set of power  $\kappa$  lies in  $\mathcal{K}$ . Then anti- $\mathcal{K}$  is defined to be the class of spaces X such that whenever  $Y \subset X$  then  $Y \in \mathcal{K}$  if and only if  $|Y| \in \operatorname{spec}(\mathcal{K})$ .

Bankston [1, Proposition 1.2] showed the following

**2.2.** Proposition. If  $\mathcal{K}$  and  $\mathcal{M}$  are classes of spaces, with  $\mathcal{K} \subset \mathcal{M}$  and spec $(\mathcal{K})$  = spec $(\mathcal{M})$ , then anti- $\mathcal{K} \supset$  anti- $\mathcal{M}$ .

Since any set containing at least two distinct points can have a non-Hausdorff topology defined on it, we have that spec({Hausdorff spaces}) =  $\{0, 1\} = 2$ . Thus X is anti-Hausdorff if and only if no two-point subspace of X is Hausdorff. For example, the set of real numbers  $\mathbb{R}$  with the left hand topology  $\mathcal{L}$ , which has as a base the family of sets  $\{(-\infty, a) : a \in \mathbb{R}\}$  is anti-Hausdorff. Observe that  $(\mathbb{R}, \mathcal{L})$  is  $T_0$ . This is a best possible example in the sense that  $T_1$  anti-Hausdorff spaces do not exist.

Let  $\mathcal{K}_i$  be the class of topological spaces having the separation property

 $T_i, i = 0, 1, 2, 3, 3\frac{1}{2}, 4, 5, \alpha, \beta, m, t,$ 

where  $T_{\alpha} = \text{discrete}, T_{\beta} = \text{indiscrete}, T_m = \text{metrizable}, \text{ and } T_t = \text{totally ordered}.$ Then

$$\mathcal{K}_0 \supset \mathcal{K}_1 \supset \mathcal{K}_3 \supset \mathcal{K}_{3\frac{1}{2}} \supset \mathcal{K}_5 \supset \mathcal{K}_m \supset \mathcal{K}_\alpha,$$

while  $\operatorname{spec}(\mathcal{K}) = \{0, 1\}$  for all these classes. Hence Proposition 1 implies that the opposite inclusions hold for the anti  $(\mathcal{K})$  classes.

### 2.3. Theorem.

- (a) anti- $\mathcal{K}_0 = \mathcal{K}_\beta$ .
- (b) anti- $\mathcal{K}_i = \mathcal{K}_t$  for  $i \in \{1, 2, 3, 3\frac{1}{2}, 4, 5, m, \alpha\}$ .
- (c) anti- $\mathcal{K}_{\beta} = \mathcal{K}_0$ .
- (d) anti- $\mathcal{K}_t = \mathcal{K}_1$ .

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#### 2.4. Corollary.

- (a) anti-(anti- $\mathcal{K}_i$ ) =  $\mathcal{K}_i$  for  $i \in \{0, \beta, t\}$ .
- (b) anti-(anti- $\mathcal{K}_i$ ) =  $\mathcal{K}_1$  for  $i \in \{1, 2, 3, 3\frac{1}{2}, 4, 5, m, \alpha\}$

If  $\mathcal{P}$  is any of the properties, finiteness, compactness, countable compactness, sequential compactness or pseudocompactness then  $\operatorname{spec}(\mathcal{K}) = \omega$ .

If  $\mathcal{P}$  is  $\omega$ -compactness or Lindelofness then  $\operatorname{spec}(\mathcal{K}) = \Omega$ .

Reilly and Vamanamurthy [25, Theorem 3] proved the following result.

2.5. Theorem. The following inclusions are all proper.

- (a)  $anti-pseudocompact \subset anti-countably compact \subset anti-compact \subset anti-finite.$
- (b) anti-compact  $\subset$  anti-sequentially compact.
- (c) anti-countably compact  $\subset$  anti-sequentially compact.
- (d) anti-Lindelof  $\subset$  anti- $\sigma$ -compact.
- (e) anti-compact  $\subset$  anti- $\sigma$ -compact.

They also established [25, Theorem 8] that a space is anti-(anti-compact) if and only if it is hereditarily compact, and that a similar result holds when "compact" is replaced by "Lindelof".

The question of repeated iteration of the anti- $(\cdot)$  operation has been considered by Matier and McMaster [16, 18]. Indeed, McMaster and his students have studied several aspects of this topic [16, 17, 18, 19, 20, 21].

## 3. Spaces in which every countably infinite subset is discrete

Such spaces were considered by Potoczny [23]. They are called *cid* spaces, and are closely related to the *cf* spaces. Indeed, Potozny [23] showed that any Hausdorff cid space is anti-compact. Reilly and Vamanamurthy [26] showed that the Hausdorff condition is superfluous, that is every cid space is anti-compact. The Arens-Fort space shows that the converse is false.

