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On Soft Generalized Topological Spaces

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Abstract – The main purpose of this paper is to introduce soft generalized topology on a soft set. The definitions of subspace soft generalized topology and soft continuity of soft functions are introduced. Some basic concepts in soft generalized topological spaces are also defined and studied their properties.

Keywords – *Soft sets, generalized topology, soft generalized topology, Soft μ-closure, soft(μ,η)-continuous functions.*

1. Introduction

Molodtsov [10] in 1999, initiated the concept of soft set theory as a mathematical tool for modeling uncertainties. A soft set is a collection of approximate descriptions of an object. Later other researchers like Maji et al. [8] have further improved the theory of soft sets. NaimCagman et al. [4] modified the definition of soft sets which is similar to that of Molodtsov. Csaszar [6] in 2002 introduced the concept of generalized topology and also studied some of its basic properties. Let X be a nonempty set and ξ be a collection of subsets of X. Then ξ is called a generalized topology (briefly GT) on X if and only if $\emptyset \in$ ξ and $G_i \in$ ξ for i ∈ J implies $\bigcup_{i\in J} G_i \in$ ξ. In this paper, we begin with the basic definitions and results related to soft set theory which are useful for subsequent sections. Basic notions and concepts of soft generalized topological spaces such as soft basis, subspace soft generalized topology, soft μ-interior, soft μ-closure, soft μ-neighborhood, soft μ-limit point, soft μ-boundary, soft μ-exterior and soft continuity of soft functions are defined and studied their basic properties. We then define soft generalized topology on an initial soft set and see that soft generalized topology gives a parameterized family of generalized topologies on the initial universe.

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2. Preliminaries

In this section we recall some definitions and results defined and discussed in [4, 7, 8, 10]. Throughout this paper U denotes initial universe, E denotes the set of all possible parameters, $\mathcal{P}$(U) is the power set of U and A is a nonempty subset of E.

Definition 2.1.A soft set F_{A} on the universe U is defined by the set of ordered pairs $F_{\mathrm{A}} = \{(e, f_A(e)) \ / \ e \in E, f_A(e) \in \mathcal{P}(U)\}$, where $f_A : E \to \mathcal{P}(U)$ such that $f_A(e)= \emptyset$ if $e \notin$ A. Here f_A is called an approximate function of the soft set F_{A}. The value of $f_A(e)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. The set of all soft sets over U with E as the parameter set will be denoted by $\mathrm{S(U)_E}$ or simply S(U).

Definition 2.2. Let $F_{\mathrm{A}} \in$S(U). If $f_A(e) = \emptyset$ for all $e \in$ E, then $F_{\mathrm{A}}$is called an empty soft set, denoted by $F_{\emptyset}$. $f_A(e) = \emptyset$ means that there is no element in U related to the parameter e in E. Therefore we do not display such elements in the soft sets as it is meaningless to consider such parameters.

Definition 2.3. Let $F_{\mathrm{A}} \in$S(U). If $f_A(e) =$ U for all $e \in$ E, then F_{A} is called an A-universal soft set, denoted by $F_{\tilde{\mathrm{A}}}$. If A = E, then the A-universal soft set is called an universal soft set, denoted by $F_{\tilde{\mathrm{E}}}$.

Definition 2.4. Let $F_{\mathrm{A}}, F_{\mathrm{B}} \in$S(U). Then F_{B} is a soft subset of F_{A}, denoted by $F_{\mathrm{B}} \subseteq F_{\mathrm{A}}$, if $f_B(e) \subseteq f_A(e)$, for all $e \in$ E.

Definition 2.5. Let $F_{\mathrm{A}}, F_{\mathrm{B}} \in$S(U). Then F_{B} and F_{A} are soft equal, denoted by $F_{\mathrm{B}} = F_{\mathrm{A}}$, if $f_B(e) = f_A(e)$, for all $e \in$ E.

Definition 2.6. Let $F_{\mathrm{A}}, F_{\mathrm{B}} \in$S(U). Then, the soft union of $F_{\mathrm{A}}$and F_{B},denoted by $F_A \cup F_{\mathrm{B}}$, is defined by the approximate function $f_{A \cup B}(e) = f_A(e) \cup f_B(e)$.

Definition 2.7. Let $F_{\mathrm{A}}, F_{\mathrm{B}} \in$S(U). Then, the soft intersection of $F_{\mathrm{A}}$and F_{B}, denoted by $F_A \cap F_{\mathrm{B}}$, is defined by the approximate function $f_{A \cap B}(e) = f_A(e) \cap f_B(e)$.

Definition 2.8. Let $F_{\mathrm{A}}, F_{\mathrm{B}} \in$S(U). Then, the soft difference of $F_{\mathrm{A}}$and F_{B},denoted by $F_A \setminus F_{\mathrm{B}}$, is defined by the approximate function $f_{A \setminus B}(e) = f_A(e) \setminus f_B(e)$.

Definition 2.9. Let $F_{\mathrm{A}} \in$S(U). Then, the soft complement of F_{A},denoted by $(F_{\mathrm{A}})^c$, is defined by the approximate function $f_{A^c}(e) = (f_A(e))^c$, where $(f_A(e))^c$ is the complement of the set $f_A(e)$, that is, $(f_A(e))^c = U \setminus f_A(e)$for all $e \in$ E.

Cleary $((F_{\mathrm{A}})^c)^c = F_{\mathrm{A}}and(F_{\emptyset})^c = F_{\tilde{\mathrm{E}}}$.

Definition 2.10. Let $F_{\mathrm{A}} \in$S(U). The soft power set of F_{A}, denoted by $\mathcal{P}(F_{\mathrm{A}})$, is defined by $\mathcal{P}(F_{\mathrm{A}}) = \{F_{A_i} \ / \ F_{A_i} \subseteq F_A, i \in J \subseteq N\}$.

Theorem 2.11. Let $F_{\mathrm{A}}, F_{\mathrm{B}}, F_{\mathrm{C}} \in$S(U). Then,

(1) $F_A \cup F_{\mathrm{A}} = F_{\mathrm{A}}$.

(2) $F_A \cap F_A = F_A$.
(3) $F_A \cup F_\emptyset = F_A$.
(4) $F_A \cap F_\emptyset = F_\emptyset$.
(5) $F_A \cup F_{\tilde{E}} = F_{\tilde{E}}$.
(6) $F_A \cap F_{\tilde{E}} = F_A$.
(7) $F_A \cup (F_A)^c = F_{\tilde{E}}$.
(8) $F_A \cap (F_A)^c = F_\emptyset$.
(9) $F_A \cup F_B = F_B \cup F_A$.
(10) $F_A \cap F_B = F_B \cap F_A$.
(11) $(F_A \cup F_B)^c = (F_A)^c \cap (F_B)^c$.
(12) $(F_A \cap F_B)^c = (F_A)^c \cup (F_B)^c$.
(13) $(F_A \cup F_B) \cup F_C = F_A \cup (F_B \cup F_C)$.
(14) $(F_A \cap F_B) \cap F_C = F_A \cap (F_B \cap F_C)$.
(15) $F_A \cup (F_B \cap F_C) = (F_A \cup F_B) \cap (F_A \cup F_C)$.
(16) $F_A \cap (F_B \cup F_C) = (F_A \cap F_B) \cup (F_A \cap F_C)$.

Definition 2.12.[7] Let $S(U)_E$ and $S(V)_K$ be the families of all soft sets over U and V, respectively. Let $\varphi : U \rightarrow V$ and $\chi : E \rightarrow K$ be two mappings. The soft mapping φ_χ: $S(U)_E \rightarrow S(V)_K$ is defined as:

(1) Let F_A be a soft set in $S(U)_E$. The image of F_A under the soft mapping φ_χ is the soft set over V, denoted by $\varphi_\chi(F_A)$ and is defined by $\varphi_\chi(f_A)(k) = \begin{cases} \bigcup_{e \in \chi^{-1}(k) \cap A} \varphi(f_A(e)), & \text{if } \chi^{-1}(k) \cap A \neq \emptyset; \\ \emptyset, & \text{otherwise} \end{cases}$ for all $k \in K$.

