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Research Article

The Inflationary with Inverse Power-Law Potential in Tsallis Entropy

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Abstract

In this article, we focus on the inflation dynamics of the early Universe using an inverse power law potential scalar field ($V_{(\phi)} = V_0\phi^{-n}$) within the framework of Tsallis entropy. First, we derive the modified Friedmann equations from the non-additive Tsallis entropy by applying the first law of thermodynamics to the apparent horizon of the Friedmann–Robertson–Walker (FRW) Universe. We assume that the inflationary era of the Universe consists of two phases; the slow roll inflation phase and the kinetic inflation phase. We obtained the scalar spectral index n_s and tensor-to-scalar ratio r and compared our results with the latest Planck data for these phases. By choosing the appropriate values for the Tsallis parameters, which bounded by $\beta < 2$, and the inverse power-term of the potential n, we determined that the inflation era of the Universe in Tsallis entropy can only occur in the second phase (kinetic inflation), while the slow-roll inflation phase is incompatible with the Planck data.

Keywords: *Tsallis entropy; inflationary; inverse power-law potential.*

1. Aims and Scope

In recent years many astrophysical and cosmological studies confirm that an accelerated expansion of the Universe is taking place [1- 5]. According to our standard big bang theory there are two possible stages of accelerated expansion: one of them the early stage of the evolution of the Universe namely the inflation and the other is a current stage that we live in. In the framework of this scenario, it is possible to explain this cosmic acceleration in two ways. The first is to continue Einstein's General Theory of Relativity and introduce new energy components such as dark energy [6- 9] and an inflation field [10]. The second is to introduce a modified gravity such as F(R) gravity [11- 17], F(G) gravity [18], Galileon gravity [19-20] and Weyl gravity [21-22] etc.

On the other hand, thermodynamic gravity has been extensively discussed for a long time [23-24]. In the light of these studies, we can say that there are many indications that the concepts of gravity and entropy are related within the framework of the standard thermodynamics, which based on Boltzmann-Gibbs entropy, such as black hole mechanics [25], the Bekenstein-Hawking formula [26- 28] and the holographic principle [29-30]. However, in the framework of the Boltzmann-Gibbs entropy the energy of system is generally extensive and the entropy is additive. Therefore, it is generally known that the entropy of an entire system is not equal to the sum of the entropies of its subsystems. In another word, if we generalize this case to our Universe, the entropy of the whole Universe not necessarily equal the entropies of its subsystems. Hence, it is well known that standard Boltzmann-Gibbs additive entropy should be generalized, especially in the case of non-additive systems such as gravitational systems. [31-33]. This such approach makes use of the extended entropy instead of the additive systems. Recently, a generalized form of non-additive entropy has been proposed by Tsallis and Cirto [34] and assuming that the Universe is a spherically symmetrical system, they showed that the Tsallis entropy is proportional to a power of the horizon field, namely $S_A \sim A^\beta$. İn this context, they argued that the microscopic mathematical expression of the thermodynamical entropy of a black holes should be as

$$S_A = \gamma A^\beta \ , \tag{1}$$

which is known as the Tsallis entropy, where β is the non-additive Tsallis entropy parameter, $\gamma > 0$ is a costant and A is the black hole horizon area $A = 4\pi r^2$. Herein, in the context of Tsallis cosmology scenario the Bekenstein-Hawking area law formula ($S_A = \frac{A}{4G}$) for the black hole entropy, slightly modified as $S_A = \gamma A^\beta$. It is clear that for $\beta = 1$ it reduces to the Bekenstein-Hawking conventional form or corresponding to the standard entropy. It can be seen that the cosmological application of the above (1) non-additive thermodynamics leads to the new modified Friedmann equations since it contains extra terms that appear for the first time in the general case when the Tsallis generalized entropy becomes the usual one.

In this paper, we examined the inflationary dynamics of the very early Universe (high energy era) within the framework of apparent horizon thermodynamics with the inverse power-law potential function. In this sense, first of all, we considered the Tsallis entropy corrections in the Friedmann equations and investigated the inflation dynamics of this framework also assuming phases of slow-roll inflation and kinetic inflation. We then specifically considered the Tsallis entropy as the horizon entropy and the scalar field as the matter content inside of the horizon. Taking these

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assumptions into account, we investigated whether the above aforementioned phases can be achieved appropriately or not. It is worth noting that here we considered the slow-roll parameter expressions and the spectral index of the scalar perturbations valid for the canonical scalar field theory. This is a concise study of inflationary dynamics for the Tsallis' corrected General Theory of Relativity case.

