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AUTHORS: Liangcai ZHANG,Wujie SHI

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NONCOMMUTING GRAPH CHARACTERIZATION OF SOME SIMPLE GROUPS WITH CONNECTED PRIME GRAPHS

Liangcai Zhang and Wujie Shi

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ABSTRACT. Let G be a nonabelian group and associate a noncommuting graph $\nabla(G)$ with G as follows: The vertex set of $\nabla(G)$ is $G \setminus Z(G)$, where $Z(G)$ is the center of G , and two vertices are adjacent by an edge whenever they do not commute. In 2006, A. Abdollahi, S. Akbari and H. R. Maimani put forward a conjecture called *AAM's Conjecture* in [1] as follows: If M is a finite nonabelian simple group and G is a group such that $\nabla(G) \cong \nabla(M)$, then $G \cong M$. Even though this conjecture is known to hold for all simple groups with nonconnected prime graphs and the alternating group A_{10} (see [11]), it is still unknown for all simple groups with connected prime graphs except A_{10} . In the present paper, we prove that the conjecture is also true for $L_4(8)$, the projective special linear group of degree 4 over the finite field of order 8. The new method used in this paper also works well in the case $L_4(4)$, $L_4(7)$, $U_4(7)$, etc.

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1. Introduction

It is well known that *the prime graph* $\Gamma(G)$ associated with a finite group G is defined as follows: The vertex set of $\Gamma(G)$ is $\pi(G)$, the set of prime divisors of the order of G . The primes p and q , considered as vertices of $\Gamma(G)$, are adjacent by an edge (we write $p \sim q$) if and only if G contains an element of order pq . Denote by $t(G)$ the number of connected components of $\Gamma(G)$ (see [12]).

Given a finite group G , we construct its *noncommuting graph* $\nabla(G)$ as follows: The vertex set of $\nabla(G)$ is $G \setminus Z(G)$, where $Z(G)$ is the center of G , and two vertices are adjacent by an edge whenever they do not commute (see [1, 8]).

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For a graph X , we denote the sets of vertices and edges of X by $V(X)$ and $E(X)$, respectively. Two graphs X and Y are said to be *isomorphic* if there exists a bijective map $\phi: V(X) \rightarrow V(Y)$ such that x and y are adjacent in X if and only if $\phi(x)$ and $\phi(y)$ are adjacent in Y . If two graphs X and Y are isomorphic, we denote it by $X \cong Y$. It is easy to see that if $X \cong Y$, then $|V(X)| = |V(Y)|$ and $|E(X)| = |E(Y)|$.

In 2006, A. Abdollahi, S. Akbari and H. R. Maimani put forward a conjecture in [1] as follows.

AAM's Conjecture: If M is a finite nonabelian simple group and G is a group such that $\nabla(G) \cong \nabla(M)$, then $G \cong M$.

In [11], it has been proved that AAM's Conjecture is true for all finite simple groups with nonconnected prime graphs and A_{10} , where A_{10} is the alternating group of degree 10. In the present paper, we will give another example to show that AAM's Conjecture is also true for some simple groups with connected prime graphs. In fact, we prove that if G is a finite group such that $\nabla(G) \cong \nabla(L_4(8))$, then $G \cong L_4(8)$, where $L_4(8)$ is the projective special linear group of degree 4 over the finite field of order 8. The new method used in this paper also works well in the case $L_4(4)$, $L_4(7)$, $U_4(7)$, etc.

All further unexplained notations are standard and we refer the reader to [1, 8].

2. Preliminaries and Lemmas

For any group G , we denote by $\pi_e(G)$ the set of orders of its elements. The set $\pi_e(G)$ is *closed* and *partially ordered* by the divisibility relation. Hence, it is uniquely determined by $\mu(G)$, the subset of its elements which are *maximal* under the divisibility relation.

Lemma 2.1. ([7], Lemma 1) *Let $L_4(q)$ be a projective special linear simple group, where $q = 2^m$ and m is a natural number. Then $\mu(L_4(q)) = \{(q^2 + 1)(q + 1), q^3 - 1, 2(q^2 - 1), 4(q - 1)\}$. In particular, $\mu(L_4(8)) = \{3^2 \cdot 5 \cdot 13, 7 \cdot 73, 2 \cdot 3^2 \cdot 7, 2^2 \cdot 7\}$.*

Let n be a natural number. We say that a finite group G is a K_n -group if $|\pi(G)| = n$. Now we quote some useful results on simple K_n -groups.

Lemma 2.2. ([3], Theorem 1) *Let G be a finite simple K_3 -group. Then G is isomorphic to one of the following simple groups: $A_5, A_6, L_2(7), L_2(8), L_2(17), L_3(3), U_3(3)$ or $U_4(2)$.*

Lemma 2.3. ([10], Theorem 2) *Let G be a finite simple K_4 -group. Then G is isomorphic to one of the following simple groups:*

1. $A_7, A_8, A_9, A_{10}, M_{11}, M_{12}, J_2, L_2(16), L_2(25), L_2(49), L_2(81), L_3(4), L_3(5), L_3(7), L_3(8), L_3(17), L_4(3), S_4(4), S_4(5), S_4(7), S_4(9), S_6(2), O_8^+(2), G_2(3), U_3(4), U_3(5), U_3(7), U_3(8), U_3(9), U_4(3), U_5(2), Sz(8), Sz(32), {}^3D_4(2), {}^2F_4(2)'$;
2. $L_2(r)$, where r is a prime satisfying the equation $r^2 - 1 = 2^a \cdot 3^b \cdot u^c$, where $a \geq 1, b \geq 1, c \geq 1$, and $u > 3$ is a prime;
3. $L_2(2^m)$, where $m \geq 1$ satisfies the equations $2^m - 1 = u$ and $2^m + 1 = 3t^b$, where $t > 3, u$ are primes and $b \geq 1$;
4. $L_2(3^m)$, where $m \geq 1$ satisfies the equations $3^m - 1 = 2u^b$ and $3^m + 1 = 4t$, or $3^m - 1 = 2u$ and $3^m + 1 = 4t^c$, where u, t are odd primes and $b \geq 1, c \geq 1$.

