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On New Decompositions Via Soft Sets

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Abstract - In present paper we define some soft operations such as B - soft set, D_c^α - soft set, C - soft set. We investigate some properties of defined soft operations and show implications of each other. Also we introduce (τ_1, τ_2) - C - soft cts, (τ_1, τ_2) - B - soft cts and (τ_1, τ_2) - D_c^α - soft cts. We obtain some decompositions of soft continuity via these soft continuities. As a consequence the relations of some soft continuities are shown with the diagram.

Keywords - *B - soft set, D_c^α - soft set, C - soft set, (τ_1, τ_2) - C - soft cts, (τ_1, τ_2) - B - soft cts, (τ_1, τ_2) - D_c^α - soft cts.*

1 Introduction

Some set theories such as theory of fuzzy sets [12], rough sets [10], intuitionistic fuzzy sets [2], vague sets [3] etc. can be cope with vague notions. But, these theories are not efficient to solve encountered some difficulties. There are some unclear problems in economics, medical science, social science, finance etc. Then, what is the reason of encountered problems? It is possible the insufficiency of the parametrization tool of the theories. In 1999, Molodtsov [9] defined the idea of soft set theory as a general mathematical tool for coping with these difficulties. In 2001, Maji, Biswas and Roy [7] introduced the concept of fuzzy soft set and [8] intuitionistic fuzzy soft set. In 2003, Maji et al. [6] defined the theoretical concepts of the soft set theory and investigated some properties of these concepts. In 2009, Ali et al. [1] investigated several operations on soft sets and defined some new notions such as the restricted intersection etc. In 2011, Naz et al. [11] defined some notions such as soft topological space, soft interior, soft closure etc. Furthermore in 2011 Hussain et al. [4] studied some properties of soft topological spaces. In 2012, Zorlutuna et al. [13] introduced the image (inverse image) of soft set under a function and soft continuity. Also in 2014, Kandil et al. [5] defined some soft operations such as semi open soft, pre open soft, α - open soft, β - open soft and investigated some properties of these soft operations.

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In this paper we define some soft operations such as B - soft set, D_c^α - soft set, C - soft set. We investigate some properties of defined soft operations and show implications of each other. Also we introduce (τ_1, τ_2) - C - soft cts, (τ_1, τ_2) - B - soft cts and (τ_1, τ_2) - D_c^α - soft cts. We obtain some decompositions of soft continuity via these soft continuities. we prove that a function is a soft continuous if and only if it is both α - soft continuous and C - soft continuous. Similarly, we prove that we prove that a function is a soft continuous if and only if it is both pre - soft continuous (resp. α - soft continuous) and B - soft continuous (resp. D_c^α - soft continuous). As a consequence the relations of some soft continuities are shown with the diagram.

2 Preliminary

In this section we recall some known definitions and theorems.

Definition 2.1. [9] *A pair (F, A), where F is mapping from A to $P(X)$, is called a soft set over X. The family of all soft sets on X denoted by $SS(X)_E$.*

Definition 2.2. [6] *Let (F, A) and (G, B) be two soft sets over a common universe X. Then (F, A) is said to be a soft subset of (G, B) if $A \subseteq B$ and $F(e) \subseteq G(e)$, for all $e \in A$. This relation is denoted by $(F, A)\widetilde{\subseteq}(G, B)$.*
(F, A) is said to be soft equal to (G, B) if $(F, A)\widetilde{\subseteq}(G, B)$ and $(G, B)\widetilde{\subseteq}(F, A)$. This relation is denoted by $(F, A) = (G, B)$.

