

## PAPER DETAILS

TITLE: Effect of Gas-Properties Evaluation Method on the Optimum Point of Gas Turbine Cycles

AUTHORS: Christos FRANGOPOULOS, George DIMOPOULOS

PAGES: 95-102

ORIGINAL PDF URL: <https://dergipark.org.tr/tr/download/article-file/65664>

## Effect of Gas-Properties Evaluation Method on the Optimum Point of Gas Turbine Cycles

Christos A. Frangopoulos\* and George G. Dimopoulos<sup>+</sup>

National Technical University of Athens

Department of Naval Architecture and Marine Engineering

Heroon Polytechniou 9, 157 73 Zografou Greece

Phone: +30-210-7721108, Fax: +30-210-7721117

E-mail: caf@naval.ntua.gr

### Abstract

Recent work has revealed that the assumption regarding the behavior of gases (perfect, ideal, real) and, consequently, the way their properties are evaluated may alter critically the picture obtained about the performance of gas turbine systems. This fact prompted an investigation of how the aforementioned assumption may affect the optimal design point of gas turbine systems. The present study is restricted to a comparison between the ideal and perfect gas assumption. Three systems have been selected for study and three optimization problems have been formulated and solved for each system: two thermodynamic and one thermoeconomic. The results demonstrate that the method used for the evaluation of properties of gases has a very significant effect on the optimal point of each system.

**Keywords:** *Gas properties, gas turbine cycles, optimization, thermoeconomics.*

### 1. Introduction

It is common knowledge that the efficiency of a simple gas turbine cycle increases monotonically with the maximum cycle temperature for the constant pressure ratio (Haywood 1987). In order to be more specific, the efficiency of the air standard cycle (assumption of perfect gas with no change of mass flow rate due to fuel addition and no pressure losses in the ducts and the combustion chamber) is given by the equation:

$$\eta_A = \frac{\eta_C \eta_T \eta_J \tau_3 - (r^k - 1)}{\eta_C (\tau_3 - 1) - (r^k - 1)} \quad (1)$$

where

$$\eta_J = 1 - \frac{1}{r^k} \quad (2)$$

is the Joule cycle efficiency, i.e. the ideal cycle with isentropic compression and expansion and no losses. Starting with equation (1) it is easily proved that, if the turbine temperature is increased, keeping the pressure ratio constant, the thermal efficiency of the air standard cycle increases continuously and asymptotically it reaches the limit:

$$\lim_{\tau_3 \rightarrow \infty} \eta_A = \eta_T \eta_J \quad (3)$$

It was tacitly assumed that the general trend was the same even if a change of specific heat capacity or of the mass flow rate due to fuel addition was considered. Thus, it was a surprise to read in Horlock (2003) that, if the assumption of a working substance of constant quality and quantity is relaxed, then the behavior changes drastically; for a constant pressure ratio, the efficiency initially increases with the turbine-inlet temperature, it reaches a maximum value and then it decreases. Detailed studies of these effects appear in Horlock (2000 and 2001) and Guha (2003). This remark prompted the investigation reported here.

Many publications on optimization of gas turbine cycles, e.g. Frangopoulos (1988, 1992 and 1994), Valero et al. (1994), are based on the assumption of perfect gas with different values for the specific heats of air and exhaust gases, in order to decrease the inaccuracy. After the aforementioned, the question arises: "How is the optimum point affected if the properties of gases are evaluated with a higher accuracy?" An answer to this question is attempted in the following, using as examples three different system configurations.

\*Author to whom correspondence should be addressed  
The article is published with the permission of the organizers of ECOS 2005, where it was first presented.

## 2. Evaluation of Gas Properties

A clarification of terminology is useful at this point. The specific heat capacities of a real gas are functions of both temperature and pressure:

$$c_p = c_p(p, T), \quad c_v = c_v(p, T) \quad (4)$$

For an *ideal gas*, they are functions of the temperature only:

$$c_p = c_p(T), \quad c_v = c_v(T) \quad (5)$$

For a *perfect gas*, they are constant:

$$c_p = \text{const.}, \quad c_v = \text{const.} \quad (6)$$

This ‘textbook material’ is repeated here because it is often written in related publications that ‘real-gas’ effects are studied, while in fact the gases are considered ideal. Thus, the reader should be careful.

