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Optimal Allocation of Heat Exchanger Inventory Associated with Fixed Power Output or Fixed Heat Transfer Rate Input

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Abstract

The purpose of this study is to determine the optimal distribution of the heat transfer surface area or conductance among the Stirling engine heat exchangers when the minimum of the total heat transfer surface area of the heat exchangers is sought. The optimization procedure must fulfill one of the following constraints: (1) fixed power output of the engine, (2) fixed heat transfer rate available at the source, or (3) fixed power output and heat transfer rate at the source. Internal and external irreversibilities of the Stirling engine are considered. An analytic approach, when heat transfer occurs at small temperature differences at the heat reservoirs, provides several restrictions with regard to variables of the model. A sensitivity analysis of the minimum of the total heat transfer surface area of the heat exchangers with respect to these variables and parameters is presented. The results show optimal temperatures of the working fluid and optimum allocation of heat exchanger inventory.

Key words: stirling engine, optimization, heat exchangers, regenerator

1. Introduction

The allocation of the heat exchanger inventory is an important issue in the design and optimization of thermal machines. Besides the fundamental operating point of cyclic heat engines that is of maximum power (Bejan, 1996, 1997, Feidt, 1995, 1996, Blank et al., 1994, Costea, 1997, Costea et al., 1995, 1999, De Vos, 1992, Petrescu et al., 1996, 2000a, Ibrahim and Klein, 1989, Organ, 1987, 1992, Popescu et al., 1994, Radcenco et al., 1993, Reader, 1991) other optimization criteria can be chosen when size or economic constraints exist.

Previous investigation of the optimal allocation of the heat exchanger inventory (Bejan, 1996) for imposed total heat transfer area has been focused on a generic thermal engine with external irreversibilities. The model we propose here for the optimization of a Stirling engine also includes the regenerator in the total heat transfer area to be distributed. The model has been developed for a Stirling engine cycle with external and internal irreversibilities, namely the heat transfer across temperature gap at the source and the sink, the heat losses between the source and the sink and imperfect heat regeneration.

The minimum of the total heat transfer area of the heat exchangers is sought when the power output of the engine or the heat transfer rate at the source is imposed. In a previous paper (Feidt et al., 2000) have presented the mathematical model and some preliminary results showing the effect of the heat transfer rate at the source when the power output of the engine is fixed, as well as that of the power output when the heat transfer rate at the source is fixed, on the optimum cycle

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temperatures and allocation of heat exchanger inventory.

This paper closely examines the existence conditions of the minimum size of heat exchanger inventory. Thus a sensitivity study with respect to potential variables for each case is performed. In addition to that, analytical results are derived by considering the approximation of small temperature differences at the source and the sink. Finally, by solving numerically the non linear equations of the model for the case of imposed power output of the engine or heat transfer rate at the source, results are graphically presented. They show the optimum state point and area distribution among the heat exchangers, the effect of some model parameters, and operational limits where they exist.

2. Small Temperature Differences Approach

The irreversible Stirling engine cycle on which the model is based is illustrated in Figure 1. The diagram shows the three sources of irreversibility that the model accounts for; namely, the heat transfer across a finite temperature gap at the source and the sink, machine thermal losses and incomplete heat regeneration. Thus, additional heat from the source, Q_{H1}, is needed in the process due to incomplete regeneration. Similarly, the unregenerated heat is shown being rejected in the process, Q_{L1}. The heat losses of the engine, Q_{loss}, are represented between the source and sink temperatures.

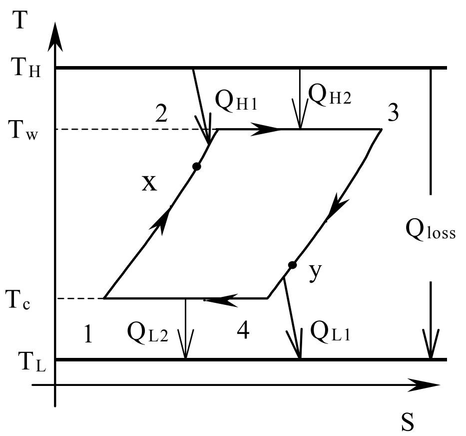


Figure 1. Stirling engine cycle with irreversibilities.

The regenerator that was modeled as a counterflow heat exchanger is added in the model to the four heat exchangers represented by heat addition or rejection in *Figure 1.*

Another modeling feature consists of replacing the constant temperature of the source by an available heat transfer rate, $\dot{\Phi}$. Thus, the energy balance equation is added to the model and that will also allow the optimization of the source temperature.

The mathematical model has been previously presented (Feidt et al, 2000). It consists of the first and second law statements for the cycle, the thermal balance at the source and the heat transfer equation in each additional heat exchanger H1 and L1 (see Annex A). Eventually, these equations are used to express the total heat transfer area as a function of the model variables and parameters in order to search its minimum value when the power output of the engine or the heat transfer rate available at the source is imposed (see Annex B).

The optimization of the total heat transfer area of the engine is more complicated than the optimization of engine power or efficiency because of the interdependence of variables and parameters. A preliminary study of the existence of min γNTU_T with respect to the fluid and source temperatures and number of heat transfer units of the regenerator in the approximation of small temperature differences at the heat reservoirs was considered helpful. Actually, this approach fits very well for Stirling engines operating near ambient temperatures.

By using small temperature differences at the two reservoirs (denoted by $x = \theta_H - \theta_w$ and $y = \theta_c - 1$) in the expression of the total number of heat transfer corresponding to the total heat exchange inventory already introduced by Feidt et al. (2000), one can derive analytical expressions for the variables.