In this regard, we assumed that the inflationary epoch of the Universe could consist of two phenomenological phases before the quintessence era begin, namely slow-roll and kinetic inflation. With such considerations, we calculated the observable parameters (the scalar spectral index n_s and tensor-to-scalar ratio r). Here, we used the exponent of the inverse power law term of the potential ($V_{(\phi)} = V_0\phi^{-n}$) n and the Tsallis parameter within the limits of $0 < n < 1$, $\beta < 2$, respectively, with the e-folding number N. With these parameters, we determined how the inflationary period of the Universe behaves with respect to the Tsallis entropy scenario. In this context, we compared our results with the observational consistency of the scalar spectral of the primordial curvature perturbations n_s and the tensor-to-scalar ratio r with the latest Planck 2020 observable indices [35] for both phases aforementioned above.

Consequently, our investigations shows that in the context of the Tsallis entropy scenario under the inverse power-law potential scalar field function the slow-roll inflation phase incompatible with Planck 2020 observable satellite data since we have some β parameter constraints ($\beta < 2$). However, we observed that the kinetic inflation phase exists, which is in good agreement with Planck observable data.

This paper is organized as follows; in the next section applying the first law of thermodynamics to the apparent horizon of the Universe, we derive the modified Friedmann equations from Tsallis entropy. In the section 3, we analyzed the early inflationary dynamics of Tsallis cosmology. In the section 4, we discuss the early inflationary dynamics of Tsallis cosmology with the inverse power-law potential function, $V_{(\phi)} = V_0\phi^{-n}, n > 0$, under the slow-roll condition and we determined that the slow-roll phase incompatible with Planck's observable data since the some restrict of the Tsallis entropy does not allow. In the section 5, we explored the kinetic energy (second phase) phase of the inflationary era of the Universe. We proceed in this section by considering the second phase of inflation driven by an inverse power-law potential. However, we seen that the Tsallis parameter β take the large negative values and therefore the entropy decreases in the kinetic inflation phase. We conclude final section with a discussion.

2. Modified Friedmann Equations in Tsallis Cosmology

In this section, we derive the modified Friedmann equations assuming a homogeneous and isotropic flat FRW Universe from the Tsallis Cosmology. Following [36],

$$ds^2 = h_{\mu\nu}dx^\mu dx^\nu - \tilde{r}^2[d\theta^2 + sin^2\theta d\phi^2], \quad (2)$$

where $h_{\mu\nu} = (-1, a^2 / 1 - kr^2), \tilde{r} = a(t)$ and $x^0 = t, x^1 = r$ represents the two dimensional metric. The parameter k is introduced to explain the spatial curvature of the metric and takes values $k = -1, 0, 1$ in case of the closed, spatially flat, an open Universe, respectively. In the framework of the thermodynamics laws, we assume that the boundary of the Universe is the apparent horizon and with its radius

$$\tilde{r}_A = \frac{1}{\sqrt{H^2+k/a^2}}, \quad (3)$$

where $H \equiv \frac{\dot{a}}{a}$ and the dot represents the time derivative. Here the surface gravity and the apparent horizon has a related temperature [37, 38]

$$T = \frac{1}{2\pi r_A}. \quad (4)$$

We now suppose that the energy and matter content of the Universe is in the form of perfect fluid and represented by a scalar field ϕ . The corresponding Lagrangian is given by

$$\mathcal{L}_\phi = X - V_\phi \text{ and } X = -\frac{1}{2}h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi, \quad (5)$$

where X and V_ϕ are the kinetic energy and the potential terms, respectively. The corresponding stress-energy tensor in four dimensions reads

$$T_{\mu\nu} = (\rho_\phi + p_\phi)u_\mu u_\nu + ph_{\mu\nu}, \quad (6)$$

where ρ_ϕ and p_ϕ represent the energy density and pressure, respectively. These are expressed as follows

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V_\phi, \quad (7)$$

$$p_\phi = \frac{\dot{\phi}^2}{2} - V_\phi. \quad (8)$$

In turn, the conservation equation, $\nabla_\mu T^{\mu\nu} = 0$, which shows the continuity equation

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0, \quad (9)$$

combining (7), (8) and (9), we obtain the Klein–Gordon equation of the canonical scalar field form

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0. \quad (10)$$

The work density is defined as [39]

$$W = -\frac{1}{2}T^{\mu\nu}h_{\mu\nu} \quad (11)$$

$$W = \frac{1}{2}(\rho_\phi - p_\phi), \quad (12)$$

where the equation (12) is in the simple form. The first law of thermodynamics at the apparent horizon is

$$dE = TdS + WdV, \quad (13)$$

where $E = \rho V$ is the total energy content of the Universe of 3- dimensional spherical volume $V = 4\pi\tilde{r}_A^3/3$ and horizon surface area $A = 4\pi\tilde{r}_A^2$.

