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A SOLVABLE SYSTEM OF NONLINEAR DIFFERENCE EQUATIONS

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Abstract

In this paper, we show that the following systems of nonlinear difference equations

$$x_{n+1} = \frac{x_n y_n + a}{x_n + y_n}, y_{n+1} = \frac{y_n z_n + a}{y_n + z_n}, z_{n+1} = \frac{z_n x_n + a}{z_n + x_n} \text{ for } n \in \mathbb{N}_0$$

where $a \in [0, \infty)$ and the initial values x_0, y_0, z_0 are real numbers, can be solved in explicit form. Also, we investigate the asymptotic behavior of the solutions by using these formulae and give some numerical examples which verify our theoretical result.

Keywords: Asymptotic behavior; explicit solution; nonlinear difference equation; system.
MSC 2010: 39A10.

1 Introduction

Recently, studying nonlinear difference equations and their systems have taken much attention see [1-22] and the references therein. That is because nonlinear difference equations and their systems have appeared in many scientific areas such as biology, physics, economics, etc.

Li and Zhu [10] studied the globally asymptotic stability of the nonlinear difference equation

$$x_{n+1} = \frac{x_n x_{n-1} + a}{x_n + x_{n-1}} \text{ for } n \in \mathbb{N}_0, \tag{1}$$

where $a \in [0, \infty)$ and the initial values are positive real numbers.

Abu-Saris et al. [1] investigated the globally asymptotically stability of the nonlinear difference equation

$$x_{n+1} = \frac{x_n x_{n-k} + a}{x_n + x_{n-k}} \text{ for } n \in \mathbb{N}_0, \tag{2}$$

where k is a nonnegative integer, $a \in [0, \infty)$ and the initial values are positive real numbers.

Motivated by all above mentioned study, in this paper, we show that the following systems of nonlinear difference equations

$$x_{n+1} = \frac{x_n y_n + a}{x_n + y_n},\ y_{n+1} = \frac{y_n z_n + a}{y_n + z_n},\ z_{n+1} = \frac{z_n x_n + a}{z_n + x_n} \text{ for } n \in \mathbb{N}_0, \tag{3}$$

where $a \in [0, \infty)$ and the initial values x_0, y_0, z_0 are real numbers, can be solved in explicit form. Also, we investigate the asymptotic behavior of the solutions by using these formulae and give some numerical examples which verify our theoretical result.

2 Solvability and the general solution of the system

In this section, we show that system (3) can be solved for both the case $a = 0$ and the case $a > 0$. We also obtain its general solution in explicit form.

2.1 Case $a = 0$

In this case, system (3) is in the form of

$$x_{n+1} = \frac{x_n y_n}{x_n + y_n},\ y_{n+1} = \frac{y_n z_n}{y_n + z_n},\ z_{n+1} = \frac{z_n x_n}{z_n + x_n} \text{ for } n \in \mathbb{N}_0. \tag{4}$$

The changes of variables

$$x_n = \frac{1}{u_n},\ y_n = \frac{1}{v_n},\ z_n = \frac{1}{w_n}, \tag{5}$$

where $x_n y_n z_n \neq 0$ for every $n \in \mathbb{N}_0$, reduce system (3) to the linear system of difference equations

$$u_{n+1} = u_n + v_n,\ v_{n+1} = v_n + w_n,\ w_{n+1} = w_n + u_n \text{ for } n \in \mathbb{N}_0. \tag{6}$$

By summing the equations of (6), we have

$$u_{n+1} + v_{n+1} + w_{n+1} = 2\left(u_n + v_n + w_n\right) \text{ for } n \in \mathbb{N}_0, \tag{7}$$

whose solution is

$$u_n + v_n + w_n = 2^n K_0 \text{ for } n \in \mathbb{N}_0, \tag{8}$$

where $K_0 = u_0 + v_0 + w_0$, from (6) and (8) we can write

$$v_n + w_{n+1} = 2^n K_0. \tag{9}$$

From (6) and (9), one can obtain the equations

$$v_{n+2} - v_{n+1} + v_n = 2^n K_0, \tag{10}$$

and

$$v_{n+1} = v_n - v_{n-1} + 2^{n-1} K_0. \tag{11}$$

A particular solution of (11) is

$$\frac{2^n K_0}{3}. \tag{12}$$

From (11) and (12), we have

$$v_{n+1} - \frac{2^{n+1} K_0}{3} = v_n - \frac{2^n K_0}{3} - \left(v_{n-1} - \frac{2^{n-1} K_0}{3}\right). \tag{13}$$

Let

$$r_n = v_n - \frac{2^n K_0}{3} \text{ for } n \in \mathbb{N}_0. \tag{14}$$

Then, from (13) and (14), we obtain the equation

$$r_{n+1} = r_n - r_{n-1} \text{ for } n \in \mathbb{N}_1, \tag{15}$$

whose solution is given by

$$r_{6n+i} = r_i \quad \text{for} \quad i \in \{0, 1, 2, 3, 4, 5\} \tag{16}$$

which is periodic with period 6. Therefore, (14) and (16) imply that

$$r_0 = v_0 - \frac{K_0}{3} = -\frac{u_0 - 2v_0 + w_0}{3}, \tag{17}$$

$$r_1 = v_1 - \frac{2K_0}{3} = v_0 + w_0 - \frac{2K_0}{3} = -\frac{2u_0 - v_0 - w_0}{3}, \tag{18}$$