In the present work, for the perfect gas model the values

$$c_{pa} = 1.004 \text{ kJ/kgK}, \quad c_{pg} = 1.170 \text{ kJ/kgK}$$

have been considered. For the ideal gas model, it is considered that air consisting of  $N_2$ ,  $O_2$ ,  $CO_2$  and  $H_2O$  is compressed and then reacts with a fuel having the general composition  $C_\alpha H_\beta$  in a complete combustion to produce exhaust gases consisting of  $N_2$ ,  $O_2$ ,  $CO_2$  and  $H_2O$ . For simplicity, minor constituents (such as  $CO$ ,  $NO_x$ , etc.) due to additional reactions, dissociation, impurities or other reasons are not considered here. The properties of each species are evaluated by the following equations obtained from Gordon et al. (1994) and McBride et al. (2002):

$$\frac{\tilde{c}_{p0}}{\tilde{R}} = a_1 \cdot T^{-2} + a_2 \cdot T^{-1} + a_3 + a_4 \cdot T + a_5 \cdot T^2 + a_6 \cdot T^3 + a_7 \cdot T^4 \quad (7)$$

$$\frac{\tilde{h}_0}{\tilde{R}} = -a_1 \cdot T^{-1} + a_2 \cdot \ln(T) + a_3 \cdot T + \frac{1}{2} a_4 \cdot T^2 + \frac{1}{3} a_5 \cdot T^3 + \frac{1}{4} a_6 \cdot T^4 + \frac{1}{5} a_7 \cdot T^5 + b_1 \quad (8)$$

$$\frac{\tilde{s}_0}{\tilde{R}} = -\frac{1}{2} a_1 \cdot T^{-2} - a_2 \cdot T^{-1} + a_3 \cdot \ln(T) + a_4 \cdot T + \frac{1}{2} a_5 \cdot T^2 + \frac{1}{3} a_6 \cdot T^3 + \frac{1}{4} a_7 \cdot T^4 + b_2 - \ln(P) \quad (9)$$

The numerical values of the parameters  $a_i$  depend on the species and the temperature range, and are given in McBride et al. (2002).

The properties of air and exhaust gases are evaluated with the assumption that they are ideal

mixtures; for example, the molar heat capacity is calculated by the equation:

$$\tilde{c}_p = \sum_i x_i \tilde{c}_{pi} \quad (10)$$

Thus, the perfect gas assumption of previous works has been replaced here with the ideal gas assumption. The effect of pressure is still considered negligible for the pressure ranges used in the systems that are studied here, as justified by values obtained for the compressibility factor. Furthermore, models for thermodynamic properties of real gases are not presently available for all the constituents and all the temperature and pressure ranges appearing in gas turbine cycles. Therefore, the real gas effect is left for future investigation.

## 3. Systems Studied

### 3.1 Description of the systems

Three systems have been selected in order to study the effect of the method used for property evaluation on the optimal design point.

System I consists of a simple, open-cycle gas turbine (*Figure 1*). An approach for its thermodynamic and thermoeconomic optimization based on the perfect gas assumption has been presented in Frangopoulos (1988 and 1992).

System II is a cogeneration plant consisting of a regenerative gas turbine with an exhaust gas boiler producing saturated steam (*Figure 2*) of a given quality and quantity; it is the system of the CGAM problem (Valero et al. 1994 and Frangopoulos 1994).

System III is an inter-cooled, regenerative gas turbine with a twin spool gas generator and a power turbine (*Figure 3*).

### 3.2 Mathematical Models of the Systems

#### 3.2.1 Thermodynamic model of System I

The air temperature at the exit of the compressor and the exhaust gas temperature at the exit of the turbine are evaluated by the equations:

$$T_2 = T_1 \cdot \left[ 1 + \left( \frac{k_{12}-1}{r_C^{k_{12}}} - 1 \right) \cdot \frac{1}{\eta_C} \right] \quad (11)$$

$$T_4 = T_3 \cdot \left[ 1 - \left( 1 - r_T^{\frac{1-k_{34}}{k_{34}}} \right) \cdot \eta_T \right] \quad (12)$$

where

$$k_{ij} = \frac{\tilde{c}_{pij}}{\tilde{c}_{pij} - \tilde{R}} \quad (13)$$

$$\tilde{c}_{p,ij} = \frac{1}{T_j - T_i} \int_{T_i}^{T_j} \tilde{c}_p(T) dT \quad (14)$$

Since  $k_{12}$  and  $k_{34}$  depend on the temperatures  $T_2$  and  $T_4$ , respectively, equations (11) and (12) are used in an iterative procedure in order to obtain the temperatures  $T_2$  and  $T_4$ .

An energy balance in the combustion chamber gives the equation:

$$f H_u \eta_B = (1+f)(h_3 - h_0) - (h_2 - h_0) \quad (15)$$

which can be solved for  $f$ , if the temperature  $T_3$  is given. The composition of the exhaust gases is determined by the reaction of combustion for any specified fuel. Since the temperature  $T_3$  and the composition of exhaust gases are interrelated through the temperature-dependent properties of the constituents, an iterative procedure is also applied here.