The following results have been obtained for the two cases that were envisaged:

- for the case with fixed power output of the engine

$$\gamma NTU_T = \ln a_R + \frac{1}{rU_{H2}} \frac{\theta_H - x}{\theta_H - x - y - 1} \frac{\dot{W}_r + \dot{S}_{Tr}(y+1)}{x} +$$

$$+ \frac{1}{rU_{L1}} \ln b_R + \frac{1}{rU_{L2}} \frac{y+1}{\theta_H - x - y + 1} \cdot$$

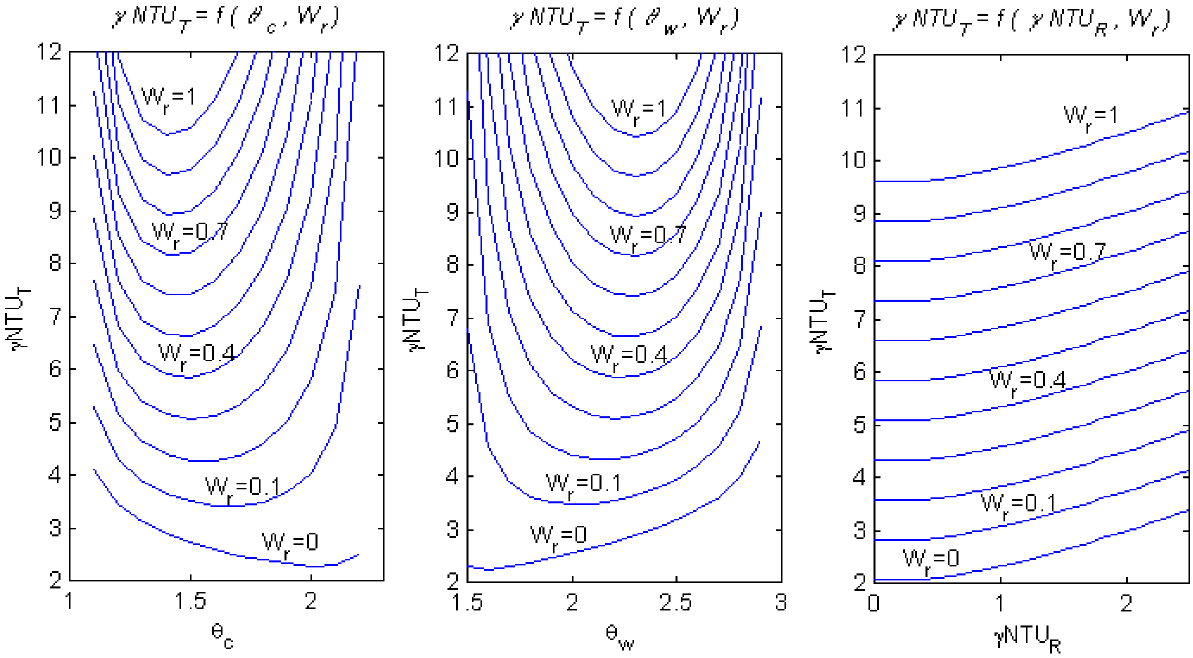
$$\cdot \frac{\dot{W}_r + \dot{S}_{Tr}(\theta_H - x)}{y} + \frac{1}{rU_R} \gamma NTU_R \quad (1)$$

where:

$$a_R = 1 + \frac{1}{1 + \gamma NTU_R} \frac{\theta_H - x - y - 1}{x} \quad (2)$$

$$b_R = 1 + \frac{1}{1 + \gamma NTU_R} \frac{\theta_H - x - y - 1}{y} \quad (3)$$

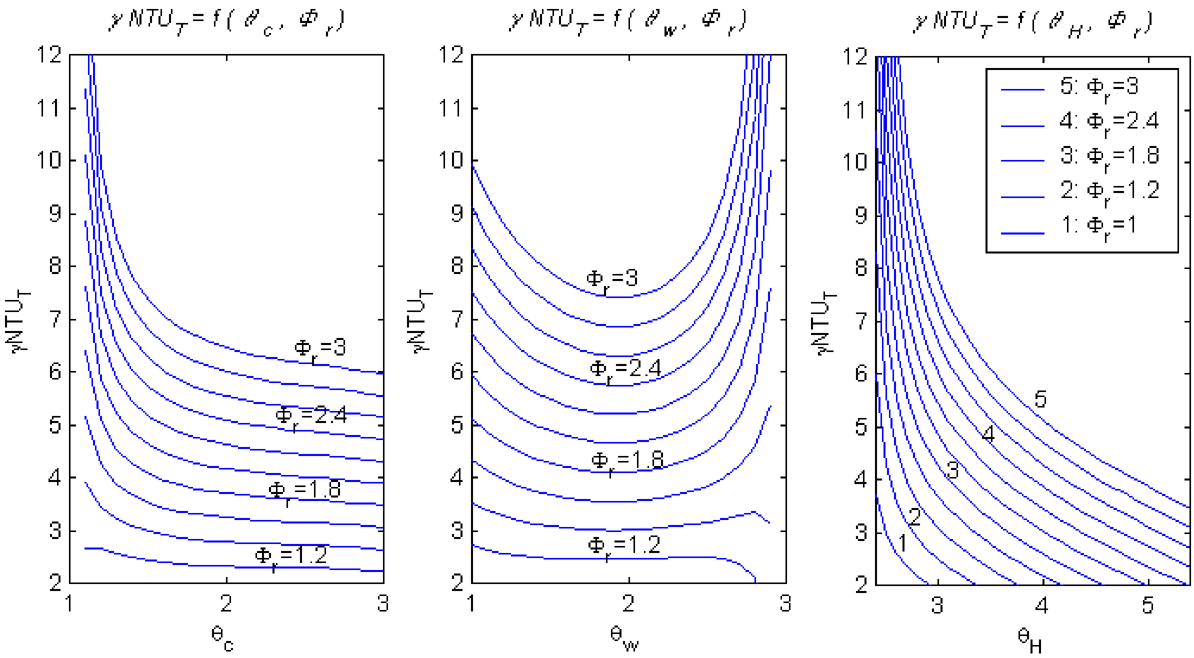
$$rU_Z = \frac{U_Z}{U_{H1}} \qquad (4)$$



a **b** **c**

($\theta_w = 2.3$, $\theta_H = 3$, $\gamma NTU_R = 1.85$) ($\theta_c = 1.4$, $\theta_H = 3$, $\gamma NTU_R = 1.85$) ($\theta_c = 1.4$, $\theta_w = 2.3$, $\theta_H = 3$)