Definition 2.3. [6] *Let (F, A) be a soft set over X. Then (F, A) is said to be a null soft set if $F(e) = \emptyset$, for all $e \in A$. This denoted by $\widetilde{\emptyset}$.*

Definition 2.4. [6] *Let (F, A) be a soft set over X. Then (F, A) is said to be an absolute soft set if $F(e) = X$, for all $e \in A$. This denoted by $\widetilde{X}$.*

Definition 2.5. [5] *Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then (F, E) is said to be*

1. *pre - open soft set if $(F, E)\widetilde{\subseteq}int(cl(F, E))$;*
2. *semi - open soft set if $(F, E)\widetilde{\subseteq}cl(int(F, E))$;*
3. *α - open soft set if $(F, E)\widetilde{\subseteq}int(cl(int(F, E)))$;*
4. *β - open soft set if $(F, E)\widetilde{\subseteq}cl(int(cl(F, E)))$.*

The set of all pre - open soft sets (resp. semi - open soft sets, α - open soft sets, β - open soft sets)denoted by $POS(X)$ (resp. $SOS(X)$, $\alpha OS(X)$, $\beta OS(X)$).

Theorem 2.6. [5] *Let X be an initial universe and E be a set of parameters. Let τ be a soft topology on X. If either (F, E) is a semi - open soft or (G, E) is a semi - open soft set, $int(cl((F, E)\widetilde{\cap}(G, E))) = int(cl(F, E))\widetilde{\cap}int(cl(G, E))$.*

Definition 2.7. [11] *Let τ be the collection of soft sets over X. Then τ is said to be a soft topology on X if*

1. *$\widetilde{\emptyset}$, $\widetilde{X} \in \tau$;*

2. *the intersection of any two soft sets in* τ *belongs to* τ*;*

3. *the union of any number of soft sets in* τ *belongs to* τ*.*

The triple (X, τ, E) *is called a soft topological space over X. The members of* τ *are said to be* $\tau-$ *soft open sets or soft open sets in X. A soft set over X is said to be soft closed in X if its complement belongs to* τ*. The set of all soft open sets over* X *denoted by* $OS(X, \tau, E)$ *or* $OS(X)$ *and the set of all soft closed sets denoted by* $CS(X, \tau, E)$ *or* $CS(X)$*.*

Definition 2.8. *Let* (X, τ, E) *be a soft topological space and* (F, E) *be a soft set over* X*. Then*

1. [13] *the soft interior of* (F, E) *is the soft set* $int(F, E) = \widetilde{\cup}\{(G, E) : (G, E)$ *is soft open and* $(G, E)\widetilde{\subseteq}(F, E)\}$*;*

2. [11] *the soft closure of* (F, E) *is the soft set* $cl(F, E) = \widetilde{\cap}\{(H, E) : (H, E)$ *is soft closed and* $(F, E)\widetilde{\subseteq}(H, E)\}$*.*

Theorem 2.9. [4] *Let* (X, τ, E) *be a soft topological space over* X *and* (F, E) *and* (G, E) *are soft sets over* X*. Then*

1. $int\widetilde{\emptyset} = \widetilde{\emptyset}$ *and* $int\widetilde{X} = \widetilde{X}$*.*

2. $int(F, E)\widetilde{\subseteq}(F, E)$*.*

3. $int(int(F, E)) = int(F, E)$*.*

4. (F, E) *is a soft open set if and only if* $int(F, E) = (F, E)$*.*

5. $(F, E)\widetilde{\subseteq}(G, E)$ *implies* $int(F, E)\widetilde{\subseteq}int(G, E)$*.*

6. $int(F, E)\widetilde{\cap}int(G, E) = int((F, E)\widetilde{\cap}(G, E))$*.*

7. $int(F, E)\widetilde{\cup}int(G, E)\widetilde{\subseteq}int((F, E)\widetilde{\cup}(G, E))$*.*

Theorem 2.10. [4] *Let* (X, τ, E) *be a soft topological space over* X*,* (F, E) *and* (G, E) *are soft sets over* X*. Then*

1. $cl(\widetilde{\emptyset}) = \widetilde{\emptyset}$ *and* $cl(\widetilde{X}) = \widetilde{X}$*.*

2. $(F, E)\widetilde{\subseteq}cl(F, E)$*.*

3. (F, E) *is a closed set if and only if* $(F, E) = cl(F, E)$*.*

4. $cl(cl(F, E)) = cl(F, E)$*.*

5. $(F, E)\widetilde{\subseteq}(G, E)$ *implies* $cl(F, E)\widetilde{\subseteq}cl(G, E)$*.*