In the sequel we denote by $\pi(n)$ the set of prime divisors of a natural number n .

Lemma 2.4. ([5], Theorem A) *Let q be a prime power. Then each finite simple K_5 -group is isomorphic to one of the following simple groups:*

1. $L_2(q)$, where q satisfies $|\pi(q^2 - 1)| = 4$;
2. $L_3(q)$, where q satisfies $|\pi((q^2 - 1)(q^3 - 1))| = 4$;
3. $U_3(q)$, where q satisfies $|\pi((q^2 - 1)(q^3 + 1))| = 4$;
4. $O_5(q)$, where q satisfies $|\pi(q^4 - 1)| = 4$;
5. $Sz(2^{2m+1})$, where $|\pi((2^{2m+1} - 1)(2^{4m+2} + 1))| = 4$;
6. $R(q)$, where $q = 3^{2m+1}$ satisfies $|\pi(q^2 - 1)| = 3$ and $|\pi(q^2 - q + 1)| = 1$;
7. $A_{11}, A_{12}, M_{22}, J_3, HS, He, M^cL, L_4(4), L_4(5), L_4(7), L_5(2), L_5(3), L_6(2), O_7(3), O_9(2), S_6(3), S_8(2), U_4(4), U_4(5), U_4(7), U_4(9), U_5(3), U_6(2), O_8^+(3), O_8^-(2), {}^3D_4(3), G_2(4), G_2(5), G_2(7), G_2(8)$.

Lemma 2.5. ([5], Theorem B) *Let q be a prime power. Then each finite simple K_6 -group is isomorphic to one of the following simple groups:*

1. $L_2(q)$, where q satisfies $|\pi(q^2 - 1)| = 5$;
2. $L_3(q)$, where q satisfies $|\pi((q^2 - 1)(q^3 - 1))| = 5$;
3. $L_4(q)$, where q satisfies $|\pi((q^2 - 1)(q^3 - 1)(q^4 - 1))| = 5$;
4. $U_3(q)$, where q satisfies $|\pi((q^2 - 1)(q^3 + 1))| = 5$;
5. $U_4(q)$, where q satisfies $|\pi((q^2 - 1)(q^3 + 1)(q^4 - 1))| = 5$;
6. $O_5(q)$, where q satisfies $|\pi(q^4 - 1)| = 5$;
7. $G_2(q)$, where q satisfies $|\pi(q^6 - 1)| = 5$;
8. $Sz(2^{2m+1})$, where $|\pi((2^{2m+1} - 1)(2^{4m+2} + 1))| = 5$;
9. $R(3^{2m+1})$, where $|\pi((3^{2m+1} - 1)(3^{6m+3} + 1))| = 5$;
10. $A_{13}, A_{14}, A_{15}, A_{16}, M_{23}, M_{24}, J_1, Suz, Ru, Co_2, Co_3, Fi_{22}, HN, L_5(7), L_6(3), L_7(2), O_7(4), O_7(5), O_7(7), O_9(3), S_6(4), S_6(5), S_6(7), S_8(3), U_5(4), U_5(5), U_5(9), U_6(3), U_7(2), F_4(2), O_8^+(4), O_8^+(5), O_8^+(7), O_{10}^+(2), O_8^-(3), O_{10}^-(2), {}^3D_4(4), {}^3D_4(5)$.

Now we quote some preliminary results from elementary number theory which will play an important role in proving Lemma 2.9.

Lemma 2.6. ([4], Theorems 2.3.3 and 4.6.3) *Let a, x, m and n be natural numbers. Then the following assertions hold.*

1. *If p is a prime number, then $a^p \equiv a \pmod{p}$.*
2. *If $\text{g.c.d.}\{m, n\} = d$, then $\text{g.c.d.}\{x^m - 1, x^n - 1\} = x^d - 1$. In particular, $m \mid n$ if and only if $x^m - 1 \mid x^n - 1$.*

Lemma 2.7. ([13]) *If p is a prime and $n \geq 2$ is a natural number, then there exists a prime z such that $z \mid p^n - 1$ and $z \nmid p^m - 1$ for $1 \leq m < n$ unless either*

1. *$n = 6$ and $p = 2$, or*
2. *$n = 2$ and $p = 2^q - 1$ is a Mersenne prime, where q is natural number.*

Lemma 2.8. ([6]) *Let $n \geq 2$ and f be two natural numbers and $q = p^f$, where p is a prime number. Then*

1. *$\text{Out}(L_n(q)) \cong Z_{(n, q-1)} : Z_f : Z_2$ if $n \geq 3$;*
2. *$\text{Out}(L_2(q)) \cong Z_{(2, q-1)} \times Z_f$.*

Lemma 2.9. *Let G be a finite nonabelian simple group such that $\pi(G) \subseteq \{2, 3, 5, 7, 13, 73\}$. Then G is isomorphic to one of the groups listed in Table 1. In particular,*

1. *$3 \in \pi(G)$ if and only if $G \neq Sz(8)$;*
2. *$\pi(\text{Out}(G)) \subseteq \{2, 3\}$ if $G \neq S_6(2)$.*