The system efficiency is given by the equation:

$$\eta_I = \frac{\dot{W}}{\dot{m}_f H_u} = \frac{(1+f)(h_3 - h_4) - (h_2 - h_1)}{f H_u} \quad (16)$$

The specific work is also of interest:

$$w = \frac{\dot{W}}{\dot{m}_a} \quad (17)$$

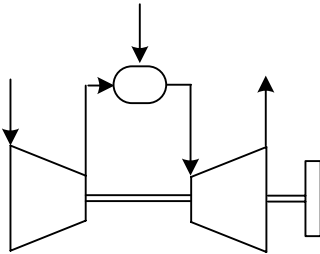


Figure 1. System I

### 3.2.2 Thermodynamic model of system II

Equations (11)-(15) and (17) are valid for System II, with a proper adjustment of certain numerical indexes. The effectiveness of the air pre-heater is given by the equation:

$$\varepsilon_X = \frac{h_3 - h_2}{h_{a5} - h_2} \quad (18)$$

The subscript 'a5' is used in equation (18) in order to make it clear that  $h_{a5}$  is the enthalpy of air at temperature  $T_5$ . There is no ambiguity about  $h_2$  and  $h_3$ . The following efficiencies are defined for this system.

Net shaft-power efficiency:

$$\eta_{II} = \frac{\dot{W}}{\dot{m}_f H_u} = \frac{(1+f)(h_4 - h_5) - (h_2 - h_1)}{f H_u} \quad (19)$$

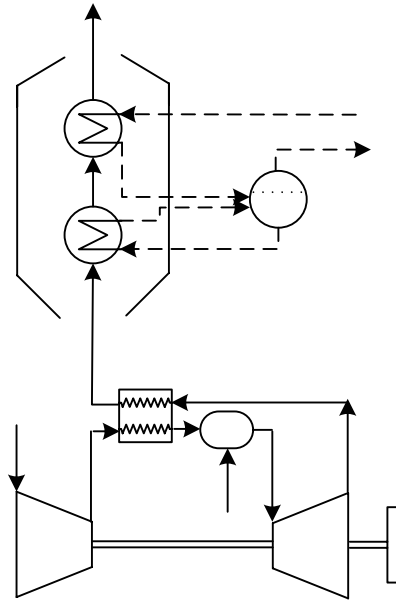


Figure 2. System II

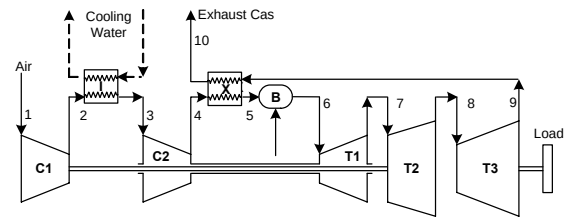


Figure 3. System III

Efficiency of providing the useful heat:

$$\eta_{II,Q} = \frac{\dot{Q}}{\dot{m}_f H_u} = \frac{\dot{m}_s (h_9 - h_8)}{\dot{m}_f H_u} \quad (20)$$

Total efficiency:

$$\eta_{II,tot} = \eta_{II} + \eta_{II,Q} = \frac{\dot{W} + \dot{Q}}{\dot{m}_f H_u} \quad (21)$$

The model of System II consists of many more equations, which are given in Ref. [9], but they are not repeated here due to space limitations.

### 3.2.3 Thermodynamic model of system III

For the compression, combustion and expansion processes, equations similar to those of System I are used. The effectiveness of the air pre-heater is given by an equation similar to equation (18), with proper adjustment of the numerical indexes. In addition, the following equalities are taken into consideration:

$$\dot{W}_{C1} = \dot{W}_{T2}, \quad \dot{W}_{C2} = \dot{W}_{T1} \quad (22)$$

The division of the pressure ratio between the low-pressure and high-pressure spool is

determined by an iterative procedure so that equation. (22) is satisfied.

The system efficiency is given by the equation:

$$\eta_{III} = \frac{\dot{W}}{\dot{m}_f H_u} = \frac{(1+f)(h_8 - h_9)}{f H_u} \quad (23)$$

The specific work is given by equation. (17).

### 3.2.4 Thermoeconomic models of the systems

The cost functions for Systems I and II appear in Frangopoulos (1988, 1992 and 1994) and Valero et al. (1994). For System III, equations available for Systems I and II have been properly modified and used. Space limitations do not allow giving the complete set of equations here.

## 4. Performance of Systems with Alternative Methods for Evaluation of Properties

For the performance evaluation and for the optimization of the systems, certain values have been considered for the various parameters involved, which are given in TABLE I.

The first step in this investigation has been the study of the effect of properties evaluation on the simple cycle efficiency (System I). The results depicted in Figure 4 are revealing: the perfect gas assumption, as expected, gives an efficiency continuously increasing with the turbine inlet temperature. With the ideal gas assumption and properties evaluated by equations. (7)-(10), the efficiency exhibits a maximum at a temperature of about 1600 K (for the parameter values considered here). Thus, the related results of Guha (2003) are reproduced to a very close approximation (small differences are due to different values of parameters and to the different sources of equations for evaluation of gas properties).

With the perfect gas model, the specific work increases continuously with turbine inlet temperature for a certain pressure ratio. This trend remains the same with the ideal gas model too.

In Figures 5 and 6, the effect of the gas model assumption on the system efficiency and specific work as functions of pressure ratio is shown. It is clarified that the graphs of Figures 5B and 6B correspond to the system of Figure 2 but without the exhaust gas boiler.