$$r_2 = r_1 - r_0 = -\frac{u_0 + v_0 - 2w_0}{3}, \tag{19}$$

$$r_3 = r_2 - r_1 = -r_0 = \frac{u_0 - 2v_0 + w_0}{3}, \tag{20}$$

$$r_4 = r_3 - r_2 = -r_1 = \frac{2u_0 - v_0 - w_0}{3}, \tag{21}$$

$$r_5 = r_4 - r_3 = -(r_1 - r_0) = \frac{u_0 + v_0 - 2w_0}{3}, \tag{22}$$

and so

$$v_{6n} = 2^{6n}\frac{K_0}{3} + r_0 = u_0\left(\frac{2^{6n}-1}{3}\right) + v_0\left(\frac{2^{6n}+2}{3}\right) + w_0\left(\frac{2^{6n}-1}{3}\right), \tag{23}$$

$$v_{6n+1} = 2^{6n+1}\frac{K_0}{3} + r_1 = u_0\left(\frac{2^{6n+1}-2}{3}\right) + v_0\left(\frac{2^{6n+1}+1}{3}\right) + w_0\left(\frac{2^{6n+1}+1}{3}\right), \tag{24}$$

$$v_{6n+2} = 2^{6n+2}\frac{K_0}{3} + r_2 = u_0\left(\frac{2^{6n+2}-1}{3}\right) + v_0\left(\frac{2^{6n+2}-1}{3}\right) + w_0\left(\frac{2^{6n+2}+2}{3}\right), \tag{25}$$

$$v_{6n+3} = 2^{6n+3}\frac{K_0}{3} + r_3 = u_0\left(\frac{2^{6n+3}+1}{3}\right) + v_0\left(\frac{2^{6n+3}-2}{3}\right) + w_0\left(\frac{2^{6n+3}+1}{3}\right), \tag{26}$$

$$v_{6n+4} = 2^{6n+4}\frac{K_0}{3} + r_4 = u_0\left(\frac{2^{6n+4}+2}{3}\right) + v_0\left(\frac{2^{6n+4}-1}{3}\right) + w_0\left(\frac{2^{6n+4}-1}{3}\right), \tag{27}$$

$$v_{6n+5} = 2^{6n+5}\frac{K_0}{3} + r_5 = u_0\left(\frac{2^{6n+5}+1}{3}\right) + v_0\left(\frac{2^{6n+5}+1}{3}\right) + w_0\left(\frac{2^{6n+5}-2}{3}\right). \tag{28}$$

By using the formulae (23)-(28) in the second equation of (6), one can find the formulae

$$w_{6n} = u_0\left(\frac{2^{6n}-1}{3}\right) + v_0\left(\frac{2^{6n}-1}{3}\right) + w_0\left(\frac{2^{6n}+2}{3}\right), \tag{29}$$

$$w_{6n+1} = u_0\left(\frac{2^{6n+1}+1}{3}\right) + v_0\left(\frac{2^{6n+1}-2}{3}\right) + w_0\left(\frac{2^{6n+1}+1}{3}\right), \tag{30}$$

$$w_{6n+2} = u_0\left(\frac{2^{6n+2}+2}{3}\right) + v_0\left(\frac{2^{6n+2}-1}{3}\right) + w_0\left(\frac{2^{6n+2}-1}{3}\right), \tag{31}$$

$$w_{6n+3} = u_0\left(\frac{2^{6n+3}+1}{3}\right) + v_0\left(\frac{2^{6n+3}+1}{3}\right) + w_0\left(\frac{2^{6n+3}-2}{3}\right), \tag{32}$$

$$w_{6n+4} = u_0\left(\frac{2^{6n+4}-1}{3}\right) + v_0\left(\frac{2^{6n+4}+2}{3}\right) + w_0\left(\frac{2^{6n+4}-1}{3}\right) \tag{33}$$

and

$$w_{6n+5} = u_0\left(\frac{2^{6n+5}-2}{3}\right) + v_0\left(\frac{2^{6n+5}+1}{3}\right) + w_0\left(\frac{2^{6n+5}+1}{3}\right). \tag{34}$$

Similarly, by using the formulae (29)-(34) in the third equation of (6), one can obtain the formulae

$$u_{6n} = u_0\left(\frac{2^{6n}+2}{3}\right) + v_0\left(\frac{2^{6n}-1}{3}\right) + w_0\left(\frac{2^{6n}-1}{3}\right), \tag{35}$$

$$u_{6n+1} = u_0\left(\frac{2^{6n+1}+1}{3}\right) + v_0\left(\frac{2^{6n+1}+1}{3}\right) + w_0\left(\frac{2^{6n+1}-2}{3}\right), \tag{36}$$

$$u_{6n+2} = u_0\left(\frac{2^{6n+2}-1}{3}\right) + v_0\left(\frac{2^{6n+2}+2}{3}\right) + w_0\left(\frac{2^{6n+2}-1}{3}\right), \tag{37}$$

$$u_{6n+3} = u_0\left(\frac{2^{6n+3}-2}{3}\right) + v_0\left(\frac{2^{6n+3}+1}{3}\right) + w_0\left(\frac{2^{6n+3}+1}{3}\right), \tag{38}$$

$$u_{6n+4} = u_0\left(\frac{2^{6n+4}-1}{3}\right) + v_0\left(\frac{2^{6n+4}-1}{3}\right) + w_0\left(\frac{2^{6n+4}+2}{3}\right) \tag{39}$$

and

$$u_{6n+5} = u_0\left(\frac{2^{6n+5}+1}{3}\right) + v_0\left(\frac{2^{6n+5}-2}{3}\right) + w_0\left(\frac{2^{6n+5}+1}{3}\right). \tag{40}$$