6. $cl((F, E)\widetilde{\cup}(G, E)) = cl(F, E)\widetilde{\cup}cl(G, E)$*.*

7. $cl((F, E)\widetilde{\cap}(G, E))\widetilde{\subseteq}cl(F, E)\widetilde{\cap}cl(G, E)$*.*

Definition 2.11. [13] *Let* $SS(X)_A$ *and* $SS(Y)_B$ *be families of soft sets,* $u : X \to Y$ *and* $p : A \to B$ *be mappings. Then the mapping* $f_{pu} : SS(X)_A \to SS(Y)_B$ *is defined as:*

1. *Let $(F, A) \in SS(X)_A$. The image of (F, A) under f_{pu}, written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that $f_{pu}(F)(y) = \bigcup_{x \in p^{-1}(y) \cap A} u(F(x))$ if $p^{-1}(y) \cap A \neq \emptyset$ and $f_{pu}(F)(y) = \emptyset$ if $p^{-1}(y) \cap A = \emptyset$ for all $y \in B$.*

2. *Let $(G, B) \in SS(Y)_B$. The inverse image of (G, B) under f_{pu}, written as $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that $f_{pu}^{-1}(G)(x) = u^{-1}(G(p(x)))$ if $p(x) \in B$ and $f_{pu}^{-1}(G)(x) = \emptyset$ if $p(x) \notin B$ for all $x \in A$.*

Definition 2.12. [13] *Let (X, τ, A) and $(Y, \tau*, B)$ be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then The function f_{pu} is called soft continuous (soft - cts or τ - soft continuous) if $f_{pu}^{-1}(G, B) \in \tau$ for all $(G, B) \in \tau*$.*

Definition 2.13. [5] *Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then*

1. *The function f_{pu} is called a pre - soft continuous function (Pre - cts soft) if $f_{pu}^{-1}(G, B) \in POS(X)$ for all $(G, B) \in OS(Y)$.*

2. *The function f_{pu} is called a α - soft continuous function (α - cts soft) if $f_{pu}^{-1}(G, B) \in \alpha OS(X)$ for all $(G, B) \in OS(Y)$.*

3. *The function f_{pu} is called a semi - soft continuous function (semi - cts soft) if $f_{pu}^{-1}(G, B) \in SOS(X)$ for all $(G, B) \in OS(Y)$.*

4. *The function f_{pu} is called a β - soft continuous function (β - cts soft) if $f_{pu}^{-1}(G, B) \in \beta OS(X)$ for all $(G, B) \in OS(Y)$.*

3 B-Soft Sets, D_c^α-Soft Sets and C-Soft Sets

In this section we define semi - preclosed soft set, t - soft set, $\alpha*$- soft set, regular open soft set, B - soft set, D_c^α - soft set and C - soft set. Also we investigate some properties of these soft sets.

Definition 3.1. *Let X be an initial universe and E be a set of parameters. Let τ be a soft topology on X. Then $(F, E) \in SS(X)_E$ is said to be*

1. *semi - preclosed soft set if $int(cl(int(F, E))) \widetilde{\subseteq} (F, E)$;*

2. *t - soft set if $int(F, E) = int(cl(F, E))$;*

3. *$\alpha*$ - soft set if $int(cl(int(F, E))) = int(F, E)$.*

4. *regular open soft set if $(F, E) = int(cl(F, E))$.*

Definition 3.2. *Let X be an initial universe and E be a set of parameters. Let τ be a soft topology on X. Then $(F, E) \in SS(X)_E$ is said to be*

1. *B - soft set if there exist $(G, E) \in \tau$ and (H, E) is a t - soft set such that $(F, E) = (G, E) \widetilde{\cap} (H, E)$;*

2. D_c^α - *soft set if* $(F,E) \in D_c^\alpha = \{(F,E) \in SS(X)_E : int(F,E) = (F,E)\widetilde{\cap} int(cl(int(F,E)))\}$;

3. C - *soft set if there exist* $(G,E) \in \tau$ *and* (H,E) *is a* $\alpha*$ - *soft set such that* $(F,E) = (G,E)\widetilde{\cap}(H,E)$.