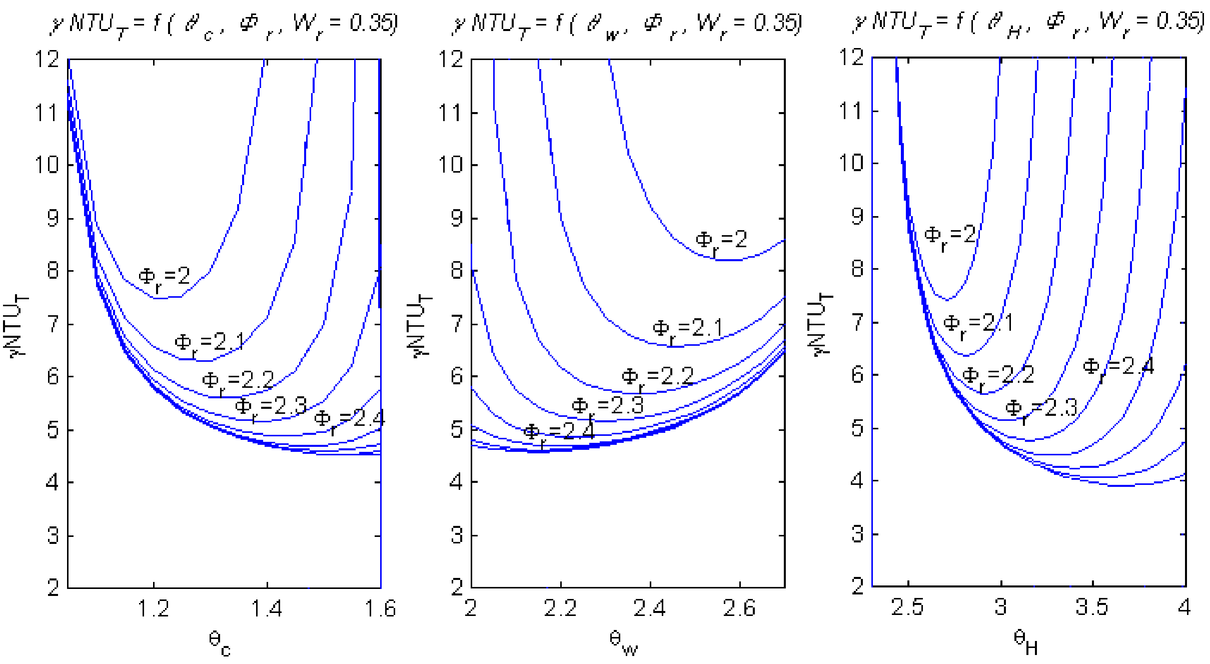
Figure 2. Sensitivity study of min γNTU_T with respect to the model variables for fixed reduced power output ($\gamma NTU_{loss} = 0.5$; $\dot{S}_{T_r} = 0.005$; $rU_Z = 1$).



a **b** **c**

($\theta_w = 2.3$, $\theta_H = 3$) ($\theta_c = 1.4$, $\theta_H = 3$) ($\theta_c = 1.4$, $\theta_w = 2.3$)

Figure 3. Sensitivity study of min γNTU_T with respect to the model variables for fixed reduced heat transferrate available at the source (γNTU_{loss} = 0.5; $\dot{S}_{T_r}$ = 0.005; rU_Z = 1; γNTU_R = 1.85).



a (θ_w = 2.3, θ_H = 3) **b** (θ_c = 1.4, θ_H = 3) **c** (θ_c = 1.4, θ_w = 2.3)

Figure 4. Sensitivity study of min γNTU_T with respect to the model variables for both power and heat flux fixed (γNTU_{loss} = 0.5; $\dot{S}_{T_r}$ = 0.005; rU_Z = 1; γNTU_R = 1.85).

$$\theta_H = \frac{T_H}{T_L} \tag{5}$$

$$\theta_w = \frac{T_w}{T_L} \tag{6}$$

$$\theta_c = \frac{T_c}{T_L} \tag{7}$$

$$\dot{W}_r = \frac{\dot{W}}{\dot{m} c_v T_L} \quad (8$$

$$\dot{S}_{Tr} = \frac{\dot{S}_T}{\dot{m} c_v T_L} \tag{9}$$

$$\gamma NTU_Z = \frac{U_Z A_Z}{\dot{m} c_v} \tag{10}$$

$$\gamma NTU_T = \frac{U_{H1} A_T}{\dot{m} c_v} \tag{11}$$

The derivatives of equation (1) with respect to θ_H and γNTU_R yield:

$$\frac{1}{\theta_H - 1 + x \cdot \gamma NTU_R - y} + \frac{1}{rU_{L1}} \frac{1}{\theta_H - 1 + y \cdot \gamma NTU_R - x} =$$

$$= \frac{1}{rU_R} \frac{1 + \gamma NTU_R}{\theta_H - 1 - x - y} \tag{12}$$

$$\theta_{Hopt} = 1 + \frac{\dot{W}_r + \dot{S}_{Tr}}{1 + 1/rU_{L1}} \left(\frac{1}{rU_{H2} \cdot x} + \frac{1}{rU_{L2} \cdot y} \right) \tag{13}$$

Equation (12) is a 3rd degree equation in γNTU_R that is expected to have at least one real solution.

Unfortunately, γNTU_T decreases with both x and y differences, so that there is no solution of the derivative of γNTU_T with respect to x and y. It means that only two variables (θ_H and γNTU_R) can be optimized.

- for the case with fixed heat transfer rate at the source

$$\gamma NTU_T = \ln a_R + \frac{1}{rU_{H2}} \frac{1}{x} \left[\Phi_r - \gamma NTU_{loss} (\theta_H - 1) - \right.$$

$$\left. - \frac{1}{1 + \gamma NTU_R} (\theta_H - x - y - 1) \right] + \frac{1}{rU_{L1}} \ln b_R + \frac{1}{rU_{L2}} \frac{y+1}{\theta_H - x} \frac{1}{y} \cdot$$

$$\cdot\left[\dot{\Phi}_r - \gamma NTU_{loss}(\theta_H - 1) - \frac{1}{1+\gamma NTU_R}(\theta_H - x - y - 1) + \right.$$

$$\left. + \dot{S}_{Tr}(\theta_H - x)\right] + \frac{1}{rU_R}\gamma NTU_R \qquad (14)$$

The analytical expressions we have obtained after derivation for small x and y are:

$$1 + \frac{1}{rU_{L1}} = \frac{1+\gamma NTU_R}{rU_R} + \frac{\theta_H - 1}{1+\gamma NTU_R}\cdot \left(\frac{1}{rU_{H2}\cdot x} + \frac{1}{rU_{L2}\cdot y}\frac{1}{\theta_H}\right) \qquad (15)$$

$$\left(1+\frac{1}{rU_{L1}}\right)\frac{1}{\theta_H - 1} = \frac{\dot{\Phi}_r}{rU_{L2}\cdot y\cdot\theta_H^2} + \left(\gamma NTU_{loss} + \frac{1}{1+\gamma NTU_R}\right)\cdot\left(\frac{1}{rU_{H2}\cdot x} + \frac{1}{rU_{L2}\cdot y}\cdot\frac{1}{\theta_H^2}\right) \qquad (16)$$

where:

$$\dot{\Phi}_r = \frac{\dot{\Phi}}{\dot{m}c_v T_L} \qquad (17)$$

Similar to the previous case, min of γNTU_T was found with respect to the same variables, θ_H and γNTU_R. The same conclusion is true for the case when both power output and heat input are fixed.