The Eqs. (1), (4), (9), (11) and (12) into the (13) and after some algebra, we obtain the first modified Friedmann equation in Tsallis entropy [36]

$$(H^2 + \frac{k}{a^2})^{2-\beta} = \frac{8\pi G}{3}\rho_\phi, \quad (14)$$

where we define $\gamma \equiv \frac{(2-\beta)4\pi^{1-\beta}}{4\beta G}$ and $\kappa^2 = 8\pi, G = 1$. Taking the first derivative with respect to the time of (14), using Eq. (9) and with the relation $\dot{H} = \frac{\ddot{a}}{a} - H^2$ one can get the second modified Friedmann equations [36]

$$(4-2\beta)\frac{\ddot{a}}{a}\left(H^2 + \frac{k}{a^2}\right)^{1-\beta} + (2\beta - 1)\left(H^2 + \frac{k}{a^2}\right)^{2-\beta} = -8\pi p_{\phi}. \tag{15}$$

3. Inflation in Tsallis Cosmology

Let us now proceed to the study of the inflationary era of the Universe within the framework of the general scalar theories. In this context, the expressions called slow-roll parameters takes the following values

$$\epsilon = -\frac{\dot{H}}{H^2}, \tag{16}$$

$$\eta = -\frac{\ddot{H}}{2H\dot{H}}. \tag{17}$$

It should be noted that, according to the slow-roll conditions both of these parameters take the very small values during the inflation $\epsilon \ll 1$, $\eta \ll 1$. On the other hand, applying the slow-roll condition to ensure inflation during the inflationary period depends only on $\epsilon \ll 1$. That is, this case is the only necessary and sufficient condition to existence an inflation. Therefore, when we apply slow-roll conditions ($\dot{\phi}^2 \ll 1$ and $\ddot{\phi} \ll 1$) on the Friedmann equations the Eq. (14) becomes [40]

$$H^2 \cong \left(\frac{8\pi}{3}\right)^{\frac{1}{2-\beta}} V_{(\phi)}{}^{\frac{1}{2-\beta}}. \tag{18}$$

Applying Eqs. (18) to the second Friedmann Eqs. (15) and with (10), we get the following equation [40]:

$$\dot{H} \cong -\frac{3\dot{\phi}^2}{2(2-\beta)}\left(\frac{8\pi}{3}\right)^{\frac{1}{2-\beta}} V_{(\phi)}{}^{\frac{\beta-1}{2-\beta}}. \tag{19}$$

Using the equations (18) and (19), the slow-roll parameters given by (16), (17) becomes [40]

$$\epsilon = \frac{\dot{\emptyset}^2}{2(2-\beta)} V_{(\phi)}{}^{-1}, \tag{20}$$

$$\eta = -\frac{1}{2}\left(\frac{8\pi}{3}\right)^{\frac{1}{2\beta-4}} \frac{V_{(\phi)}{}^{\frac{-1}{4-2\beta}}}{\dot{\phi}}\left[2\ddot{\phi} + \frac{\dot{\phi}^2(\beta-1)}{(2-\beta)}\frac{V_{\phi}}{V_{(\phi)}}\right]. \tag{21}$$