Proposition 3.3. *Let* X *be an initial universe and* E *be a set of parameters. Let* τ *be a soft topology on* X. *Then the following statements equivalent:*

1. (F,E) *is an* $\alpha*$ - *soft set.*

2. (F,E) *is a semi - preclosed soft set.*

3. (F,E) *is a regular open soft set.*

Proof. The proof is obvious from Definition 3.1.. □

Proposition 3.4. *Let* X *be an initial universe and* E *be a set of parameters. Let* τ *be a soft topology on* X. *Then we have the following results:*

1. *If* (F,E) *is a* t - *soft set, then* (F,E) *is an* $\alpha*$ - *soft.*

2. *semi open soft set* (F,E) *is a* t - *soft set if and only if* (F,E) *is an* $\alpha*$ - *soft set.*

3. (F,E) *is an* α - *open soft set and* (F,E) *is an* $\alpha*$ - *soft set if and only if* (F,E) *is a regular open soft.*

Proof. 1. Let (F,E) is a t - soft set, then $int(F,E) = int(cl(F,E))$. We have $int(cl(int(F,E))) = int(cl(F,E)) = int(F,E)$. Hence (F,E) is an $\alpha*$ - soft.

2. *Necessity*. Let (F,E) be semi open soft and t - soft set. Since (F,E) is a semi open soft, $cl(int(F,E)) = cl(F,E)$. Then,
$int(F,E) = int(cl(F,E)) = int(cl(int(F,E)))$. Hence (F,E) is an $\alpha*$ - soft.
Sufficiency. Let (F,E) be semi open soft and $\alpha*$ - soft set. Since (F,E) is a semi open soft, $cl(int(F,E)) = cl(F,E)$. Then,
$int(cl(F,E)) = int(cl(int(F,E))) = int(F,E)$. Hence (F,E) is a t - soft set.

3. *Necessity*. Let (F,E) be an α - open soft and $\alpha*$ - soft set. By Proposition 3.3., (F,E) is a - semi - preclosed soft. Since (F,E) is an α - open soft, we have $int(cl(int(F,E))) = (F,E)$ and so $int(cl(F,E)) = int(cl(int(F,E))) = (F,E)$.
Sufficiency. It is obvious.

□

Remark 3.5. *The following example show that*

1. *The converse of Proposition* 3.4.(1) *is not always true.*

2. *A soft open set need not be an* $\alpha*$ - *soft set.*

3. *The concept of* α - *open soft sets is different from* $\alpha*$ - *soft sets.*

Example 3.6. *Let* $X = \{a, b, c, d\}$, $E = \{e\}$, $\tau = \{\widetilde{\emptyset}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, *where* $(F_1, E), (F_2, E), (F_3, E)$ *are soft sets over* X *defined as follows:*
$(F_1, E) = \{(e, \{a\})\}$;
$(F_2, E) = \{(e, \{b, c\})\}$;
$(F_3, E) = \{(e, \{a, b, c\})\}$.
Then the soft set $(G, E) = \{(e, \{b, d\})\}$ *is an* $\alpha*$ *- soft set, but it is neither* t *- soft set nor* α *- open soft set.*
Also the soft set $(H, E) = \{(e, \{a, b, c\})\}$ *is a soft open set, but it is not* $\alpha*$ *- open soft. Since every soft open set is an* α *- open soft,* $\alpha*$ *- soft sets and* α *- open soft sets are independent of each other.*

Proposition 3.7. *Let* X *be an initial universe and* E *be a set of parameters. Let* τ *be a soft topology on* X*. Then we have the following results:*

1. *Every* $\alpha*$ *- soft set is* C *- soft.*
2. *Every soft open set is* C *- soft.*

