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Decompositions of πg -Continuity via Idealization

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Abstract - In this paper, we introduce the notions of $\pi g \alpha$ -*I*-open sets, $\pi g p$ -*I*-open sets, E_r -*I*-sets and E_r^* -*I*-sets in ideal topological spaces and investigate some of their properties and using these notions we obtain three decompositions of πg -continuity.

Keywords - $\pi g \alpha - \mathcal{I}$ continuity, $\pi g p - \mathcal{I}$ -continuity and πg -continuity.

1 Introduction and Preliminaries

In 1968, Zaitsev [25] introduced the concept of π -closed sets and in 1970, Levine [13] initiated the study of so called g-closed sets in topological spaces. The concept of g-continuity was introduced and studied by Balachandran et.al in 1991 [3]. Dontchev and Noiri [5] defined the notions of πg -closed sets and πg -continuity in topological spaces. In 1993, Palaniappan and Rao [18] introduced the notions of regular generalized closed (rg-closed) sets and rg-continuity in topological spaces. In 2000, Sundaram and Rajamani [22] obtained three different decompositions of rg-continuity by providing two types of weaker forms of continuity, namely C_r -continuity and C_r^* -continuity. Recently, Noiri et. al. [16] introduced the notions of αg - \mathcal{I} -open sets, gp- \mathcal{I} -open sets, G^* - \mathcal{I} -sets and S^* - \mathcal{I} -sets to obtain three different decompositions of πg -continuity via idealization. Recently Ravi et. al. [20] obtained three different decompositions of πg -continuity, namely E_r -continuity by providing two types of weaker forms of continuity. In this paper, we introduce the notions of $\pi g \alpha$ - \mathcal{I} -open sets, $\pi g p$ - \mathcal{I} -open sets, E_r - \mathcal{I} -sets to obtain the further decompositions of sets.

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 πg -continuity. Let (X, τ) be a topological space. An ideal is defined as a nonempty collection \mathcal{I} of subsets of X satisfying the following two conditions:

- (i) If $A \in \mathcal{I}$ and $B \subseteq A$, then $B \in \mathcal{I}$
- (ii) If $A \in \mathcal{I}$ and $B \in \mathcal{I}$, then $A \cup B \in \mathcal{I}$.

For a subset $A \subseteq X$, $A^*(\mathcal{I}) = \{x \in X/U \cap A \notin \mathcal{I} \text{ for each neighborhood } U \text{ of } x\}$ is called the local function of A with respect to \mathcal{I} and τ [10]. We simply write A^* instead of $A^*(\mathcal{I})$ in case there is no chance for confusion. X^* is often a proper subset of X. For every ideal topological space (X, τ, \mathcal{I}) there exists a topology $\tau^*(\mathcal{I})$, finer than τ , generated by $\beta(\mathcal{I}, \tau) = \{U \setminus I : U \in \tau \text{ and } I \in \mathcal{I}\}$, but in general $\beta(\mathcal{I}, \tau)$ is not always a topology [24]. Also, $cl^*(A) = A \cup A^*$ defines a Kuratowski closure operator for $\tau^*(\mathcal{I})$ [24]. Additionally, $cl^*(A) \subseteq cl$ (A) for any subset A of X [8]. Throughout this paper, X denotes the ideal topological space (X, τ, \mathcal{I}) and also cl(A) and int(A) denote the closure of A and the interior of A in (X, τ) , respectively.

Definition 1.1. A subset A of (X, τ) is said to be

- 1. α -open [15] if $A \subseteq int(cl(int(A)))$,
- 2. preopen [14] if $A \subseteq int(cl(A))$,
- 3. regular open [21] if A = int(cl(A)),
- 4. π -open [25] if the finite union of regular open sets,
- 5. πg -open [5] iff $F \subseteq int(A)$ whenever $F \subseteq A$ and F is π -closed in (X, τ) ,
- 6. π gp-open [19] iff $F \subseteq pint(A)$ whenever $F \subseteq A$ and F is π -closed in (X, τ) ,
- 7. $\pi g \alpha$ -open [2] iff $F \subseteq \alpha int(A)$ whenever $F \subseteq A$ and F is π -closed in (X, τ) ,
- 8. a t-set [23] if int(A) = int(cl(A)),
- 9. an α^* -set [7] if int(A) = int(cl(int(A))),
- 10. a E_r -set [20] if $A = U \cap V$, where U is πg -open and V is a t-set in (X, τ) ,
- 11. a E_r^* -set [20] if $A = U \cap V$, where U is πg -open and V is an α^* -set in (X, τ) .