Now, by using the formulae in (23)-(40) into (5), we get the general solution of (4) as follows:

$$x_{6n} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n}+2\right) + x_0z_0\left(2^{6n}-1\right) + x_0y_0\left(2^{6n}-1\right)}, \tag{41}$$

$$x_{6n+1} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n+1}+1\right) + x_0z_0\left(2^{6n+1}+1\right) + x_0y_0\left(2^{6n+1}-2\right)}, \tag{42}$$

$$x_{6n+2} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n+2}-1\right) + x_0z_0\left(2^{6n+2}+2\right) + x_0y_0\left(2^{6n+2}-1\right)}, \tag{43}$$

$$x_{6n+3} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n+3}-2\right) + x_0z_0\left(2^{6n+3}+1\right) + x_0y_0\left(2^{6n+3}+1\right)}, \tag{44}$$

$$x_{6n+4} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n+4}-1\right) + x_0z_0\left(2^{6n+4}-1\right) + x_0y_0\left(2^{6n+4}+2\right)}, \tag{45}$$

$$x_{6n+5} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n+5}+1\right) + x_0z_0\left(2^{6n+5}-2\right) + x_0y_0\left(2^{6n+5}+1\right)}, \tag{46}$$

$$y_{6n} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n}-1\right) + x_0z_0\left(2^{6n}+2\right) + x_0y_0\left(2^{6n}-1\right)}, \tag{47}$$

$$y_{6n+1} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n+1}-2\right) + x_0z_0\left(2^{6n+1}+1\right) + x_0y_0\left(2^{6n+1}+1\right)}, \tag{48}$$

$$y_{6n+2} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n+2}-1\right) + x_0z_0\left(2^{6n+2}-1\right) + x_0y_0\left(2^{6n+2}+2\right)}, \tag{49}$$

$$y_{6n+3} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n+3}+1\right) + x_0z_0\left(2^{6n+3}-2\right) + x_0y_0\left(2^{6n+3}+1\right)}, \tag{50}$$

$$y_{6n+4} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n+4}+2\right) + x_0z_0\left(2^{6n+4}-1\right) + x_0y_0\left(2^{6n+4}-1\right)}, \tag{51}$$

$$y_{6n+5} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n+5}+1\right) + x_0z_0\left(2^{6n+5}+1\right) + x_0y_0\left(2^{6n+5}-2\right)}, \tag{52}$$

$$z_{6n} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n}-1\right) + x_0z_0\left(2^{6n}-1\right) + x_0y_0\left(2^{6n}+2\right)}, \tag{53}$$

$$z_{6n+1} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n+1}+1\right) + x_0z_0\left(2^{6n+1}-2\right) + x_0y_0\left(2^{6n+1}+1\right)}, \tag{54}$$

$$z_{6n+2} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n+2}+2\right) + x_0z_0\left(2^{6n+2}-1\right) + x_0y_0\left(2^{6n+2}-1\right)}, \tag{55}$$

$$z_{6n+3} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n+3}+1\right) + x_0z_0\left(2^{6n+3}+1\right) + x_0y_0\left(2^{6n+3}-2\right)}, \tag{56}$$

$$z_{6n+4} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n+4}-1\right) + x_0z_0\left(2^{6n+4}+2\right) + x_0y_0\left(2^{6n+4}-1\right)}, \tag{57}$$

$$z_{6n+5} = \frac{3x_0y_0z_0}{y_0z_0\left(2^{6n+5}-2\right) + x_0z_0\left(2^{6n+5}+1\right) + x_0y_0\left(2^{6n+5}+1\right)}. \tag{58}$$

2.2 Case $a > 0$

In this case, system (3) can be written in the forms of

$$x_{n+1} + \sqrt{a} = \frac{x_n y_n + a + \sqrt{a} x_n + \sqrt{a} y_n}{x_n + y_n}, \tag{59}$$

$$y_{n+1} + \sqrt{a} = \frac{y_n z_n + a + \sqrt{a} y_n + \sqrt{a} z_n}{y_n + z_n}, \tag{60}$$

$$z_{n+1} + \sqrt{a} = \frac{z_n x_n + a + \sqrt{a} z_n + \sqrt{a} x_n}{z_n + x_n} \tag{61}$$

and

$$x_{n+1} - \sqrt{a} = \frac{x_n y_n + a - \sqrt{a} x_n - \sqrt{a} y_n}{x_n + y_n}, \tag{62}$$

$$y_{n+1} - \sqrt{a} = \frac{y_n z_n + a - \sqrt{a} y_n - \sqrt{a} z_n}{y_n + z_n}, \tag{63}$$

$$z_{n+1} - \sqrt{a} = \frac{z_n x_n + a - \sqrt{a} z_n - \sqrt{a} x_n}{z_n + x_n}. \tag{64}$$

From (59)-(61) and (62)-(64), we get the system

$$\frac{X_{n+1}^{+}}{X_{n+1}^{-}} = \frac{X_n^{+}}{X_n^{-}}\frac{Y_n^{+}}{Y_n^{-}}, \; \frac{Y_{n+1}^{+}}{Y_{n+1}^{-}} = \frac{Y_n^{+}}{Y_n^{-}}\frac{Z_n^{+}}{Z_n^{-}}, \; \frac{Z_{n+1}^{+}}{Z_{n+1}^{-}} = \frac{Z_n^{+}}{Z_n^{-}}\frac{X_n^{+}}{X_n^{-}} \tag{65}$$

where

$$x_n + \sqrt{a} = X_n^{+}, \; y_n + \sqrt{a} = Y_n^{+}, \; z_n + \sqrt{a} = Z_n^{+}, \tag{66}$$

and

$$x_n - \sqrt{a} = X_n^{-}, \; y_n - \sqrt{a} = Y_n^{-}, \; z_n - \sqrt{a} = Z_n^{-}. \tag{67}$$

for $(x_n + y_n)(y_n + z_n)(z_n + x_n) \neq 0$ and $(x_n \pm \sqrt{a})(y_n \pm \sqrt{a})(z_n \pm \sqrt{a}) \neq 0$ for $n \in \mathbb{N}_0$. System (65) can easily be solved. By iterating (65) for $n \geq 0$, we get