The some observable indices like the spectral index for the primordial curvature perturbation, (n_s) and the tensor-to-scalar ratio (r) can be defined by the slow-roll parameters at the leading order as:

$$n_s \cong 1 - 6\epsilon + 2\eta, \tag{22}$$

$$r = 16\epsilon. \tag{23}$$

It is well known the slow-roll parameters are determined by the value of the inflation scalar field $\emptyset$, where the comoving scale $k = aH$ exits the horizon during inflation, and k shows the comoving wave vectors [41]. It is worth noting here that to solve for $\emptyset$, the slow-roll approach is often used to calculate the e-foldings number (N), which describes the amount of inflation between two times, t_i and t_f, after the horizon exit and it defined as

$$N = \int_{t_i}^{t_f} H(t)dt, \tag{24}$$

where t_i and t_f are the quantities indicate that the beginning and ending times of the inflation period, respectively, and also t_f denotes the end of inflation defined by $\epsilon(\phi)_{end} = 1$. Here, we consider $t_i = t_c$ as the horizon crossing time and also equivalently the scalar field $\phi_i = \phi_c$. So, the above equation (24) can be expressed as

$$N = \int_{\phi_i}^{\phi_f} H\dot{\phi}^{-1}d\phi. \tag{25}$$

It is worth noting here that about 40 to 60 e-foldings numbers ($N = 40 - 60$) are required to solve the most well known flatness and horizon problems of our cosmological model. [42- 43].

4. Inverse Power-Law Scalar Potentials in the Tsallis Slow-Roll Inflation

We consider the inverse power-law potential which arises in supersymmetric theories and are given by the following form

$$V_{(\phi)} = V_0\phi^{-n}, \tag{26}$$

where both $V_0 > 0$ and $n > 0$ are constants parameters. In the standard inflationary cosmology such potentials leads to the intermediate inflation with scale factor $a(t) \propto exp(At^f)$ where $A > 0$ and $0 < f = 4/(n+4) < 1$ [44-47] are completely incompatible by Planck 2015 data [48].

In order to examine this potential term of the scalar field $(V_{(\phi)} = V_0\phi^{-n})$, we need to defined $\dot{\phi}$ (velocity) by using slow-roll inflation scenario. So the above Eqs. (10) can be write as

$$\dot{\phi} \cong -\frac{V_{\phi}}{3H}. \tag{27}$$

And insertion the Eqs. (18) into (27), the following is obtained:

$$\dot{\phi} = \frac{nV_0^{\frac{3-2\beta}{4-2\beta}}}{3\left(\frac{8\pi}{3}\right)^{\frac{1}{4-2\beta}}}\phi^{\frac{2\beta n+2\beta-3n-4}{4-2\beta}}. \tag{28}$$

It is well known that inflation ends when $\epsilon(\phi_{end}) \sim 1$. Hence, using Eqs. (28) and (26), we get the following expression

$$\phi_f = \left[\frac{6(2-\beta)\left(\frac{8\pi}{3}\right)^{\frac{1}{2-\beta}}}{n^2 V_0^{\frac{1-\beta}{2-\beta}}}\right]^{\frac{2-\beta}{2\beta+\beta n-4-n}}. \tag{29}$$

Similarly, we get the e-foldings number (N) from the relevant equations above as follows:

$$N = \frac{3(2-\beta)\left(\frac{8\pi}{3}\right)^{\frac{1}{2-\beta}}}{n(n-\beta n-2\beta+4)V_0^{\frac{3-2\beta}{4-2\beta}}}\left[\phi_f{}^{\frac{n-\beta n-2\beta+4}{2-\beta}} - \phi_c{}^{\frac{n-\beta n-2\beta+4}{2-\beta}}\right]. \tag{30}$$

From this Eq. (30) one can get the following expression

$$\phi_c = \left[\frac{V_0^{\frac{1-\beta}{2-\beta}}\left(\frac{8\pi}{3}\right)^{\frac{-1}{2-\beta}}}{6(2-\beta)}\right]^{\frac{2-\beta}{n-\beta n-2\beta+4}} \left[n^2 - 2nN(n-\beta n-2\beta+4)\right]^{\frac{2-\beta}{n-\beta n-2\beta+4}}. \tag{31}$$

This equation (31) expresses the scalar field at the horizon crossing. We can now write expressions corresponding to the observable indices, the scalar spectral index n_s (22) and the tensor-scalar ratio r (23). These equations can be expressed in terms of power term n and e-foldings number N as follows:

$$n_s = 1 - \left[\frac{6n-4(n+2)(2-\beta)}{n-2N(n-\beta n-2\beta+4)}\right], \tag{32}$$

$$r = \frac{16n}{n-2N(n-\beta n-2\beta+4)}. \tag{33}$$

It can be seen that the slow-roll indices depend only on the power term n and the Tsallis parameter β, and it is independent of V_0, as we expected. The Planck 2020 data [35], which set the following limits on n_s and r

$$n_s = 0.9649 \pm 0.0042 \text{ (68\%CL)} \tag{34}$$

from Planck TT, TE, EE + lowE + lensing,

$$r < 0.064 \text{ (95\%CL)} \tag{35}$$

from Planck TT, TE, EE + lensing + lowEB.