The complements of the above mentioned open sets are called their respective closed sets. The preinterior pint(A) (resp. α -interior, α int(A)) of A is the union of all preopen sets (resp. α -open sets) contained in A. The α -closure α cl(A) of A is the intersection of all α -closed sets containing A.

Lemma 1.2. [1] If A is a subset of X, then

- 1. $pint(A) = A \cap int(cl(A)),$
- 2. $\alpha int(A) = A \cap int(cl(int(A)))$ and $\alpha cl(A) = A \cup cl(int(cl(A)))$.

Definition 1.3. A subset A of an ideal topological space (X, τ, \mathcal{I}) is said to be

1. pre- \mathcal{I} -open [4] if $A \subseteq int(cl^*(A))$,

Journal of New Results in Science 7 (2014) 72-80

- 2. α - \mathcal{I} -open [6] if $A \subseteq int(cl^*(int(A)))$,
- 3. $a \ t-\mathcal{I}-set \ [6] \ if \ int(cl^*(A)) = int(A),$
- 4. an α^* - \mathcal{I} -set [6] if $int(cl^*(int(A))) = int(A)$.

Also, we have $\alpha - \mathcal{I} - int(A) = A \cap int(cl^*(int(A)))$ [16] and $p - \mathcal{I} - int(A) = A \cap int(cl^*(A))$ [16], where $\alpha - \mathcal{I} - int(A)$ denotes the $\alpha - \mathcal{I}$ interior of A in (X, τ, \mathcal{I}) which is the union of all $\alpha - \mathcal{I}$ -open sets of (X, τ, \mathcal{I}) contained in A. $p - \mathcal{I} - int(A)$ has similar meaning.

Remark 1.4. The following hold in a topological space.

- 1. Every πg -open set is πgp -open but not conversely.[19]
- 2. Every πg -open set is $\pi g \alpha$ -open but not conversely.[2]

2 $\pi g \alpha$ -*I*-Open Sets and $\pi g p$ -*I*-Open Sets

Definition 2.1. A subset A of an ideal topological space (X, τ, \mathcal{I}) is called

- 1. $\pi g \alpha \mathcal{I}$ -open if $F \subseteq \alpha \mathcal{I}$ -int(A) whenever $F \subseteq A$ and F is π -closed in X.
- 2. πgp - \mathcal{I} -open if $F \subseteq p$ - \mathcal{I} -int(A) whenever $F \subseteq A$ and F is π -closed in X.

Proposition 2.2. For a subset of an ideal topological space, the following hold:

- 1. Every $\pi g \alpha$ - \mathcal{I} -open set is $\pi g \alpha$ -open.
- 2. Every πgp - \mathcal{I} -open set is πgp -open.
- 3. Every $\pi g \alpha$ -open set is $\pi g p$ -open.

Proof. (1) Let A be an $\pi g \alpha - \mathcal{I}$ -open set. Let $F \subseteq A$ and F is π -closed in X. Then, $F \subseteq \alpha - \mathcal{I}$ -int $(A) = A \cap (int(cl^*(int(A)))) \subseteq A \cap int(cl(int(A))) = \alpha int(A)$. This shows that A is $\pi g \alpha$ -open.

(2) Let A be πgp - \mathcal{I} -open set. Let $F \subseteq A$ and F is π -closed in X. Then, $F \subseteq p$ - \mathcal{I} -int(A) = $A \cap int(cl^*(A)) \subseteq A \cap int(cl(A)) = pint(A)$. This shows that A is πgp -open. (3) It follows from the definitions.

Remark 2.3. The converses of Proposition 2.2 are not true, in general.

Example 2.4. Let $X = \{a, b, c, d\}, \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Then $\{a, b, d\}$ is $\pi g \alpha$ -open but not an $\pi g \alpha$ - \mathcal{I} -open set.

Example 2.5. In Example 2.4, $\{a, b, d\}$ is πgp -open but not a πgp - \mathcal{I} -open set.

Example 2.6. Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, \{a\}, \{e\}, \{a, e\}, \{c, d\}, \{a, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}, X\}$. Then $\{b, c, e\}$ is πgp -open set but not an $\pi g\alpha$ -open.