As it will be shown in the next section, the results of this approach are only representatives for this particular operating regime.

3. Sensitivity Study of the General Model

To prove the existence of the minimum of γNTU_T with respect to the model variables, a sensitivity study was performed. *Figure 2* illustrates the variation of γNTU_T versus the working fluid temperature at the cold- and hot-end and the number of heat transfer units corresponding to the regenerator. Clearly, the minimum of γNTU_T can be achieved for all values considered for the imposed power output of the engine only for the reduced temperatures θ_c (diagram a) and θ_w (diagram b). Moreover, the optimal values of the two temperatures corresponding to minimum of γNTU_T are decreasing, respectively increasing as the demand for power is going up. Thus, the reduced temperature $\theta_{c,\ opt}$ decreases from 2 to 1.4 (diagram a), and the reduced temperature $\theta_{w,\ opt}$ increases from 1.6 to 2.3 (diagram b) when the imposed reduced power output of the engine increases from 0 to 1. This variation trend of the two optimal temperatures is completely expected because its effect is to increase the difference of the extreme temperatures of the working gas, θ_c and θ_w, and the cycle area as well. Diagram c shows that there is a minimum of γNTU_T for very small values of γNTU_R.

The model answer with respect to the reduced temperature of the source θ_H is not giving optimum values and thus it is not illustrated here. Actually all the curves of γNTU_T corresponding to the values of 0< $\dot{W}_r$ <1 are decreasing when θ_H increases.

Analog diagrams from Figures 3-4 are relevant for the minimum of γNTU_T existence. For imposed heat transfer rate at the source (Figure 3), it appears that only one of the temperatures can be optimized, namely θ_w (diagram b), and only for values of $\dot{\Phi}_r$ higher than 1.2. For $\dot{\Phi}_r$ < 1.2 the value of γNTU_T becomes too small to include γNTU's of all heat exchangers of the engine. This limitation has important consequences on the numerical solution of the problem.

The others two diagrams of *Figure 3* (a and c) show that γNTU_T decreases monotonically with the cold-end reduced temperature of the gas and the reduced temperature of the source for all values considered for the imposed heat transfer rate at the source.

The most exciting result occurs in *Figure 4*, in spite of the fact that both parameters, $\dot{\Phi}_r$ and $\dot{W}_r$ are fixed. Minimum of γNTU_T is shown with all three temperatures. One can see that the cold-end reduced temperature value corresponding to min γNTU_T and that of the source (*Figure 4* a and c) increase from 1.25 to 1.45, respectively from 2.7 to 3.2 when $\dot{\Phi}_r$ increases from 2 to 2.4. Contrarily, the hot-end reduced temperature value corresponding to min γNTU_T (*Figure 4* b) decreases from 2.55 to 2.2 with the heat flux. Remember that the reduced power output of the engine is fixed. Thus, when the heat transfer rate at the source increases, the difference of the extreme optimal temperatures of the working gas, $\theta_{c,\ opt}$ and $\theta_{w,\ opt,}$ must decrease in order to keep constant the useful effect of the engine. Also, when the heat transfer rate at the source increases, the difference $\theta_{H,\ opt}$ - $\theta_{w,\ opt}$ increases since more heat must be injected in the cycle.

A general feature for all curves from Figure 4 is the reduction of min γNTU_T when the heat flux increases. This behavior is quite expected since the power output of the engine is fixed and the heat input increases.

There is no optimum with respect to γNTU_R for the cases from *Figures 3* and *4*.

To conclude, the minimum of the heat exchanger inventory exists under certain conditions regarding the model variables. Hence the number of variables and parameters is different upon the specific conditions of each case. It results in three variables (θ_c, θ_w, γNTU_R) and one parameter (θ_H) from *Figure 2*, only one variable (θ_w)

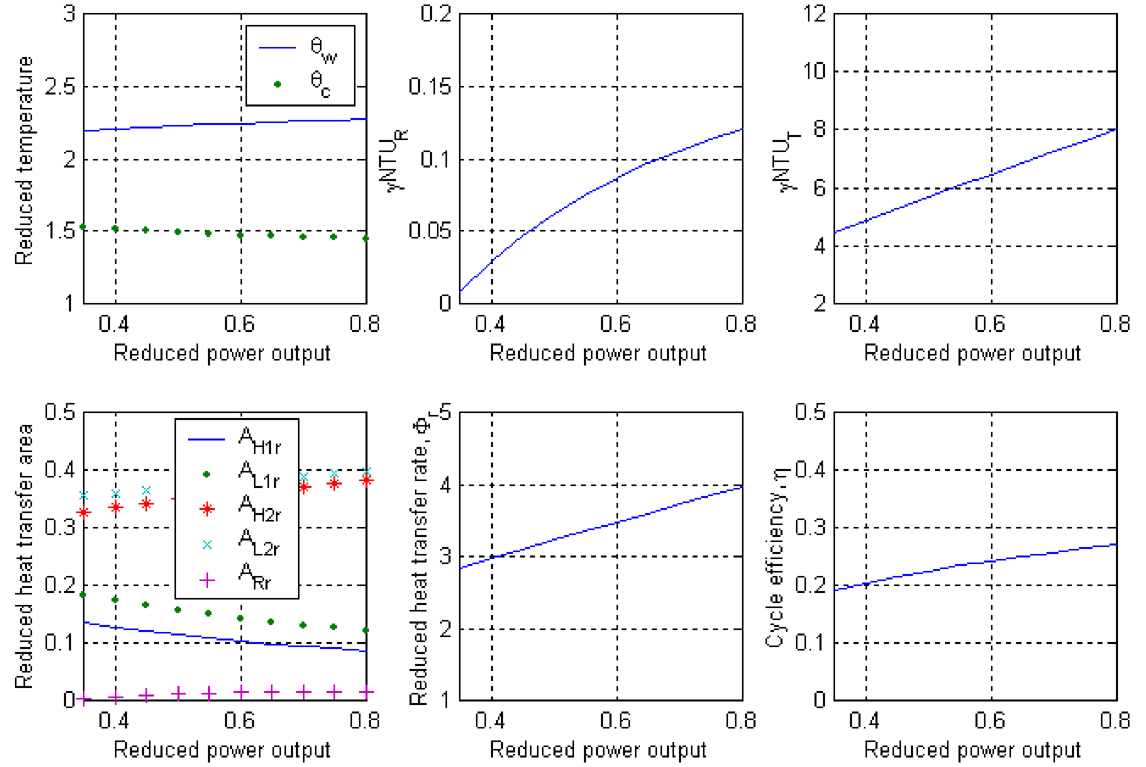


Figure 5. Performance of the Stirling engine for reduced power output of the engine imposed (θ_H = 3; γNTU_{loss} = 0.5; $\dot{S}_{T_r}$ = 0.005; rU_Z = 1).

