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ERROR ANALYSIS OF NON-ITERATIVE FRICTION FACTOR FORMULAS RELATIVE TO COLEBROOK-WHITE EQUATION FOR THE CALCULATION OF PRESSURE DROP IN PIPES

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Abstract

Pressure drop in pipes can be calculated by using Darcy-Weisbach formula. In order to use this formula, Darcy friction factor should be known. The best approximation to Darcy friction factor for turbulent flow is given by Colebrook-White equation. This equation can only be solved by numerical root finding methods. There are several other approximate equations to Darcy friction factor with some relative error compared to Colebrook-White equation. In some of these equations the percentage error is so small that they can be used directly in place of Colebrook equation. In this study relative error of several equations are re-evaluated.

BORULARDAKİ SÜRTÜNME KAYIPLARI ANALİZİNDE DARCY-WEISBACH SÜRTÜNME KATSAYISI HESAPLARINDA COLEBROOK-WHITE DENKLEMİ YERİNE GEÇECEK DÖNGÜSEL OLMAYAN ÇÖZÜMLÜ DENKLEMLERİN HATA ANALİZİ

Özetçe

Borulardaki sürtünme basınç kayıpları Darcy-Weisbach formula ile hesaplanır. Bu basınç kaybını hesaplamak için f Darcy sürtünme *katsayısının hesaplanması gereklidir. Türbülanslı akışlarda Darcy sürtünme katsayısının hesaplanmasında en geçerli yöntem Colebrook-White denklemidir, ancak bu denklem sayısal kök bulma yöntemleri kullanılarak çözülebilen bir denklemdir. Colebrook-White denklemine yaklaşım yapan ve direk olarak çözülebilen çeşitli denklemler mevcuttur. Bu denklemlerin bazılarının Colebrook-White denklemiyle kıyaslandığında hata yüzdeleri çok küçük olduğundan, direk olarak bu denklemin yerine kulanılmaları mümkündür. Yazımızda çeşitli Darcy sürtünme faktörü denklemlerinin Colebrook – White denklemine gore göreceli hatası irdelenmiştir.*

Keywords: Darcy –Weisbach pressure drop formula, Pressure drop in pipes, Colebrook equation, Friction factors.
Anahtar Kelimeler: Darcy-Weichbach basınç düşümü, Boru içi basınç düşümü, Colebrook denklemi.

1. INTRODUCTION

The pressure loss in pipe flow is calculated by using Darcy-Weisbach equation. The equation is given as:

$$\Delta P = f\,\frac{L}{D}\rho\frac{V^2}{2} \tag{1}$$

In equation (1) *ΔP* is the pressure drop, *f* is Darcy friction factor, *D* is the hydraulic diameter of the pipe and *V* is the average velocity. Darcy friction factor *f* depends on the flow regime. For fully developed laminar flow (Reynolds number *Re* < 2100) friction factor can be determined from Hagen-Poiseuille equation as

$$f = \frac{64}{\mathrm{Re}} \tag{2}$$

Where, Re Reynolds number. The definition of the Re number can be given as:

$$\mathrm{Re} = \frac{\rho V D}{\mu} \tag{3}$$

Where ρ is the density and μ is the dynamic viscosity of the fluid. In equation (2) the friction factor changes inversely with Reynolds number. For transition region ($2100 \leq Re \leq 4000$) and turbulent region ($Re \geq 4000$) in smooth as well as rough pipe the friction factor can be described by Colebrook-White equation.

2. FRICTION FACTOR EQUATIONS AND ERROR ANALYSIS

Colebrook-White equation (1937)[4]
Colebrook-White equation can be defined as

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{(\varepsilon / D)}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right] \tag{4}$$

