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Discontinuous Contact Problem of Elastic Two Layers Loaded with Two Rigid Rectangular Blocks

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Abstract

In this study, unlike the literature, the discontinuous contact problem of two elastic layers resting on a loaded elastic semi-infinite plane with two rigid rectangular blocks is analyzed analytically. P and Q loads are transferred to the layers through blocks. Sheet weights were included in the problem. When the load value λ applied to the system exceeds the critical load value λ_{cr}, discontinuities occur on the contact surfaces. The problem is reduced to a singular integral equation using Fourier integral transform techniques in case of discontinuous contact. Singular integral equation is solve using Gauss-Chebyshev integral formulation. These discontinuities have been examined for the change in distance between blocks, block widths and changes in load ratios. Moreover, the swelling rates occurring during the separations are presented in graphics. In addition, the results obtained have been solved and compared with the help of ANSYS package program using the Finite Element Method.

Keywords: Discontinuous contact, elasticity, finite element analysis, integral equations

İki Rijit Dikdörtgen Blok ile Yüklenen Elastik İki Tabakanın Süreksiz Temas Problemi

Öz

Bu çalışmada, literatürden farklı olarak, iki rijit dikdörtgen blok yüklü elastik yarı sonsuz bir düzlem üzerinde duran iki elastik tabakanın süreksiz temas problemi analitik olarak analiz edilmiştir. P ve Q yükleri bloklar aracılığıyla tabakalara aktarılır. Problemde tabaka ağırlıkları dahil edilmektedir. Sisteme uygulanan yük değeri λ, kritik yük değerini λ_{cr} aştığında temas yüzeylerinde süreksizlikler meydana gelmektedir. Problem süreksiz temas durumunda Fourier integral dönüşüm teknikleri kullanılarak tekil bir integral denkleme indirgenir. Tekil integral denklemi Gauss-Chebyshev integral formülü kullanılarak çözülür. Bu süreksizlikler bloklar arası mesafe, blok genişlikleri ve yük oranlarındaki değişimler açısından incelenmiştir. Yine ayrılmalar sırasında meydana gelen kabarma oranları grafiklerle verilmektedir. Ayrıca elde edilen sonuçlar Sonlu Elemanlar Metodunun kullanıldığı ANSYS paket programı yardımıyla çözülmüş ve karşılaştırılmıştır.

Anahtar Kelimeler: Süreksiz temas, elastisite, sonlu elemanlar analizi, integral denklemler

INTRODUCTION

The biggest reason why contact problems continue to attract attention today is that most of the mechanical system components are in contact with each other. Knowing the contact character, length and the stress distribution on the contact area in these systems facilitates material design and production for engineers. In fact, the components have weights, and, in this case, it is necessary to include the mass forces for each component, but in practice two types of problems arise. In the first type of problems, the effect of weight is considered, and the separation takes place in a finite region. When the applied load is less than a certain critical value, the contact is continuous, and when the load exceeds a certain value, separations occur between the interfaces. Frictionless contact problem between elastic plane and elastic layer (Keer

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and Chantaramungkorn, 1972), the problem of frictionless contact in the elastic layer sitting on two elastic quarter planes (Erdogan and Ratwani, 1974), frictionless contact problem of an elastic layer under an axially symmetrical load resting on a rigid plane. (Gecit, 1978), the problem of continuous and discontinuous contact between the semi-infinite plane and an elastic layer (Çakıroğlu and Çakıroğlu, 1991), the contact problem of anisotropic layers sitting on an elastic semi-infinite plane and loaded by a rigid rectangular block is investigated. (Urquhart and Pindera ,1994). On the other hand, Özşahin has solved the frictionless contact problem of the system consisting of layers with different properties loaded with two rigid flat blocks (Özşahin, 2007.) Studies in the literature can be shown as an example of the first type of problems (Argatov 2013; Bora P. 2016; Çömez 2010; ETLİ 2021; Oner at al., 2014; Zhupanska 2011). In the second type of problems, the effect of weight is neglected, and the separation zone is in infinite length. These types of problems are called "receding contact" (Adiyaman at al., 2018; Adıbelli at al., 2013; Comez at al., 2004; El-Borgi at al., 2006; Kahya at al., 2007; Yan and Li , 2015). In contact problems, it is also possible to encounter various studies on functionally graded (FG) layers using functionally graded materials (FGM) varying from one surface of the material to another. When these studies are examined, we might encounter separation contact problems (Adıbelli at al., 2013; El-Borgi at al., 2006; Rhimi at al, 2011) and in which the weight effect is neglected and contact problems including the weight effect (Adiyaman and Öner, 2017; Çömez and Guler, 2017; Comez, 2013; Dag at al., 2009; Giannakopoulos and Pallot, 2000; Polat and Özsahin, 2018; Volkov at al., 2013; Yang and Ke, 2008). Yaylacı et al. analyzed a separating contact problem with analytical and finite element method comparatively (Yaylaci at al., 2014). Öner et al. comparatively investigated the continuous contact problem of a functionally graded layer resting on an elastic semi-infinite plane (Oner at. al., 2017). Kaya and Polat the continuous contact problem of the FG layer sitting on the semi-infinite plane is investigated comparatively (Kaya and Polat, 2019). Kaya et al. investigated the continuous contact problem in an FG layer loaded with three flat rigid blocks and fitted on an elastic semi-infinite plane using the finite element method (Polat and Kaya 2018). In the related literature, it is possible to encounter many problems solved using the finite element method (Abhilash and Murthy 2014; Bendine and Polat 2020; Birinci at al.,2015; Güler at al., 2017; Kaya at al., 2020; Polat at al., 2019).