Proof. It is obvious since $\widetilde{X}$ is both soft open and $\alpha*$ - soft set. □

Remark 3.8. *The converse of Proposition* 3.7. *is not always true as shown in the next example.*

Example 3.9. *Consider Example* 3.6. *Then the soft set* $(G, E) = \{(e, \{a, b, c\})\}$ *is a* C *- soft set, but it is not* $\alpha*$ *- soft set. Also the soft set* $(H, E) = \{(e, \{b, d\})\}$ *is a* C *- soft set, but it is not soft open.*

Proposition 3.10. *Let* X *be an initial universe and* E *be a set of parameters. Let* τ *be a soft topology on* X*. Then we have the following relations:*

$$B \text{ - soft set} \Rightarrow C \text{ - soft set} \Rightarrow D_c^\alpha \text{ - soft set}$$

Proof. Since every t - soft set is an $\alpha*$ - soft set, every B - soft set is a C - soft set. Let (F, E) be C - soft set. Then there exist $(G, E) \in \tau$ and (H, E) which is $\alpha*$ - soft set such that $(F, E) = (G, E)\widetilde{\cap}(H, E)$. Since $int(F, E)$ is a semi open soft set, we obtain
$int(cl(int(F, E))) = int(cl(int((G, E)\widetilde{\cap}(H, E)))) = int(cl(G, E)) \widetilde{\cap}$
$int(cl(int(H, E))) = int(cl(G, E)) \widetilde{\cap}\, int(H, E)$.
Hence we obtain
$(F, E)\widetilde{\cap} int(cl(int(F, E))) = ((G, E)\widetilde{\cap}(H, E)) \widetilde{\cap}\, int(cl(G, E)) \widetilde{\cap}\, int(H, E) = (G, E)\widetilde{\cap}$
$int(H, E) = int(H, E)$.
Therefore (F, E) is a D_c^α - soft set. □

Remark 3.11. *The converse of Proposition* 3.10. *is not always true as shown in the next examples.*

Example 3.12. *Consider Example* 3.6. *Then the soft set* $(G, E) = \{(e, \{b, d\})\}$ *is a* C *- soft set, but it is not* B *- soft.*

Example 3.13. *Let* $X = \{a, b, c, d\}$, $E = \{e\}$, $\tau = \{\widetilde{\emptyset}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)\}$, *where* $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)$ *are soft sets over* X *defined as follows:*
$(F_1, E) = \{(e, \{a\})\}$;
$(F_2, E) = \{(e, \{b\})\}$;
$(F_3, E) = \{(e, \{a, b\})\}$;
$(F_4, E) = \{(e, \{a, c\})\}$;
$(F_5, E) = \{(e, \{a, c, b\})\}$.
Then the soft set $(H, E) = \{(e, \{a, d\})\}$ *is a* D_c^{α} *- soft set, but it is not* C *- soft set.*

Theorem 3.14. *Let* X *be an initial universe and* E *be a set of parameters. Let* τ *be a soft topology on* X. *Then* (F, E) *is a soft open set if and only if it is both* α *- open soft and* C *- open soft.*

Proof. Necessity. It is obvious.
Sufficiency. Let (F, E) be both α - open soft and C - soft set. Since (F, E) is a C - soft, there exist $(G, E) \in \tau$ and (H, E) which is $\alpha*$ - soft open such that $(F, E) = (G, E)\widetilde{\cap}(H, E)$. Since (F, E) is a α - open soft, we obtain
$(F, E)\widetilde{\subseteq} int(cl(int(F, E))) = int(cl(int((G, E)\widetilde{\cap}(H, E)))) = int(cl(G, E)) \widetilde{\cap} int(cl(int(H, E))) = int(cl(G, E)) \widetilde{\subseteq} int(H, E)$. Therefore $(F, E) = (G, E)\widetilde{\cap}(H, E) \widetilde{\subseteq} (G, E)\widetilde{\cap} [int(cl(G, E)) \widetilde{\cap} int(H, E)] = (G, E)\widetilde{\cap} int(H, E) \widetilde{\subseteq} (F, E)$. As a consequence, $(F, E) = (G, E)\widetilde{\cap} int(H, E)$ and $(F, E) \in \tau$. □