Where ϵ/D is the relative roughness which is the ratio of the mean height of roughness of the pipe to the pipe diameter. As seen from equation (4) the friction factor is function of Reynolds number and pipe roughness (ϵ). Colebrook-White equation cannot be solved directly due to it's an implicit form as the value of f appears on both side of the equation. In order to solve equation (4) a numerical root finding method, for example Newton-Raphson method can be used.
Assume

$$X = \frac{1}{\sqrt{f}} \tag{5}$$

$$f(X) = X + 2\log_{10}\left[\frac{(\varepsilon / D)}{3.7} + \frac{2.51}{\mathrm{Re}} X\right] \tag{6}$$

$$\frac{df(X)}{dX} = 1 + 2\frac{\frac{2.51}{\mathrm{Re}}}{\left[\frac{(\varepsilon / D)}{3.7} + \frac{2.51}{\mathrm{Re}} X\right]} \tag{7}$$

$$X_{k+1} = X_k - \frac{[f(X)]_k}{\left[\frac{df(X)}{dX}\right]_k} \qquad k = 0,......,n \tag{8}$$

Equation (8) is required to be solved iteratively. In order to solve the equation, an initial guess is needed. If the value of the first guess is diverge from the true root value too much equation might converge very slowly or might not converge at all. In order to find the first guess value one of the approximate formulas given below can be used. For example Haaland equation can be used to give the first estimation value for the Colebrook-White equation. The result of some of the approximation equations listed is very close to the result obtained from Colebrook-White equation. When the error level of the new equation is relatively small, requirement of using Colebrook-White equation can be eliminated all together. Calculation of pipe pressure drop usually calculated by computers. Replacing an iterative root finding problem with a directly calculatable equation could save computer calculation time. The simler equation can be easily computed by using simpler computational environment such as programmable calculators or spreadsheet programs such as MS Excel. Some of the Colebrook-White equation approximation formulas are listed below.

Haaland equation (1983) [23]

A good approximate equation shown. Haaland equation valid for turbulent flow (Re>2300)

$$\frac{1}{\sqrt{f}} = -1.8\log_{10}\left[\left(\frac{(\varepsilon/D)}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right] \tag{9}$$

Moody equation (1944) [9]

Developed a relationship that is valid for all ranges of Reynolds numbers and relative roughness as follows.

$$f = 5.5x10^{-3}\left[1+\left(2x10^{4}(\varepsilon / D)+\frac{10^{6}}{\mathrm{Re}}\right)^{1/3}\right] \qquad (10)$$

Wood equation (1966) [18]
Its validation region for Re >10000 , and $10^{-5} < (\varepsilon / D) < 0.04$

$$f = a + b\,\mathrm{Re}^{-C} \qquad (11)$$

Where

$$a = 0.53(\varepsilon / D) + 0.094(\varepsilon / D)^{0.225} \qquad (12)$$

$$b = 88(\varepsilon / D)^{0.44} \qquad (13)$$

$$C = 1.62(\varepsilon / D)^{0.134} \qquad (14)$$

Churchill equation (1977) [3]
This equation is valid for all ranges of Reynolds numbers.

$$f = 8\left[\left(\frac{8}{\mathrm{Re}}\right)^{12} + (A+B)^{-3/2}\right]^{1/12} \qquad (15)$$

Where

$$A = \left[-2\log_{10}\left(\frac{(\varepsilon / D)}{3.7} + \left(\frac{7}{\mathrm{Re}}\right)^{0.9}\right)\right]^{16} \qquad (16)$$

$$B = \left(\frac{37530}{\mathrm{Re}}\right)^{16} \qquad (17)$$

Chen equation (1979) [2]
Chen proposed the following equation for friction factor covering all the ranges of Reynolds number and relative roughness.

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\left(\frac{(\varepsilon / D)}{3.7065}\right) - \frac{5.0452A}{\mathrm{Re}}\right] \tag{18}$$

Where

$$A = \log_{10}\left(\frac{(\varepsilon / D)^{1.1098}}{2.8257} + \frac{5.8506}{\mathrm{Re}^{0.8981}}\right) \tag{19}$$

Swamee-Jain equation (1976) [14]
Its validation region for 5000 >Re>10^7, 0.00004<(ε / D)<0.05. Swamee-Jain have developed the following equation to the Darcy friction factor.

$$f = \frac{0.25}{A^2} \tag{20}$$

Where

$$A = \log_{10}\left(\frac{(\varepsilon / D)}{3.7} + \frac{5.74}{\mathrm{Re}^{0.9}}\right) \tag{21}$$