G	$ G $	$\text{Out}(G)$	G	$ G $	$\text{Out}(G)$
A_5	$2^2 \cdot 3 \cdot 5$	2	$L_2(7)$	$2^3 \cdot 3 \cdot 7$	2
A_6	$2^3 \cdot 3^2 \cdot 5$	2^2	$U_3(3)$	$2^5 \cdot 3^3 \cdot 7$	2
$L_2(8)$	$2^3 \cdot 3^2 \cdot 7$	3	$L_3(3)$	$2^4 \cdot 3^3 \cdot 13$	2
$U_4(2)$	$2^6 \cdot 3^4 \cdot 5$	2	A_7	$2^3 \cdot 3^2 \cdot 5 \cdot 7$	2
A_8	$2^6 \cdot 3^2 \cdot 5 \cdot 7$	2	$L_3(4)$	$2^6 \cdot 3^2 \cdot 5 \cdot 7$	D_{12}
A_9	$2^6 \cdot 3^4 \cdot 5 \cdot 7$	2	$L_2(49)$	$2^4 \cdot 3 \cdot 5^2 \cdot 7^2$	2^2
A_{10}	$2^7 \cdot 3^4 \cdot 5^2 \cdot 7$	2	$U_3(5)$	$2^4 \cdot 3^2 \cdot 5^3 \cdot 7$	S_3
$U_4(3)$	$2^7 \cdot 3^6 \cdot 5 \cdot 7$	D_8	J_2	$2^7 \cdot 3^3 \cdot 5^2 \cdot 7$	2
$O_8^+(2)$	$2^{12} \cdot 3^5 \cdot 5^2 \cdot 7$	S_3	$S_6(2)$	$2^9 \cdot 3^4 \cdot 5 \cdot 7$	1
$S_4(7)$	$2^8 \cdot 3^2 \cdot 5^2 \cdot 7^4$	2	$L_2(25)$	$2^3 \cdot 3 \cdot 5^2 \cdot 13$	2^2
$L_2(13)$	$2^2 \cdot 3 \cdot 5 \cdot 13$	2	$U_3(4)$	$2^6 \cdot 3 \cdot 5^2 \cdot 13$	4
$L_4(3)$	$2^7 \cdot 3^6 \cdot 5 \cdot 13$	2^2	$S_4(5)$	$2^6 \cdot 3^2 \cdot 5^4 \cdot 13$	2
${}^2F_4(2)'$	$2^{11} \cdot 3^3 \cdot 5^2 \cdot 13$	2	$L_2(27)$	$2^2 \cdot 3^3 \cdot 7 \cdot 13$	6
$G_2(3)$	$2^6 \cdot 3^6 \cdot 7 \cdot 13$	2	$L_2(64)$	$2^6 \cdot 3^2 \cdot 5 \cdot 7 \cdot 13$	6
$L_3(9)$	$2^7 \cdot 3^6 \cdot 5 \cdot 7 \cdot 13$	2^2	$Sz(8)$	$2^6 \cdot 5 \cdot 7 \cdot 13$	3
$G_2(4)$	$2^{12} \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 13$	2	$S_4(8)$	$2^{12} \cdot 3^4 \cdot 5 \cdot 7^2 \cdot 13$	6
$S_6(3)$	$2^9 \cdot 3^9 \cdot 5 \cdot 7 \cdot 13$	2	$O_7(3)$	$2^9 \cdot 3^9 \cdot 5 \cdot 7 \cdot 13$	2
${}^3D_4(2)$	$2^{12} \cdot 3^4 \cdot 7^2 \cdot 13$	3	$U_4(5)$	$2^7 \cdot 3^4 \cdot 5^6 \cdot 7 \cdot 13$	2^2
$O_8^+(3)$	$2^{12} \cdot 3^{12} \cdot 5^2 \cdot 7 \cdot 13$	S_4	$L_3(8)$	$2^9 \cdot 3^2 \cdot 7^2 \cdot 73$	6
$U_3(9)$	$2^5 \cdot 3^6 \cdot 5^2 \cdot 73$	4	$L_2(3^6)$	$2^3 \cdot 3^6 \cdot 5 \cdot 7 \cdot 13 \cdot 73$	2×6
${}^3D_4(3)$	$2^6 \cdot 3^{12} \cdot 7^2 \cdot 13^2 \cdot 73$	3	$G_2(9)$	$2^8 \cdot 3^{12} \cdot 5^2 \cdot 7 \cdot 13 \cdot 73$	2
$S_4(27)$	$2^6 \cdot 3^{12} \cdot 5 \cdot 7^2 \cdot 13^2 \cdot 73$	6	$L_4(8)$	$2^{18} \cdot 3^4 \cdot 5 \cdot 7^3 \cdot 13 \cdot 73$	6

Table 1 Finite Nonabelian Simple Groups with $\pi(G) \subseteq \{2, 3, 5, 7, 13, 73\}$

Proof. If G is a simple K_3 -group, then $G \cong A_5, L_2(7), L_2(8), A_6, L_3(3), U_3(3)$ or $U_4(2)$ by Lemma 2.2.

If G is a simple K_4 -group, then $G \cong L_2(13), A_7, L_2(25), L_2(27), A_8, A_9, Sz(8), L_2(49), U_3(4), U_3(5), U_3(9), L_3(4), J_2, S_6(2), A_{10}, U_4(3), G_2(3), S_4(5), L_4(3), {}^2F_4(2)', L_3(8), S_4(7), O_8^+(2), {}^3D_4(2)$ by Lemma 2.3.

If G is a simple K_5 -group or a simple K_6 -group, then $\pi(G) \subseteq \{2, 3, 5, 7, 13, 73\}$. By Lemmas 2.4 and 2.5, G is isomorphic to one of the groups listed in Lemmas 2.4 and 2.5. Now we assert all candidates for G must be $L_2(64), L_2(3^6), L_3(9), G_2(4), G_2(9), S_4(8), S_4(27), S_6(3), O_7(3), U_4(5), O_8^+(3), {}^3D_4(3)$ and $L_4(8)$.

Suppose G is isomorphic to $L_2(q)$, where q is a prime power satisfying $|\pi(q^2 - 1)| = 4$ or 5 . Let $q = p^s$, where p is a prime number and s is a natural number. Then $|G| = |L_2(q)| = \frac{q(q^2-1)}{(2,q-1)} = \frac{p^s(p^{2s}-1)}{(2,p^s-1)}$ (see [2]). Therefore $p \nmid |G|$. Since $\pi(G) \subseteq \{2, 3, 5, 7, 13, 73\}$, it follows that p must be one of the numbers $2, 3, 5, 7, 13$ or 73 .

Suppose $p = 2$. Since $\{3, 5, 7, 13, 73\} \subseteq \pi(\prod_{i=1}^{12} (2^i - 1))$, we assert that $2s \leq 12$. In fact, if $2s > 12$, then there exists a prime z such that $z \mid 2^{2s} - 1$ and $z \nmid 2^i - 1$ for $1 \leq i < 2s$ by Lemma 2.7. On one side, we have that $z \in \pi(G) \subseteq \{2, 3, 5, 7, 13, 73\}$ since $z \mid 2^{2s} - 1$. On the other side, we have that $z \notin \{2, 3, 5, 7, 13, 73\} \supseteq \pi(G)$ since $z \nmid 2^i - 1$ for $1 \leq i < 2s$. This is again a contradiction. Therefore $1 \leq s \leq 6$.