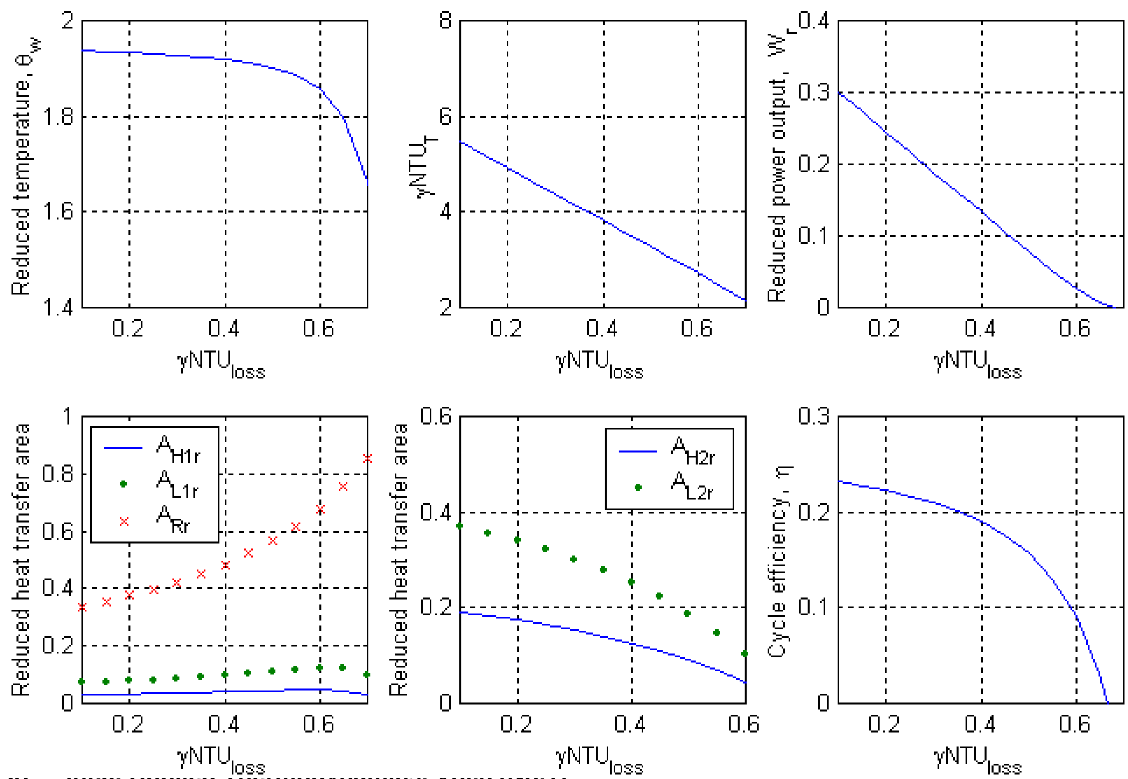


Figure 6. Performance of the Stirling engine when reduced power output of the engine and regenerator are are imposed (θ_H = 3; γNTU_{loss} = 0.5; $\dot{S}_{T_r}$ = 0.005; rU_Z = 1; γNTU_R = 1.85).

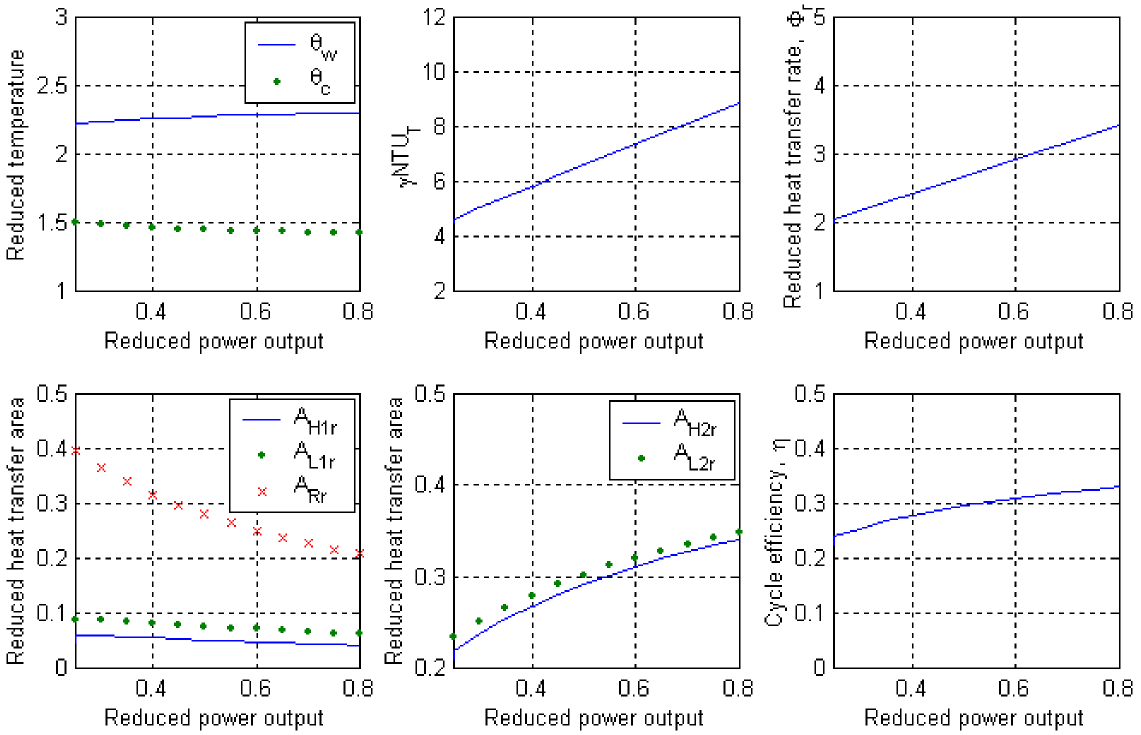


Figure 7. Reduced heat transfer area distribution and engine performance for fixed heat transfer rate at the source (θ_c = 1.4; θ_H = 3; γNTU_{loss} = 0.5; $\dot{S}_{T_r}$ = 0.005; rU_Z = 1; γNTU_R = 1.85).