The coordinates of the optimum points in Figures 5 and 6 are given in TABLES II and III, respectively.

TABLE I. VALUES OF PARAMETERS.

| System I                            | System II                        | System III                       |
|-------------------------------------|----------------------------------|----------------------------------|
| $r_B = 0.975$                       | $r_B = 0.975$                    | $r_{C1} = r_{C2}$                |
| $\eta_B = 0.99$                     | $\eta_B = 0.99$                  | $r_I = 0.98$                     |
| $\eta_m = 0.99$                     | $r_{Xa} = 0.975$                 | $T_3 = T_I$                      |
|                                     | $r_{Xg} = 0.965$                 | $r_B = 0.975$                    |
| Air                                 | $\Delta T_{Xmin} = 20 \text{ K}$ | $\eta_B = 0.99$                  |
| $N_2: 77.82\%$                      | $\eta_m = 0.99$                  | $r_{Xa} = 0.975$                 |
| $O_2: 20.68\%$                      | $r_R = 0.95$                     | $r_{Xg} = 0.965$                 |
| $CO_2: 0.03\%$                      | $\Delta T_{Pmin} = 15 \text{ K}$ | $\Delta T_{Xmin} = 20 \text{ K}$ |
| $H_2O: 1.47\%$                      | $T_{7min} = 373.15 \text{ K}$    | $\eta_m = 0.99$                  |
| $T_{amb} = T_0 = 25^\circ \text{C}$ | $\dot{m}_{st} = 14 \text{ kg/s}$ | $r_R = 0.95$                     |
| $P_{amb} = P_0$                     | $P_8 = 20 \text{ bar}$           |                                  |
| $= 1.01325 \text{ bar}$             | $T_8 = 298.15 \text{ K}$         |                                  |
| Fuel: $CH_4$                        | $T_{8P} = T_9 - 15 \text{ K}$    |                                  |
| $H_u = 50000 \text{ kJ/kg}$         | $P_9 = 20 \text{ bar (sat.)}$    |                                  |

TABLE II. COORDINATES OF THE OPTIMUM POINTS OF FIGURE 5.

| Cycle                       | Perfect gas |          | Ideal Gas |          |
|-----------------------------|-------------|----------|-----------|----------|
|                             | $r^*$       | $\eta^*$ | $r^*$     | $\eta^*$ |
| Simple                      | 27          | 0.3457   | 37        | 0.3841   |
| Regenerative                | 7           | 0.4045   | 6         | 0.4574   |
| Intercooled<br>Regenerative | 13          | 0.4497   | 11        | 0.4918   |

TABLE III. COORDINATES OF THE OPTIMUM POINTS OF FIGURE 6.

| Cycle                       | Perfect gas |        | Ideal Gas |        |
|-----------------------------|-------------|--------|-----------|--------|
|                             | $r^*$       | $w^*$  | $r^*$     | $w^*$  |
| Simple                      | 12          | 344.64 | 14        | 398.04 |
| Regenerative                | 14          | 328.93 | 18        | 372.50 |
| Intercooled<br>Regenerative | 42          | 442.85 | 63        | 513.06 |

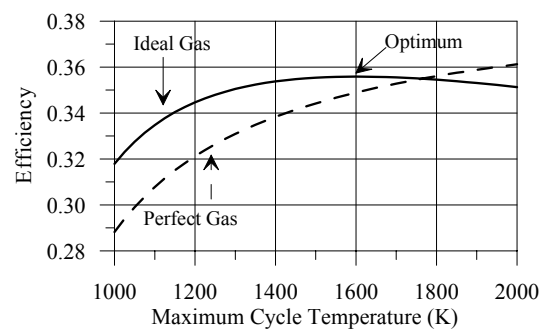


Figure 4. Efficiency of System I as a function of turbine inlet temperature with two gas models: perfect gas, ideal gas;  $\eta_C = 0.90$ ,  $\eta_T = 0.92$ ,  $r = 10$ .