If $1 \leq s \leq 5$, then G is a simple K_n -group according to its order, where $n = 3$ or 4 , an obvious contradiction. If $s = 6$, then $|G| = \frac{2^6(2^{12}-1)}{(2,2^6-1)} = 2^6 \cdot 3^2 \cdot 5 \cdot 7 \cdot 13$, hence a candidate $L_2(64)$ arises in this subcase.

We omit the remaining proof, which is made by a case by case analysis by a similar argument as done above in term of the changes of the values of p and the types of the involved groups. If necessary, one may apply Lemmas 2.4–2.8. Finally, we find that our assertion is true as desired. \square

In the sequel a completely reducible group will be called a CR -group. The center of a CR -group is the direct product of the abelian factors in the decomposition. Hence a CR -group is centerless, that is has trivial center, if and only if it is a direct product of nonabelian simple groups. The following lemma determines the structure of the automorphism group of a centerless CR -group.

Lemma 2.10. ([9], Theorem 3.3.20) *Let R be a finite centerless CR -group and write $R = R_1 \times R_2 \times \cdots \times R_k$ where R_i is a direct product of n_i isomorphic copies of a simple group H_i , and H_i and H_j are not isomorphic if $i \neq j$. Then $\text{Aut}(R) \cong \text{Aut}(R_1) \times \text{Aut}(R_2) \times \cdots \times \text{Aut}(R_k)$ and $\text{Aut}(R_i) \cong \text{Aut}(H_i) \wr S_{n_i}$ where in this wreath product $\text{Aut}(H_i)$ appears in its right regular representation and the*

symmetric group S_{n_i} in its natural permutation representation. Moreover these isomorphisms induce isomorphisms $\text{Out}(R) \cong \text{Out}(R_1) \times \text{Out}(R_2) \times \cdots \times \text{Out}(R_k)$ and $\text{Out}(R_i) \cong \text{Out}(H_i) \wr S_{n_i}$.

At the end of this section, we quote two lemmas on noncommuting graph. Let g be an element of a finite group G . We denote by g^G the conjugacy class of G containing g . Also, we denote by $|g^G|$ the size of the conjugacy class g^G .

Lemma 2.11. ([8], Lemma 2) *Let G be a finite group such that $Z(G) = 1$. If H is a group such that $\nabla(H) \cong \nabla(G)$, then $|Z(H)|(|C_G(g_i)| - 1)$ and $|Z(H)|(|g_i^G| - 1)$ for every $g_i \in G^*$, where $G^* = G \setminus \{1\}$ and $1 \leq i \leq |G^*|$. In particular, if one of the following two conditions holds:*

1. $\text{g.c.d.}\{|C_G(g_1)| - 1, |C_G(g_2)| - 1, \dots, |C_G(g_{|G^*|})| - 1\} = 1$, or
 2. $\text{g.c.d.}\{|g_1^G| - 1, |g_2^G| - 1, \dots, |g_{|G^*|}^G| - 1\} = 1$,
- then $|H| = |G|$.

Lemma 2.12. ([11], Lemma 2.4) *Let G and H be finite groups. If $\nabla(H) \cong \nabla(G)$, then*

$$|C_H(x) \setminus Z(H)| = |C_G(\phi(x)) \setminus Z(G)|$$

for all $x \in H \setminus Z(H)$, where ϕ is a graph isomorphism from $\nabla(H)$ to $\nabla(G)$.

3. A new characterization of $L_4(8)$ by its noncommuting graph

Theorem : *Let $L_4(8)$ be the projective special linear group of degree 4 over the finite field of order 8. If G is a finite group with $\nabla(G) \cong \nabla(L_4(8))$, then $G \cong L_4(8)$.*

Proof. First we suppose that $M := L_4(8)$. Now we want to prove that $G \cong M$. We divide the proof into the following seven lemmas.

Lemma 3.1. *If $\nabla(G) \cong \nabla(M)$, then $|G| = |M|$. In particular, $Z(G) = 1$.*

Proof. By Lemma 2.11, it is sufficient to find a pair of elements in M , say u and v , such that $\text{g.c.d.}\{|C_M(u)| - 1, |C_M(v)| - 1\} = 1$ or $\text{g.c.d.}\{|u^M| - 1, |v^M| - 1\} = 1$. By Lemma 2.1, it follows that $\mu(M) = \{3^2 \cdot 5 \cdot 13, 7 \cdot 73, 2 \cdot 3^2 \cdot 7, 2^2 \cdot 7\}$. Therefore there exist two elements in M , say x and y , such that $o(x) = 3^2 \cdot 5 \cdot 13$ and $o(y) = 7 \cdot 73$ respectively. Since $|M| = 2^{18} \cdot 3^4 \cdot 5 \cdot 7^3 \cdot 13 \cdot 73$ and $\{2 \cdot 5, 2 \cdot 13, 2 \cdot 73, 3 \cdot 73, 5 \cdot 7, 5 \cdot 73, 7 \cdot 13, 13 \cdot 73\} \cap \pi_e(M) = \emptyset$ by Lemma 2.1, it follows that $|C_M(x)| = 3^i \cdot 5 \cdot 13$ and $|C_M(y)| = 7^j \cdot 73$, where $2 \leq i \leq 4$, $1 \leq j \leq 3$.

In the sequel, we will use the following symbols, where x and y are those elements mentioned above.

- (1) See Table 2: $Z_x := |C_M(x)|$; $N_x := \frac{|M|}{Z_x} = |x^M|$; $N'_x := N_x - 1$.
 (2) See Table 3: $Z_y := |C_M(y)|$; $N_y := \frac{|M|}{Z_y} = |y^M|$; $N'_y := N_y - 1$.

Z_x	N'_x
$3^2 \cdot 5 \cdot 13$	229 · 257966867
$3^3 \cdot 5 \cdot 13$	31 · 635208737
$3^4 \cdot 5 \cdot 13$	$3^2 \cdot 5 \cdot 145862747$

Table 2 Possible Values of Z_x and N'_x

From Table 2, we have $\pi(N'_x) \subseteq \{3, 5, 31, 229, 257966867, 635208737, 145862747\}$.