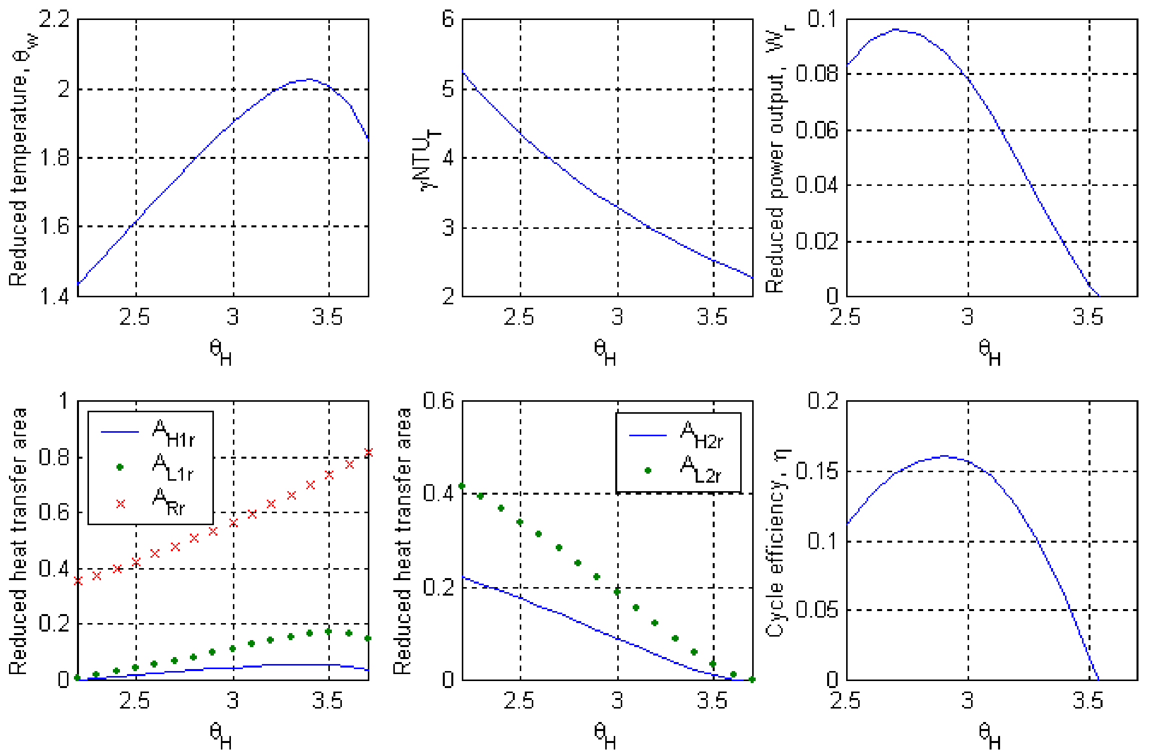


Figure 8. The effect of the thermal losses when the reduced heat transfer rate at the source is fixed (θ_c = 1.4; θ_H = 3; $\dot{\Phi}_r$ = 0.5; $\dot{S}_{T_r}$ = 0.005; rU_Z = 1; γNTU_R = 1.85).

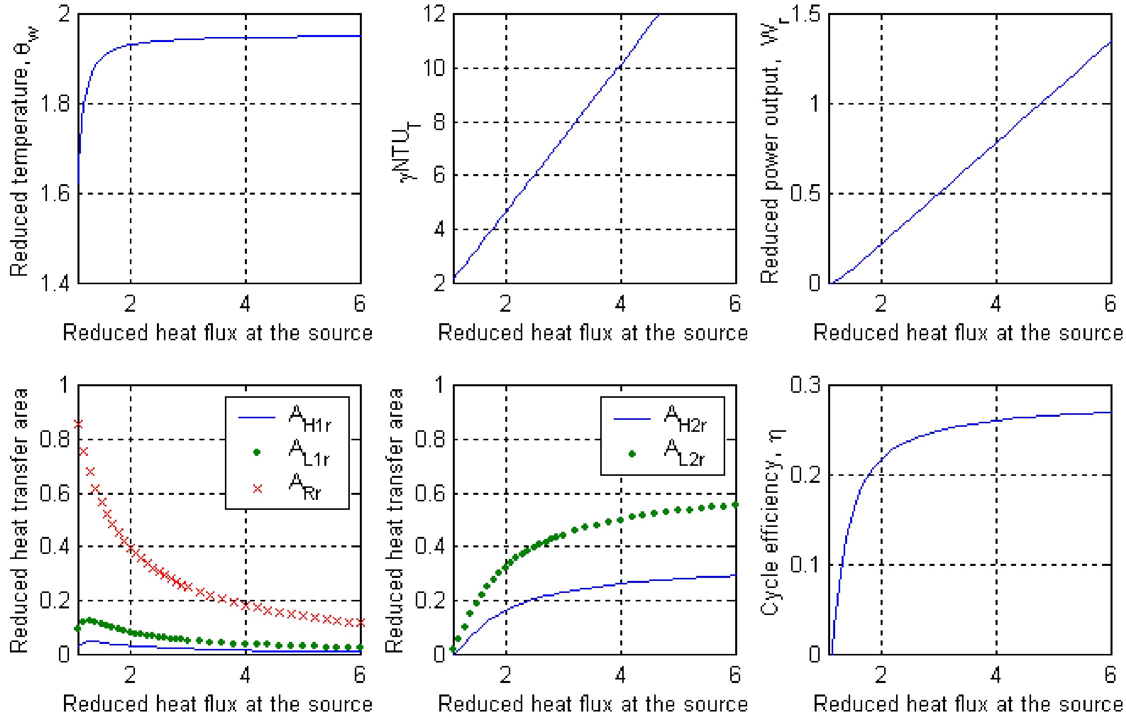


Figure 9. The effect of the reduced source temperature when the reduced heat transfer rate at the source is fixed (θ_c = 1.4; $\dot{\Phi}_r$ = 1.5; γNTU_{loss} = 0.5; $\dot{S}_{T_r}$ = 0.005; rU_Z = 1; γNTU_R = 1.85).

and three parameters (θ_c, θ_H, γNTU_R) from *Figure 3*, and again three variables (θ_c, θ_w, θ_H) and one parameter (γNTU_R) from *Figure 4*. The way the parameter values are chosen is mostly decided by the numerical calculation, where some solutions may not have physical significance.