$$\begin{aligned}
&\frac{X_1^{+}}{X_1^{-}} = \frac{X_0^{+}}{X_0^{-}}\frac{Y_0^{+}}{Y_0^{-}}, \frac{Y_1^{+}}{Y_1^{-}} = \frac{Y_0^{+}}{Y_0^{-}}\frac{Z_0^{+}}{Z_0^{-}}, \frac{Z_1^{+}}{Z_1^{-}} = \frac{Z_0^{+}}{Z_0^{-}}\frac{X_0^{+}}{X_0^{-}} \\
&\frac{X_2^{+}}{X_2^{-}} = \frac{X_0^{+}}{X_0^{-}}\left(\frac{Y_0^{+}}{Y_0^{-}}\right)^2\frac{Z_0^{+}}{Z_0^{-}}, \frac{Y_2^{+}}{Y_2^{-}} = \frac{Y_0^{+}}{Y_0^{-}}\left(\frac{Z_0^{+}}{Z_0^{-}}\right)^2\frac{X_0^{+}}{X_0^{-}}, \frac{Z_2^{+}}{Z_2^{-}} = \frac{Z_0^{+}}{Z_0^{-}}\left(\frac{X_0^{+}}{X_0^{-}}\right)^2\frac{Y_0^{+}}{Y_0^{-}} \\
&\frac{X_3^{+}}{X_3^{-}} = \left(\frac{X_0^{+}}{X_0^{-}}\right)^2\left(\frac{Y_0^{+}}{Y_0^{-}}\right)^3\left(\frac{Z_0^{+}}{Z_0^{-}}\right)^3, \frac{Y_3^{+}}{Y_3^{-}} = \left(\frac{Y_0^{+}}{Y_0^{-}}\right)^2\left(\frac{Z_0^{+}}{Z_0^{-}}\right)^3\left(\frac{X_0^{+}}{X_0^{-}}\right)^3, \frac{Z_3^{+}}{Z_3^{-}} = \left(\frac{Z_0^{+}}{Z_0^{-}}\right)^2\left(\frac{X_0^{+}}{X_0^{-}}\right)^3\left(\frac{Y_0^{+}}{Y_0^{-}}\right)^3 \\
&\quad\vdots
\end{aligned} \tag{68}$$

From (68), we conclude that the solution of system (65) is in the form of

$$\frac{X_n^{+}}{X_n^{-}} = \left(\frac{X_0^{+}}{X_0^{-}}\right)^{a_n}\left(\frac{Y_0^{+}}{Y_0^{-}}\right)^{b_n}\left(\frac{Z_0^{+}}{Z_0^{-}}\right)^{c_n}, \tag{69}$$

$$\frac{Y_n^{+}}{Y_n^{-}} = \left(\frac{Y_0^{+}}{Y_0^{-}}\right)^{a_n}\left(\frac{Z_0^{+}}{Z_0^{-}}\right)^{b_n}\left(\frac{X_0^{+}}{X_0^{-}}\right)^{c_n}, \tag{70}$$

$$\frac{Z_n^{+}}{Z_n^{-}} = \left(\frac{Z_0^{+}}{Z_0^{-}}\right)^{a_n}\left(\frac{X_0^{+}}{X_0^{-}}\right)^{b_n}\left(\frac{Y_0^{+}}{Y_0^{-}}\right)^{c_n}. \tag{71}$$

From (69)-(71), we obtain

$$\frac{X_0^+}{X_0^-}=\left(\frac{X_0^+}{X_0^-}\right)^{a_0}\left(\frac{Y_0^+}{Y_0^-}\right)^{b_0}\left(\frac{Z_0^+}{Z_0^-}\right)^{c_0}, \tag{72}$$

$$\frac{Y_0^+}{Y_0^-}=\left(\frac{Y_0^+}{Y_0^-}\right)^{a_0}\left(\frac{Z_0^+}{Z_0^-}\right)^{b_0}\left(\frac{X_0^+}{X_0^-}\right)^{c_0}, \tag{73}$$

$$\frac{Z_0^+}{Z_0^-}=\left(\frac{Z_0^+}{Z_0^-}\right)^{a_0}\left(\frac{X_0^+}{X_0^-}\right)^{b_0}\left(\frac{Y_0^+}{Y_0^-}\right)^{c_0} \tag{74}$$

from which it follows that $a_0=1, b_0=c_0=0$. By using (69)-(71) in (65), we have the system

$$\left(\frac{X_0^+}{X_0^-}\right)^{a_{n+1}}\left(\frac{Y_0^+}{Y_0^-}\right)^{b_{n+1}}\left(\frac{Z_0^+}{Z_0^-}\right)^{c_{n+1}}=\left(\frac{X_0^+}{X_0^-}\right)^{a_n+c_n}\left(\frac{Y_0^+}{Y_0^-}\right)^{a_n+b_n}\left(\frac{Z_0^+}{Z_0^-}\right)^{b_n+c_n} \tag{75}$$

for $n\in\mathbb{N}_0$, which implies that

$$a_{n+1}=a_n+c_n,\ c_{n+1}=c_n+b_n,\ b_{n+1}=b_n+a_n. \tag{76}$$

By taking $a_n=u_n$, $c_n=v_n$, $b_n=w_n$ and $a_0=1, b_0=c_0=0$, the solution of (76) can be obtained from the solution of (6) as follows:

$$a_{6n}=\frac{2^{6n}+2}{3},\ a_{6n+1}=\frac{2^{6n+1}+1}{3},\ a_{6n+2}=\frac{2^{6n+2}-1}{3}, \tag{77}$$

$$a_{6n+3}=\frac{2^{6n+3}-2}{3},\ a_{6n+4}=\frac{2^{6n+4}-1}{3},\ a_{6n+5}=\frac{2^{6n+5}+1}{3}. \tag{78}$$