Proposition 2.7. For a subset of an ideal topological space, the following hold:

1. Every $\pi g \alpha$ - \mathcal{I} -open set is $\pi g p$ - \mathcal{I} -open.

- 2. Every πg -open set is $\pi g p$ - \mathcal{I} -open.
- 3. Every πg -open set is $\pi g \alpha$ - \mathcal{I} -open.
- *Proof.* 1. Let A be $\pi g \alpha \mathcal{I}$ -open. Then, for any π -closed set F with $F \subseteq A$, we have $F \subseteq \alpha \mathcal{I}$ -int $(A) = A \cap \operatorname{int}(\operatorname{cl}^*(\operatorname{int}(A))) \subseteq A \cap \operatorname{int}(\operatorname{cl}^*(A)) = p \mathcal{I}$ -int(A) which implies that A is $\pi g p \mathcal{I}$ -open.
 - 2. Let A be an πg -open set. Then, for any π -closed set F with $F \subseteq A$, we have $F \subseteq \operatorname{int}(A) \subseteq \operatorname{int}((\operatorname{int}(A))^*) \cup \operatorname{int}(A) = \operatorname{int}((\operatorname{int}(A))^*) \cup \operatorname{int}(\operatorname{int}(A)) \subseteq \operatorname{int}((\operatorname{int}(A))^* \cup \operatorname{int}(A)) = \operatorname{int}(\operatorname{cl}^*(\operatorname{int}(A)))$. That is, $F \subseteq A \cap \operatorname{int}(\operatorname{cl}^*(\operatorname{int}(A))) = \alpha \mathcal{I} \operatorname{int}(A) = A \cap \operatorname{int}(\operatorname{cl}^*(\operatorname{int}(A))) \subseteq A \cap \operatorname{int}(\operatorname{cl}^*(A)) = p \mathcal{I} \operatorname{int}(A)$ which implies that A is $\pi g p \mathcal{I} \operatorname{open}$.
 - 3. Let A be an πg -open set. Then, for any π -closed set F with $F \subseteq A$, we have $F \subseteq int(A) \subseteq int((int(A))^*) \cup int(A) = int((int(A))^*) \cup int(int(A)) \subseteq int((int(A))^* \cup int(A)) = int(cl^*(int(A)))$. That is, $F \subseteq A \cap int(cl^*(int(A))) = \alpha \mathcal{I} int(A)$ which implies that A is $\pi g \alpha \mathcal{I}$ -open.

Remark 2.8. The converses of Proposition 2.7 are not true, in general.

Example 2.9. Let $X = \{a, b, c, d, e\}, \tau = \{\emptyset, X, \{b\}, \{e\}, \{b, e\}, \{c, d\}, \{b, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\}$ and $\mathcal{I} = \{\emptyset, \{b\}, \{e\}, \{b, e\}\}$. Then $\{a, d, e\}$ is πgp - \mathcal{I} -open but not an $\pi g\alpha$ - \mathcal{I} -open set.

Example 2.10. In Example 2.9, $\{a, d, e\}$ is πgp - \mathcal{I} -open but not a πg -open set.

Example 2.11. Let $X = \{a, b, c, d\}, \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then $\{a, b, d\}$ is $\pi g \alpha \cdot \mathcal{I}$ -open but not a πg -open set.

Remark 2.12. By Remark 1.4, Propositions 2.2 and 2.7, we have the following diagram. In this diagram, there is no implication which is reversible as shown by examples above.

3 E_r - \mathcal{I} -Sets and E_r^* - \mathcal{I} -Sets

Definition 3.1. A subset A of an ideal topological space (X, τ, \mathcal{I}) is called

- 1. a E_r - \mathcal{I} -set if $A = U \cap V$, where U is πg -open and V is a t- \mathcal{I} -set,
- 2. a E_r^* - \mathcal{I} -set if $A = U \cap V$, where U is πg -open and V is an α^* - \mathcal{I} -set.

We have the following proposition:

Proposition 3.2. For a subset of an ideal topological space, the following hold:

1. Every t- \mathcal{I} -set is an α^* - \mathcal{I} -set [6] and a E_r - \mathcal{I} -set.

- 2. Every α^* - \mathcal{I} -set is a E_r^* - \mathcal{I} -set.
- 3. Every E_r - \mathcal{I} -set is a E_r^* - \mathcal{I} -set.
- 4. Every πg -open set is a E_r -set.
- 5. Every E_r -set is a E_r - \mathcal{I} -set and a E_r^* -set.
- 6. Every E_r^* -set is a E_r^* - \mathcal{I} -set.

From Proposition 3.2, We have the following diagram.

Remark 3.3. The converses of implications in Diagram II need not be true as the following examples show.