Reilly and Vamanamurthy [26] showed that every infinite cid space is  $T_1$ . Recall that a space is a P space if each  $G_{\delta}$  set is open. They proved [26, Theorem 10] that every  $T_1$ P space is cid, so there are plenty of cid spaces.

Another analogue between  $T_1$  spaces and cid spaces relates to their position in the lattice of topologies. Reilly and Vamanamurthy [26, Theorem 11] showed that the cocountable topology on an infinite set X is minimal cid in the lattice of topologies on X.

Ganster, Reilly and Vamanamurthy [6] were able to characterize the cid property as a separation-like property, as follows. A space X is cid if and only if for each countably infinite subset C of X and for each point  $p \notin C$  there are open sets U and V such that  $p \in U, C \subset V, p \notin V$ , and  $C \cap U = \phi$ .

Similar results for more general cardinalities have been obtained by Grant and Reilly [10].

## 4. Bitopological spaces

If X is a set and  $\tau_1$  and  $\tau_2$  are topologies on X, then the triple  $(X, \tau_1, \tau_2)$  is defined to be a *bitopological space*. It seems that this term was first used by Kelly [12] in his classical paper on the topic.

Bitopological spaces arise naturally whenever one considers a non-symmetrical topological structure. This is because the original structure and its conjugate each generate (usually different) topologies on the underlying set. Examples are quasi-metrics, quasiproximities, quasi-topological groups and quasi-uniformities.

There is a well-developed theory of separation properties for bitopological spaces. Some authors distinguish between weak and strong versions of most of these properties. For example, the bitopological Hausdorff property may be defined as follows:  $(X, \tau_1, \tau_2)$  is strong (weak) pairwise Hausdorff if for each pair of distinct points x and y in X there are disjoint open sets  $U \in \tau_1$  and  $V \in \tau_2$  such that  $x \in U, y \in V$  (U contains one point and V contains the other point). Weston [29] first defined this notion in its strong form and used the term "consistent." Kelly [12] first used the term pairwise Hausdorff.

Fletcher Hoyle and Patty [5] defined  $(X, \tau_1, \tau_2)$  to be strong pairwise  $T_0$  if for each pair of distinct points x, y in X there is either a  $\tau_1$  open set U such that  $x \in U$  and  $y \notin U$  or a  $\tau_2$  open set V such that  $y \in V$  and  $x \notin V$ . The weak version of this property is defined as follows:  $(X, \tau_1, \tau_2)$  is weak pairwise  $T_0$  if for each pair of distinct points of X there is a set which is either  $\tau_1$  open or  $\tau_2$  open containing one of the points but not the other. Similarly,  $(X, \tau_1, \tau_2)$  is defined to be strong (weak) pairwise  $T_1$  if for each pair of distinct points x and y in X there are open sets  $U \in \tau_1$  and  $V \in \tau_2$  such that  $x \in U, y \notin U$  and  $y \in V, x \notin V$  (either  $[x \in U, y \notin U, y \in V \text{ and } x \notin V]$  or  $[y \in U, x \notin U, x \in V \text{ and}$  $y \notin V$ ]).

In the bitopological space  $(X, \tau_1, \tau_2)$ , Kelly [12] defined  $\tau_1$  to be *regular* with respect to  $\tau_2$  if for each point x in X and each  $\tau_1$  closed set P such that  $x \notin P$  there is a  $\tau_1$  open set U and a  $\tau_2$  open set V disjoint from U such that  $x \in U$  and  $P \subset V$ . Then  $(X, \tau_1, \tau_2)$ is *strong* (*weak*) pairwise regular if  $\tau_1$  is regular with respect to  $\tau_2$  and (or)  $\tau_2$  is regular with respect to  $\tau_1$ . In  $(X, \tau_1, \tau_2), \tau_1$  is defined to be *completely regular* with respect to  $\tau_2$ if for each  $\tau_1$  closed set C and each point  $x \notin C$  there is a real valued function f on Xinto [0, 1] such that f(x) = 0, f(C) = 1, and f is  $\tau_1$  upper semi-continuous and  $\tau_2$  lower semi-continuous. Furthermore,  $(X, \tau_1, \tau_2)$  is strong (*weak*) pairwise completely regular if  $\tau_1$  is completely regular with respect to  $\tau_2$  and (or)  $\tau_2$  is completely regular with respect to  $\tau_1$ .