(2) Let G_B be a soft set in $S(V)_K$. The inverse image of G_B under the soft mapping φ_χ is the soft set over U, denoted by $\varphi_\chi{}^{-1}(G_B)$ and is defined by

$$\varphi_\chi{}^{-1}(g_B)(e) = \begin{cases} \varphi^{-1}(g_B(\chi(e))), & \text{if } \chi(e) \in B; \\ \emptyset, & \text{otherwise} \end{cases} \text{for all } e \in E.$$

The soft mapping φ_χ is called injective, if φ and χ are injective. The soft mapping φ_χ is called surjective, if φ and χ are surjective.

Definition 2.13. Let φ_χ: $S(U)_E \rightarrow S(V)_K$ and τ_σ: $S(V)_K \rightarrow S(W)_L$, then the soft composition of the soft mappings φ_χ and τ_σ, denoted by $\varphi_\chi \, o \, \tau_\sigma$, is defined by $\varphi_\chi \, o \, \tau_\sigma = (\varphi \, o \, \tau)_{(\chi \, o \, \sigma)}$.

Theorem 2.14. [7]Let $S(U)_E$ and $S(V)_K$ be the families of all soft sets over U and V, respectively. Let $F_A, F_B, F_{A_i} \in S(U)_E$ and $G_A, G_B, G_{B_i} \in S(V)_K$. For a soft mappings φ_χ: $S(U)_E \rightarrow S(V)_K$ and τ_σ: $S(V)_K \rightarrow S(W)_L$ the following statements are true:

(1) If $F_B \subseteq F_A$, then $\varphi_\chi(F_B) \subseteq \varphi_\chi(F_A)$.
(2) $\varphi_\chi(\bigcup_{i \in J} F_{A_i}) = \bigcup_{i \in J} \varphi_\chi(F_{A_i})$.

(3) $\varphi_\chi(\bigcap_{i\in J} F_{A_i}) \subseteq \bigcap_{i\in J} \varphi_\chi(F_{A_i})$, equality holds if φ_χ is injective.
(4) $F_A \subseteq {\varphi_\chi}^{-1}(\varphi_\chi(F_A))$, equality holds if φ_χ is injective.
(5) $\varphi_\chi({\varphi_\chi}^{-1}(F_A)) \subseteq F_A$, equality holds if φ_χ is surjective.
(6) If $G_B \subseteq G_A$, then ${\varphi_\chi}^{-1}(G_B) \subseteq {\varphi_\chi}^{-1}(G_A)$.
(7) ${\varphi_\chi}^{-1}((G_B)^c) = ({\varphi_\chi}^{-1}(G_B))^c$
(8) ${\varphi_\chi}^{-1}(\bigcup_{i\in J} G_{B_i}) = \bigcup_{i\in J} {\varphi_\chi}^{-1}(G_{B_i})$.
(9) ${\varphi_\chi}^{-1}(\bigcap_{i\in J} G_{B_i}) = \bigcap_{i\in J} {\varphi_\chi}^{-1}(G_{B_i})$.
(10) $(\tau_\sigma\ o\ \varphi_\chi)^{-1} = {\varphi_\chi}^{-1} o\ {\tau_\sigma}^{-1}$.

3. Soft generalized topological spaces

Definition 3.1. Let $F_{\mathrm{A}} \in$S(U). A Soft Generalized Topology (SGT) on F_{A}, denoted by μ or μ_{F_A} is a collection of soft subsets of F_{A} having the following properties:

(1) $F_\emptyset \in \mu$
(2) $\{F_{A_i} \subseteq F_A \ / \ i \in \mathrm{J} \subseteq \mathrm{N}\} \subseteq \mu \Rightarrow \bigcup_{i\in \mathrm{J}} F_{A_i} \in \mu$

The pair (F_{A}, μ) is called a Soft Generalized Topological Space (SGTS)

Observe that $F_{\mathrm{A}} \in \mu$ must not hold.

Definition 3.2. Let F_A $\in$S(U) and μ be the collection of all possible soft subsets of F_A, then μ is a SGT on F_A, and is called the discrete SGT on F_A.

Definition 3.3. A soft generalized topology μ on F_A is said to be strong if $F_{\mathrm{A}} \in \mu$.

Definition3.4. Let (F_{A}, μ) be a SGTS. Then, every element of μ is called a soft μ–open set.

Note: clearly $F_\emptyset$ is a soft μ–open set.

Definition 3.5. Let (F_A, μ_1) and (F_A, μ_2) be SGTS's. Then

(1) If $\mu_2 \supseteq \mu_1$, then μ_2 is soft finer than μ_1
(2) If $\mu_2 \supset \mu_1$, then μ_2 is soft strictly finer than μ_1
(3) If either $\mu_2 \supseteq \mu_1$ or $\mu_1 \supseteq \mu_2$ then μ_1 is comparable with μ_2.

Theorem 3.6. Let F_A be a soft set and $\{\mu_j\}_{\mathrm{j}\in\mathrm{J}}$ be an indexed family of SGT's on F_A. Then $\bigcap_{j\in J} \mu_j$ is a SGT on F_{A} and each μ_j, j $\in$ J is soft finer than $\bigcap_{j\in J} \mu_j$.

Proof. Since each μ_j, j $\in$ J is a SGT on F_A, the empty soft set $F_\emptyset$ belongs to each μ_j, j $\in$ J and so $F_\emptyset \in \bigcap_{j\in J} \mu_j$. Let $\{F_{B_i}\}_{\mathrm{i}\in\mathrm{I}}$ be a family of soft sets in $\bigcap_{j\in J} \mu_j$. Then each F_{B_i} belongs to each μ_j. But μ_j, being a SGT on F_A, is closed under arbitrary soft unions. So $\bigcup_{i\in I} F_{B_i} \in \mu_j$ for each j $\in$ J. Thus $\bigcup_{i\in I} F_{B_i} \in \bigcap_{j\in J} \mu_j$. Hence $\bigcap_{j\in J} \mu_j$ is a SGT on F_{A}. Clearly each μ_j, j $\in$ J is soft finer than $\bigcap_{j\in J} \mu_j$. ■

Remark 3.7. Let $(F_A, \mu_1)$and (F_A, μ_2)be SGTS's on F_{A}. Then $(F_A,\ \mu_1 \cup\ \mu_2)$ may not be a SGTSon F_{A}.

Example 3.8. Let U = {h_1, h_2, h_3, h_4, h_5}, A = E = {e_1, e_2}, F_{A} = {(e_1, {h_1, h_2, h_3, h_4}), (e_2, {h_2, h_3})}. Let (F_A, μ_1) and (F_A, μ_2)be two SGTS's on F_{A} where μ_1= $\{F_\emptyset, F_{A_1}, F_{A_2}\}$, μ_2 = $\{F_\emptyset, F_{A_3}, F_{A_4}\}$, where F_{A_1} = {(e_1, {h_1, h_2}), (e_2, {h_2})}, F_{A_2} = {(e_1, {h_1}), (e_2, {h_2})}, F_{A_3} = {(e_1, {h_3, h_4})}, F_{A_4} = {(e_1, {h_3, h_4}), (e_2, {h_3})}. Now define $\mu = \mu_1 \cup \mu_2$ = $\{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}\}$. Then $F_{A_1} \cup F_{A_3}$= {(e_1, {h_1, h_2, h_3, h_4}), (e_2, {h_2})} $\notin \mu$. Hence μ is not a SGT on F_{A}.

Theorem 3.9. Let F_{A} be a soft set and η be a family of soft subsets of F_{A}. Then there exists a unique SGT μ on F_{A} such that it is the smallest SGT on F_{A} containing η.