Example 3.4. In Example 2.4, $\{a, b, d\}$ is E_r - \mathcal{I} -set but not a E_r -set.

Example 3.5. In Example 2.4, $\{a, b, c\}$ is E_r - \mathcal{I} -set but not a t- \mathcal{I} -set.

Example 3.6. Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, X, \{b\}, \{e\}, \{b, e\}, \{c, d\}, \{b, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\}$. Then $\{a\}$ is E_r -set but not a πg -open set.

Example 3.7. In Example 3.6, $\{a, b, d, e\}$ is E_r^* -set but not a E_r -set.

Example 3.8. In Example 2.10, $\{a, b, d, e\}$ is E_r^* - \mathcal{I} -set but not a E_r - \mathcal{I} -set.

Example 3.9. In Example 2.4, $\{a, b, d\}$ is E_r^* - \mathcal{I} -set but not a E_r^* -set.

Example 3.10. In Example 2.4, $\{a, b, c\}$ is E_r^* - \mathcal{I} -set but not an α^* - \mathcal{I} -set.

Example 3.11. Let $X = \{a, b, c\}, \tau = \{\emptyset, X, \{a, b\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then $\{b\}$ is α^* - \mathcal{I} -set but not a t- \mathcal{I} -set.

Remark 3.12. Examples 3.13 and 3.14 show that E_r - \mathcal{I} -sets and E_r^* -sets are independent of each other.

Example 3.13. Let $X = \{a, b, c, d, e\}, \tau = \{\emptyset, X, \{b\}, \{e\}, \{b, e\}, \{c, d\}, \{b, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\}$ and $\mathcal{I} = \{\emptyset, \{b\}, \{e\}, \{b, e\}\}$. Then $\{a, b, d, e\}$ is E_r^* -set but not a E_r - \mathcal{I} -set.

Example 3.14. In Example 2.4, $\{a, b, d\}$ is E_r - \mathcal{I} -set but not a E_r^* -set.

Proposition 3.15. A subset A of X is πg -open if and only if it is both πgp - \mathcal{I} -open and a E_r - \mathcal{I} -set in X.

Proof. Necessity is trivial. We prove the sufficiency. Assume that A is πgp - \mathcal{I} -open and a E_r - \mathcal{I} -set in X. Let $F \subseteq A$ and F is π -closed in X. Since A is a E_r - \mathcal{I} -set in X, A = U $\cap V$, where U is πg -open and V is a t- \mathcal{I} -set. Since A is πgp - \mathcal{I} -open, $F \subseteq p$ - \mathcal{I} -int(A) = $A \cap \operatorname{int}(\operatorname{cl}^*(A)) = (U \cap V) \cap \operatorname{int}(\operatorname{cl}^*(U \cap V)) \subseteq (U \cap V) \cap \operatorname{int}(\operatorname{cl}^*(U) \cap \operatorname{cl}^*(V)) = (U$ $\cap V) \cap \operatorname{int}(\operatorname{cl}^*(U)) \cap \operatorname{int}(\operatorname{cl}^*(V))$. This implies $F \subseteq \operatorname{int}(\operatorname{cl}^*(V)) = \operatorname{int}(V)$ since V is a t- \mathcal{I} -set. Since F is π -closed, U is πg -open and $F \subseteq U$, we have $F \subseteq \operatorname{int}(U)$. Therefore, $F \subseteq \operatorname{int}(U) \cap \operatorname{int}(V) = \operatorname{int}(U \cap V) = \operatorname{int}(A)$. Hence A is πg -open in X. **Corollary 3.16.** A subset A of X is πg -open if and only if it is both $\pi g \alpha$ - \mathcal{I} -open and a E_r - \mathcal{I} -set in X.

Proof. This is an immediate consequence of Proposition 3.15.

Proposition 3.17. A subset A of X is πg -open if and only if it is both $\pi g \alpha$ - \mathcal{I} -open and a E_r^* - \mathcal{I} -set in X.