The number of heat transfer units corresponding to the regenerator was taken as γNTU_R = 1.85 where it was the considered parameter. This value results from computation of regenerative heat losses (Petrescu et al., 2000b, 2001; Florea, 1999) that have been validated by comparison with experimental data available for several operating Stirling engines (Fujii, 1990; Stine and Diver, 1994).

4. Discussions

Some results of computation based on this model are shown in *Figures 5* to *9*. Generally the figures illustrate the optimum working fluid temperatures, the minimum of γNTU_T corresponding to the total heat transfer area of the heat exchangers, the area distribution between the five heat exchangers, the power output or the heat flux at the source, and the cycle efficiency, all in non dimensional form. Every point in the diagrams corresponds to optimum operation of the Stirling engine at min γNTU_T or min γNTU_T with fixed γNTU_R.

When the reduced power output of the engine is imposed and the regenerator area is part of the optimization procedure (*Figure 5*) one can see that the heat exchange inventory is mainly distributed in the second heat exchanger at the source and sink, A_{H2r} and A_{L2r}, while the regenerator area is small. Actually, the answer of the model to the minimum of γNTU_T search is a Stirling engine with almost zero regeneration. This could be surprising for a Stirling engine, but we have to remember that maximum performance is not the objective here. Actually, one could say that the heat exchange area concentrated at the sink is dominating, (A_{L1r} and A_{L2r}). The reason why this occurs can be found in the smooth variation of the working fluid temperatures θ_c and θ_w, and in the relatively small rise of the heat flux input. It means that if more power output of the engine is requested more surface must be allocated at the sink heat exchangers. In addition to that, an optimum heat transfer area of the regenerator was obtained in this case, even if it is very small. If one considers that the cost of high temperature surface (heater and regenerator) is high, one might say that these results are quite remarkable from an economical viewpoint. Nevertheless, note that the cycle efficiency has a reasonable value.

In order to allow the comparison of the results for all the cases we have studied, *Figure 6* shows the same diagrams as *Figure 5*, but for an imposed value of γNTU_R. The comparison of the corresponding properties in the diagrams of *Figures 5* and *6* could be relevant for a choice of a non-optimal operation regime. The effect of imposing the regenerator surface is positive on the heat transfer rate at the source that decreases due to a better heat regeneration.

There is no significant change of γNTU_T, because the imposed value of γNTU_R only determines a redistribution of the heat area between the heat exchangers. The gain mainly consists of efficiency increasing by 7-8%, and also decreasing of the heat transfer rate at the source. Perhaps an economic analysis could decide whether the distribution in favor of the regenerator is less expensive, if there is a significant price difference between heat exchange surface at the source and regenerator.

In *Figure 7*, the imposed property to the system is changed by considering fixed heat transfer rate at the source. The heat exchanger inventory keeps the same variation as in *Figure 6*, but with other values. The important change is registered by the reduced area of the second heat exchanger at the sink which is now almost twice of that of the second heat exchanger at the source. It means that in order to absorb more heat flux at the source when the regenerator area is fixed, the engine needs more heat transfer surface for the exchanger in contact with the sink, A_{L2}. Also, the last diagram of *Figure 7* could help the designer to choose an "economic" value of the heat transfer rate at the source. Thus upon the cycle efficiency smooth variation for high values of $\dot{\Phi}_r$, it is not worth to increase the heat flux available at the source above $\dot{\Phi}_r = 4$.

For the same case of fixed heat transfer rate available at the source ($\dot{\Phi}_r = 1.5$), *Figures 8* and 9 illustrate a sensitivity study with respect to two of the model parameters, namely the thermal losses from the source to the sink γNTU_{loss}, and the source temperature θ_H respectively. Both of them indicate operational limits imposed by the power output and the efficiency when they become zero ($\gamma NTU_{loss} < 0.65$, $\theta_H < 3.55$). Moreover, these curves from *Figure 9* suggest the value of the heat source temperature to be chosen as ($2.7 < \theta_H < 3$). Also the reduced heat transfer areas A_{H2} and A_{L2} that become zero for $\theta_H > 3.55$ confirm the limitation for θ_H. Another interesting feature is related to the fact that the more these two parameters increase, the more regenerator surface is needed.

In summary, for both cases of imposed property we have studied (fixed output, fixed input) the dominating heat transfer area was that of the sink heat exchangers (A_{L1}.and A_{L2}). Nevertheless, this result is true if we do not consider the regenerator area that was fixed (*Figures 6-7*). The same conclusion is by far true for the non-constrained optimization (γNTU_R variable).

Although the minimum of the total heat exchange inventory was sought, it is worth to say that the values of the efficiency shown in *Figure 6* are close to those indicated for the NS-03T Stirling engine (Fujii 1990) operating at maximum power ($\eta = 0.303$). As *Figure 6* illustrates the case with fixed power output of the engine, we may conclude that our results are in good agreement with experimental data.

5. Conclusion

An optimization model searching for the operational conditions for a Stirling engine and the optimal allocation of the heat transfer inventory when the optimization goal is the minimum of the total heat exchange surface has been presented.

The existence of this minimum was proved analytically by an approach of small temperature differences at the heat reservoirs and a sensitivity study with respect to potential variables of the model, respectively. We have found this model more complicated than that of maximum power or efficiency because of the interdependence of the operational conditions.

Besides the optimal temperatures for the working fluid and optimal distribution of the heat transfer area among the machine heat exchangers, the model also has indicated operational limitations with respect to some parameters.

For both conditions imposed on the engine, namely fixed power output or heat transfer rate at the source, the sink heat transfer area was shown to be preponderant. This result is really attractive for the designer due to the lower cost of the low-temperature heat transfer surface. However, the thermoeconomic analysis will enhance the optimal design.