$$b_{6n}=\frac{2^{6n}-1}{3},\ b_{6n+1}=\frac{2^{6n+1}+1}{3},\ b_{6n+2}=\frac{2^{6n+2}+2}{3}, \tag{79}$$

$$b_{6n+3}=\frac{2^{6n+3}+1}{3},\ b_{6n+4}=\frac{2^{6n+4}-1}{3},\ b_{6n+5}=\frac{2^{6n+5}-2}{3}, \tag{80}$$

$$c_{6n}=\frac{2^{6n}-1}{3},\ c_{6n+1}=\frac{2^{6n+1}-2}{3},\ c_{6n+2}=\frac{2^{6n+2}-1}{3}, \tag{81}$$

$$c_{6n+3}=\frac{2^{6n+3}+1}{3},\ c_{6n+4}=\frac{2^{6n+4}+2}{3},\ c_{6n+5}=\frac{2^{6n+5}+1}{3}. \tag{82}$$

Consequently, from (69)-(71) and (77)-(82), we have the following formulae:

$$x_{6n}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n}+2}{3}}\left(Y_0^+\right)^{\frac{2^{6n}-1}{3}}\left(Z_0^+\right)^{\frac{2^{6n}-1}{3}}+\left(X_0^-\right)^{\frac{2^{6n}+2}{3}}\left(Y_0^-\right)^{\frac{2^{6n}-1}{3}}\left(Z_0^-\right)^{\frac{2^{6n}-1}{3}}}{\left(X_0^+\right)^{\frac{2^{6n}+2}{3}}\left(Y_0^+\right)^{\frac{2^{6n}-1}{3}}\left(Z_0^+\right)^{\frac{2^{6n}-1}{3}}-\left(X_0^-\right)^{\frac{2^{6n}+2}{3}}\left(Y_0^-\right)^{\frac{2^{6n}-1}{3}}\left(Z_0^-\right)^{\frac{2^{6n}-1}{3}}}, \tag{83}$$

$$x_{6n+1}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n+1}+1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+1}+1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+1}-2}{3}}+\left(X_0^-\right)^{\frac{2^{6n+1}+1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+1}+1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+1}-2}{3}}}{\left(X_0^+\right)^{\frac{2^{6n+1}+1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+1}+1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+1}-2}{3}}-\left(X_0^-\right)^{\frac{2^{6n+1}+1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+1}+1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+1}-2}{3}}}, \tag{84}$$

$$x_{6n+2}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n+2}-1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+2}+2}{3}}\left(Z_0^+\right)^{\frac{2^{6n+2}-1}{3}}+\left(X_0^-\right)^{\frac{2^{6n+2}-1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+2}+2}{3}}\left(Z_0^-\right)^{\frac{2^{6n+2}-1}{3}}}{\left(X_0^+\right)^{\frac{2^{6n+2}-1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+2}+2}{3}}\left(Z_0^+\right)^{\frac{2^{6n+2}-1}{3}}-\left(X_0^-\right)^{\frac{2^{6n+2}-1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+2}+2}{3}}\left(Z_0^-\right)^{\frac{2^{6n+2}-1}{3}}}, \tag{85}$$

$$x_{6n+3}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n+3}-2}{3}}\left(Y_0^+\right)^{\frac{2^{6n+3}+1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+3}+1}{3}}+\left(X_0^-\right)^{\frac{2^{6n+3}-2}{3}}\left(Y_0^-\right)^{\frac{2^{6n+3}+1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+3}+1}{3}}}{\left(X_0^+\right)^{\frac{2^{6n+3}-2}{3}}\left(Y_0^+\right)^{\frac{2^{6n+3}+1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+3}+1}{3}}-\left(X_0^-\right)^{\frac{2^{6n+3}-2}{3}}\left(Y_0^-\right)^{\frac{2^{6n+3}+1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+3}+1}{3}}}, \tag{86}$$

$$x_{6n+4}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n+4}-1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+4}-1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+4}+2}{3}}+\left(X_0^-\right)^{\frac{2^{6n+4}-1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+4}-1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+4}+2}{3}}}{\left(X_0^+\right)^{\frac{2^{6n+4}-1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+4}-1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+4}+2}{3}}-\left(X_0^-\right)^{\frac{2^{6n+4}-1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+4}-1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+4}+2}{3}}}, \quad (87)$$

$$x_{6n+5}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n+5}+1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+5}-2}{3}}\left(Z_0^+\right)^{\frac{2^{6n+5}+1}{3}}+\left(X_0^-\right)^{\frac{2^{6n+5}+1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+5}-2}{3}}\left(Z_0^-\right)^{\frac{2^{6n+5}+1}{3}}}{\left(X_0^+\right)^{\frac{2^{6n+5}+1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+5}-2}{3}}\left(Z_0^+\right)^{\frac{2^{6n+5}+1}{3}}-\left(X_0^-\right)^{\frac{2^{6n+5}+1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+5}-2}{3}}\left(Z_0^-\right)^{\frac{2^{6n+5}+1}{3}}}, \quad (88)$$

$$y_{6n}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n}-1}{3}}\left(Y_0^+\right)^{\frac{2^{6n}+2}{3}}\left(Z_0^+\right)^{\frac{2^{6n}-1}{3}}+\left(X_0^-\right)^{\frac{2^{6n}-1}{3}}\left(Y_0^-\right)^{\frac{2^{6n}+2}{3}}\left(Z_0^-\right)^{\frac{2^{6n}-1}{3}}}{\left(X_0^+\right)^{\frac{2^{6n}-1}{3}}\left(Y_0^+\right)^{\frac{2^{6n}+2}{3}}\left(Z_0^+\right)^{\frac{2^{6n}-1}{3}}-\left(X_0^-\right)^{\frac{2^{6n}-1}{3}}\left(Y_0^-\right)^{\frac{2^{6n}+2}{3}}\left(Z_0^-\right)^{\frac{2^{6n}-1}{3}}}, \quad (89)$$