Proof. Necessity is trivial. We prove the sufficiency. Assume that A is $\pi g \alpha \cdot \mathcal{I}$ -open and a $E_r^* \cdot \mathcal{I}$ -set in X. Let $F \subseteq A$ and F is π -closed in X. Since A is a $E_r^* \cdot \mathcal{I}$ -set in $X, A = U \cap V$, where U is πg -open and V is an $\alpha^* \cdot \mathcal{I}$ -set. Now since F is π -closed, $F \subseteq U$ and Uis πg -open, $F \subseteq \operatorname{int}(U)$. Since A is $\pi g \alpha \cdot \mathcal{I}$ -open, $F \subseteq \alpha \cdot \mathcal{I}$ -int $(A) = A \cap \operatorname{int}(\operatorname{cl}^*(\operatorname{int}(A)))$ $= (U \cap V) \cap \operatorname{int}(\operatorname{cl}^*(\operatorname{int}(U \cap V))) = (U \cap V) \cap \operatorname{int}(\operatorname{cl}^*(\operatorname{int}(U) \cap \operatorname{int}(V))) \subseteq (U \cap V) \cap$ $\operatorname{int}(\operatorname{cl}^*(\operatorname{int}(U)) \cap \operatorname{cl}^*(\operatorname{int}(V))) = (U \cap V) \cap \operatorname{int}(\operatorname{cl}^*(\operatorname{int}(V))) = (U \cap V)$ $\cap \operatorname{int}(\operatorname{cl}^*(\operatorname{int}(U))) \cap \operatorname{int}(V)$, since V is an $\alpha^* \cdot \mathcal{I}$ -set. This implies $F \subseteq \operatorname{int}(V)$. Therefore, $F \subseteq \operatorname{int}(U) \cap \operatorname{int}(V) = \operatorname{int}(U \cap V) = \operatorname{int}(A)$. Hence A is πg -open in X.

Remark 3.18. 1. The concepts of πgp - \mathcal{I} -open sets and E_r - \mathcal{I} -sets are independent of each other.

- 2. The concepts of $\pi g \alpha \mathcal{I}$ -open sets and $E_r \mathcal{I}$ -sets are independent of each other.
- 3. The concepts of $\pi g \alpha \mathcal{I}$ -open sets and $E_r^* \mathcal{I}$ -sets are independent of each other.

Example 3.19. Let $X = \{a, b, c, d\}, \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then

(1) $\{b, c, d\}$ is E_r - \mathcal{I} -set but not a πgp - \mathcal{I} -open.

(2) In Example 3.13, $\{a, b, d, e\}$ is πgp - \mathcal{I} -open but not a E_r - \mathcal{I} -set.

Example 3.20. Let $X = \{a, b, c, d\}, \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then

(1) $\{b, c, d\}$ is E_r - \mathcal{I} -set but not an $\pi g \alpha$ - \mathcal{I} -open set.

(2) $\{a, b, d\}$ is $\pi g \alpha - \mathcal{I}$ -open set but not a E_r - \mathcal{I} -set.

Example 3.21. Let $X = \{a, b, c, d\}, \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then

(1) $\{b, c, d\}$ is E_r^* - \mathcal{I} -set but not an $\pi g \alpha$ - \mathcal{I} -open set.

(2) $\{a, b, d\}$ is $\pi g \alpha - \mathcal{I}$ -open set but not a $E_r^* - \mathcal{I}$ -set.

4 Decompositions of πg -Continuity

Definition 4.1. A mapping $f: (X, \tau) \to (Y, \sigma)$ is said to be πg -continuous [5] (resp. $\pi g p$ -continuous [19], $\pi g \alpha$ -continuous [20], E_r -continuous [20] and E_r^* -continuous [20]) if $f^{-1}(V)$ is πg -open (resp. $\pi g p$ -open, $\pi g \alpha$ -open, E_r -set and E_r^* -set) in (X, τ) for every open set V in (Y, σ) .

Definition 4.2. A mapping $f: (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be $\pi g \alpha \cdot \mathcal{I}$ -continuous (resp. $\pi g p \cdot \mathcal{I}$ -continuous, $E_r \cdot \mathcal{I}$ -continuous and $E_r^* \cdot \mathcal{I}$ -continuous) if for every $V \in \sigma$, $f^{-1}(V)$ is $\pi g \alpha \cdot \mathcal{I}$ -open (resp. $\pi g p \cdot \mathcal{I}$ -open, a $E_r \cdot \mathcal{I}$ -set and a $E_r^* \cdot \mathcal{I}$ -set) in (X, τ, \mathcal{I}) .

From Propositions 3.15 and 3.17 and Corollary 3.16 we have the following decompositions of πg -continuity.