Nomenclature

A	heat transfer area, [m^2]
A_{Zr}	reduced heat transfer area

c_v	specific heat at constant volume, [J kg^{-1} K^{-1}]
NTU	number of heat transfer units
$\dot{Q}$	heat transfer rate, [W]
rU	ratio of overall heat transfer coefficients
$\dot{S}_T$	total internal entropy generation, [J s^{-1} K^{-1}]
T	temperature, [K]
U	overall heat transfer coefficient, [W m^{-2} K^{-1}]
$\dot{W}$	power output, [W]
x	reduced temperature difference at the source
y	reduced temperature difference at the sink
Z	notation indicating the heat exchanger
$\dot{\Phi}$	heat transfer rate available at the source, [W]

Greek symbols

ε	heat exchanger effectiveness
γ	specific heat ratio
η	first law efficiency
θ	temperature ratio

Subscripts

c	cold - end of the engine
cf	counterflow
H	source
H_1	first heat exchanger - source
H_2	second heat exchanger - source
L	sink
L_1	first heat exchanger - sink
L_2	second heat exchanger - sink
loss	heat loss between the source and sink
R	regenerator
r	reduced, dimensionless
T	total
w	hot - end of the engine

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Annex A. The mathematical model

The mathematical model is based on the first and second law statements for the cycle (equations (A.1) and (A.2)), the thermal balance at the source (equation (A.3)) and the heat transfer equations in the additional heat exchanger at the source (equation (A.4)), and at the sink (equation (A.5)), respectively:

$$\dot{W} = U_{H2} A_{H2} (T_H - T_w) + U_{L2} A_{L2} (T_L - T_c) \quad (A.1)$$

$$\frac{U_{H2} A_{H2} (T_H - T_w)}{T_w} + \frac{U_{L2} A_{L2} (T_L - T_c)}{T_c} = -\dot{S}_T \quad (A.2)$$

$$\dot{m} c_v (T_w - T_x) + U_{H2} A_{H2} (T_H - T_w) = \dot{\Phi} - (U A)_{loss} (T_H - T_L) \quad (A.3)$$

$$\dot{m} c_v (T_w - T_x) = U_{H1} A_{H1} \frac{(T_H - T_x) - (T_H - T_w)}{\ln \frac{T_H - T_x}{T_H - T_w}} \quad (A.4)$$

$$\dot{m} c_v (T_y - T_L) = U_{L1} A_{L1} \frac{(T_y - T_L) - (T_c - T_L)}{\ln \frac{T_y - T_L}{T_c - T_L}} \quad (A.5)$$

These equations are completed by:

- the incomplete regeneration factor given by

$$\eta_R = \frac{T_x - T_c}{T_w - T_c} = \frac{T_w - T_y}{T_w - T_c} \quad (A.6)$$

- the effectiveness (Incropera and DeWitt 1996) of the regenerator when it is modeled by a counter-flow heat exchanger,

$$\varepsilon_{R,cf} = 1 - \frac{1}{1 + NTU_R} \quad (A.7)$$

Actually, the incomplete regeneration factor has the same definition expression as the regenerator effectiveness.

- the total heat transfer area of the machine

$$A_T = A_{H1} + A_{H2} + A_{L1} + A_{L2} + A_R \quad (A.8)$$

Note that for the assumed high efficiency of the regenerators the heat amonunts Q_{H1} and Q_{L1} are small and their contribution to entropy generation are neglected

In order to render the model more general and facilitate the extension of the analysis, the calculations have been performed in non-dimensional form of the equations. Thus, the temperatures were divided by the sink temperature, T_L, respectively the heat transfer rates and the power output, by the product $\dot{m} c_v T_L$, and the entropy generation term by $\dot{m} c_v$. The term “reduced” was added to each non-dimensional value.

Equations (A.1)÷(A.7) allow to express the intermediate variables of the model, i.e. T_x and T_y, and the heat transfer area of the heat exchangers of equation (A.8) as functions of only four variables (Costea 1997):

⇒ the reduced temperature of the source θ_H,

⇒ the reduced temperature of the gas at the hot-end of the engine θ_w,

⇒ the reduced temperature of the gas at the cold-end of the engine θ_c,

⇒ the number of heat transfer units in the regenerator, γNTU_R.

The total heat transfer area A_T is the objective function to be minimized, subject to constraint.

Annex B. The optimization analysis of the Stirling engine

The optimization analysis of the Stirling engine have focused on two different constraints, namely fixed available heat transfer rate at the source or fixed power output of the engine. The result consisted in two non-linear equations given by:

- for the case with fixed heat transfer rate at the source

$$NTU_T = \frac{1}{\gamma}\left\{ \ln a_R + \frac{1}{ruH}\frac{1}{\theta_H - \theta_w}\left[\dot{\Phi}_r - \gamma NTU_{loss}(\theta_H - 1) - \right.\right.$$

$$\left. - \frac{1}{1+NTU_R}(\theta_w - \theta_c)\right] + \frac{1}{ruL1}\ln b_R + \frac{1}{ruL2}\frac{\theta_c}{\theta_w}\frac{1}{\theta_c - 1}\left[\dot{\Phi}_r - \right.$$

$$\left. - \gamma NTU_{loss}(\theta_H - 1) - \frac{1}{1+NTU_R}(\theta_w - \theta_c) + \dot{S}_{Tr}\,\theta_w\right] +$$

$$\left. + \frac{1}{ruR}\gamma NTU_R \right\} \qquad (B.1)$$

$$\dot{\Phi}_r = \text{fixed} \qquad (B.2)$$

- for the case with fixed power output of the engine

$$NTU_T = \frac{1}{\gamma}\left\{ \ln a_R + \frac{1}{ruH}\frac{\theta_w}{\theta_w - \theta_c}\frac{1}{\theta_H - \theta_w}\left(\dot{W}_r + \dot{S}_{Tr}\,\theta_c\right) + \right.$$

$$+ \frac{1}{ruL1}\ln b_R + \frac{1}{ruL2}\frac{\theta_c}{\theta_w - \theta_c}\frac{1}{\theta_c - 1}\left(\dot{W}_r + \dot{S}_{Tr}\,\theta_w\right) +$$

$$\left. + \frac{1}{ruR}\gamma NTU_R \right\} \qquad (B.3)$$

$$\dot{W}_r = \text{fixed} \qquad (B.4)$$

By using the Lagrangian undetermined multiplier to solve the two systems the minimum of the total heat transfer area of the machine was obtained for each case.