Z_y	N'_y
$7 \cdot 73$	67629219839
$7^2 \cdot 73$	$71 \cdot 239 \cdot 431 \cdot 1321$
$7^3 \cdot 73$	$7 \cdot 83 \cdot 269 \cdot 8831$

Table 3 Possible Values of Z_y and N'_y

From Table 3, we have $\pi(N'_y) \subseteq \{7, 71, 83, 239, 269, 431, 1321, 8831, 67629219839\}$.

If Z_x is equal to one of the values listed in Table 2, then N'_x satisfies the condition $\pi(N'_x) \cap \pi(N'_y) = \emptyset$ for any y satisfying $o(y) = 7 \cdot 73$ by Tables 2 and 3. Let $u = x$ and $v = y$, where N'_x and N'_y satisfy the condition mentioned above. Then $\text{g.c.d.}\{|u^M| - 1, |v^M| - 1\} = \text{g.c.d.}\{N'_x, N'_y\} = 1$, which implies that $|G| = |M|$ by Lemma 2.11. Since $|G \setminus Z(G)| = |M \setminus Z(M)|$ by the definition of the noncommuting graph, it follows immediately that $Z(G) = Z(M) = 1$. \square

Lemma 3.2. *If $\nabla(G) \cong \nabla(M)$, then $5 \cdot 73 \notin \pi_e(G)$ and $13 \cdot 73 \notin \pi_e(G)$.*

Proof. Let ϕ be a graph isomorphism from $\nabla(M)$ to $\nabla(G)$. If $5 \cdot 73 \in \pi_e(G)$, then there exists an element, say x , such that $o(x) = 5$. Therefore, $5 \cdot 73 \mid |C_G(x)|$. By Lemmas 2.12 and 3.1, $5 \cdot 73 \mid |C_M(\phi^{-1}(x))|$. If $2 \in \pi(o(\phi^{-1}(x)))$, then there exists a natural number i such that $x_1 := (\phi^{-1}(x))^i \in M$ is of order 2. Then $5 \cdot 73 \mid |C_M(x_1)|$, too. Let $y_1 \in C_M(x_1)$ such that $o(y_1) = 5$. Hence $o(x_1 y_1) = 2 \cdot 5$. Thus $2 \cdot 5 \in \pi_e(M)$, which is a contradiction since $2 \approx 5$ in $\Gamma(M)$. Therefore $2 \notin \pi(o(\phi^{-1}(x)))$. By a similar argument, we have that $\{3, 5, 7, 13, 73\} \cap \pi(o(\phi^{-1}(x))) = \emptyset$, too. Thus $\pi(M) \cap \pi(o(\phi^{-1}(x))) = \emptyset$. It follows that $\phi^{-1}(x) = 1$, a contradiction.

By a similar argument as done above, we get that $13 \cdot 73 \notin \pi_e(G)$. \square

Lemma 3.3. *If $\nabla(G) \cong \nabla(M)$, then G is nonsolvable.*

Proof. Suppose that G is solvable. Since $|G| = |M|$ by Lemma 3.1, it follows that G has a Hall $\{5, 73\}$ -subgroup H of order $5 \cdot 73$. Therefore H is a cyclic subgroup, which implies that $5 \cdot 73 \in \pi_e(G)$. This is a contradiction by Lemma 3.2. \square

Lemma 3.4. *Let K be the maximal normal solvable subgroup of G . Then K is a $\{2, 3, 7\}$ -subgroup.*

Proof. First we assume that $\{5, 13, 73\} \subseteq \pi(K)$. Let T be a Hall $\{5, 73\}$ -subgroup of K . It is easy to see that T is a cyclic subgroup of order $5 \cdot 73$ since $|G| = 2^{18} \cdot 3^4 \cdot 5 \cdot 7^3 \cdot 13 \cdot 73$ by Lemma 3.1. Thus $5 \cdot 73 \in \pi_e(K) \subseteq \pi_e(G)$, a contradiction.

Now we suppose that $\{p, q, r\} = \{5, 13, 73\}$.

Next we assume that $\{p, q\} \subseteq \pi(K)$ and $r \notin \pi(K)$. Let T be a Hall $\{p, q\}$ -subgroup of K . It is easy to see that T is a cyclic subgroup of order $p \cdot q$. If $\{p, q\} \neq \{5, 13\}$, then $p \cdot q \in \pi_e(K) \subseteq \pi_e(G)$, a contradiction. If $\{p, q\} = \{5, 13\}$, then K is a $\{2, 3, 5, 7, 13\}$ -subgroup. Let $R_p \in \text{Syl}_p(K)$. By Frattini argument $G = KN_G(R_p)$. Therefore, the normalizer $N_G(R_p)$ contains an element of order 73, say x . Obviously, $\langle x \rangle R_p$ is a subgroup of G of order $73 \cdot p$, which is abelian. Hence $73 \cdot p \in \pi_e(G)$, a contradiction.

Finally, if $r \in \pi(K)$ and $\{p, q\} \cap \pi(K) = \emptyset$, then K is a $\{2, 3, 7, r\}$ -subgroup. Let $R_r \in \text{Syl}_r(K)$. By Frattini argument $G = KN_G(R_r)$. Therefore, the normalizer $N_G(R_r)$ contains two elements of orders p and q , say x and y , respectively. Obviously, $\langle x \rangle R_r$ and $\langle y \rangle R_r$ are subgroups of G of orders $p \cdot r$ and $q \cdot r$, respectively. It is clear that both of them are abelian. Hence $\{p \cdot r, q \cdot r\} \subseteq \pi_e(G)$, a contradiction. \square

In the sequel, we will use the following notation. Given a finite group G , denote by $\text{Soc}(G)$ the socle of G , which is the subgroup generated by the set of all minimal normal subgroups of G .