In this study, the problem of discontinuous contact for two elastic layers resting on an elastic semi-infinite plane is investigated. The rectangular rigid block problem has important applications in soil mechanics, especially in predicting the safety of foundations. The blocks can be taken as foundations placed on elastic layers. By installing the foundations at certain distances, it is possible to prevent overlapping of pressures of different settlements. In the current studies in the literature, the number of layers, loading conditions and basic types differ, and although the solution methods are similar, the analytical solution must be done separately for each. With this study, the general solution of the discontinuous contact problems in a layered medium resting on an elastic base will be obtained, and a computer program based on this solution will provide results regarding any geometry and loading situation to be entered by the user.

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SOLUTION OF THE PROBLEM

In this study, the discontinuous contact problem is investigated for two layers with frictionless surfaces. The mass forces of the layers are taken into account. It lies along the x-axis in the range of layers and semi-infinite plane (-∞, +∞). The problem is solved for the plane state; unit thickness is taken in the z-axis direction.

The discontinuous contact problem has been studied separately for two cases. The first of these is the discontinuity occurring at the interface of the two elastic layers, and the second is the discontinuity at the interface of the lower layer and the semi-infinite plane.

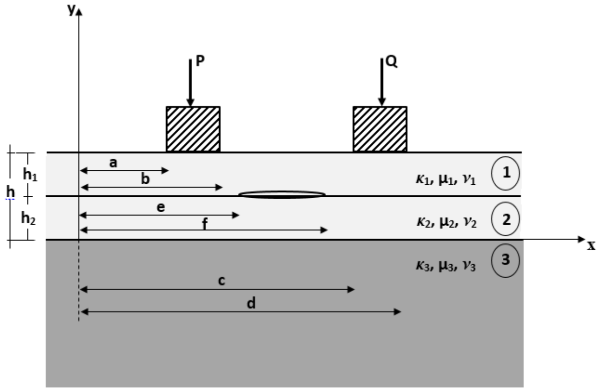

Figure 1. Discontinuous contact between layers

Discontinuity at the Interface of the Layers

In order for separation to occur at the interface of the layers, load value (λ_1) must take values greater than critical load value (λ_{cr1}) that will create the first separation on this surface. $\lambda_1 > \lambda_{cr_1}$ It is seen that the layers are separated from each other. It is taken equal to an unknown function such as the derivative of the vertical displacement difference in the interval (e, f). The integral of this function will give the separation between layers in the interval (e, f).

The boundary conditions can be written as:

$$\sigma_{y_1}(x,h) = \begin{cases} -p(x) & a < x < b \\ -q(x) & c < x < d \\ 0 & -\infty < x < a,\ b < x < c,\ d < x < \infty \end{cases} \quad (1a)$$

$$\tau_{xy_1}(x,\ h) = 0 \quad -\infty < x < \infty \quad (1b)$$

$$\tau_{xy_1}(x,\ h_2) = 0 \quad -\infty < x < \infty \quad (1c)$$

$$\tau_{xy_2}(x,\ 0) = 0 \quad -\infty < x < \infty \quad (1d)$$

$$\tau_{xy_2}(x,\ h_2) = 0 \quad -\infty < x < \infty \quad (1e)$$

$$\sigma_{y_2}(x,\ h_2) = \sigma_{y_1}(x,\ h_2) = 0 \quad e < x < f \quad (1f)$$

$$\frac{\partial}{\partial x}[v_2(x,h_2) - v_1(x,h_2)] = \begin{cases} \omega(x) & e < x < f \\ 0 & -\infty < x < e,\ f < x < \infty \end{cases}$$

$$\tau_{xy_3}(x,\ 0) = 0 \quad -\infty < x < \infty \quad (1h)$$

$$\frac{\partial}{\partial x}[v_2(x,\ 0) - v_3(x,\ 0)] = 0 \quad -\infty < x < \infty$$

(1i)

$$\sigma_{y_2}(x,\ 0) = \sigma_{y_3}(x,\ 0) \quad -\infty < x < \infty \quad (1j)$$

$$\frac{\partial}{\partial x}[v_1(x,h)] = 0 \quad a < x < b \quad (1k)$$

$$\frac{\partial}{\partial x}[v_1(x,h)] = 0 \quad c < x < d \quad (1l)$$

As equilibrium conditions for the problem

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$$\int_a^b p(x)dx = P^I \qquad \int_c^d q(x)dx = Q^I \tag{2a-b}$$

can be written.

By solving the set of equations obtained as a result of applying stress and displacement expressions under the boundary conditions given in equations unknown coefficients are obtained depending on the unknown contact stresses p (x), q (x) and *ω(x)*.

$$\sigma_{y_1}(x,h_2) - \rho_1 g h_1 = 0 \quad e < x < f \tag{3}$$

In order to find these functions, boundary conditions (1k), (1l) and Equation (3) are used.