Proof. Consider the collection of all SGT's on F_{A} which contains η (as subsets of $\mathcal{P}(F_A)$). This family is non-empty, for the discrete SGT (i.e, the entire power set $\mathcal{P}(F_A)$) surely contains η. Now let μ be the intersection of the members of this collection. By theorem3.6, μ is a SGT on F_{A}, it contains η and clearly it is the smallest SGT containing η, for any such SGT will be a member of the collection of SGT's just considered and hence soft finer than its intersections viz, μ. Uniqueness of μ is trivial.■

Definition 3.10. Let (F_{A}, μ) be a SGTS. A sub family $\mathfrak{B}$ of μ is said to be a soft basis for μ if every member of μ can be expressed as the soft union of some members of $\mathfrak{B}$.

Theorem 3.11. Let (F_{A}, μ)be a SGTS and $\mathfrak{B} \subseteq \mu$. Then $\mathfrak{B}$ is a soft basis for μ if and only if for each soft μ-open setF_G, and each $\alpha \in F_G$, there exists $F_B \in \mathfrak{B}$ such that $\alpha \in F_B$ and $F_B \subseteq F_G$.

Proof. First suppose that $\mathfrak{B}$ is a soft basis for μ. Let F_G be a soft μ-open set and $\alpha \in F_G$. Then F_G can be written as the soft union of some members of $\mathfrak{B}$, say, $F_G = \bigcup_{\mathrm{i} \in \mathrm{J}} F_{B_i}$ where J is an index set and $F_{B_i} \in \mathfrak{B}$, $\forall$ i. Since $\alpha \in F_G$, there exists j $\in$ J such that $\alpha \in F_{B_j}$. Take this F_{B_j} as the set F_B required in the assertion.

Conversely, suppose that the given condition holds. Let F_D be a soft μ-open set of F_A. For each $\alpha \in F_D$, there exists $F_{B_\alpha} \in \mathfrak{B}$ such that $\alpha \in F_{B_\alpha}$ and $F_{B_\alpha} \subseteq F_D$. Clearly $F_D = \bigcup_{\alpha \in F_D} F_{B_\alpha}$. Thus every member of μ can be expressed as the soft union of some members of $\mathfrak{B}$. Hence $\mathfrak{B}$ is a soft basis for μ.■

Theorem 3.12. Two distinct SGT's can never have the same family of soft subsets as a soft basis for both of them.

Proof. Let μ_1 and μ_2 be two SGT's on a soft set F_A and each have $\mathfrak{B}$ as a soft basis. If $F_G \in \mu_1$, then F_G can be expressed as the soft union of some members of $\mathfrak{B}$; these members are also members of μ_2, since $\mathfrak{B} \subseteq \mu_2$. But μ_2, being a SGT, is closed under arbitrary soft unions, $F_G \in \mu_2$. Thus $\mu_1 \subseteq \mu_2$. Similarly $\mu_2 \subseteq \mu_1$ and hence $\mu_1 = \mu_2$.■

Theorem3.13. Let (F_A, μ_1) and (F_A, μ_2) be two SGTS's for a soft set F_A having soft basis $\mathfrak{B}_1$ and $\mathfrak{B}_2$ respectively. Then μ_2 is soft finer than μ_1 if and only if every member of $\mathfrak{B}_1$ can be expressed as the soft union of some members of $\mathfrak{B}_2$.

Theorem 3.14. Let F_A be a soft set and $\mathfrak{B}$ be a family of its soft subsets. Then there exists a SGT on F_A with $\mathfrak{B}$ as a soft basis.

Proof. Let $\mathfrak{B}$= $\{F_{B_i}/ F_{B_i} \subseteq F_A, i \in J\} \cup F_\emptyset \subseteq \mathcal{P}(F_A)$. Define μ = $\{F_G \subseteq F_A$ / $\forall$ $\alpha \in F_G$, there exists $F_{B_i} \in \mathfrak{B}$ such that $\alpha \in F_{B_i}$ and $F_{B_i} \subseteq F_G\}$.i.e,μ = $\{\bigcup F_{B_i}/ F_{B_i} \in \mathfrak{B}\}$. We assert that μ is a SGT on F_A. Clearly $F_\emptyset \in \mu$. Let $\{F_{H_j}\}_{j\in J} \in \mu$ and assume that$F_H = \bigcup_{j\in J} F_{H_j}$. To show $F_H \in \mu$. Now, since each $F_{H_j} \in \mu, F_{H_j} = \bigcup_{i\in I} F_{B_i}, F_{B_i} \in \mathfrak{B}$. Then $F_H = \bigcup_{j\in J}(\bigcup_{i\in I} F_{B_i}) \in \mu$. Hence μ is a SGTon F_A. By theorem 3.11, it follows that $\mathfrak{B}$ is a soft basis for μ.■

Definition 3.15. Let (F_A, μ) be a SGTS and$F_B \subseteq F_A$. Then the collection $\mu_{F_B} = \{F_D \cap F_B$ / $F_D \in \mu\}$ is called a Subspace Soft Generalized Topology (SSGT) on F_B. The pair (F_B, μ_{F_B}) is called a Soft Generalized Topological Subspace (SGTSS) of F_A.

Theorem 3.16. Let (F_A, μ)be a SGTS and $F_B \subseteq F_A$. Then a SSGT on F_B is a SGT.

Proof. Since $F_\emptyset \in \mu, F_\emptyset \cap F_B = F_\emptyset \in \mu_{F_B}$. Suppose $\{F_{G_i}\}_{i\in J} \in \mu_{F_B}$. Since each $F_{G_i} \in \mu_{F_B} \Rightarrow F_{G_i} = F_{D_i} \cap F_B$ where $F_{D_i} \in \mu$. Now consider $\bigcup_{i\in J} F_{G_i} = \bigcup_{i\in J}(F_{D_i} \cap F_B) = (\bigcup_{i\in J} F_{D_i}) \cap F_B \in \mu_{F_B}$, since μ is closed under arbitrary soft unions.■

Theorem 3.17. Let (F_A, μ)be a SGTS. If $\mathfrak{B}$ is a soft basis for μ, then the collection $\mathfrak{B}_{F_B} = \{F_{D_i} \cap F_B / F_{D_i} \in \mathfrak{B}, i \in J\}$ is a soft basis for the SSGT μ_{F_B}on F_B.

Proof. Let F_G be an arbitrary element of the SSGT on F_B. Then $F_G = F_H \cap F_B$ where $F_H \in \mu$.Because $F_H \in \mu$, F_H can be expressed as the soft union of some elements of $\mathfrak{B}$.i.e, $F_H = \bigcup_{F_{D_i} \in \mathfrak{B}} F_{D_i}$. Therefore$F_G = (\bigcup_{F_{D_i} \in \mathfrak{B}} F_{D_i}) \cap F_B = \bigcup_{F_{D_i} \in \mathfrak{B}}(F_{D_i} \cap F_B)$. Thus each element of the SSGT μ_{F_B} on F_B is the soft union of members of $\mathfrak{B}_{F_B}$. Hence $\mathfrak{B}_{F_B}$ is a soft basis for the SSGT on F_B.■

Definition3.18. Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then the soft μ-interior of F_B denoted by $(F_B)^o$ is defined as the soft union of all soft μ-open subsets of F_B.

Note that $(F_B)^o$ is the largest soft μ-open set that is contained in F_B.

Theorem 3.19. Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then F_B is a soft μ-open set if and only if F_B= $(F_B)^o$.

Proof. Assume that F_B is a soft μ-open set. Then the largest soft μ-open set contained in F_B is F_B itself. Therefore $(F_B)^o$= F_B.