Remark 3.15. α *- open soft sets and* C *- soft sets are independent of each other.*

Example 3.16. *Consider Example* 3.6. *Then the soft set* $(G, E) = \{(e, \{b, d\})\}$ *is a* C *- soft set, but it is not* α *- open soft.*

Example 3.17. *Let* $X = \{a, b, c\}$, $E = \{e\}$, $\tau = \{\widetilde{\emptyset}, \widetilde{X}, (F, E)\}$, *where* (F, E) *is a soft set as follow:*
$(F, E) = \{(e, \{b\})\}$.
Then the soft set $(G, E) = \{(e, \{b, c\})\}$ *is an* α *- open soft set, but it is not* C *- soft.*

Theorem 3.18. *Let* X *be an initial universe and* E *be a set of parameters. Let* τ *be a soft topology on* X. *Then* (F, E) *is a soft open set if and only if it is both pre open soft and* B *- open soft.*

Proof. Necessity. It is obvious.
Sufficiency. Let (F, E) be both pre open soft and B - soft set. Since (F, E) is a B - soft, there exist $(G, E) \in \tau$ and (H, E) which is t - soft open such that $(F, E) = (G, E)\widetilde{\cap}(H, E)$. Since (F, E) is a pre open soft, we obtain
$(F, E)\widetilde{\subseteq} int(cl(F, E)) = int(cl((G, E)\widetilde{\cap}(H, E))) = int(cl(G, E)) \widetilde{\cap} int(cl(H, E)) = int(cl(G, E)) \widetilde{\subseteq} int(H, E)$.
Therefore $(F, E) = (G, E)\widetilde{\cap}(H, E) \widetilde{\subseteq} (G, E)\widetilde{\cap} [int(cl(G, E)) \widetilde{\cap} int(H, E)] = (G, E)\widetilde{\cap} int(H, E) \widetilde{\subseteq} (F, E)$. As a consequence, $(F, E) \in \tau$. □

Remark 3.19. *Pre open soft sets and* B *- soft sets are independent of each other.*

Example 3.20. *Let* $X = \{a, b, c, d\}$, $E = \{e\}$, $\tau = \{\widetilde{\emptyset}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E\}$, *where* $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E)$ *are soft sets over* X *as follows:*

$(F_1, E) = \{(e, \{d\})\}$;
$(F_2, E) = \{(e, \{b, c\})\}$;
$(F_3, E) = \{(e, \{b, c, d\})\}$;
$(F_4, E) = \{(e, \{a, c\})\}$;
$(F_5, E) = \{(e, \{a, c, d\})\}$;
$(F_6, E) = \{(e, \{a, b, c\})\}$;
$(F_7, E) = \{(e, \{c\})\}$.
Then the soft set $(G, E) = \{(e, \{b, d\})\}$ *is a* B *- soft set, but it is not pre open soft set.*

Example 3.21. *Consider Example* 3.17. *Then the soft set* $(G, E) = \{(e, \{b\})\}$ *is a pre open soft set, but it is not* B *- soft set.*

Theorem 3.22. *Let* X *be an initial universe and* E *be a set of parameters. Let* τ *be a soft topology on* X. *Then* (F, E) *is a soft open set if and only if it is both* α *- open soft and* D_c^α *- open soft.*

Proof. Necessity. It is obvious.
Sufficiency. Let (F, E) be both α - open soft and D_c^α - soft set. Since (F, E) is a D_c^α - soft, $int(F, E) = (F, E)\widetilde{\cap} int(cl(int(F, E)))$. Since (F, E) is a α - open soft, we obtain $(F, E)\widetilde{\subseteq} int(cl(int(F, E)))$. Then
$(F, E)\widetilde{\cap}(F, E) = (F, E) \widetilde{\subseteq} int(cl(int(F, E))) \widetilde{\cap} (F, E)$.
As a consequence, $(F, E)\widetilde{\subseteq} int(F, E)$ and $(F, E) \in \tau$. □