$$-\frac{1}{\pi}\frac{1}{\mu_1}\int_a^b p(t_1)dt_1\left[k_1(x_1,t_1) + \frac{1+\kappa_1}{4}\frac{1}{(t_1 - x_1)}\right]$$
$$-\frac{1}{\pi}\frac{1}{\mu_1}\int_c^d q(t_2)dt_2\left[k_1(x_1,t_2) + \frac{1+\kappa_1}{4}\frac{1}{(t_2 - x_1)}\right] = 0 \tag{4a}$$
$$a < x_1 < b$$

$$-\frac{1}{\pi}\frac{1}{\mu_1}\int_a^b p(t_1)dt_1\left[k_1(x_2,t_1) + \frac{1+\kappa_1}{4}\frac{1}{(t_1 - x_2)}\right]$$
$$-\frac{1}{\pi}\frac{1}{\mu_1}\int_c^d q(t_2)dt_2\left[k_1(x_2,t_2) + \frac{1+\kappa_1}{4}\frac{1}{(t_2 - x_2)}\right] = 0 \tag{4b}$$
$$c < x_2 < d$$

$$-\frac{1}{\pi}\int_a^b k_2(x_3,t_1)\, p(t_1)dt_1 - \frac{1}{\pi}\int_c^d k_2(x_3,t_2)q(t_2)dt_2$$
$$-\frac{\mu_1}{\pi}\int_e^f [k_4(x_3,t_3) - \frac{4(1+\kappa_1)}{(1+\kappa_1)+(1+\kappa_2)\frac{\mu_2}{\mu_1}}\frac{1}{t_3 - x_3}] \tag{4c}$$
$$\omega(t_3)dt_3 - \rho_1 g h_1 = 0, \qquad e<x_3<f$$

$\rho_k g$ is the mass force in the y-axis direction, ρ_k and g are the density of the layer and the acceleration of gravity, respectively. μ_k and κ_k shows the shear stress modulus and elastic material constants. It is known that κ_k material constants of elastic layers are $\kappa_k = (3-4\upsilon_k)$ if plane is in deformation, and $\kappa_k = (3-\upsilon_k)/(1+\upsilon_k)$ if the plane is in stress. υ_k shows the Poisson ratio (k=1,2,3). Index 1 and 2 represent elastic layers, while the index 3 represents the elastic semi-infinite plane.

The Δ and k_1, k_2, k_4 kernels mentioned in the equations can be seen in the reference (Bora, 2016)

The index of integral Equations (4a) and (4b) is +1. Conversely, the index of the singular integral Equation (4c) is -1 because of the physical necessity of uniform contact at the e and f endpoints (Erdogan and Gupta 1972)The univalence condition can be written as follows.

$$\int_e^f \omega(x)dx = 0 \tag{5}$$

If we define the dimensionless quantities below,

$$x_3 = \frac{f-e}{2}r_3 + \frac{f+e}{2} \qquad t_3 = \frac{f-e}{2}s_3 + \frac{f+e}{2} \tag{6a}$$

$$g_3(s_3) = \mu_1\omega\left(\frac{f-e}{2}s_3 + \frac{f+e}{2}\right) / P / h \tag{6b}$$

Other equations can be seen in reference (Bora, 2016).

Discontinuity at the Interface of the Elastic Semi-Infinite Plane with the Lower-layer

For separation to occur between the semi-infinite plane and the lower-layer, the load (λ_2) must be greater than the load (λ_{cr_2}) that will cause the initial separation at the interface. In case of $\lambda_2 > \lambda_{cr_2}$, the elastic semi-infinite plane and the lower-layer are separated from each other and the derivative of the vertical displacement difference in the interval (k, l) is taken equal to an unknown function such as $\varphi(x)$. The integral of this function will give the separation between the elastic semi-infinite plane in the interval (k, l) and the lower-layer.

When the boundary conditions are rearranged; The other boundary conditions remain the same but the changing boundary conditions are written as follows.

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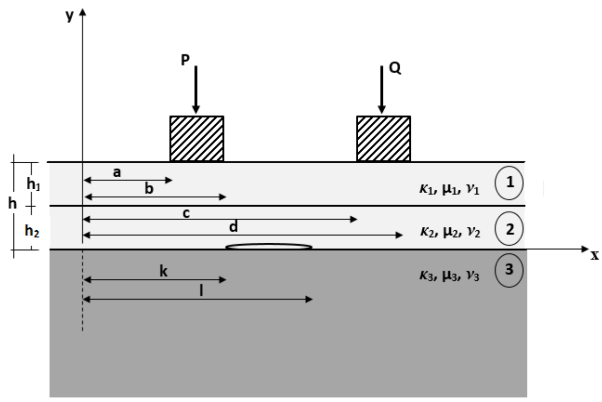

Figure 2. Discontinuous contact between the elastic semi-infinite plane and the lower-layer

$$\sigma_{y_2}(x,\ h_2)=\sigma_{y_1}(x,\ h_2) \quad -\infty<x<\infty \tag{7a}$$

$$\frac{\partial}{\partial x}[v_2(x,\ h_2)-v_1(x,\ h_2)]=0 \quad -\infty<x<\infty \tag{7b}$$

$$\frac{\partial}{\partial x}[v_2(x,0)-v_3(x,0)]=\begin{cases}\varphi(x) & k<x<l\\ 0 & -\infty<x<k,\ l<x<\infty\end{cases} \tag{7c}$$

$$\sigma_{y_2}(x,\ 0)=\sigma_{y_3}(x,\ 0)=0 \quad k<x<l \tag{7d}$$

Equilibrium conditions for the problem;

$$\int_a^b p(x)dx=P^{II} \qquad \int_c^d q(x)dx=Q^{II} \tag{8a-b}$$

can be written as given above.