Conversely, assume that F_B= $(F_B)^o$. Since $(F_B)^o$ is the soft union of all soft μ-open subsets of F_B and μ is closed under arbitrary soft union, $(F_B)^o$ is soft μ-open. If $(F_B)^o$= F_B, then F_B is a soft μ-open set.■

Theorem 3.20. Let (F_{A}, μ) be a SGTS and $F_G, F_H \subseteq F_{\mathrm{A}}$. Then

(1) $((F_G)^o)^o = (F_G)^o$
(2) $F_G \subseteq F_H \Rightarrow (F_G)^o \subseteq (F_H)^o$
(3) $(F_G)^o \cap (F_H)^o \supseteq (F_G \cap F_H)^o$
(4) $(F_G)^o \cup (F_H)^o \subseteq (F_G \cup F_H)^o$
(5) $(F_G)^o \subseteq F_G$

Proof.

(1) Let $(F_G)^o = F_D$, then $F_D \in \mu$ if and only if $F_D = (F_D)^o$. Therefore $((F_G)^o)^o = (F_G)^o$
(2) Let $F_G \subseteq F_H$. Since the soft μ-interior of a soft set is the largest soft μ-open set contained in that soft set. Therefore $(F_G)^o \subseteq F_G$ and $(F_H)^o \subseteq F_H$. The largest soft μ-open set that is contained in F_H is $(F_H)^o$. Hence $F_G \subseteq F_H \Rightarrow (F_G)^o \subseteq (F_H)^o$.
(3) $F_G \cap F_H \subseteq F_G$ and $F_G \cap F_H \subseteq F_H$. Then $(F_G \cap F_H)^o \subseteq (F_G)^o$ and $(F_G \cap F_H)^o \subseteq (F_H)^o$. Therefore $(F_G \cap F_H)^o \subseteq (F_G)^o \cap (F_H)^o$.
(4) $(F_G)^o \subseteq F_G$ and $(F_H)^o \subseteq F_H$. Then $(F_G)^o \cup (F_H)^o \subseteq F_G \cup F_H$. $(F_G \cup F_H)^o$ is the largest soft μ-open set that is contained in $F_G \cup F_H$. Hence $(F_G)^o \cup (F_H)^o \subseteq (F_G \cup F_H)^o$.
(5) Trivial. ■

Definition 3.21. Let (F_{A}, μ) be a SGTS and $F_{\mathrm{B}} \subseteq F_{\mathrm{A}}$. Then F_{B} is said to be a soft μ-closed set if its soft complement $(F_{\mathrm{B}})^c$ is a soft μ-open set.

Theorem 3.22. Let (F_{A}, μ) be a SGTS and $F_{\mathrm{B}} \subseteq F_{\mathrm{A}}$. Then the following conditions hold:

(1) The universal soft set $F_{\tilde{\mathrm{E}}}$ is soft μ–closed.
(2) Arbitrary soft intersections of the soft μ–closed sets are soft μ–closed.

Proof.

(1) $(F_{\tilde{\mathrm{E}}})^c = F_{\emptyset} \in \mu$. That is, the soft complement of the universal soft set $F_{\tilde{\mathrm{E}}}$ is the soft empty set $F_{\emptyset}$ and $F_{\emptyset} \in \mu$. Therefore $F_{\tilde{\mathrm{E}}}$ is soft μ–closed
(2) $\{F_{G_i}\}_{i\in J}$ be a given collection of soft μ–closed sets. To show $\bigcap_{i\in J} F_{G_i}$ is soft μ–closed. Now $(\bigcap_{i\in J} F_{G_i})^c = \bigcup_{i\in J} (F_{G_i})^c \in \mu$, since each F_{G_i} is soft μ-closed, its soft complement $(F_{G_i})^c$ is soft μ-open. Therefore $\bigcap_{i\in J} F_{G_i}$ is a soft μ-closed set. ■

Theorem 3.23. Let (F_{A}, μ) be a SGTS and $(F_{\mathrm{B}}, \mu_{F_B})$ a SGTSS of F_{A}. Then,

(1) F_{G} is soft μ_{F_B}-open if and only if $F_{\mathrm{G}} = F_{\mathrm{H}} \cap F_{\mathrm{B}}$ for some soft μ-open set F_{H}.
(2) F_{G} is soft μ_{F_B}-closed if and only if $F_{\mathrm{G}} = F_{\mathrm{H}} \cap F_{\mathrm{B}}$ for some soft μ-closed set F_{H}.

Proof.

(1) Follows from the definition of a SSGT.

(2) $F_G$is soft μ_{F_B}-closed $\Leftrightarrow F_G = F_B \setminus F_M$ for some $F_M \in \mu_{F_B}$. Now $F_M \in \mu_{F_B} \Rightarrow F_M = F_D \cap F_B$ for some $F_D \in \mu$. Therefore F_G is soft μ_{F_B}-closed $\Leftrightarrow F_G = F_B \setminus (F_D \cap F_B)$for some $F_D \in \mu \Leftrightarrow F_G = F_B \setminus F_D \Leftrightarrow F_G = F_B \cap (F_D)^c \Leftrightarrow F_G = F_B \cap F_H$ where $F_H = (F_D)^c$ is a soft μ-closed set. Hence F_G is soft μ_{F_B}-closed if and only if $F_G = F_H \cap F_B$ for some soft μ-closed set F_H.■

Definition 3.24. Let (F_A, μ)be a SGTS and $F_B \subseteq F_A$. Then the soft μ-closure of F_B, denoted by$c(F_B)$is defined as the soft intersection of all soft μ-closed super sets of F_B.

Note that $c(F_B)$ is the smallest soft μ-closed set that containing F_B.

Theorem 3.25. Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$.$F_B$is a soft μ-closed set if and only if F_B = $c(F_B)$.

Proof. The proof is trivial.

Theorem 3.26. Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then $(F_B)^o \subseteq F_B \subseteq c(F_B)$

Proof. Indeed$(F_B)^o = \cup \{F_{B_i} \,/\, F_{B_i} \in \mu, F_{B_i} \subseteq F_B, i \in J\}$.

Then $f_{B_i}(e) \subseteq f_B(e)and\cup_{i \in J} f_{B_i}(e) \subseteq f_B(e), \forall\, e \in$ E.

So$(F_B)^o \subseteq F_B$.$c(F_B) = \cap \{F_{B_i} \,/\, {F_{B_i}}^c \in \mu, F_B \subseteq F_{B_i}, i \in J\}$. Then $f_B(e) \subseteq f_{B_i}(e)$and

$f_B(e) \subseteq \cap f_{B_i}(e), \forall\, e \in$ E. So $F_B \subseteq c(F_B)$. Hence $(F_B)^o \subseteq F_B \subseteq c(F_B)$.■

Theorem 3.27. Let (F_A, μ) be a SGTS and $F_G, F_H \subseteq F_A$. Then

(1) $c(c(F_G)) = c(F_G)$
(2) $F_G \subseteq F_H \Rightarrow c(F_G) \subseteq c(F_H)$
(3) $c(F_G) \cap c(F_H) \supseteq c(F_G \cap F_H)$
(4) $c(F_G) \cup c(F_H) \subseteq c(F_G \cup F_H)$

Proof.

(1) Let $c(F_G)$= F_D. Then F_D is a soft μ-closed set. Therefore F_D and $c(F_D)$ are soft equal. i.e, F_D= $c(F_D)$. Hence $c(c(F_G)) = c(F_G)$.
(2) Let $F_G \subseteq F_H$. By the definition of soft μ-closure, $F_G \subseteq c(F_G)$ and $F_H \subseteq c(F_H)$ and the smallest soft μ-closed set that containing $F_G$is $c(F_G)$. Hence$c(F_G) \subseteq c(F_H)$.
(3) $c(F_G)$and$c(F_H)$ are soft μ-closed sets. So their soft intersection $c(F_G) \cap c(F_H)$ is a soft μ-closed set. Since $c(F_G \cap F_H)$ is the smallest soft μ-closed set that containing $F_G \cap F_H$ and $F_G \cap F_H \subseteq c(F_G) \cap c(F_H)$, $c(F_G) \cap c(F_H) \supseteq c(F_G \cap F_H)$.
(4) Since $F_G \subseteq F_G \cup F_H$and $F_H \subseteq F_G \cup F_H$, $c(F_G) \subseteq c(F_G \cup F_H)$ and $c(F_H) \subseteq c(F_G \cup F_H)$. Therefore $c(F_G) \cup c(F_H) \subseteq c(F_G \cup F_H)$.■

Theorem 3.28. Let (F_A, μ)be a SGTS and $F_B \subseteq F_A$. Then the following hold:

(1) If α $\in c(F_{\mathrm{B}})$, then every soft μ-open set F_{G} containing α soft intersect F_{B}.
(2) Supposing the SGT on F_{A} is given by a soft basis $\mathfrak{B}$. If α $\in c(F_{\mathrm{B}})$, then every soft basis element F_{H} containing α soft intersect F_{B}.