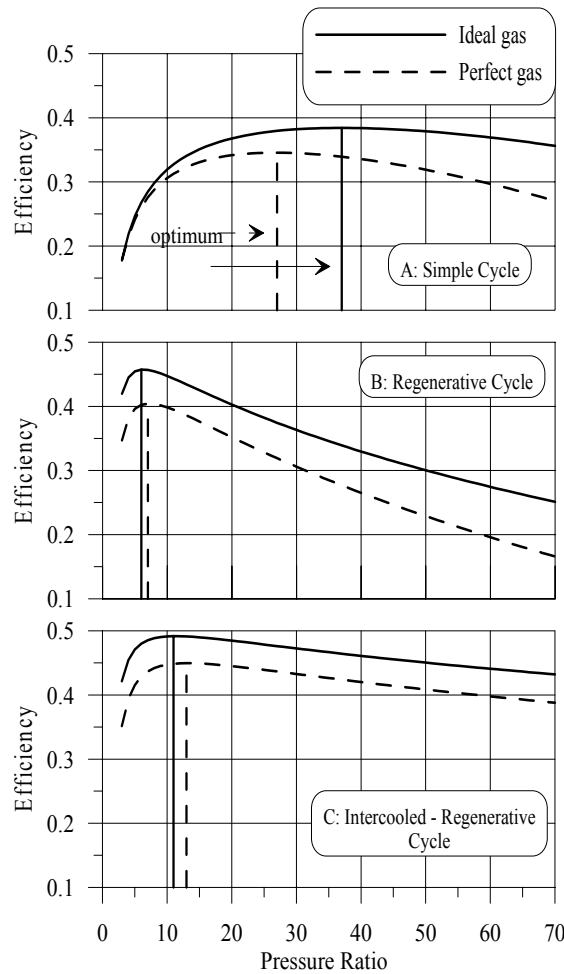


Figure 5. Efficiency as a function of pressure ratio with two gas models: perfect gas, ideal gas;  $\eta_C=0.85$ ,  $\eta_T=0.88$ ,  $\tau_{max}=5$ .

A change from the perfect to the ideal gas model changes the pressure ratio for maximum efficiency by +37.04%, -14.29% and -15.38% for the simple cycle, the regenerative cycle and the intercooled-regenerative cycle, respectively. The optimum efficiency increases by 11.11%, 13.08% and 10.10%, respectively.

A change from the perfect to the ideal gas model increases the pressure ratio for maximum specific work by 16.67%, 28.57% and 50.00% for the simple cycle, the regenerative cycle and the intercooled-regenerative cycle, respectively. The optimum specific work increases by 15.49%, 13.25% and 15.85%, respectively.

It is noted that for Figures 5 and 6, component efficiencies more or less realistic have been used. For those values of efficiencies, the temperature of maximum cycle efficiency increases to an extremely unrealistic value. In order to keep the maximum point in a realistic temperature, we have considered more optimistic component efficiencies in Figure 4.

## 5. Optimization of Systems with Alternative Methods for Evaluation of Properties

Three distinct optimization problems have been formulated and solved for each system:

- (a) Maximization of the cycle efficiency:

$$\max \eta_i, \quad i = I, II, III \quad (24)$$

- (b) Maximization of the specific work:

$$\max w_i, \quad i = I, II, III \quad (25)$$

- (c) Minimization of the annualized cost rate:

$$\min Z_i, \quad i = I, II, III \quad (26)$$

$$Z = FCR \cdot \phi \cdot \sum_n C_n + c_f \cdot \dot{m}_f \cdot H_u \cdot t \quad (27)$$

$$n = C, B, T, I, X, R$$

For the air standard cycle mentioned in the Introduction, the first two optimization problems have a well-known closed-form analytic solution (Guha 2003, Frangopoulos 1988 and 1992). Changing the model from constant to variable quantity and quality and simultaneously introducing pressure losses make the analytic solution very difficult or impossible. An analytic solution for certain cases based on various simplifying assumptions has been attempted in Horlock et al. (2000) but, as mentioned in Guha (2003), the errors introduced by these assumptions may be critical. Therefore, the optimization problems are solved numerically here.

### 5.1 Optimization of system I

The following independent variables have been considered for the three optimization problems:

$$\mathbf{x}_{I,\eta} = (r), \quad \mathbf{x}_{I,w} = (r), \quad (28)$$

$$\mathbf{x}_{I,Z} = (\eta_C, r, \tau_3, \eta_T)$$

The problems (a) and (b) have been solved for the thermoeconomic optimum values of  $\eta_C^*$ ,  $\tau_3^*$ , and  $\eta_T^*$  obtained from the solution of problem (c). The results are given in TABLE IV, where the optimum values of the objective functions are written in bold numbers.

The most significant effects of the change from perfect to ideal gas are the following: The optimum pressure ratio for the problems (a), (b) and (c) increases by 34.48%, 9.69% and 21.05%, respectively. The optimum efficiency of problem (a) increases by 9.85%. The optimum specific work of problem (b) increases by 13.58%. The optimum annualized cost rate of problem (c) decreases by 7.78%.

## 5.2 Optimization of system II

The following independent variables have been considered for the three optimization problems:

$$\begin{aligned} \mathbf{x}_{II,\eta} &= (r), & \mathbf{x}_{II,w} &= (r), \\ \mathbf{x}_{II,Z} &= (\eta_C, r, \tau_3, \eta_T, \varepsilon_X) \end{aligned} \quad (29)$$

It is noted that the useful heat rate  $\dot{Q}$  is fixed; consequently, maximization of  $\eta_{II}$  is equivalent to maximization of  $\eta_{II,tot}$ .

## 5.3 Optimization of system III

The following independent variables have been considered for the three optimization problems:

$$\begin{aligned} \mathbf{x}_{III,\eta} &= (r, \varepsilon_X), & \mathbf{x}_{III,w} &= (r, \varepsilon_X), \\ \mathbf{x}_{III,Z} &= (\eta_C, r, \tau_6, \eta_T, \varepsilon_X) \end{aligned} \quad (30)$$

The problems (a) and (b) have been solved for the thermoeconomic optimum values of  $\eta_C^*$ ,  $\tau_6^*$  and  $\eta_T^*$  obtained from the solution of problem (c).