By solving the set of equations obtained as a result of applying stress and displacement expressions under the boundary conditions given in equations unknown coefficients are obtained depending on the unknown contact stresses p (x), q (x) and φ (x).

$$\sigma_{y_2}(x,0)-(\rho_1 g h_1+\rho_2 g h_2)=0 \quad k<x<l \tag{9}$$

In order to find these functions, boundary conditions and Equation (9) are used.

$$-\frac{1}{\pi}\frac{1}{\mu_1}\int_a^b p(t_1)dt_1\left[k_1(x_1,t_1)+\frac{1+\kappa_1}{4}\frac{1}{(t_1-x_1)}\right]-\frac{1}{\pi}\frac{1}{\mu_1}\int_c^d q(t_2)dt_2\left[k_1(x_1,t_2)+\frac{1+\kappa_1}{4}\frac{1}{(t_2-x_1)}\right]=0 \tag{10a}$$

$$a<x_1<b$$

$$-\frac{1}{\pi}\frac{1}{\mu_1}\int_a^b p(t_1)dt_1\left[k_1(x_2,t_1)+\frac{1+\kappa_1}{4}\frac{1}{(t_1-x_2)}\right]-\frac{1}{\pi}\frac{1}{\mu_1}\int_c^d q(t_2)dt_2\left[k_1(x_2,t_2)+\frac{1+\kappa_1}{4}\frac{1}{(t_2-x_2)}\right]=0 \tag{10b}$$

$$c<x_2<d$$

$$-\frac{1}{\pi}\int_a^b k_3(x_4,t_1)\,p(t_1)dt_1-\frac{1}{\pi}\int_c^d k_3(x_4,t_2)q(t_2)dt_2-\frac{\mu_2}{\pi}\int_k^l\Bigg[k_5(x_4,t_4)-\frac{4\,{}^{\mu_3}\!/_{\mu_2}}{(1+\kappa_3)+{}^{\mu_3}\!/_{\mu_2}(1+\kappa_2)}\frac{1}{t_4-x_4}\Bigg]\varphi(t_4)dt_4-\rho_1 g h_1-\rho_2 g h_2=0, \tag{10c}$$

$$k<x_4<l$$

The k_3, k_5, kernels mentioned in the equations can be seen in the reference (Bora, 2016).

The index of integral Equations (10a) and (10b) is +1. Also, the index of the singular integral Equation (10c) is -1 due to the physical requirement of uniform

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contact at the k and l endpoints (Erdogan, F. and Gupta 1972) The univalence condition can be written as follows.

$$\int_k^l \varphi(x)dx = 0 \tag{11}$$

If we define the dimensionless quantities below,

$$x_4 = \frac{l-k}{2} r_4 + \frac{l+k}{2} \qquad t_4 = \frac{l-k}{2} s_4 + \frac{l+k}{2} \tag{12a}$$

$$g_4(s_4) = \mu_2 \varphi\left(\frac{l-k}{2} s_4 + \frac{l+k}{2}\right) / P/h \tag{12b}$$

Other equations can be seen in reference (Bora P. 2016)

RESULTS AND DISCUSSION

In the solution of the problem, the appropriate Gauss-Chebyshev integration formulas are used. The effects of the change of distance between blocks on elastic layers and elastic semi-infinite plane interfaces have been studied (Initial separation points, separation distances and distances where the interaction ends). In addition, the effects of change in block widths and change in load ratios on departure distances and swells are graphically presented. Analytical solutions were compared with the solutions obtained by finite element method and the results were found to be very close.

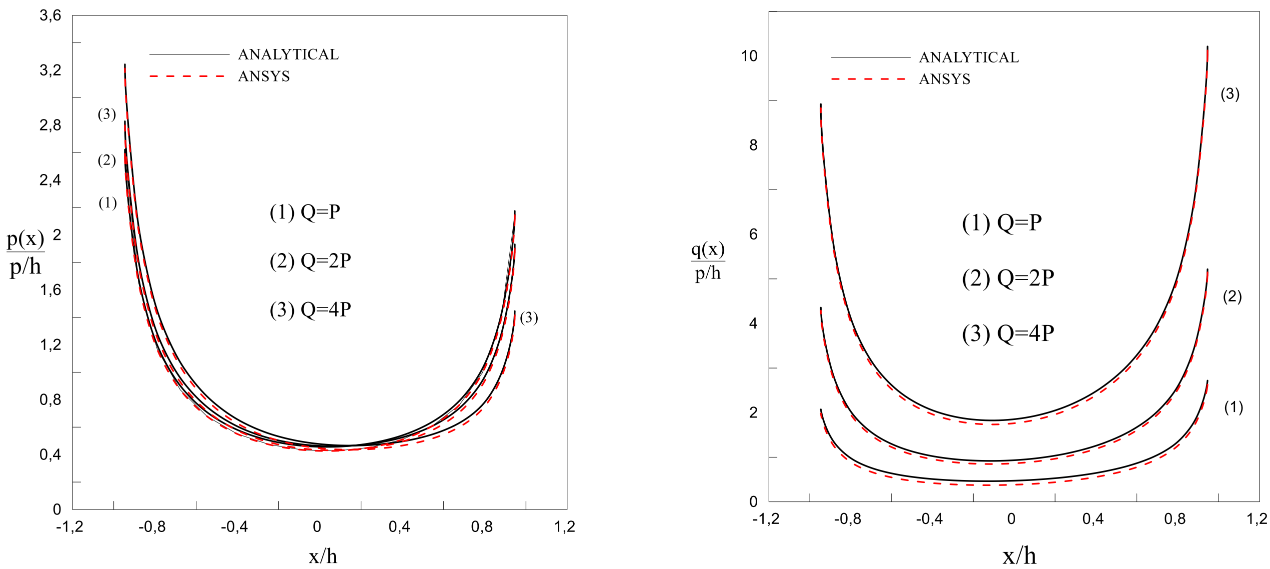

Figure 3a-b.1.and 2. Stress distribution under the blocks ($\mu_2/\mu_1 = 2$,$\mu_3/\mu_2 = 0.5$, a/h=3, (b-a)/h=0.5,(d-c)/h=0.5,(c-b)/h=1)

In Figures 3 a-b, when the stresses under the first block are examined, it is seen that as the load value on the second block increases, the stresses under the first block also increase. Considering the interaction between the blocks, the lowest stress value occurs at the corner of the first block that is close to the second block. When the contact stress under the second block examined, it is seen that the contact stresses under the second block increase if the load is twice or four times. The stresses have their greatest value at the block edges. The graphs were obtained by both analytical and finite element methods. It has been observed that the results obtained with both methods are consistent.