Lemma 3.5. *Let $\overline{G} := G/K$ and $S := \text{Soc}(\overline{G})$. Then the following assertions are true:*

- (1) *If $r \in \{5, 13, 73\}$ and $r \notin \pi(S)$, then $r \notin \pi(\text{Aut}(S))$.*
- (2) *If $S \neq M$, then $\pi(S) = \{2, 3, 7\}$.*
- (3) *S is isomorphic to one of the following groups: $L_2(7), L_2(8), U_3(3), L_2(7) \times L_2(7), L_2(7) \times L_2(8), L_2(7) \times U_3(3), L_2(8) \times L_2(8), L_2(7) \times L_2(7) \times L_2(7), L_2(7) \times L_2(7) \times L_2(8)$ or M .*

Proof. Since G is nonsolvable by Lemma 3.3, we have that S is a centerless CR -group. Put $S = P_1 \times P_2 \times \cdots \times P_m$, where P_i 's are nonabelian finite simple groups. Since $|G| = |M| = 2^{18} \cdot 3^4 \cdot 5 \cdot 7^3 \cdot 13 \cdot 73$ by Lemma 3.1, it is obvious that P_i is isomorphic to one of the groups listed in Table 1. Moreover, $\{5, 13, 73\} \subseteq \pi(\overline{G})$ by Lemma 3.4.

In the sequel, we assume that $\{p, q, r\} = \{5, 13, 73\}$.

(1) Assume the first assertion to be false, then $r \mid |Aut(S)|$. Since $Inn(S) \cong S$, we have that $r \nmid |Inn(S)|$. Because $Aut(S)/Inn(S) = Out(S)$, it follows that $r \mid |Out(S)|$. But $Out(S) = Out(S_1) \times Out(S_2) \times \cdots \times Out(S_k)$, where the groups S_j 's are direct products of some isomorphic copies of the simple groups belonging to the set $\{P_1, P_2, \dots, P_m\}$ such that $S = P_1 \times P_2 \times \cdots \times P_m = S_1 \times S_2 \times \cdots \times S_k$. Therefore $r \mid |Out(S_j)|$ for some j , where $1 \leq j \leq k$. Suppose that S_j is a direct product of t isomorphic copies of a simple group P_i , where $P_i \in \{P_1, P_2, \dots, P_m\}$. By Lemma 2.10, we obtain that $|Out(S_j)| = |Out(P_i)|^t \cdot t!$. Since $\pi(P_i) \subseteq \pi(S) \subseteq \{2, 3, 7, p, q\}$, we have that $\pi(Out(P_i)) \subseteq \{2, 3\}$ or $|Out(P_i)| = 1$ by Lemma 2.9, which implies that $r \nmid |Out(P_i)|$. Therefore $r \nmid t!$, which implies that $t \geq r \geq 5$. If $P_i \not\cong Sz(8)$, then $3 \mid |P_i|$ by Lemma 2.9, which implies that $3^5 \mid |S_j| \mid |S| \mid |G|$, a contradiction. If $P_i \cong Sz(8)$, then $5 \mid |P_i|$ by Lemma 2.9, which implies that $5^5 \mid |S_j| \mid |S| \mid |G|$, a contradiction. Hence $r \notin \pi(Aut(S))$, as desired.

(2) First, we assume that $\{p, q\} \subseteq \pi(S)$ and $r \notin \pi(S)$. It is easy to see that $r \notin \pi(Aut(S))$ by the first assertion. Thus $r \in \pi(C_{\overline{G}}(S))$ since $\overline{G}/C_{\overline{G}}(S) \lesssim Aut(S)$. This implies that $\{p \cdot r, q \cdot r\} \subseteq \pi_e(G)$, a contradiction.

Next, we assume that $r \in \pi(S)$ and $\{p, q\} \cap \pi(S) = \emptyset$. It is easy to see that $\{p, q\} \cap \pi(Aut(S)) = \emptyset$ by the first assertion. Thus $\{p, q\} \subseteq \pi(C_{\overline{G}}(S))$ since $\overline{G}/C_{\overline{G}}(S) \lesssim Aut(S)$. This implies that $\{p \cdot r, q \cdot r\} \subseteq \pi_e(G)$, a contradiction.

Finally, we assume that $\{p, q, r\} \subseteq \pi(S)$. Since $|G| = |M| = 2^{18} \cdot 3^4 \cdot 5 \cdot 7^3 \cdot 13 \cdot 73$, it follows that $S \cong M$ by Lemma 2.9. However, this contradicts our hypothesis.

(3) Recall that $\pi(S) = \{2, 3, 7\}$ if $S \neq M$ by the second assertion. Therefore, using Table 1, we have that S is isomorphic to one of the following groups: $L_2(7), L_2(8), U_3(3), L_2(7) \times L_2(7), L_2(7) \times L_2(8), L_2(7) \times U_3(3), L_2(8) \times L_2(8), L_2(7) \times L_2(7) \times L_2(7), L_2(7) \times L_2(7) \times L_2(8)$ or M . \square

Lemma 3.6. *Let S be a group mentioned in Lemma 3.5(3). Then the following assertions are true.*

(1) $|S| \mid \frac{|\overline{G}|}{|C_{\overline{G}}(S)|} \mid |Aut(S)|$.

(2) If $S \neq M$, then $\{5, 13, 73\} \subseteq \pi(C_{\overline{G}}(S))$ and $C_{\overline{G}}(S)$ is nonsolvable.

Proof. (1) Since $S \cap C_{\overline{G}}(S) = Z(S) = 1$, we have that $S \cong S/S \cap C_{\overline{G}}(S) \cong C_{\overline{G}}(S)S/C_{\overline{G}}(S) \leq \overline{G}/C_{\overline{G}}(S) \lesssim Aut(S)$. Thus $|S| \mid \frac{|\overline{G}|}{|C_{\overline{G}}(S)|} \mid |Aut(S)|$.