Table 1.The table shows some of the values of n_s and r with increasing values of n when N has large values ($N \geq 220$).

$V_{(\phi)} = V_0\phi^{-n}$					
$\beta = 1.9$					
$N \geq 220$	$n = 0.6$	$n = 0.7$	$n = 0.8$	$n = 0.9$	$n = 1$
n_s	0.9829	0.9835	0.9839	0.9842	0.9844
r	0.0639	0.0589	0.0557	0.0534	0.0517

From the table 1, it can be seen that the slow-roll inflation does not occur as the observable indices lies outside the range allowed by the Planck 2020 data [35]. This is a notable difference with the power-law potential based on Tsallis entropy [40], where the results allow for the slow-roll inflation phase. The Table 1 shows that the observable indices approach the Planck observation limits with very large values $N \geq 220$ and the large value of $\beta, \beta = 1.9$. On the other hand, the point to remember here is that the Tsallis parameter is limited to a specific values ($\beta < 2$), due to the positive definition of the energy density. For example, choosing $\beta = 2.13$ with an corresponding value of $n = 0.2$, then we would get a value in a range that allowed by Planck data [35], but this case leads to the emergence of negative energy density.

5. Inverse Power-Law Scalar Potentials in the Tsallis Kinetic Inflation

In this section, we move on to kinetic inflation, the second phase of the early inflation of the Universe after the slow-roll inflation. So we will now proceed to study this kinetic phase of the Universe, which has kinetic energy and assuming that kinetic energy is a function of the scalar field potential. In this context, we assume that the kinetic energy has the following form

$$\dot{\phi}^2 = nV_{(\phi)}. \tag{36}$$

Using Eqs. (36), the Eqs. (18) and (19) becomes;

$$H^2 \cong \left(\frac{8\pi}{3}\right)^{\frac{1}{2-\beta}} \left(\frac{n+2}{2}\right)^{\frac{1}{2-\beta}} \phi^{\frac{-n}{2-\beta}}, \tag{37}$$

$$\dot{H} \cong -\frac{3\dot{\phi}^2}{2(2-\beta)} \left(\frac{8\pi}{3}\right)^{\frac{1}{2-\beta}} \left(\frac{n+2}{2}\right)^{\frac{\beta-1}{2-\beta}} \phi^{\frac{n-\beta n}{2-\beta}}. \tag{38}$$

Now we can write the new slow-roll parameters considering these Eqs. (37) and (38) as follows

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{3n}{(2-\beta)(n+2)}, \tag{39}$$

$$\eta = -\frac{\ddot{H}}{2H\dot{H}} = \frac{n^{\frac{3}{2}}}{(4-2\beta)} \left(\frac{8\pi}{3}\right)^{\frac{-1}{4-2\beta}} \left(\frac{n+2}{2}\right)^{\frac{-1}{4-2\beta}} \phi^{\frac{2\beta+n\beta-n-4}{4-2\beta}}. \tag{40}$$

Here, at the end of kinetic inflation this parameter take the value $\eta(\phi)_{end} = 1$, so we obtain as follows

$$\phi_f = \left[\frac{(4-2\beta)}{n^{\frac{3}{2}}} \left(\frac{8\pi}{3}\right)^{\frac{1}{4-2\beta}} \left(\frac{n+2}{2}\right)^{\frac{1}{4-2\beta}}\right]^{\frac{4-2\beta}{2\beta+\beta n-4-n}}. \tag{41}$$

Using the relevant equations above and taking integration over the scalar field (25), we can get the e-foldings number as

$$N = \frac{(4-2\beta)\left(\frac{8\pi}{3}\right)^{\frac{1}{4-2\beta}}\left(\frac{n+2}{2}\right)^{\frac{1}{4-2\beta}}}{n^{\frac{1}{2}}(n-\beta n-2\beta+4)} \left[\phi_f^{\frac{n-\beta n-2\beta+4}{4-2\beta}} - \phi_c^{\frac{n-\beta n-2\beta+4}{4-2\beta}}\right]. \tag{42}$$