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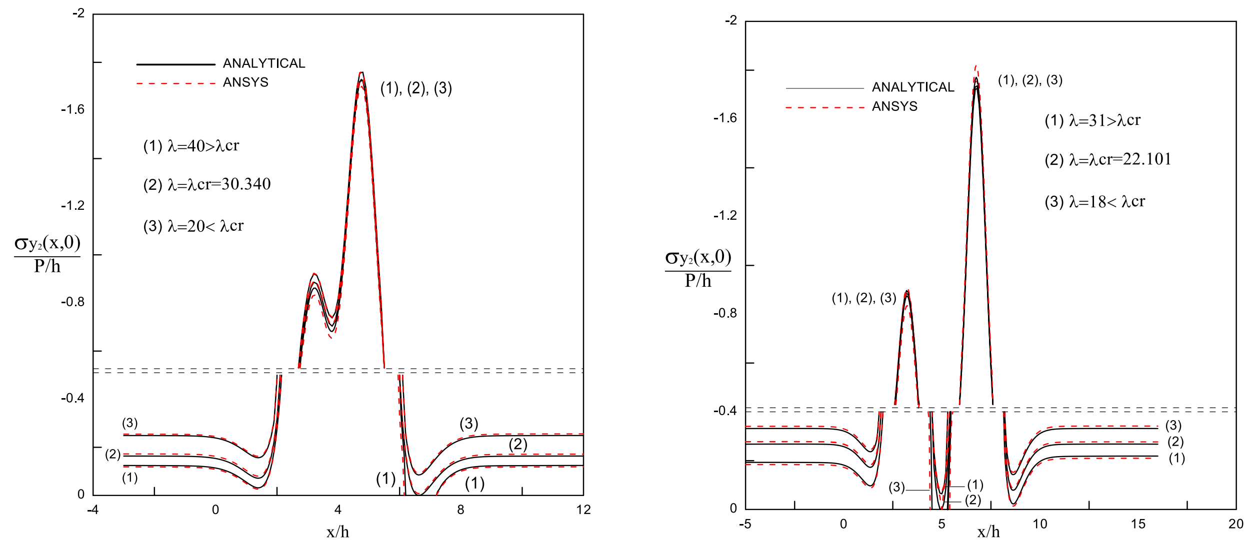

Figure 4 a-b. In continuous contact ($\lambda < \lambda_{cr}$) and discontinuous contact, in case of ($\lambda > \lambda_{cr}$) (c-b)/h=1 and (c-b)/h=3, theoretical and numerical results of the dimensionless stress distribution of $\sigma_y(x,0)/(P/h)$, (μ_2/μ_1=1, μ_3/μ_2=1, a/h=3, (b-a)/h=0.5, (d-c)/h=05, Q=2P

Table 1. The variation of critical load factor (λ_{cr}) values with distance between blocks ((c-b) / h) (Q = 2P, μ_2/μ_1=2, μ_3/μ_2=2, a / h = 3, (b-a) / h = (d-c) / h = 1) at the elastic semi-infinite plane interface with the lower-layer

$\frac{(c-b)}{h}$	BLOCK I				BLOCK II			
	$\lambda_{cr_{sol}}$	$x_{cr_{sol}}$	$\lambda_{cr_{sağ}}$	$x_{cr_{sağ}}$	$\lambda_{cr_{sol}}$	$x_{cr_{sol}}$	$\lambda_{cr_{sağ}}$	$x_{cr_{sağ}}$
0.5	71.2228	1.2556					48.7765	7.2451
1	82.4970	1.2284					47.0447	7.7476
3	92.9404	1.2317					48.3093	9.7452
5	94.4009	1.2402	**46.2553**	7.2439	**46.2553**	7.2439	48.4878	11.743
6.0647	**94.6074**	1.2430	**94.6074**	5.2570	**48.5176**	8.3303	**48.5176**	12.799

In Figures 4a-4b, contact stress distributions are given for three different values of load factor λ. ($\lambda<\lambda_{cr}$, $\lambda=\lambda_{cr}$, $\lambda>\lambda_{cr}$) In Figure 4a, if $\lambda=20<\lambda_{cr}$, there is continuous contact and there is no separation at any point. If $\lambda=30.340=\lambda_{cr}$, there is a possibility that the first separation will occur to the right of the second block (**(c-b)/h=1**, x_{cr} =6.65). $\lambda=40>\lambda_{cr}$, a separation zone (k/h=6.260, l/h=7.2477) occurs between the lower-layer and the elastic semi-infinite plane and the stress values in this region are zero.