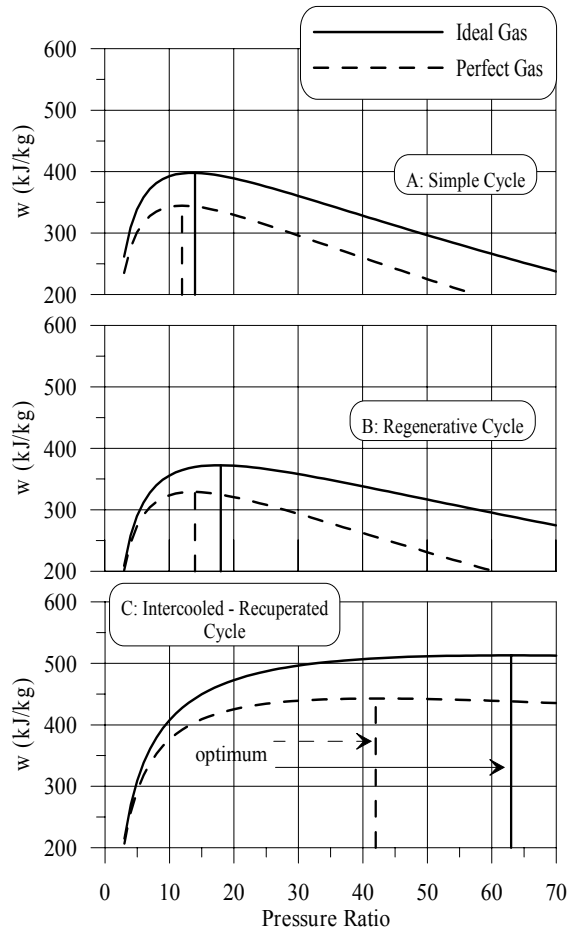


Figure 6. Specific work as a function of pressure ratio with two gas models: perfect gas, ideal gas;  $\eta_C = 0.85$ ,  $\eta_T = 0.88$ ,  $\tau_{max} = 5$ .

The results are given in TABLE VI. The most significant effects of the change from perfect to ideal gas are the following: The optimum pressure ratio for the problems (a), (b) and (c) changes by  $-44.67\%$ ,  $+19.47\%$  and  $-24.98\%$ , respectively. The optimum efficiency of problem (a) increases by  $12.24\%$ . The optimum specific work of problem (b) increases by  $15.89\%$ . The optimum annualized cost rate of problem (c) decreases by  $9.57\%$ .

It is worth noting also that the maximization of the specific work results in elimination of the air pre-heater ( $\varepsilon_X = 0$ ) in both cases (perfect and ideal gas).

## 5.4 Further remarks on system optimization

According to the analysis of Section 4, the optimization problem (a) of System I might be considered as having two independent variables:

$$\mathbf{x}_{I,\eta} = (r, \tau_3) \quad (31)$$

The solution of this problem gives extremely and unrealistically high optimum values  $r^*$  and  $\tau_3^*$  for the values of  $\eta_C$  and  $\eta_T$  considered here. This is why only the pressure ratio has been considered as an independent variable, while  $\tau_3$  is a parameter.

The effectiveness  $\varepsilon_X$  of the air pre-heater could be an independent variable for problems (a) and (b) of System II also. In such a case, maximization of the specific work would result in elimination of the air-pre-heater, changing the structure of the system. In order to keep the structure the same as in the CGAM problem, it was decided to treat  $\varepsilon_X$  as a fixed parameter for these problems.

It is interesting to note that, going from the simple cycle to the intercooled regenerative cycle, the optimum value of the annualized cost rate decreases by  $25.22\%$  (from  $9.914 \cdot 10^6 \$$  to  $7.417 \cdot 10^6 \$$ ), in spite of the fact that the system becomes more complex. The most important reasons for this decrease are the significant decrease of the pressure ratio (which decreases the capital cost of certain components) and the significant increase of the system efficiency (which decreases the fuel cost).

## Conclusion

A preliminary performance evaluation followed by the solution of three optimization problems for each one of three different gas turbine system configurations has demonstrated that a change from the perfect gas to the ideal gas model for evaluation of properties has a very significant effect on the results, which cannot be ignored. With the computing capabilities of today, the necessary calculations are conveniently performed.