$$y_{6n+1}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n+1}-2}{3}}\left(Y_0^+\right)^{\frac{2^{6n+1}+1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+1}+1}{3}}+\left(X_0^-\right)^{\frac{2^{6n+1}-2}{3}}\left(Y_0^-\right)^{\frac{2^{6n+1}+1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+1}+1}{3}}}{\left(X_0^+\right)^{\frac{2^{6n+1}-2}{3}}\left(Y_0^+\right)^{\frac{2^{6n+1}+1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+1}+1}{3}}-\left(X_0^-\right)^{\frac{2^{6n+1}-2}{3}}\left(Y_0^-\right)^{\frac{2^{6n+1}+1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+1}+1}{3}}}, \quad (90)$$

$$y_{6n+2}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n+2}-1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+2}-1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+2}+2}{3}}+\left(X_0^-\right)^{\frac{2^{6n+2}-1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+2}-1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+2}+2}{3}}}{\left(X_0^+\right)^{\frac{2^{6n+2}-1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+2}-1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+2}+2}{3}}-\left(X_0^-\right)^{\frac{2^{6n+2}-1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+2}-1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+2}+2}{3}}}, \quad (91)$$

$$y_{6n+3}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n+3}+1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+3}-2}{3}}\left(Z_0^+\right)^{\frac{2^{6n+3}+1}{3}}+\left(X_0^-\right)^{\frac{2^{6n+3}+1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+3}-2}{3}}\left(Z_0^-\right)^{\frac{2^{6n+3}+1}{3}}}{\left(X_0^+\right)^{\frac{2^{6n+3}+1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+3}-2}{3}}\left(Z_0^+\right)^{\frac{2^{6n+3}+1}{3}}-\left(X_0^-\right)^{\frac{2^{6n+3}+1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+3}-2}{3}}\left(Z_0^-\right)^{\frac{2^{6n+3}+1}{3}}}, \quad (92)$$

$$y_{6n+4}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n+4}+2}{3}}\left(Y_0^+\right)^{\frac{2^{6n+4}-1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+4}-1}{3}}+\left(X_0^-\right)^{\frac{2^{6n+4}+2}{3}}\left(Y_0^-\right)^{\frac{2^{6n+4}-1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+4}-1}{3}}}{\left(X_0^+\right)^{\frac{2^{6n+4}+2}{3}}\left(Y_0^+\right)^{\frac{2^{6n+4}-1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+4}-1}{3}}-\left(X_0^-\right)^{\frac{2^{6n+4}+2}{3}}\left(Y_0^-\right)^{\frac{2^{6n+4}-1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+4}-1}{3}}}, \quad (93)$$

$$y_{6n+5}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n+5}+1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+5}+1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+5}-2}{3}}+\left(X_0^-\right)^{\frac{2^{6n+5}+1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+5}+1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+5}-2}{3}}}{\left(X_0^+\right)^{\frac{2^{6n+5}+1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+5}+1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+5}-2}{3}}-\left(X_0^-\right)^{\frac{2^{6n+5}+1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+5}+1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+5}-2}{3}}}, \quad (94)$$

$$z_{6n}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n}-1}{3}}\left(Y_0^+\right)^{\frac{2^{6n}-1}{3}}\left(Z_0^+\right)^{\frac{2^{6n}+2}{3}}+\left(X_0^-\right)^{\frac{2^{6n}-1}{3}}\left(Y_0^-\right)^{\frac{2^{6n}-1}{3}}\left(Z_0^-\right)^{\frac{2^{6n}+2}{3}}}{\left(X_0^+\right)^{\frac{2^{6n}-1}{3}}\left(Y_0^+\right)^{\frac{2^{6n}-1}{3}}\left(Z_0^+\right)^{\frac{2^{6n}+2}{3}}-\left(X_0^-\right)^{\frac{2^{6n}-1}{3}}\left(Y_0^-\right)^{\frac{2^{6n}-1}{3}}\left(Z_0^-\right)^{\frac{2^{6n}+2}{3}}}, \quad (95)$$

$$z_{6n+1}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n+1}+1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+1}-2}{3}}\left(Z_0^+\right)^{\frac{2^{6n+1}+1}{3}}+\left(X_0^-\right)^{\frac{2^{6n+1}+1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+1}-2}{3}}\left(Z_0^-\right)^{\frac{2^{6n+1}+1}{3}}}{\left(X_0^+\right)^{\frac{2^{6n+1}+1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+1}-2}{3}}\left(Z_0^+\right)^{\frac{2^{6n+1}+1}{3}}-\left(X_0^-\right)^{\frac{2^{6n+1}+1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+1}-2}{3}}\left(Z_0^-\right)^{\frac{2^{6n+1}+1}{3}}}, \quad (96)$$

$$z_{6n+2}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n+2}+2}{3}}\left(Y_0^+\right)^{\frac{2^{6n+2}-1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+2}-1}{3}}+\left(X_0^-\right)^{\frac{2^{6n+2}+2}{3}}\left(Y_0^-\right)^{\frac{2^{6n+2}-1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+2}-1}{3}}}{\left(X_0^+\right)^{\frac{2^{6n+2}+2}{3}}\left(Y_0^+\right)^{\frac{2^{6n+2}-1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+2}-1}{3}}-\left(X_0^-\right)^{\frac{2^{6n+2}+2}{3}}\left(Y_0^-\right)^{\frac{2^{6n+2}-1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+2}-1}{3}}}, \quad (97)$$