Proof.

(1) First prove that if α $\in c(F_{\mathrm{B}})$, then every soft μ-open set F_{G} containing α soft intersect F_{B}. It is equivalent to prove that if there exists a soft μ-open set F_{G} containing α that does not soft intersect F_{B}, then α $\notin c(F_{\mathrm{B}})$. Assume that $F_{\mathrm{G}} \cap F_{\mathrm{B}} = F_{\emptyset}$, where $F_{\mathrm{G}} \in \mu$, α $\in F_{\mathrm{G}}$. Then $(F_G)^c$ is a soft μ-closed set and $F_{\mathrm{B}} \subseteq (F_G)^c$. By the definition of soft μ-closure, $c(F_{\mathrm{B}})$ is the smallest soft μ-closed set containing F_{B}. Therefore$c(F_{\mathrm{B}}) \subseteq (F_G)^c$and therefore α cannot be in $c(F_{\mathrm{B}})$.
(2) If α $\in c(F_{\mathrm{B}})$, then every soft μ-open set F_{G} containing α soft intersect F_{B}. i.e, If α $\in c(F_{\mathrm{B}})$, then $F_{\mathrm{G}} \cap F_{\mathrm{B}} \neq F_{\emptyset}$, $\forall\, F_{\mathrm{G}} \in \mu$, and α $\in F_{\mathrm{G}}$. Since F_{G} is soft μ-open, it can be expressed as the soft union of some members of the soft basis. So every soft basis element F_{H} containing α soft intersect F_{B}.■

Remark 3.29. The converse of the above theorem need not be true.

Theorem 3.30. Let (F_{A}, μ) be a SGTS and $F_{\mathrm{B}}, F_{\mathrm{G}} \subseteq F_{\mathrm{A}}$. Then

(1) $c((F_{\mathrm{B}})^c) = ((F_{\mathrm{B}})^o)^c$
(2) $((F_{\mathrm{G}})^c)^o = (c(F_{\mathrm{G}}))^c$
(3) $(F_B)^o = (c((F_{\mathrm{B}})^c))^c$
(4) $c(F_{\mathrm{G}}) = (((F_{\mathrm{G}})^c)^o)^c$
(5) $(F_B \setminus F_{\mathrm{G}})^o \subseteq (F_B)^o \setminus (F_G)^o$

Proof.

(1) $(F_B)^o = \cup\{F_{B_i} \,/\, F_{B_i} \in \mu, F_{B_i} \subseteq F_B, i \in J\}$. $((F_B)^o)^c = \cap\,\{(F_{B_i})^c \,/\, F_{B_i} \in \mu, (F_B)^c \subseteq (F_{B_i})^c, i \in J\}$=$c((F_{\mathrm{B}})^c)$, by thedefinition of soft μ-closure.
(2) Consider the definitions of soft μ-closure and soft μ-interior, $c(F_{\mathrm{G}}) = \cap\,\{F_{G_i} \,/\, (F_{G_i})^c \in \mu, F_G \subseteq F_{G_i}, i \in J\}$. $(c(F_{\mathrm{G}}))^C = [\cap\,\{F_{G_i} \,/\, (F_{G_i})^c \in \mu, F_G \subseteq F_{G_i}, i \in J\}]^C = \cup\,\{(F_{G_i})^c \,/\, (F_{G_i})^c \in \mu, (F_{G_i})^c \subseteq (F_G)^c, i \in J\} = ((F_G)^c)^o$.
(3) Obtained by taking the soft complements of (1)
(4) Obtained by taking the soft complements of (2)
(5) $(F_B \setminus F_{\mathrm{G}})^o = (F_B \cap (F_{\mathrm{G}})^c)^o \subseteq (F_B)^o \cap ((F_{\mathrm{G}})^c)^o = (F_B)^o \cap (c(F_{\mathrm{G}}))^c \subseteq (F_B)^o \cap ((F_G)^o)^c \subseteq (F_B)^o \setminus (F_G)^o$.■

Definition 3.31. Let (F_{A}, μ)be a SGTS and α $\in F_{\mathrm{A}}$. If there is a soft μ-open set F_{B} such that α $\in F_{\mathrm{B}}$, then F_{B} is called a soft μ-open neighborhood or soft μ-nbd of α. The set of all soft μ-nbds of α, denoted by ψ(α), is called the family of soft μ-nbds of α. i.e, ψ(α)= $\{F_{\mathrm{B}} \,/\, F_{\mathrm{B}} \in \mu$, α $\in F_{\mathrm{B}}\}$.

Theorem 3.32. Let (F_{A}, μ) be a SGTS and $F_{\mathrm{G}}, F_{\mathrm{H}} \subseteq F_{\mathrm{A}}$. Then

(1) if $F_G \in \psi(\alpha)$, then $\alpha \in F_G$
(2) if $F_G \in \psi(\alpha)$ and $F_G \subseteq F_H$ where $F_H \in \mu$, then $F_H \in \psi(\alpha)$
(3) F_G is a soft μ-open if and only if F_G contains a soft μ-nbd of each of its points.

Proof.

(1) Since F_G is a soft μ -nbd of α, F_G is a soft μ -open set such that $\alpha \in F_G$.
(2) Assume that $F_G \subseteq F_H$ and $F_H \in \mu$. If F_G is a soft μ-nbd of α, then $\alpha \in F_G \Rightarrow \alpha \in F_H$. Therefore F_H is a soft μ-nbd of α.
(3) Suppose F_G is soft μ-open. Then $\alpha \in F_G \Rightarrow F_G$ is a soft μ-nbd of each $\alpha \in F_G$. Conversely, if each $\alpha \in F_G$ has a soft μ-nbd $F_{H_\alpha} \subseteq F_G$, then $F_G = \{\alpha \,/\, \alpha \in F_G\} \subseteq \bigcup_{\alpha \in F_G} F_{H_\alpha} \subseteq F_G$. Or $F_G = \bigcup_{\alpha \in F_G} F_{H_\alpha}$. This implies that F_G is the soft union of soft μ-open sets. Thus F_G is a soft μ-open set.■

Definition 3.33. Let (F_A, μ) be a SGTS, $F_B \subseteq F_A$ and $\alpha \in F_A$. If every soft μ-nbd of α soft intersect F_B in some point other than α itself, then α is called soft μ-limit point of F_B. The set of all soft μ-limit points of F_B is denoted by $(F_B)'$. In other words, if (F_A, μ) is a SGTS, $F_B, F_G \subseteq F_A$ and $\alpha \in F_A$. Then $\alpha \in (F_B)'$ if and only if $F_G \cap (F_B \setminus \{\alpha\}) \neq F_\emptyset$ for all $F_G \in \psi(\alpha)$.

Remark 3.34. If $\alpha \in c(F_B \setminus \{\alpha\})$, then by theorem 3.28.(1), $F_G \cap (F_B \setminus \{\alpha\}) \neq F_\emptyset$ for every soft μ-open set F_G containing α, which implies $\alpha \in (F_B)'$.