In Figure 4b, if $\lambda=18<\lambda_{cr}$, there is continuous contact. It is understood that for $\lambda=22.101=\lambda_{cr}$ value, the first separation will occur between blocks (**(c-b)/h=3**, x_{cr} =4.961). In case of $\lambda=31>\lambda_{cr}$, a separation zone (k/h = 4.5912, l/h = 5.3001) is formed at the interface of the elastic semi-infinite plane with the lower-layer and the stress values in this region are

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zero. When the two figures are compared, it is seen that if the distance between the blocks increases, the first separation occurs between the blocks. And the initial separation load and separation zone are smaller. In addition, it is seen in both graphs that the results obtained by analytical and finite element methods are quite close.

Table 1 shows the effect of inter-block distance variation on the initial separation load and initial separation distance at the interface of the lower-layer and the elastic semi-infinite plane. Accordingly, for small values of (c-b) / h ((c-b) / h) <3), two separation zones may occur depending on λ. Since Q / h≥P / h, the first separation zone is on the right side of the second block. In this case, if λ is large enough, the second separation zone may occur to the left of the first block. If the distance between blocks ((c-b) / h>3) is further increased, another separation zone occurs between the blocks. And this zone is probably the first separation zone. When the distance between the layers continues to increase ((c-b) / h> 5), there is a possibility that four separation zones will occur depending on the load factor λ. The first separation zone is again formed between the blocks and after a certain value of (c-b) / h ((c-b) /h=6.0647), the interaction between the blocks is lost. If the distance between two blocks is greater than a limit value, each block can be considered separately.

Figure 5a shows effects of the change in distance between the blocks on the separation zone between the layers. Accordingly, for small values of (c-b) / h, there is a possibility that two separation zones depending on ((c-b) / h) <3) will occur, while Q / h≥ is P / h, the first separation zone is on the right side of the second block. If the distance between blocks ((c-b) / h = 3) is further increased, it appears that there may also be a separation zone between blocks. And this zone is probably the first separation zone. If the distance between the blocks is continued to be increased ((c-b) / h> 5), it is understood that four separation zones can occur depending on the load factor λ. In this case, the possible first separation zone occurs near the second block and after a certain value of (c-b) / h, the interaction between the blocks is lost.

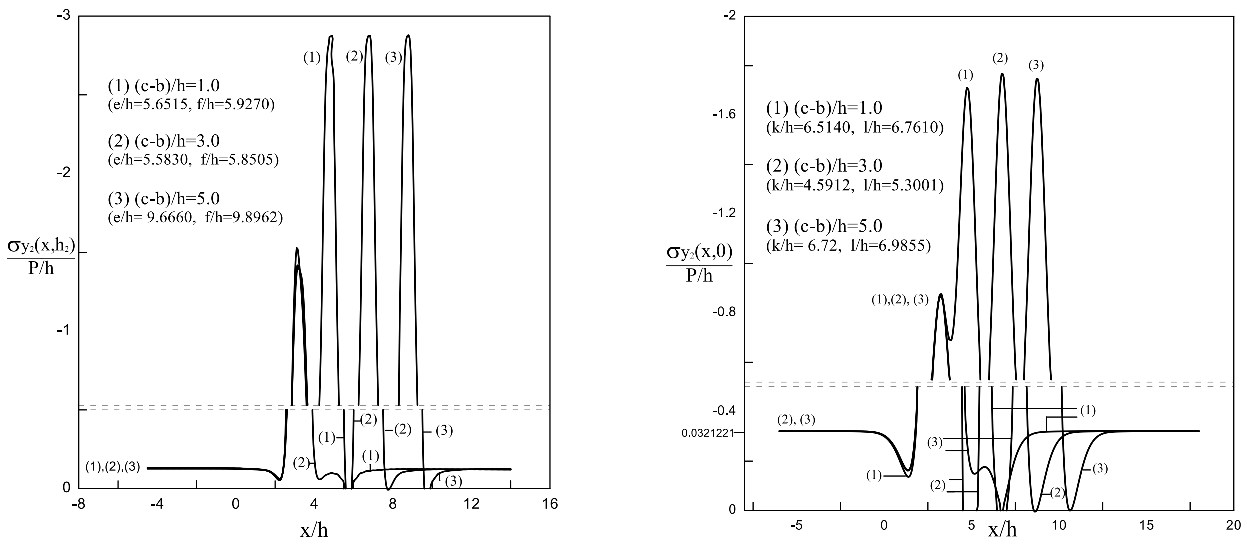

Figure 5a-b. The change in the dimensionless stress distribution of σ_y (x, h_2) / (P / h) and σ_y (x, 0) / (P / h) with distance between blocks (μ_2/μ_1 = 1, μ_3/μ_2 = 1, a / h = 3, (b-a) /h=0.5, (d-c) / h = 05, Q = 2P, λ = 30)

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Figure 5b shows the effects of the change in distance between the blocks on the separation zone between the lower-layer and the elastic semi-infinite plane. Accordingly, for small values of (c-b) / h, ((c-b) / h) <3), the possibility of two separation zones depending on λ arises, and when Q / h≥ P / h, the first separation zone is on the right side of the second block. If the distance between blocks ((c-b) / h = 3) is further increased, another separation zone arises between the blocks. And this zone is probably the first separation zone. When the distance between the blocks continues to increase ((c-b) / h> 5), the possibility of four separation zones arises depending on the load factor λ. The first detachment zone again occurs between blocks and after a certain value of (c-b) / h, the interaction between blocks disappears.