Remark 3.23. α *- open soft sets and* D_c^α *- soft sets are independent of each other.*

Example 3.24. *Consider Example* 3.13. *Then the soft set* $(H, E) = \{(e, \{a, d\})\}$ *is a* D_c^α *- soft set, but it is not* α *- open soft set.*

Example 3.25. *Consider Example* 3.17. *Then the soft set* $(H, E) = \{(e, \{b, c\})\}$ *is an* α *- open soft set, but it is not* D_c^α *- soft set.*

$$\begin{array}{ccc}
\alpha \text{ - open soft} & \longleftarrow \text{ soft open set } \longrightarrow & \text{pre open soft} \\
& \downarrow & \\
& B \text{ - soft set} & \\
& \downarrow & \\
& C \text{ - soft set} & \\
& \downarrow & \\
& D_c^\alpha \text{ - soft set} &
\end{array}$$

Diagram 1. The relations of some soft sets.

4 A Decomposition of Soft Continuity

In this section we obtain some decompositions of soft continuity.

Definition 4.1. *Let X, Y be an initial universe, $A, B \subseteq E$ be two sets of parameters, τ_1 be a soft topology over X and τ_2 be a soft topology over Y. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then f_{pu} is called*

1. *C - soft continuous (briefly, (τ_1, τ_2) - C - soft cts) if $f_{pu}^{-1}(G, B)$ is C - soft set for every $(G, B) \in \tau_2$.*
2. *B - soft continuous (briefly, (τ_1, τ_2) - B - soft cts) if $f_{pu}^{-1}(G, B)$ is B - soft set for every $(G, B) \in \tau_2$.*
3. *D_c^α - soft continuous (briefly, (τ_1, τ_2) - D_c^α - soft cts) if $f_{pu}^{-1}(G, B)$ is D_c^α - soft set for every $(G, B) \in \tau_2$.*

We obtain the following implications from Definition 4.1.

$$(\tau_1, \tau_2) \text{ - } B \text{ - soft cts}$$

$$\Downarrow$$

$$(\tau_1, \tau_2) \text{ - } C \text{ - soft cts}$$

$$\Downarrow$$

$$(\tau_1, \tau_2) \text{ - } D_c^\alpha \text{ - soft cts}$$

Theorem 4.2. *Let X, Y be an initial universe, $A, B \subseteq E$ be two sets of parameters, τ_1 be a soft topology over X and τ_2 be a soft topology over Y. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then f_{pu} is a soft continuous function if and only if it is both α - soft continuous function and C - soft continuous function.*

Proof. This is an obvious result from Theorem 3.14. □

Theorem 4.3. *Let X, Y be an initial universe, $A, B \subseteq E$ be two sets of parameters, τ_1 be a soft topology over X and τ_2 be a soft topology over Y. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then f_{pu} is a soft continuous function if and only if it is both pre - soft continuous function and B - soft continuous function.*

Proof. This is an obvious result from Theorem 3.18. □

Theorem 4.4. *Let X, Y be an initial universe, $A, B \subseteq E$ be two sets of parameters, τ_1 be a soft topology over X and τ be a soft topology over Y. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then f_{pu} is a soft continuous function if and only if it is both α - soft continuous function and D_c^α - soft continuous function.*

Proof. This is an obvious result from Theorem 3.22. □

$$\begin{array}{c} \alpha \text{ - soft cts} \longleftarrow \text{soft cts} \longrightarrow \text{pre soft cts} \\ \downarrow \\ (\tau_1, \tau_2) \text{ - } B \text{ - soft cts} \\ \downarrow \\ (\tau_1, \tau_2) \text{ - } C \text{ - soft cts} \\ \downarrow \\ (\tau_1, \tau_2) \text{ - } D_c^{\alpha} \text{ - soft cts} \end{array}$$

Diagram 2. The relations of some soft continuities.

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