Theorem 3.35. Let (F_A, μ) be a SGTS, $F_B \subseteq F_A$. Then $c(F_B) \subset F_B \cup (F_B)'$

Proof. If $\alpha \in c(F_B)$, then either $\alpha \in F_B$ or $\alpha \notin F_B$. First consider $\alpha \in F_B$. Then $\alpha \in F_B \cup (F_B)'$ and hence $c(F_B) \subset F_B \cup (F_B)'$. Next consider if $\alpha \notin F_B$. Then the soft sets F_B and $(F_B \setminus \{\alpha\})$ are soft equal. So $\alpha \in c(F_B) \Rightarrow F_G \cap F_B \neq F_\emptyset, \forall F_G \in \psi(\alpha) \Rightarrow F_G \cap (F_B \setminus \{\alpha\}) \neq F_\emptyset \Rightarrow \alpha \in (F_B)' \Rightarrow \alpha \in F_B \cup (F_B)'$. So $c(F_B) \subset F_B \cup (F_B)'$. Hence in both case $c(F_B) \subset F_B \cup (F_B)'$.■

Theorem 3.36. Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. If $(F_B)' \subseteq F_B$, then F_B is soft μ-closed.

Proof. Assume that $(F_B)' \subseteq F_B$. Then $F_B = F_B \cup (F_B)'$. But by the above theorem $F_B \cup (F_B)' \supseteq c(F_B)$. Therefore $F_B \supseteq c(F_B) \Rightarrow F_B = c(F_B)$. Hence F_B is soft μ-closed.■

Theorem 3.37. Let (F_A, μ) be a SGTS and $F_G, F_H \subseteq F_A$. Then

(1) $F_G \subseteq F_H \Rightarrow (F_G)' \subseteq (F_H)'$
(2) $(F_G \cap F_H)' \subseteq (F_G)' \cap (F_H)'$
(3) $(F_G \cup F_H)' \supseteq (F_G)' \cup (F_H)'$

Proof.

(1) Since $F_G \subseteq F_H$, $(F_G \setminus \{\alpha\}) \subseteq (F_H \setminus \{\alpha\})$. Suppose $\alpha \in (F_G)' \Rightarrow F_D \cap (F_G \setminus \{\alpha\}) \neq F_\emptyset$, $\forall F_D \in \psi(\alpha) \Rightarrow F_D \cap (F_H \setminus \{\alpha\}) \neq F_\emptyset$, $\forall F_D \in \psi(\alpha) \Rightarrow \alpha \in (F_H)'$. Hence $(F_G)' \subseteq (F_H)'$.
(2) $F_G \cap F_H \subseteq F_G$ and $F_G \cap F_H \subseteq F_H$. Then $(F_G \cap F_H)' \subseteq (F_G)'$ and $(F_G \cap F_H)' \subseteq (F_H)'$. Therefore $(F_G \cap F_H)' \subseteq (F_G)' \cap (F_H)'$.

(3) $c((F_G \cup F_H) \setminus \{\alpha\}) = c((F_G \cup F_H) \cap \{\alpha\}^c) = c((F_G \cap \{\alpha\}^c) \cup (F_H \cap \{\alpha\}^c)) \supseteq c(F_G \cap \{\alpha\}^c) \cup c(F_H \cap \{\alpha\}^c) = c(F_G \setminus \{\alpha\}) \cup c(F_H \setminus \{\alpha\})$. Therefore $\alpha \in [c(F_G \setminus \{\alpha\}) \cup c(F_H \setminus \{\alpha\})] \Rightarrow \alpha \in c((F_G \cup F_H) \setminus \{\alpha\})$. i.e, $\alpha \in c(F_G \setminus \{\alpha\})$ or $\alpha \in c(F_H \setminus \{\alpha\}) \Rightarrow \alpha \in c((F_G \cup F_H) \setminus \{\alpha\})$. i.e, $\alpha \in (F_G)'$ or $\alpha \in (F_H)' \Rightarrow \alpha \in (F_G \cup F_H)' \Rightarrow \alpha \in (F_G)' \cup (F_H)' \Rightarrow \alpha \in (F_G \cup F_H)' \Rightarrow ((F_G)' \cup (F_H)') \subseteq (F_G \cup F_H)'$.■

Definition 3.38. Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then the soft μ-boundary of F_B, denoted by $(F_B)^b$, is defined by $(F_B)^b = c(F_B) \cap c((F_B)^c)$.

Theorem 3.39. Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then

(1) $(F_B)^b \subseteq c(F_B)$
(2) $(F_B)^b = c(F_B) \setminus (F_B)^o$

Proof.

(1) $(F_B)^b = c(F_B) \cap c((F_B)^c) \Rightarrow (F_B)^b \subseteq c(F_B)$.
(2) $c(F_B) \setminus (F_B)^o = c(F_B) \cap ((F_B)^o)^c = c(F_B) \cap (\bigcup_{\substack{F_{B_i} \subseteq F_B \\ F_{B_i} \in \mu, i \in J}} F_{B_i})^c$

$= c(F_B) \cap (\bigcap_{\substack{(F_B)^c \subseteq (F_{B_i})^c \\ F_{B_i} \in \mu,\ i \in J}} (F_{B_i})^c) = c(F_B) \cap c((F_B)^c) = (F_B)^b$.■

Theorem 3.40. Let (F_A, μ) be a SGTS and $F_G, F_H \subseteq F_A$. Then the following hold:

(1) $((F_G)^b)^c = (F_G)^o \cup ((F_G)^c)^o$
(2) $c(F_G) = F_G \cup (F_G)^b$
(3) $(F_G)^o = F_G \setminus (F_G)^b$
(4) $(F_G)^b = c(F_G) \cap c((F_G)^c) = c(F_G) \setminus (F_G)^o$

Proof.

(1) $(F_G)^o \cup ((F_G)^c)^o = (((F_G)^o)^c)^c \cup ((((F_G)^c)^o)^c)^c = [((F_G)^o)^c \cap (((F_G)^c)^o)^c]^c = [c((F_G)^c) \cap c(F_G)]^c = ((F_G)^b)^c$, by theorem 3.30.(1) and (2).■
(2) $F_G \cup (F_G)^b = F_G \cup [c(F_G) \cap c((F_G)^c)] = [F_G \cup c(F_G)] \cap [F_G \cup c((F_G)^c)] = c(F_G) \cap [F_G \cup c((F_G)^c)] = c(F_G) \cap F_{\tilde{E}} = c(F_G)$.
(3) $F_G \setminus (F_G)^b = F_G \cap ((F_G)^b)^c = F_G \cap [(F_G)^o \cup ((F_G)^c)^o]$, by (1) $= [F_G \cap (F_G)^o] \cup [F_G \cap ((F_G)^c)^o = (F_G)^o \cup F_\emptyset = (F_G)^o$.
(4) Follows from definition and theorem 3.30.(1).■

Theorem 3.41. Let (F_A, μ) be a SGTS and $F_G \subseteq F_A$. Then the following hold:

(1) $(F_G)^b \cap (F_G)^o = F_\emptyset$
(2) F_G is soft μ-open iff $F_G \cap (F_G)^b = F_\emptyset$.

Theorem 3.42. Let (F_A, μ)be a SGTS and $F_G \subseteq F_A$. Then $(F_G)^b = F_\emptyset$ if and only if F_G is both soft μ-open and soft μ-closed.

Proof. Assume that $(F_G)^b = F_\emptyset$. $(F_G)^b = F_\emptyset \Rightarrow c(F_G) \cap c((F_G)^c) = F_\emptyset \Rightarrow F_G \cap ((F_G)^o)^c = F_\emptyset \Rightarrow F_G \subseteq (F_G)^o$, by theorem 3.30.(1). This implies that F_G is a soft μ-open set. Again by theorem 3.30.(1), $(F_G)^b = F_\emptyset \Rightarrow c(F_G) \cap c((F_G)^c) = F_\emptyset \Rightarrow c(F_G) \subseteq (c((F_G)^c))^c = (F_G)^o \subseteq F_G \Rightarrow c(F_G) \subseteq F_G$. This implies that F_G is a soft μ-closed set. Conversely assume that F_G is both soft μ-closed and soft μ-open. Then $(F_G)^b = c(F_G) \cap c((F_G)^c) = F_G \cap ((F_G)^o)^c = F_G \cap (F_G)^c = F_\emptyset$, by theorem 3.30.(1).■

Definition 3.43. Let (F_A, μ)be a SGTS and $F_B \subseteq F_A$. Then the soft μ-exterior of F_B is denoted by $(F_B)^e$and is defined as $(F_B)^e = ((F_B)^c)^o$.