(2) If $S \cong L_2(7) \times L_2(7), L_2(7) \times L_2(8), L_2(7) \times U_3(3), L_2(8) \times L_2(8), L_2(7) \times L_2(7) \times L_2(7)$ or $L_2(7) \times L_2(7) \times L_2(8)$, then $|Aut(S)| = |Aut(L_2(7))|^2 \cdot |S_2| = 2^9 \cdot 3^2 \cdot 7^2, |Aut(L_2(7))| \cdot |Aut(L_2(8))| = 2^7 \cdot 3^4 \cdot 7^2, |Aut(L_2(7))| \cdot |Aut(U_3(3))| = 2^{10} \cdot 3^4 \cdot 7^2, |Aut(L_2(8))|^2 \cdot |S_2| = 2^7 \cdot 3^6 \cdot 7^2, |Aut(L_2(7))|^3 \cdot |S_3| = 2^{13} \cdot 3^4 \cdot 7^3$ or $|Aut(L_2(7))|^2 \cdot |S_2| \cdot |Aut(L_2(8))| = 2^{12} \cdot 3^5 \cdot 7^3$ by Lemma 2.10, where S_r is a

symmetry group on $\{1, 2, \dots, r\}$. Thus $\pi(\text{Aut}(S)) = \{2, 3, 7\}$ if $S \neq M$. It follows that $\{5, 13, 73\} \subseteq \pi(C_{\overline{G}}(S))$ since $\overline{G}/C_{\overline{G}}(S) \lesssim \text{Aut}(S)$. Suppose $C_{\overline{G}}(S)$ is solvable. Then $C_{\overline{G}}(S)$ has a Hall $\{5, 73\}$ -subgroup T_1 . It is easy to see that T_1 is an abelian subgroup of order $5 \cdot 73$. Thus $5 \cdot 73 \in \pi_e(C_{\overline{G}}(S))$. Hence $5 \cdot 73 \in \pi_e(G)$ too, a contradiction. \square

Let n be a natural number and p a prime. In the sequel, $e(n_p)$ denotes a non-negative integer such that $p^{e(n_p)} \mid n$ but $p^{e(n_p)+1} \nmid n$.

Lemma 3.7. *S is not isomorphic to any of the following groups: $L_2(7), L_2(8), U_3(3), L_2(7) \times L_2(7), L_2(7) \times L_2(8), L_2(7) \times U_3(3), L_2(8) \times L_2(8), L_2(7) \times L_2(7) \times L_2(7)$ or $L_2(7) \times L_2(7) \times L_2(8)$. Therefore, $S \cong M = L_4(8)$.*

Proof. Suppose $S \cong L_2(7), L_2(8), U_3(3), L_2(7) \times L_2(7), L_2(7) \times L_2(8), L_2(7) \times U_3(3), L_2(8) \times L_2(8), L_2(7) \times L_2(7) \times L_2(7)$ or $L_2(7) \times L_2(7) \times L_2(8)$. In the sequel, we also assume that $\{p, q, r\} = \{5, 13, 73\}$ if p, q or r appears.

Step 1. Suppose $G_1 := C_{\overline{G}}(S)$, $\overline{G}_1 := G_1/K_1$ and $S_1 := \text{Soc}(\overline{G}_1)$, where K_1 is the maximal normal solvable subgroup of G_1 . Then the following assertions are true.

- (a) By Lemma 3.2, $\{5 \cdot 73, 13 \cdot 73\} \cap \pi_e(G_1) = \emptyset$ and $\{5 \cdot 73, 13 \cdot 73\} \cap \pi_e(\overline{G}_1) = \emptyset$.
- (b) By Lemma 3.6(2), $\{5, 13, 73\} \subseteq \pi(G_1)$ and G_1 is nonsolvable. By a similar argument in Lemma 3.4, we obtain that K_1 is a $\{2, 3, 7\}$ -subgroup, too.
- (c) Moreover, $\{5, 13, 73\} \subseteq \pi(\overline{G}_1)$ by (b) and so \overline{G}_1 is nonsolvable.
- (d) By Lemma 3.6(1), $e(|G_1|_2) \leq 15$, $e(|G_1|_3) \leq 3$, $e(|G_1|_7) \leq 2$, $e(|G_1|_5) = e(|G_1|_{13}) = e(|G_1|_{73}) = 1$.
- (e) By the choice of K_1 and (c), S_1 is a direct product of some finite nonabelian simple groups listed in Table 1.

By Table 1 and (d), we have that $\{5, 13, 73\} \cap \pi(S_1) \neq \{5, 13, 73\}$ and so $S_1 \neq M$. Now we assert that $\pi(S_1) = \{2, 3, 7\}$.

First we assume that $\{p, q\} \subseteq \pi(S_1)$ and $r \notin \pi(S_1)$. It is easy to see $r \notin \pi(\text{Aut}(S_1))$ by a similar argument in Lemma 3.5(1). Thus $r \in \pi(C_{\overline{G}_1}(S_1))$ since $\overline{G}_1/C_{\overline{G}_1}(S_1) \lesssim \text{Aut}(S_1)$. It implies that $\{p \cdot r, q \cdot r\} \subseteq \pi_e(\overline{G}_1)$, which contradicts (a).

Next we assume that $r \in \pi(S_1)$ and $\{p, q\} \cap \pi(S_1) = \emptyset$. It is easy to see $\{p, q\} \cap \pi(\text{Aut}(S_1)) = \emptyset$ by a similar argument in Lemma 3.5(1). Thus $\{p, q\} \subseteq \pi(C_{\overline{G}_1}(S_1))$ since $\overline{G}_1/C_{\overline{G}_1}(S_1) \lesssim \text{Aut}(S_1)$. It implies that $\{p \cdot r, q \cdot r\} \subseteq \pi_e(\overline{G}_1)$, which contradicts (a).

Hence we obtain that $\pi(S_1) = \{2, 3, 7\}$. It follows that $S_1 \cong L_2(7), L_2(8), U_3(3), L_2(7) \times L_2(7)$ or $L_2(7) \times L_2(8)$ by (d) and (e).

Step 2. Suppose $G_2 := C_{\overline{G_1}}(S_1)$, $\overline{G_2} := G_2/K_2$ and $S_2 := Soc(\overline{G_2})$, where K_2 is the maximal normal solvable subgroup of G_2 . Then the following assertions are true.

(f) By Lemma 3.2, $\{5 \cdot 73, 13 \cdot 73\} \cap \pi_e(G_2) = \emptyset$ and $\{5 \cdot 73, 13 \cdot 73\} \cap \pi_e(\overline{G_2}) = \emptyset$.