$$z_{6n+3}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n+3}+1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+3}+1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+3}-2}{3}}+\left(X_0^-\right)^{\frac{2^{6n+3}+1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+3}+1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+3}-2}{3}}}{\left(X_0^+\right)^{\frac{2^{6n+3}+1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+3}+1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+3}-2}{3}}-\left(X_0^-\right)^{\frac{2^{6n+3}+1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+3}+1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+3}-2}{3}}}, \quad (98)$$

$$z_{6n+4}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n+4}-1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+4}+2}{3}}\left(Z_0^+\right)^{\frac{2^{6n+4}-1}{3}}+\left(X_0^-\right)^{\frac{2^{6n+4}-1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+4}+2}{3}}\left(Z_0^-\right)^{\frac{2^{6n+4}-1}{3}}}{\left(X_0^+\right)^{\frac{2^{6n+4}-1}{3}}\left(Y_0^+\right)^{\frac{2^{6n+4}+2}{3}}\left(Z_0^+\right)^{\frac{2^{6n+4}-1}{3}}-\left(X_0^-\right)^{\frac{2^{6n+4}-1}{3}}\left(Y_0^-\right)^{\frac{2^{6n+4}+2}{3}}\left(Z_0^-\right)^{\frac{2^{6n+4}-1}{3}}}, \quad (99)$$

$$z_{6n+5}=\sqrt{a}\frac{\left(X_0^+\right)^{\frac{2^{6n+5}-2}{3}}\left(Y_0^+\right)^{\frac{2^{6n+5}+1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+5}+1}{3}}+\left(X_0^-\right)^{\frac{2^{6n+5}-2}{3}}\left(Y_0^-\right)^{\frac{2^{6n+5}+1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+5}+1}{3}}}{\left(X_0^+\right)^{\frac{2^{6n+5}-2}{3}}\left(Y_0^+\right)^{\frac{2^{6n+5}+1}{3}}\left(Z_0^+\right)^{\frac{2^{6n+5}+1}{3}}-\left(X_0^-\right)^{\frac{2^{6n+5}-2}{3}}\left(Y_0^-\right)^{\frac{2^{6n+5}+1}{3}}\left(Z_0^-\right)^{\frac{2^{6n+5}+1}{3}}}. \quad (100)$$

2.3 Behavior of the solutions of the system

In this section, we investigate the asymptotic behavior of the solutions of system (3) and give some numerical examples which verify our theoretical result. The main result of this subsection is the following theorem:

Theorem 1 *The following statements are true*
(i) If $a = 0$, then $(x_n, y_n, z_n) \to (0,0,0)$ as $n \to \infty$.
(ii) If $a > 0$, then $(|x_n|, |y_n|, |z_n|) \to (\sqrt{a}, \sqrt{a}, \sqrt{a})$ as $n \to \infty$.

Proof.
(i) From the formulae (41)-(58) the desired result immediately follows.
(ii) We prove (ii) for only x_{6n}, since the proof is similar for the other subsequences of x_n, y_n and z_n. Note that the formula (83) can be written as follows:

$$x_{6n} = \sqrt{a}\left(1 + \frac{2}{\left(\frac{x_0+\sqrt{a}}{x_0-\sqrt{a}}\right)^{\frac{2^{6n}+2}{3}}\left(\frac{y_0+\sqrt{a}}{y_0-\sqrt{a}}\right)^{\frac{2^{6n}-1}{3}}\left(\frac{z_0+\sqrt{a}}{z_0-\sqrt{a}}\right)^{\frac{2^{6n}-1}{3}} - 1}\right), n \in \mathbb{N}_0. \quad (101)$$

Hence, we consider the function

$$f(x) = \frac{x+\sqrt{a}}{x-\sqrt{a}}$$

which supplies the property

$$\begin{cases} |f(x)| < 1, \text{if } x < 0, \\ |f(x)| > 1, \text{if } x > 0. \end{cases} \quad (102)$$

That is, it is arise two specific cases from the formula (101) and the property (102):
(a) If $x_0 < 0$, $y_0 < 0$ and $z_0 < 0$, then $(x_n, y_n, z_n) \to (-\sqrt{a}, -\sqrt{a}, -\sqrt{a})$ as $n \to \infty$.
(b) If $x_0 > 0$, $y_0 > 0$ and $z_0 > 0$, then $(x_n, y_n, z_n) \to (\sqrt{a}, \sqrt{a}, \sqrt{a})$ as $n \to \infty$.
As to the other cases, we consider the sequence

$$(s_n)_{n\geq 0} = \left(\left(\frac{x_0+\sqrt{a}}{x_0-\sqrt{a}}\right)^{\frac{2^{6n}+2}{3}}\left(\frac{y_0+\sqrt{a}}{y_0-\sqrt{a}}\right)^{\frac{2^{6n}-1}{3}}\left(\frac{z_0+\sqrt{a}}{z_0-\sqrt{a}}\right)^{\frac{2^{6n}-1}{3}}\right)_{n\geq 0}.$$

If $s_n \to 0$ as $n \to \infty$, then $(x_n, y_n, z_n) \to (-\sqrt{a}, -\sqrt{a}, -\sqrt{a})$ as $n \to \infty$. If $s_n \to \infty$ as $n \to \infty$, then $(x_n, y_n, z_n) \to (\sqrt{a}, \sqrt{a}, \sqrt{a})$ as $n \to \infty$. ■

Now, we give some numerical examples to support our theoretical results related to system (3) with some restrictions on the parameter a.