Note that the soft μ-exterior of F_B is the largest soft μ-open set contained in $(F_B)^c$.

Theorem 3.44. Let (F_A, μ) be a SGTS and $F_G, F_H \subseteq F_A$. Then,

(1) $(F_B)^e = ((F_B)^c)^o$
(2) $(F_G \cup F_H)^e \subseteq (F_G)^e \cap (F_H)^e$
(3) $(F_G \cap F_H)^e \supseteq (F_G)^e \cup (F_H)^e$

Proof.

(1) Follows from definition
(2) $(F_G \cup F_H)^e = ((F_G \cup F_H)^c)^o = ((F_G)^c \cap (F_H)^c)^o \subseteq ((F_G)^c)^o \cap ((F_H)^c)^o = (F_G)^e \cap (F_H)^e$.
(3) $(F_G)^e \cup (F_H)^e = ((F_G)^c)^o \cup ((F_H)^c)^o \subseteq ((F_G)^c \cup (F_H)^c)^o = ((F_G \cap F_H)^c)^o = (F_G \cap F_H)^e$.■

Theorem 3.45. Let (F_A, μ) be a SGTS and $F_G, F_H \subseteq F_A$. Then

(1) $((F_G)^b)^c = (F_G)^o \cup (F_G)^e$.
(2) $(F_G)^o \cup (F_G)^e \cup (F_G)^b = F_{\tilde{E}}$

Proof.

(1) By theorem 3.40.(1), $((F_G)^b)^c = (F_G)^o \cup ((F_G)^c)^o$. Also $(F_G)^o \cup ((F_G)^c)^o = (F_G)^o \cup (F_G)^e$.
(2) By theorem 3.45(1), $((F_G)^b)^c = (F_G)^o \cup (F_G)^e$. Therefore $(F_G)^o \cup (F_G)^e \cup (F_G)^b = [(F_G)^o \cup (F_G)^e] \cup (F_G)^b = ((F_G)^b)^c \cup (F_G)^b = F_{\tilde{E}}$.■

Theorem 3.46. Let (F_A, μ)be a SGTS and $F_G \subseteq F_A$. Then,$(F_G)^b \cap (F_G)^e = F_\emptyset$.

Theorem 3.47. Let (F_A, μ)be a SGTS. Then the collection $\mu_e = \{f_B(e)$ / there exists $F_B \in \mu$ such that $(e, f_B(e)) \in F_B\}$ for each $e \in$ E, is a generalized topology on U.

Proof. Let $\mu_e = \{f_B(e)$ / there exists $F_B \in \mu$ such that $(e, f_B(e)) \in F_B\}$ for each $e \in$ E. Clearly $\emptyset \in \mu_e$, since $F_\emptyset \in \mu$. Now let $\{f_{B_i}(e)\}_{i \in J}$ be a collection of sets in μ_e. Then there exists soft sets $F_{B_i} \in \mu$, i ∈ J such that $(e, f_{B_i}(e)) \in F_{B_i}$. Since μ is a SGT, $\{F_{B_i}\}_{i \in J} \in \mu \Rightarrow \bigcup_{i \in J} F_{B_i} \in \mu$. i.e, $(e, \bigcup_{i \in J} f_{B_i}(e)) \in \bigcup_{i \in J} F_{B_i} \Rightarrow \bigcup_{i \in J} f_{B_i}(e) \in \mu_e$. Hence $\mu_e$is a GT on U.■

The above theorem shows that corresponding to each parameter $e \in$E, we have a GT$\mu_e$on U. Thus a SGT on F_A gives a parameterized family of GT's on U. The converse of the above theorem does not hold.

Example 3.48. Let U = {h_1, h_2, h_3, h_4}, E = {e_1, e_2, e_3}, A = {e_1, e_2} ⊆ E and F_A = {(e_1, {h_1, h_2, h_3, h_4}), (e_2, {h_2, h_3, h_4})}. Let $\mu = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_3}\}$, where F_{A_1}= {(e_1, {h_3}), (e_2, {h_2})}, F_{A_2}= {(e_1, {h_2, h_4}), (e_2, {h_2, h_4})}, F_{A_3}= {(e_1, {h_2, h_3, h_4})}. Then μ is not a SGT on F_A, because $F_{A_1} \cup F_{A_2}$= {(e_1, {h_2, h_3, h_4}), (e_2, {h_2, h_4})} $\notin \mu$. Also μ_{e_1} = {∅, {h_3}, {h_2, h_4}, {h_2, h_3, h_4}} and μ_{e_2}= {∅, {h_2},{h_2, h_4}} are GT's on U. This example shows that any collection of soft sets need not to be a SGT on F_A, even if the collection corresponding to each parameter defines a GT on U.

Theorem 3.49. Let (F_A, μ)be a SGTS and $F_B \subseteq F_A$. Then $(\mu_{F_B})_e$ is a subspace of the GT μ_e for each $e \in$ E.

Proof. If (F_A, μ) is a SGTS, then $\mu_e = \{f_D(e)$ / there exists $F_D \in \mu$ such that $(e, f_D(e)) \in F_D\}$ is a GT on U. Now for any $e \in$ E, $(\mu_{F_B})_e = \{f_G(e)$ / there exists $F_G \in \mu_{F_B}$such that $(e, f_G(e)) \in F_G\}$ $=\{f_G(e)$ / there exists $F_H \in \mu$ such that $F_G = F_H \cap F_B, (e, f_{H \cap B}(e)) \in F_H \cap F_B\} = \{f_H(e) \cap f_B(e) \,/\, f_H(e) \in \mu_e$ such that $(e, f_H(e) \cap f_B(e)) \in F_H \cap F_B\}$. i.e, every element of $(\mu_{F_B})_e$ is the intersection of an element $f_H(e)$ in μ_e with $f_B(e)$. Thus $(\mu_{F_B})_e$ is a subspace of the GTS μ_e.■

4. Soft continuous functions in SGTS

Definition 4.1. Let (F_A, μ) and (F_B, η) be two SGTS's. A soft function $\varphi_\chi : (F_A, \mu) \rightarrow (F_B, \eta)$ is said to be soft (μ, η)-continuous (briefly, soft continuous), if for each soft η-open subset $F_G$of F_B, the inverse image ${\varphi_\chi}^{-1}(F_G)$ is a soft μ-open subset of F_A.

Theorem 4.2. Every soft function from a discrete SGTS into any SGTS is soft continuous.

Proof. Let (F_A, μ) and (F_B, η) be two SGTS's. Suppose $\mu$is a discrete SGT. Let $\varphi_\chi : (F_A, \mu) \rightarrow (F_B, \eta)$be a soft function. Then for every soft η-open set$F_G$of F_B, the inverse image ${\varphi_\chi}^{-1}(F_G)$ is soft μ-open with respect to the discrete SGT μon F_A. Thus φ_χ is soft continuous.■

Theorem 4.3. Let (F_A, μ) and (F_B, η) be two SGTS's and $\varphi_\chi : (F_A, \mu) \rightarrow (F_B, \eta)$ be a soft function. Suppose the SGT η on F_B is given by a soft basis $\mathfrak{B}$. Then φ_χ is soft continuous if the inverse image of every soft basis element is soft μ-open.