TABLE 4. OPTIMIZATION RESULTS FOR SYSTEM I (SIMPLE CYCLE).

|            | Perfect gas        |                    |                                      | Ideal gas          |                    |                                      |
|------------|--------------------|--------------------|--------------------------------------|--------------------|--------------------|--------------------------------------|
|            | max $\eta_I$       | max $w_I$          | min $Z_I$                            | max $\eta_I$       | max $w_I$          | min $Z_I$                            |
| $\eta_C^*$ | 0.8445             | 0.8445             | 0.8445                               | 0.8360             | 0.8360             | 0.8360                               |
| $r^*$      | 29.0974            | 12.3112            | 18.9046                              | 39.1313            | 13.5037            | 22.8845                              |
| $\tau_3^*$ | 4.9924             | 4.9924             | 4.9924                               | 4.9887             | 4.9887             | 4.9887                               |
| $\eta_T^*$ | 0.9047             | 0.9047             | 0.9047                               | 0.9048             | 0.9048             | 0.9048                               |
| $\eta_I$   | <b>0.3686</b>      | 0.3686             | 0.3686                               | <b>0.4049</b>      | 0.3582             | 0.3915                               |
| $w_I$      | 312.98             | <b>356.44</b>      | 345.77                               | 336.15             | <b>404.84</b>      | 387.97                               |
| $Z_I$      | $1.120 \cdot 10^7$ | $1.110 \cdot 10^7$ | <b><math>1.075 \cdot 10^7</math></b> | $1.055 \cdot 10^7$ | $1.037 \cdot 10^7$ | <b><math>9.914 \cdot 10^6</math></b> |

TABLE 5. OPTIMIZATION RESULTS FOR SYSTEM II (COGENERATION SYSTEM).

|                 | Perfect gas        |                    |                                      | Ideal gas          |                    |                                      |
|-----------------|--------------------|--------------------|--------------------------------------|--------------------|--------------------|--------------------------------------|
|                 | max $\eta_{II}$    | max $w_{II}$       | min $Z_{II}$                         | max $\eta_{II}$    | max $w_{II}$       | min $Z_{II}$                         |
| $\eta_C^*$      | 0.8408             | 0.8408             | 0.8445                               | 0.8323             | 0.8323             | 0.8323                               |
| $r^*$           | 9.3053             | 13.4909            | 9.2207                               | 12.3133            | 16.3740            | 10.4584                              |
| $\tau_4^*$      | 5.0271             | 5.0271             | 5.0271                               | 5.0292             | 5.0292             | 5.0292                               |
| $\eta_T^*$      | 0.8876             | 0.8876             | 0.8876                               | 0.8864             | 0.8864             | 0.8864                               |
| $\varepsilon_X$ | 0.7675             | 0.7675             | 0.7675                               | 0.6683             | 0.6683             | 0.6683                               |
| $\eta_{II}$     | <b>0.3792</b>      | 0.3719             | 0.3791                               | <b>0.3972</b>      | 0.3933             | 0.3959                               |
| $w_{II}$        | 322.95             | <b>330.79</b>      | 322.55                               | 367.41             | <b>371.84</b>      | 360.87                               |
| $Z_{II}$        | $1.002 \cdot 10^7$ | $1.029 \cdot 10^7$ | <b><math>1.002 \cdot 10^7</math></b> | $9.577 \cdot 10^6$ | $9.734 \cdot 10^6$ | <b><math>9.541 \cdot 10^6</math></b> |

TABLE 6. OPTIMIZATION RESULTS FOR SYSTEM III (INTERCOOLED REGENERATIVE CYCLE).

|                 | Perfect gas        |                    |                                      | Ideal gas          |                    |                                      |
|-----------------|--------------------|--------------------|--------------------------------------|--------------------|--------------------|--------------------------------------|
|                 | max $\eta_{III}$   | max $w_{III}$      | min $Z_{III}$                        | max $\eta_{III}$   | max $w_{III}$      | min $Z_{III}$                        |
| $\eta_C^*$      | 0.8579             | 0.8579             | 0.8579                               | 0.8612             | 0.8612             | 0.8612                               |
| $r^*$           | 9.8729             | 41.2845            | 8.5835                               | 5.4628             | 49.3242            | 6.4390                               |
| $\tau_6^*$      | 5.0213             | 5.0213             | 5.0213                               | 5.0044             | 5.0044             | 5.0044                               |
| $\eta_T^*$      | 0.8919             | 0.8919             | 0.8919                               | 0.8878             | 0.8878             | 0.8878                               |
| $\varepsilon_X$ | 0.9602             | 0.0                | 0.9630                               | 0.9711             | 0.0                | 0.9691                               |
| $\eta_{III}$    | <b>0.4729</b>      | 0.3813             | 0.4724                               | <b>0.5308</b>      | 0.4102             | 0.530                                |
| $w_{III}$       | 383.55             | <b>458.00</b>      | 322.55                               | 325.34             | <b>530.79</b>      | 351.03                               |
| $Z_{III}$       | $8.218 \cdot 10^6$ | $1.116 \cdot 10^7$ | <b><math>8.202 \cdot 10^6</math></b> | $7.433 \cdot 10^6$ | $1.077 \cdot 10^7$ | <b><math>7.417 \cdot 10^6</math></b> |