From this Eq. one can get the following expression

$$\phi_c = \left[\frac{\left(\frac{8\pi}{3}\right)^{\frac{1}{4-2\beta}}\left(\frac{n+2}{2}\right)^{\frac{1}{4-2\beta}}}{4-2\beta}\right]^{\frac{4-2\beta}{n-2\beta-\beta n+4}} \left[n^{\frac{3}{2}} - n^{\frac{1}{2}}N(n-2\beta-\beta n+4)\right]^{\frac{4-2\beta}{n-2\beta-\beta n+4}}. \tag{43}$$

This equation represents the scalar field at the horizon crossing. The observable indices, the scalar spectral index and the tensor-scalar ratio can be expressed as

$$n_s = 1 - \left[\frac{18n\left(n-N(n-2\beta-\beta n+4)\right)-2n(2-\beta)(n+2)}{(2-\beta)(n+2)\left(n-N(n-2\beta-\beta n+4)\right)}\right], \tag{44}$$

$$r = \frac{48n}{(2-\beta)(n+2)}. \tag{45}$$

We see that the kinetic inflation phase depends on the parameters, β, n and N. In additionally, it can be clearly seen from Eqs. (44) and (45) that observational consistency is achieved for the appropriate β and n values, unlike the previous slow-roll phase.

These results are in good agreement with the Planck data [35]. We show the observable index values (n_s, r) in the tables below. Here we see that (table 2, table 3) the best values for observable indices are in the range $0 < n < 1$. As a result, we observe that with the constraint of the inverse power term n ($0 < n < 1$), the kinetic inflation occurs in a wide range of observable indices values, unlike the slow-roll inflation.

Table 2. The table shows the corresponding β values when $n = 0.1$.

$V_{(\phi)} = V_0\phi^{-n}$ $n = 0.1$ $N = 60$	$\beta = -34$	$\beta = -38$	$\beta = -42$	$\beta = -50$	$\beta = -60$
n_s	0.9761	0.9785	0.9804	0.9834	0.9861
r	0.0634	0.0571	0.0519	0.0439	0.0368

Table 3. The table shows some values of the n_s and r corresponding to the varying values of n and β .

$V_{(\phi)} = V_0\phi^{-n}$	$n = 0.01$	$n = 0.2$	$n = 0.3$	$n = 0.6$	$n = 0.6$
$N = 60$	$\beta = -2$	$\beta = -68$	$\beta = -97$	$\beta = -176$	$\beta = -180$
n_s	0.9775	0.9765	0.9762	0.9766	0.9771
$\boldsymbol{r}$	0.0597	0.0623	0.0632	0.0622	0.0608

6. Conclusion

In the above study, using the inverse power-law potential, $V_{(\phi)} = V_0\phi^{-n}$, we explored the inflation period of the early Universe in terms of Tsallis entropy. In this context, we considered the evolution of the FRW Universe and assumed that its matter content was represented by a perfectly fluid homogeneous scalar field. Afterwards, by using the first law of thermodynamics to the apparent horizon of a FRW Universe, we derived the modification of the FRW equations from the non-additive Tsallis entropy.

For both the slow-roll and kinetic inflation phases constituting the first stage of the Universe, taking into account the inverse power-law potential, we calculated the two slow-roll parameters (ϵ, η) to obtain the observational indices (r, n_s). Finally, we compared our results with the latest Planck observation data with appropriate values of the Tsallis parameter β and the inverse power-term of the potential n. In conclusion, we determined that, unlike the slow-roll inflation, the kinetic inflation phase occurs, which is well agreement in the Planck data. In the slow-roll inflation phase, according to the obtained results, we have found that the observation does not occur due to limited Tsallis parameter, $\beta < 2$ and the e-folding number requiring $N \geq 220$.

Nomenclature

β: The non-additive Tsallis entropy parameter
ϕ: Scalar field
S_A: Tsallis entropy
N: E-folding number
r: Tensor-to-scalar ratio
n_s: Scalar spectral index
T: Temperature
V_0: Constant parameter
ϵ: Slow-roll parameter
η: Slow-roll parameter
γ: Constant
n: Exponent of the inverse power law potential
k: Spatial curvature of the metric
A: Black hole horizon area
$\tilde{r}_A$: Radius
H: Hubble constant
$\mathcal{L}$: Lagrangian
$T_{\mu\nu}$: Stress-energy tensor
ρ_ϕ: Energy density
p_ϕ: Pressure
W: Work density
E: Energy
V: Volume

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