Theorem 4.3. Let (X, τ, \mathcal{I}) be an ideal topological space. For a mapping $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, the following properties are equivalent:

- 1. f is πg -continuous;
- 2. f is πgp - \mathcal{I} -continuous and E_r - \mathcal{I} -continuous;
- 3. f is $\pi g \alpha \mathcal{I}$ -continuous and E_r - \mathcal{I} -continuous;
- 4. f is $\pi g \alpha$ - \mathcal{I} -continuous and E_r^* - \mathcal{I} -continuous.
- **Remark 4.4.** 1. The concepts of πgp - \mathcal{I} -continuity and E_r - \mathcal{I} -continuity are independent of each other.
 - 2. The concepts of $\pi g \alpha \mathcal{I}$ -continuity and $E_r \mathcal{I}$ -continuity are independent of each other.
 - 3. The concepts of $\pi g \alpha$ - \mathcal{I} -continuity and E_r^* - \mathcal{I} -continuity are independent of each other.

Example 4.5. (1) Let $X = Y = \{a, b, c, d\}, \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}, \mathcal{I} = \{\emptyset, \{c\}\} and \sigma = \{\emptyset, Y, \{b, c, d\}\}.$ Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be the identity function. Then f is E_r - \mathcal{I} -continuous but not πgp - \mathcal{I} -continuous.

(2) Let $X = Y = \{a, b, c, d, e\}, \tau = \{\emptyset, X, \{b\}, \{e\}, \{b, e\}, \{c, d\}, \{b, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\}, \mathcal{I} = \{\emptyset, \{b\}, \{e\}, \{b, e\}\}$ and $\sigma = \{\emptyset, Y, \{a, b, d, e\}\}$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be the identity function. Then f is πgp - \mathcal{I} -continuous but not E_r - \mathcal{I} -continuous.

Example 4.6. (1) Let $X = Y = \{a, b, c, d\}, \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}, \mathcal{I} = \{\emptyset, \{c\}\} and \sigma = \{\emptyset, Y, \{b, c, d\}\}.$ Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be the identity function. Then f is E_r - \mathcal{I} -continuous but not $\pi g \alpha$ - \mathcal{I} -continuous.

(2) Let $X = Y = \{a, b, c, d\}, \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}, \mathcal{I} = \{\emptyset, \{c\}\} and \sigma = \{\emptyset, Y, \{a, b, d\}\}.$ Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be the identity function. Then f is $\pi g \alpha \cdot \mathcal{I}$ -continuous but not $E_r \cdot \mathcal{I}$ -continuous.

Example 4.7. In Example 4.6 (1) f is E_r^* - \mathcal{I} -continuous but not $\pi g \alpha$ - \mathcal{I} -continuous. In Example 4.6 (2) f is $\pi g \alpha$ - \mathcal{I} -continuous but not E_r^* - \mathcal{I} -continuous.

Corollary 4.8. [20] Let (X, τ, \mathcal{I}) be an ideal topological space and $\mathcal{I} = \{\emptyset\}$. For a mapping $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, the following properties are equivalent:

- 1. f is πg -continuous;
- 2. f is πgp -continuous and E_r -continuous;
- 3. f is $\pi g \alpha$ -continuous and E_r -continuous;
- 4. f is $\pi g \alpha$ -continuous and E_r^* -continuous.

Proof. Since $\mathcal{I} = \{\emptyset\}$, we have $A^* = \operatorname{cl}(A)$ and $\operatorname{cl}^*(A) = A^* \cup A = \operatorname{cl}(A)$ for any subset A of X [[6], Proposition 2.4(a)]. Therefore, we obtain (1) A is $\pi g \alpha$ - \mathcal{I} -open (resp. $\pi g p$ - \mathcal{I} -open) if and only if it is $\pi g \alpha$ -open (resp. $\pi g p$ -open) and (2) A is a E_r - \mathcal{I} -set (resp. a E_r^* - \mathcal{I} -set) if and only if it is a E_r -set (resp. a E_r^* -set). The proof follows from Theorem 4.3 immediately.

5 Conclusion

Topology as a field of mathematics is concerned with all questions directly or indirectly related to open/closed sets. Therefore, generalization of open/closed sets is one of the most important subjects in topology. Topology plays a significant role in quantum physics, high energy physics and superstring theory. In this paper, we introduce the notions of $\pi g \alpha - \mathcal{I}$ -open sets, $\pi g p - \mathcal{I}$ -open sets, $E_r - \mathcal{I}$ -sets and $E_r^* - \mathcal{I}$ -sets in ideal topological spaces and investigate some of their properties and using these notions we obtain three decompositions of $\pi g \alpha$ -continuity. Moreover, some notions of the sets and functions in topological spaces and ideal topological spaces are highly developed and used extensively in many practical and engineering problems.

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