Example 2 *We visualize the solutions of system (4) in figures (1)-(3) for $a = 0$ and for the sets of initial values: $\{x_0 = 5.2, y_0 = 0.7, z_0 = 3.1\}$, $\{x_0 = -5.2, y_0 = -0.7, z_0 = -3.1\}$, $\{x_0 = 5.2, y_0 = -0.7, z_0 = 3.1\}$, respectively.*

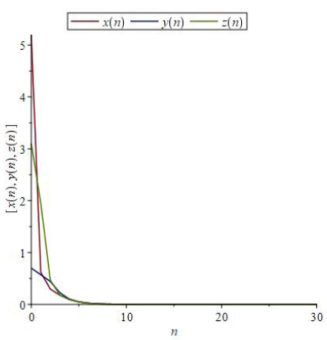

Figure 1

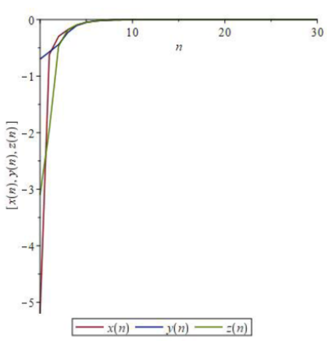

Figure 2

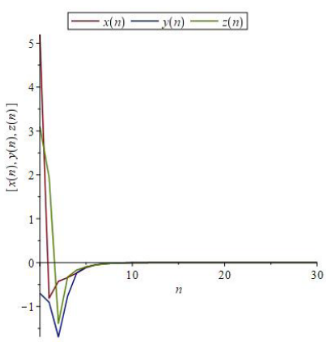

Figure 3

Example 3 *We visualize the solutions of system (3) in figures (4)-(6) for* $a = 3.14$ *and for the sets of initial values:* $\{x_0 = 0.9, y_0 = 0.7, z_0 = 2.5\}$, $\{x_0 = -0.9, y_0 = -0.7, z_0 = -2.5\}$, $\{x_0 = 0.9,\ y_0 = 0.7,\ z_0 = -2.5\}$, *respectively.*

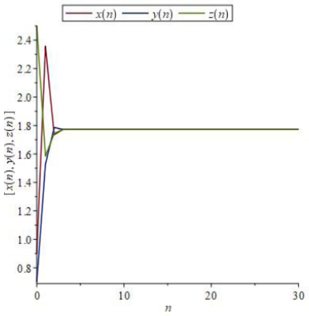

Figure 4

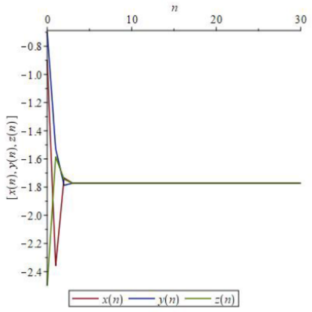

Figure 5

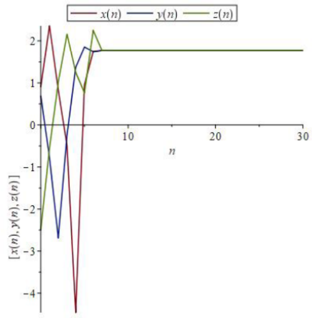

Figure 6

Example 4 *We visualize the solutions of system (3) in figures (7)-(9) for* $a = 100$ *and for the sets of initial values:* $\{x_0 = 2.9, y_0 = 5.1, z_0 = 7.8\}$, $\{x_0 = -2.9, y_0 = -5.1, z_0 = -7.8\}$, $\{x_0 = -2.9,\ y_0 = -5.1,\ z_0 = 7.8\}$, *respectively.*

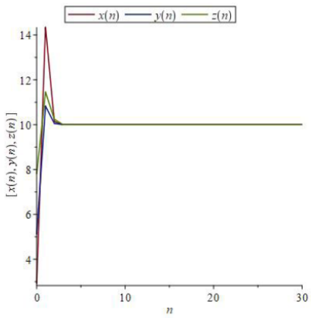

Figure 7

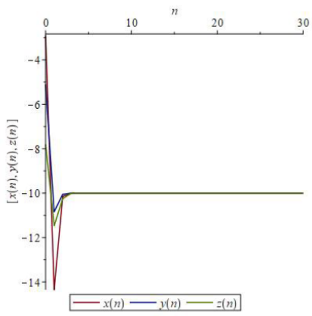

Figure 8

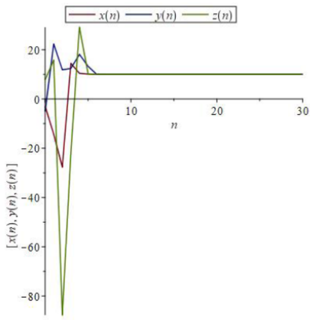

Figure 9

Example 5 *We visualize the solutions of system (3) in figures (10)-(12) for* $a = 2018$ *and for the sets of initial values:* $\{x_0 = 1.5, y_0 = 3.6, z_0 = 2.4\}$, $\{\ x_0 = -1.5, y_0 = -3.6, z_0 = -2.4\}$, $\{x_0 = -1.5,\ y_0 = 3.6,\ z_0 = -2.4\}$, *respectively.*

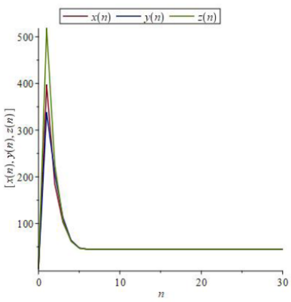

Figure 10

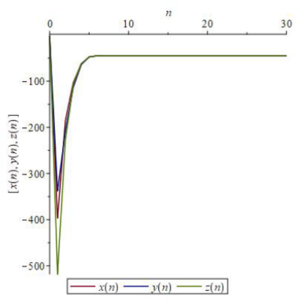

Figure 11

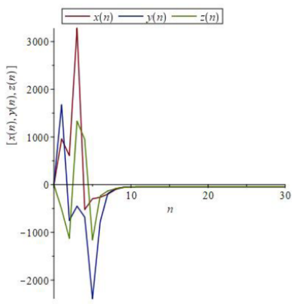

Figure 12

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