Bitopological normality was defined by Kelly [6] as follows:  $(X, \tau_1, \tau_2)$  is pairwise normal if for each  $\tau_1$  closed set A and  $\tau_2$  closed set B disjoint from A there is a  $\tau_1$ open set V containing B and a  $\tau_2$  open set U disjoint from V containing A. Consider the bitopological space  $(\mathbb{R}, \mathcal{U}, \mathcal{L})$ , where  $\mathbb{R}$  is the set of real numbers and  $\mathcal{U}$  and  $\mathcal{L}$  are the upper and lower topologies on  $\mathbb{R}$ , namely  $\mathcal{U} = \{\emptyset, \mathbb{R}, (a, \infty) : a \in \mathbb{R}\}$  and  $\mathcal{L} = \{\emptyset, \mathbb{R}, (-\infty, a) : a \in \mathbb{R}\}$ . Then  $(\mathbb{R}, \mathcal{U}, \mathcal{L})$  is pairwise normal, and satisfies the weak version of each of the other separation properties, but does not satisfy the strong form.

By adding the appropriate form of the pairwise  $T_1$  property to the higher separation properties, one obtains two hierarchies of bitopological separation properties - a weak one and a strong one. Section 2 of Kopperman [13] is a thorough discussion of bitopological separation properties.

Bitopological covering properties have proved to be much more intractable than the separation properties. Fletcher, Hoyle and Patty [5] provided an early definition of bitopological compactness. A cover  $\mathcal{U}$  of the bitopological space  $(X, \tau_1, \tau_2)$  is defined to be *pairwise open* if  $\mathcal{U} \subset \tau_1 \cup \tau_2$  and  $\mathcal{U}$  contains at least one non-empty member of  $\tau_1$  and at least one non-empty member of  $\tau_2$ . If each pairwise open cover of  $(X, \tau_1, \tau_2)$  has a finite subcover then the space  $(X, \tau_1, \tau_2)$  is defined to be *pairwise compact*. Note that  $(\mathbb{R}, \mathcal{U}, \mathcal{L})$  is pairwise compact. Cooke and Reilly [4] considered alternative definitions and characterizations of bitopological compactness. Salbany [27] has provided the most comprehensive early discussion of this topic, based on the stronger definition that  $(X, \tau_1, \tau_2)$  is *pairwise compact* if the topological space  $(X, \tau_1 \vee \tau_2)$  is compact. Salbany [27] has

introduced the bitopological analogue of the *Stone-Čech compactification*. A more recent development of these ideas is given in Sections 3 and 6 of Kopperman [13].

Bitopological spaces can be given a categorical treatment. The category **Bitop** has objects which are bitopological spaces and morphisms which are pairwise continuous functions. A function  $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$  is defined to be *pairwise continuous* if each of the functions between topological spaces  $f: (X, \tau_1) \to (Y, \sigma_1)$  and  $f: (X, \tau_2) \to (Y, \sigma_2)$  is *continuous*. Brummer [2] and Salbany [27] are two of the major contributors to such a treatment. If the topological space  $(X, \tau)$  is identified with the bitopological space  $(X, \tau, \tau)$ , then it is clear that the category **Top** is a subcategory of **Bitop**.

Recently there has been some work relating bitopological spaces to topics in theoretical computer science. For example, we cite the paper of Ciesielski, Flagg and Kopperman [3].

## 5. Continuity properties of functions

There are many examples in the literature of properties closely related to the notion of continuity of a function. In many cases the property coincides with continuity if we change the topology on either the domain or the range or both.

**5.1. Definition.** A property  $\mathcal{P}$  of functions between topological spaces is called a *continuity property* if to each pair  $(X, \sigma)$  and  $(Y, \tau)$  of topological spaces there correspond new topologies  $\sigma'$  on X and  $\tau^*$  on Y such that  $f : (X, \sigma) \to (Y, \tau)$  has property  $\mathcal{P}$  if and only if  $f : (X, \sigma') \to (Y, \tau^*)$  is continuous. Otherwise,  $\mathcal{P}$  is called a *non-continuity property*.

So a non-continuity property is something new, outside the category of topological spaces and continuous functions. However, a continuity property arises because the wrong source and/or target is taken for the morphism in the category **Top**, see Gauld, Mršević, Reilly and Vamanamurthy [9].

Studying such variations of continuity, and especially questions such as composition, restriction, preservation of appropriate classes of subsets, relationships between such properties, and equivalence to continuity under suitable conditions, from this perspective yields much insight and enhances understanding. Proofs are often made elegant, sometimes trivial.

Gauld, Greenwood and Reilly [8] have classified about 100 such properties from this point of view.

To show that a property  $\mathcal{P}$  is a continuity property we need to exhibit the topology on the domain and/or range which reduces  $\mathcal{P}$  to continuity. The following result of Gauld [7, Proposition 2] is the most effective tool to date for showing that  $\mathcal{P}$  is a non-continuity property.

**5.2. Proposition.** Let  $\mathcal{P}$  be a property of functions between topological spaces, X and Y sets, F a family of functions from X to Y, and  $g: X \to Y$  a function. Furthermore, suppose that

- (i) whatever topologies are imposed on X and Y, if each member of F is continuous then g is also continuous, and
- (ii) there are topologies on X and Y with respect to which each member of F satisfies P but g does not satisfy P.