Figure 6a shows the variation of swellings occurring at the interface of the layers with the distance between the blocks. When the change in distance between blocks is (c-b) / h = 1, (c-b) / h = 5, first separations occur on the right side of the second block, and when (c-b) / h = 3, separation occurs between blocks. As the distance between the blocks increases, the resulting swelling and separation zones get smaller. Figure 6b shows the variation of swellings at the interface of the lower-layer and elastic semi-infinite plane with the distance between the blocks. When the distance value between blocks is (c-b) / h = 1, the first separation occurs on the right side of the second block, the swelling value and the separation zone are smaller. When (c-b) / h = 3, (c-b) / h = 5, the first separations occur between the blocks. While (c-b) / h = 3, the swelling value and the separation zone grows, in the case of (c-b) / h = 5, the swelling value and separation region get smaller.

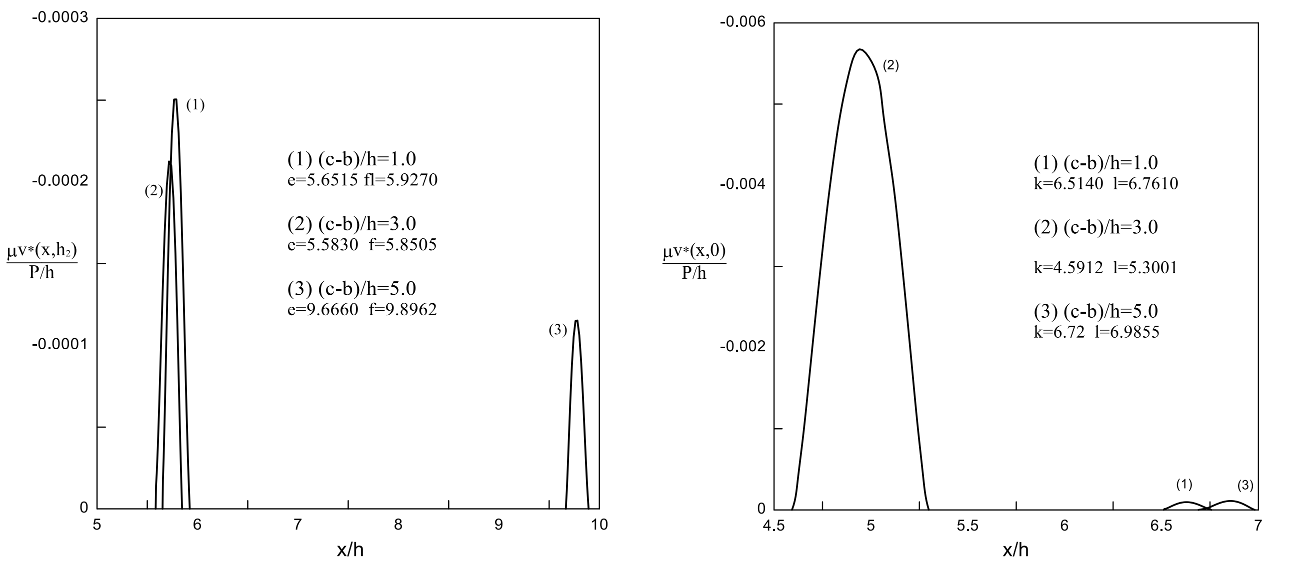

Figure 6a-b. The change in the dimensionless stress distribution of σ_y (x, h_2) / (P / h) and σ_y (x, 0) / (P / h) with distance between blocks (μ_2/μ_1 = 1, μ_3/μ_2 = 1, a / h = 3, (b-a) /h=0.5, (d-c) / h = 05, Q = 2P, λ = 30)

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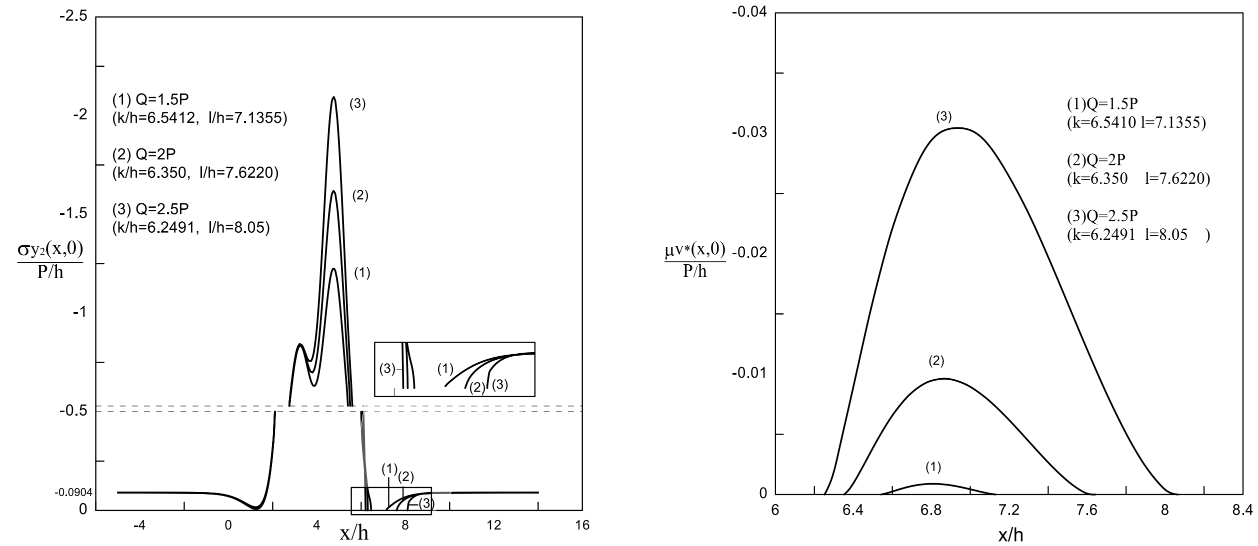