Proof. Suppose that the inverse image of every soft basis element is soft μ-open. Let F_{G} be an arbitrary η-open subset of F_{B}. Then by the definition of soft basis, F_{G}can be written as the soft union of members of the soft basis $\mathfrak{B}$ of η. i.e, $F_{\mathrm{G}} = \bigcup_{F_{\mathrm{D}} \in \mathfrak{B}} F_{\mathrm{D}}$. Then by theorem 2.14.(8), ${\varphi_\chi}^{-1}(F_{\mathrm{G}}) = {\varphi_\chi}^{-1}(\bigcup_{F_{\mathrm{D}} \in \mathfrak{B}} F_{\mathrm{D}}) = \bigcup_{F_{\mathrm{D}} \in \mathfrak{B}} {\varphi_\chi}^{-1}(F_{\mathrm{D}}) \in \mu$. Thusφ_χ is soft continuous.■

Theorem 4.4. Let (F_{A}, μ) and (F_{B}, η) be two SGTS's and $\varphi_\chi : (F_{\mathrm{A}}, \mu) \to (F_{\mathrm{B}}, \eta)$be a soft function. Then φ_χ is soft continuous if and only if for every soft η-closed subset $F_{\mathrm{H}}$of F_{B}, the soft set ${\varphi_\chi}^{-1}(F_{\mathrm{H}})$ is soft μ-closed in F_{A}.

Proof. Assume that φ_χ is soft continuous. Let F_{H} be a soft η-closed set of F_{B}. Then $(F_{\mathrm{H}})^c \in \eta$. By hypothesis and theorem 2.14.(7), ${\varphi_\chi}^{-1}((F_{\mathrm{H}})^c) \in \mu$. i.e, $[{\varphi_\chi}^{-1}(F_{\mathrm{H}})]^c \in \mu$. Thus ${\varphi_\chi}^{-1}(F_{\mathrm{H}})$ is a soft μ-closed set of F_{A}.

Conversely, assume that for every soft η-closed subset $F_{\mathrm{H}}$of F_{B}, the soft set ${\varphi_\chi}^{-1}(F_{\mathrm{H}})$ is soft μ-closed in F_{A}. Let F_{G} be a soft η-open subset of F_{B}. Then $(F_{\mathrm{G}})^c$ is soft η-closed subset of F_{B}. Therefore by hypothesis, ${\varphi_\chi}^{-1}((F_{\mathrm{G}})^c)$ is a soft μ-closed set of F_{A}. i.e,by theorem 2.14.(7), $[{\varphi_\chi}^{-1}(F_{\mathrm{G}})]^c$ is a soft μ-closed set of F_{A}. i.e, ${\varphi_\chi}^{-1}(F_{\mathrm{G}})$ is a soft μ-open set of F_{A}. Thus φ_χ is soft continuous.■

Theorem 4.5. Let (F_{A}, μ) and (F_{B}, η) be two SGTS's and $\varphi_\chi : (F_{\mathrm{A}}, \mu) \to (F_{\mathrm{B}}, \eta)$be a soft function. Then φ_χ is soft continuous if and only if for every soft subset $F_{\mathrm{G}}$of F_{A}, $\varphi_\chi(c(F_{\mathrm{G}})) \subset c(\varphi_\chi(F_{\mathrm{G}}))$

Proof. Assume that φ_χ is soft continuous. Since $c(\varphi_\chi(F_{\mathrm{G}}))$ is a soft η-closed set in F_{B}, ${\varphi_\chi}^{-1}(c(\varphi_\chi(F_{\mathrm{G}})))$ is a soft μ-closed set in F_{A} containing F_{G}. Also $c(F_{\mathrm{G}})$ is the smallest soft μ-closed set in F_{A}containing F_{G}. Hence $c(F_{\mathrm{G}}) \subset {\varphi_\chi}^{-1}(c(\varphi_\chi(F_{\mathrm{G}})))$. Therefore by theorem 2.14.(5), $\varphi_\chi(c(F_{\mathrm{G}})) \subset c(\varphi_\chi(F_{\mathrm{G}}))$.

Conversely, assume that (F_{A}, μ) and $(F_{\mathrm{B}}, \eta)$are two SGTS's and $\varphi_\chi : (F_{\mathrm{A}}, \mu) \to (F_{\mathrm{B}}, \eta)$ be a soft function. Suppose for every soft subset $F_{\mathrm{G}}$of F_{A}, $\varphi_\chi(c(F_{\mathrm{G}})) \subset c(\varphi_\chi(F_{\mathrm{G}}))$. Assume$F_{\mathrm{H}}$ is a soft η-closed subset of F_{B}. To show that ${\varphi_\chi}^{-1}(F_{\mathrm{H}})$ is soft μ-closed in F_{A}, it suffices to show that the soft μ-closure of ${\varphi_\chi}^{-1}(F_{\mathrm{H}})$ is contained in ${\varphi_\chi}^{-1}(F_{\mathrm{H}})$. If $\alpha \in c({\varphi_\chi}^{-1}(F_{\mathrm{H}}))$,then by hypothesis and by theorem 2.14.(5), $\varphi_\chi(\alpha) \in \varphi_\chi(c({\varphi_\chi}^{-1}(F_{\mathrm{H}}))) \subset c[\varphi_\chi({\varphi_\chi}^{-1}(F_{\mathrm{H}}))] \subset c(F_{\mathrm{H}}) = F_{\mathrm{H}}$ so that $\alpha \in {\varphi_\chi}^{-1}(F_{\mathrm{H}})$. Thus $c({\varphi_\chi}^{-1}(F_{\mathrm{H}})) \subset {\varphi_\chi}^{-1}(F_{\mathrm{H}})$ as desired. By theorem4.4.,φ_χ is soft continuous.■

Theorem 4.6. Let $(F_{\mathrm{A}}, \mu), (F_{\mathrm{B}}, \eta)$ and $(F_{\mathrm{C}}, \lambda)$ be SGTS's. Then the following hold:

(1) If F_{G} is a soft subspace of F_{A}, then the soft function $\varphi_\chi : F_{\mathrm{G}} \to F_{\mathrm{A}}$defined by $\varphi_\chi(\alpha) = \alpha$ is soft continuous.

(2) If the soft functions $\varphi_\chi : F_{\mathrm{A}} \to F_{\mathrm{B}}$ and $\tau_\sigma: F_{\mathrm{B}} \to F_{\mathrm{C}}$ are soft continuous, then the soft composite function $\tau_\sigma\ o\ \varphi_\chi$ is also soft continuous.

Proof.

(1) Suppose $F_{\mathrm{H}}$is a soft μ-open subset of F_{A}, then ${\varphi_\chi}^{-1}(F_{\mathrm{H}}) = F_{\mathrm{H}} \cap F_{\mathrm{G}}$ which is soft μ_{F_G}-open in F_{G}, by definition of the SSGT. Hence φ_χ is soft continuous.

(2) If F_{H} is a soft λ-open subset of F_{C}, then since τ_σ is soft continuous, ${\tau_\sigma}^{-1}(F_{\mathrm{H}})$ is a soft η-open subset of F_{B}. Again as φ_χ is soft continuous,${\varphi_\chi}^{-1}({\tau_\sigma}^{-1}(F_{\mathrm{H}}))$ is a soft μ-open subset of F_{A}. But we have $(\tau_\sigma\ o\ \varphi_\chi)^{-1} = {\varphi_\chi}^{-1} o\ {\tau_\sigma}^{-1}$, by theorem2.14.(10). So $(\tau_\sigma\ o\ \varphi_\chi)^{-1}(F_{\mathrm{H}})$is soft μ-open subset of F_{A} whenever F_{H} is soft λ-open subset of F_{C}. Hence $\tau_\sigma\ o\ \varphi_\chi$ is soft continuous.■

5. Conclusion

In the present work, we introduced the concept of SGTS which is defined on an initial soft set and gave basic definitions and theorems of this concept. We proved that SGT gives a parameterized family of generalized topologies on the initial universe. We hope that the findings in this paper will help researcher enhance and promote the further study on SGT.

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