#### Then $\mathcal{P}$ is not a continuity property.

One example which illustrates this situation is the case of semi-regularization, see Mršević, Reilly and Vamanamurthy [22]. If A is a subset of  $(X, \tau)$ , then A is called *regular open* if A = int(clA).

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Let  $RO(X, \tau)$  denote the collection of all regular open subsets of  $(X, \tau)$ .

Then  $RO(X, \tau)$  forms the base of another topology on X, called the *semi-regularization* of  $\tau$ , and it is denoted by  $\tau_s$ .

We note the following properties;

- (1)  $\tau_s \subset \tau$
- (2) If  $(X, \tau)$  has the property that  $\tau_s = \tau$ , then  $(X, \tau)$  is called a semi-regular space.
- (3) If A and B are disjoint open sets in  $(X, \tau)$ , then  $\tau \operatorname{int}(\tau \operatorname{cl} A)$  and  $\tau \operatorname{int}(\tau \operatorname{cl} B)$  are disjoint open sets in  $(X, \tau_s)$  containing A and B respectively.
- (4) Semi-regularization topologies are preserved by products.
- (5) Let  $(Y, \sigma)$  be regular. If  $f : (X, \tau) \to (Y, \sigma)$  is continuous, then  $f : (X, \tau_s) \to (Y, \sigma)$  is continuous.
- $(6) \ (\tau_s)_s = \tau_s.$

**5.3. Definition.** A function  $f : (X, \tau) \to (Y, \sigma)$  is

- (i) almost continuous (AC),
- (ii)  $\delta$ -continuous ( $\delta C$ ),
- (iii) super continuous (SC),

if for each  $x \in X$  and each  $\sigma$  open set V containing f(x) there is a  $\tau$  open set U containing x such that

- (i)  $f(U) \subset \sigma \operatorname{int}(\sigma \operatorname{cl} V)$ ,
- (ii)  $f(\tau \operatorname{int}(\tau \operatorname{cl} U)) \subset \sigma \operatorname{int}(\sigma \operatorname{cl} V),$
- (iii)  $f(\tau \operatorname{int}(\tau \operatorname{cl} U)) \subset V$ , respectively.

**5.4. Proposition.** Let  $f : (X, \tau) \to (Y, \sigma)$  be a function. Then:

- (i) f is AC iff  $f: (X, \tau) \to (Y, \sigma_s)$  is continuous
- (ii) f is  $\delta C$  if  $f: (X, \tau_s) \to (Y, \sigma_s)$  is continuous
- (iii) f is SC iff  $f: (X, \tau_s) \to (Y, \sigma)$  is continuous.

## 6. An example

#### 6.1. Definition.

(1) A point  $z \in \mathbb{R}$  is a *point of density* of a measurable set  $M \subset \mathbb{R}$  if

$$\lim_{h \to 0^+} \frac{1}{2h} \lambda \{ M \cap (z - h, z + h) \} = 1,$$

where  $\lambda$  is Lebesgue measure.

- (2) A measurable set  $M \subset \mathbb{R}$  is called *d-open* if each point of M is a point of density of M. The collection of all *d*-open sets forms a topology on  $\mathbb{R}$ , called the *density topology* on  $\mathbb{R}$ .
- (3) A function f defined on a neighbourhood of a point  $z \in \mathbb{R}$  is approximately continuous at z if there is a measurable set  $M \subset \mathbb{R}$  such that z is a point of density of M and

$$\lim_{\substack{x \to z \\ x \in M}} f(x) = f(z).$$

Tall [28] and Lukeš, Maly and Zajiček [15] have studied the density topology in some detail.

We wish to highlight the following properties of the density topology on  $\mathbb{R}$ .

- (1) It is finer than the usual topology [15, Remark p. 148].
- (2) It is cf (or anti-compact) the only compact subspaces are the finite ones [15, Theorem 6.9].
- (3) It is cid every countably infinite subset is discrete [15, Theorem 6.9].

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- (4) It is completely regular and Hausdorff [15, Theorem 6.9].
- (5) It is not normal. [15, Theorem 6.9]
- (6) It satisfies the Lusin-Menchoff property, which is the pairwise normality property of an associated bitopological space, namely (ℝ, usual topology, density topology) [15, Theorem 6.9].
- (7) A function f is approximately continuous at a point z if and only if it is density-continuous at z, or f: ( $\mathbb{R}$ , usual)  $\rightarrow \mathbb{R}$  is approximately continuous iff f: ( $\mathbb{R}$ , density)  $\rightarrow \mathbb{R}$  is continuous [15, Theorem 6.6].

The authors of the monograph [15] claim that the use of the Lusin-Menchoff property is "the main topic of this work".

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