Figure 7a-b. σ_y (x, 0) / (P / h) variation of dimensionless stress distribution and variation of swelling with load ratio ($\mu_2/\mu_1 = 2$, $\mu_3/\mu_2 = 0.5$, a / h = 3, (b-a) / h = 0.5, (d-c) / h = 0.5, (c-b) / h = 1, $\lambda = 55$)

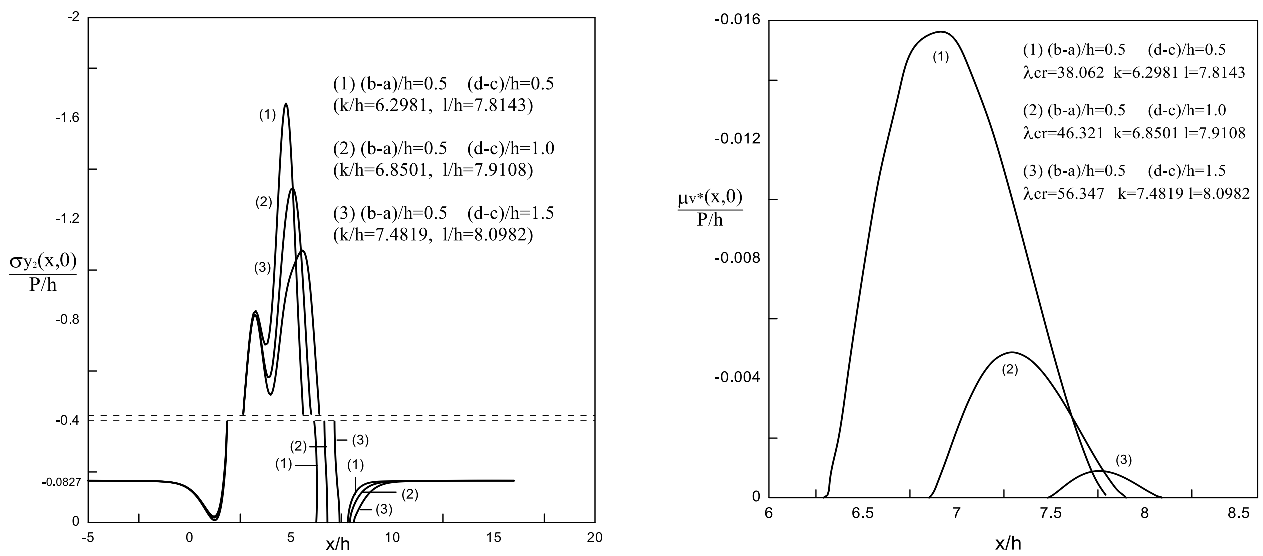

Figure 8a-b σ_y (x, 0) / (P / h) change of dimensionless stress distribution with block width, swelling ($\mu_2/\mu_1 = 2$, $\mu_3/\mu_2 = 0.5$, a / h = 3, (c-b) / h = 1, $\lambda = 60$)

The variation of the separation zone according to the load rate is examined in Figure 7a. As seen in the figure, the separation becomes easier and the separation zone increases as the load ratio increases. When the load is increased, the end point of separation increases in l / h, while the starting point of separation decreases in k / h and approaches a fixed value. As seen in Figure 7b, when the load ratio is increased, the separation zone at the interface of the

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lower-layer and the elastic semi-endless plane grows and the swelling in this region increases.

In Figure 8a, as the second block width increases, the change in the separation zones at the interface of the lower layer and the elastic semi-infinite plane is observed. Accordingly, as the block width increases, the separation zone becomes smaller. When block widths are increased, initial separation distances and initial separation loads also increase.

Figure 8b shows the variation of swellings at the interface of the lower-layer and elastic semi-endless plane with the width of the 2nd block. According to the figure, as the block width is increased, the separation zone becomes smaller and the swelling occurring in this area decreases. As the block width is increased, the separation zone moves away from the y initial axis.

The following conclusions can be drawn from the study.

CONCLUSION

In this study, the discontinuous contact problem of two layers with different material properties, loaded with two rigid flat blocks and resting on an elastic semi-infinite plane was solved using linear elasticity theory and the finite element method (FEM). The rectangular rigid block problem has important applications in soil mechanics, especially in predicting the safety of foundations. The blocks can be taken as foundations placed on elastic layers.
Larger openings occur when the blocks are close to each other. It can be said that block interactions directly affect the openings. As the distance between blocks increases, separation may occur in more than one area. Usually, the first separation occurs between blocks. Increasing the distance causes the initial separation load and the separation zone to decrease. When the distance exceeds a certain value, the effect of the blocks on each other disappears. While the change in load ratios causes small changes in the initial separation distances, it causes an increase in the size of the stress and separation zone.

When the first block width is kept constant and the second block width is increased, the first separation distances and the first separation loads increase while the separation zone (swelling) becomes smaller.

In contact problems, the analytical solution is complex and requires lengthy mathematical calculations. The finite element method (FEM) is practical and offers a fast solution.

When compared with the analytical solution results, it is seen that the error rates of the separation distances and swells obtained with FEM are at acceptable levels. For this reason, it can be said that the finite element method (FEM) is an alternative solution to analytical solutions in the solution of discontinuous contact problems.

In later studies, the layers can be functionally graded, the stresses occurring in this case can be calculated, compared with the stresses in the case of linearity, and the advantages and disadvantages can be discussed.

CONFLICT OF INTEREST

The authors declared no conflict of interest regarding this article.

RESEARCH AND PUBLICATION ETHICS STATEMENT

The authors declare that this study complies with research and publication ethics.

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