## Nomenclature

|               |                                                                                                           |
|---------------|-----------------------------------------------------------------------------------------------------------|
| $C_n$         | capital cost of component n (installed)                                                                   |
| $c_f$         | unit cost of fuel                                                                                         |
| $c_p$         | specific heat capacity at constant pressure                                                               |
| $\tilde{c}_p$ | molar heat capacity at constant pressure                                                                  |
| $c_v$         | specific heat capacity at constant volume                                                                 |
| $\tilde{c}_v$ | solar heat capacity at constant volume                                                                    |
| FCR           | fixed charge rate                                                                                         |
| $f$           | fuel to air ratio: $f = \dot{m}_f / \dot{m}_a$                                                            |
| $H_u$         | lower heating value of the fuel                                                                           |
| $k$           | specific heat ratio: $k = c_p / c_v$                                                                      |
| $\dot{m}_a$   | air mass flow rate                                                                                        |
| $\dot{m}_f$   | fuel mass flow rate                                                                                       |
| $\dot{m}_g$   | exhaust gas mass flow rate                                                                                |
| $P$           | pressure                                                                                                  |
| $r$           | pressure ratio                                                                                            |
| $\dot{Q}$     | useful heat rate of the cogeneration system (production of steam)                                         |
| $\tilde{R}$   | universal gas constant                                                                                    |
| $T$           | absolute temperature [K]                                                                                  |
| $T_1$         | compressor inlet temperature                                                                              |
| $T_3$         | turbine inlet temperature (simple cycle)                                                                  |
| $t$           | period of operation during a year                                                                         |
| $\dot{W}$     | net power to the load                                                                                     |
| $w$           | specific work, as defined by Eq. (17)                                                                     |
| $\mathbf{x}$  | set of independent variables for optimization                                                             |
| $x_i$         | molar fraction of species i in a mixture                                                                  |
| $Z$           | annualized cost rate of a system, in \$ (including capital as well as operation and maintenance expenses) |

## Greek letters

|          |                                         |
|----------|-----------------------------------------|
| $\gamma$ | $\gamma = (k - 1) / k$                  |
| $\eta$   | efficiency                              |
| $\eta_B$ | efficiency of the combustor             |
| $\eta_C$ | isentropic efficiency of the compressor |
| $\eta_J$ | efficiency of the Joule cycle           |
| $\eta_m$ | mechanical efficiency                   |
| $\eta_T$ | isentropic efficiency of the turbine    |
| $\tau_i$ | temperature ratio: $\tau_i = T_i / T_1$ |
| $\phi$   | maintenance factor                      |

## Subscripts

|   |                                |
|---|--------------------------------|
| A | air standard gas turbine cycle |
| a | air                            |
| B | combustor                      |
| C | compressor                     |
| f | fuel                           |
| g | exhaust gases                  |

|   |                                        |
|---|----------------------------------------|
| I | intercooler                            |
| T | turbine                                |
| R | exhaust gas boiler                     |
| X | air preheater                          |
| 0 | standard conditions: 25°C, 1.01325 bar |

## Superscripts

|   |               |
|---|---------------|
| * | optimum value |
|---|---------------|

## References

- Frangopoulos C.A., 1988, "Optimal Design of a Gas Turbine Plant by a Thermoeconomic Approach", In: 2<sup>nd</sup> International Symposium and Exposition on Turbo-machinery, Combined-Cycle Technologies and Cogeneration, Montreaux, Switzerland.
- Frangopoulos C.A., 1992, "Thermoeconomic versus thermodynamic design optimization of a simple gas turbine plan. Part A': Thermodynamic optimization. Part B': Thermoeconomic optimization" *The International Journal of Mechanical Engineering Education*; 20(3):149–168.
- Frangopoulos, C.A., 1994, "Application of the Thermoeconomic Functional Approach to the CGAM Problem" *Energy*; 19(3): 323-342.
- Gordon S., McBride B.J., 1994, "Computer program for calculation of complex chemical equilibrium compositions and applications; I. Analysis" NASA reference publication 1311, National Aeronautics and Space Administration.
- Guha A., 2003, "Effects of internal combustion and non-perfect gas properties on the optimum performance of gas turbine", *Proceedings of the Institute of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*; 217: 1085-1099.
- Haywood R.W., 1987, *Analysis of engineering cycles*. 3<sup>rd</sup> ed. Oxford: Pergamon Press.
- Horlock J.H., 2003, *Advanced gas turbine cycles*. Amsterdam: Pergamon.
- Horlock J.H., Woods W.A., 2000, "Determination of the optimum performance of gas turbines", *Proceedings of the Institute of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*; 214: 243-255.
- Horlock J.H., 2001, *Erratum* (for Horlock 2000). *Proceedings of the Institute of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*; 215: 1378.
- McBride B.J., Zehe M.J., 2002, "Gordon S. NASA Glenn coefficients for calculating thermodynamic properties of individual species" NASA/TP-2002-211556, National Aeronautics and Space Administration.
- Valero A. et al., 1994, "CGAM Problem: Definition and conventional solution" *Energy*; 19(3): 279-286.