(g) By a similar argument in Lemma 3.6, $\{5, 13, 73\} \subseteq \pi(G_2)$ and G_2 is non-solvable. By a similar argument in Lemma 3.4, we obtain that K_2 is a $\{2, 3, 7\}$ -subgroup, too.

(h) Moreover, $\{5, 13, 73\} \subseteq \pi(\overline{G_2})$ by (g) and so $\overline{G_2}$ is nonsolvable.

(i) $e(|G_2|_2) \leq 12$, $e(|G_2|_3) \leq 2$, $e(|G_2|_7) \leq 1$, $e(|G_2|_5) = e(|G_2|_{13}) = e(|G_2|_{73}) = 1$ since $|S_1| \mid \frac{|G_1|}{|C_{\overline{G_1}}(S_1)|} \mid |Aut(S_1)|$.

(j) By the choice of K_2 and (i), S_2 is a direct product of some finite nonabelian simple groups listed in Table 1.

By Table 1 and (i), we have that $\{5, 13, 73\} \cap \pi(S_2) \neq \{5, 13, 73\}$ and so $S_2 \neq M$. Now we assert that $\pi(S_2) = \{2, 3, 7\}$.

First we assume that $\{p, q\} \subseteq \pi(S_2)$ and $r \notin \pi(S_2)$. It is easy to see $r \notin \pi(Aut(S_2))$ by a similar argument in Lemma 3.5(1). Thus $r \in \pi(C_{\overline{G_2}}(S_2))$ since $\overline{G_2}/C_{\overline{G_2}}(S_2) \lesssim Aut(S_2)$. It implies that $\{p \cdot r, q \cdot r\} \subseteq \pi_e(\overline{G_2})$, which contradicts (f).

Next we assume that $r \in \pi(S_2)$ and $\{p, q\} \cap \pi(S_2) = \emptyset$. It is easy to see $\{p, q\} \cap \pi(Aut(S_2)) = \emptyset$ by a similar argument in Lemma 3.5(1). Thus $\{p, q\} \subseteq \pi(C_{\overline{G_2}}(S_2))$ since $\overline{G_2}/C_{\overline{G_2}}(S_2) \lesssim Aut(S_2)$. It implies that $\{p \cdot r, q \cdot r\} \subseteq \pi_e(\overline{G_2})$, which contradicts (f).

Hence we get that $\pi(S_2) = \{2, 3, 7\}$ by (i). It follows that $S_1 \cong L_2(7)$ or $L_2(8)$ by (i) and (j).

Step 3. Suppose $G_3 := C_{\overline{G_2}}(S_2)$, $\overline{G_3} := G_3/K_3$ and $S_3 := Soc(\overline{G_3})$, where K_3 is the maximal normal solvable subgroup of G_3 . Then the following assertions are true.

(k) By Lemma 3.2, $\{5 \cdot 73, 13 \cdot 73\} \cap \pi_e(G_3) = \emptyset$ and $\{5 \cdot 73, 13 \cdot 73\} \cap \pi_e(\overline{G_3}) = \emptyset$.

(l) By a similar argument in Lemma 3.6, $\{5, 13, 73\} \subseteq \pi(G_3)$ and G_3 is non-solvable. By a similar argument in Lemma 3.4, we obtain that K_3 is a $\{2, 3, 7\}$ -subgroup, too.

(m) Moreover, $\{5, 13, 73\} \subseteq \pi(\overline{G_3})$ by (l) and so $\overline{G_3}$ is nonsolvable.

(n) $e(|G_3|_2) \leq 9$, $e(|G_3|_3) \leq 1$, $e(|G_3|_7) = 0$, $e(|G_3|_5) = e(|G_3|_{13}) = e(|G_3|_{73}) = 1$ since $|S_2| \mid \frac{|G_2|}{|C_{\overline{G_2}}(S_2)|} \mid |Aut(S_2)|$.

(o) By the choice of K_3 and (n), S_3 is a direct product of some finite nonabelian simple groups listed in Table 1. In particular, S_3 is nonsolvable.

By Table 1 and (n), we have that $\{5, 13, 73\} \cap \pi(S_3) \neq \{5, 13, 73\}$ and so $S_3 \neq M$.

First we assume that $\{p, q\} \subseteq \pi(S_3)$ and $r \notin \pi(S_3)$. It is easy to see $r \notin \pi(\text{Aut}(S_3))$ by a similar argument in Lemma 3.5(1). Thus $r \in \pi(C_{\overline{G_3}}(S_3))$ since $\overline{G_3}/C_{\overline{G_3}}(S_3) \lesssim \text{Aut}(S_3)$. It implies that $\{p \cdot r, q \cdot r\} \subseteq \pi_e(\overline{G_3})$, which contradicts (k).

Next we assume that $r \in \pi(S_3)$ and $\{p, q\} \cap \pi(S_3) = \emptyset$. It is easy to see $\{p, q\} \cap \pi(\text{Aut}(S_3)) = \emptyset$ by a similar argument in Lemma 3.5(1). Thus $\{p, q\} \subseteq \pi(C_{\overline{G_3}}(S_3))$ since $\overline{G_3}/C_{\overline{G_3}}(S_3) \lesssim \text{Aut}(S_3)$. It implies that $\{p \cdot r, q \cdot r\} \subseteq \pi_e(\overline{G_3})$, which contradicts (k).

Hence we get that $\pi(S_3) = \{2, 3\}$ by (n) and so S_3 is solvable, which contradicts (o).

By Steps 1-3, we have obtained that $S \cong M$. By Lemma 3.6(1), it follows that $|\overline{G}| = |G| = |M|$. Hence $K = 1$ and $G \cong M = L_4(8)$. \square

By Lemmas 3.1-3.7, we complete the proof of Theorem. \square

Remark 3.8. *Although we have not found a general method to deal with all simple groups on AAM's conjecture, it is evident that the method used in the present paper also works well in the cases $L_4(4)$, $L_4(7)$, $U_4(7)$, etc.*

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Liangcai Zhang

College of Mathematics and Physics
Chongqing University
Shapingba, Chongqing 400044
People's Republic of China
e-mail: zlc213@163.com

Wujie Shi

School of Mathematical Sciences
Suzhou University
Suzhou, Jiangsu 215006
People's Republic of China
e-mail: wjshi@suda.edu.cn