Zigrang - Sylvester equation (1982) [20]
Developed a relationship that is valid for all ranges of Reynolds numbers and relative roughness as follows

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\left(\frac{(\varepsilon / D)}{3.7}\right) - \frac{5.02B}{\mathrm{Re}}\right] \tag{22}$$

$$A = \log_{10}\left(\frac{(\varepsilon / D)}{3.7} + \frac{13}{\mathrm{Re}}\right) \tag{23}$$

$$B = \log_{10}\left(\frac{(\varepsilon / D)}{3.7} - \frac{5.02A}{\mathrm{Re}}\right) \tag{24}$$

Serghides equation (1984) [22]
Serghides equation is an approximation of the implicit Colebrook–White equation. It is valid for all ranges of Reynolds numbers and relative roughness as follows.

$$\frac{1}{\sqrt{f}} = A - \frac{(B-A)^2}{C-2B+A} \tag{25}$$

Where

$$A = -2\log_{10}\left[\left(\frac{(\varepsilon/D)}{3.7}\right) + \frac{12}{\text{Re}}\right] \tag{26}$$

$$B = -2\log_{10}\left[\left(\frac{(\varepsilon/D)}{3.7}\right) + \frac{2.51A}{\text{Re}}\right] \tag{27}$$

$$C = -2\log_{10}\left[\left(\frac{(\varepsilon/D)}{3.7}\right) + \frac{2.51B}{\text{Re}}\right] \tag{28}$$

Goudar- Sonnad equation (2008)[21]
Goudar-Sonnad equation is an approximation of the implicit Colebrook–White equation. This equation is valid for all ranges of Reynolds numbers and relative roughness. It has the following form.

$$\frac{1}{\sqrt{f}} = a\left[\ln\left(\frac{d}{q}\right) + \delta_{CFA}\right] \tag{29}$$

Where

$$a = \frac{2}{\ln(10)} \tag{30}$$

$$b = \frac{(\varepsilon / D)}{3.7} \tag{31}$$

$$d = \frac{\ln(10)}{5.02}\mathrm{Re} \tag{32}$$

$$s = bd + \ln(d) \tag{33}$$

$$q = s^{(s/(s+1))} \tag{34}$$

$$g = bd + \ln(d/q) \tag{35}$$

$$z = \left(\frac{q}{g}\right) \tag{36}$$

$$\delta_{LA} = \frac{g}{g+1} z \tag{37}$$

$$\delta_{CFA} = \delta_{LA}\left(1 + \frac{z/2}{(g+1)^2 + (z/3)(2g-1)}\right) \tag{38}$$

Romeo equation (2002) [11]
Developed a relationship that is valid for all ranges of Reynolds numbers and relative roughness as follows.

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\left(\frac{(\varepsilon / D)}{3.7065}\right) - \frac{5.0272B}{\mathrm{Re}}\right] \tag{39}$$

$$A = \log_{10}\left[\left(\frac{(\varepsilon / D)}{7.7918}\right)^{0.9924} + \left(\frac{5.3326}{208.815 + \mathrm{Re}}\right)^{0.9345}\right] \tag{40}$$

$$B = \log_{10}\left(\frac{(\varepsilon / D)}{3.827} - \frac{4.567A}{\mathrm{Re}}\right) \tag{41}$$

In order to determine the relative error of all these approximate formulas, relative error of each equation is with respect to Colebrook-White equation is calculated by using the following equation:

$$Relative\ Error = \frac{\left(f_{Colebrook-White} - f\right)}{f_{Colebrook-White}} \times 100 \tag{42}$$

3. RESULTS

The results for the relative error are shown graphically. Results obtained from error analysis are briefly explained below.

- In transition region relative error is too high; as the effect of turbulence increased (for high Reynolds number) the relative error will be decreased. If the approximation formulas are scaled in the order of relative error, Best results are obtained from Goudar-Sonnad equation, Serghides equation, Romeo equation; Ziagrang equation and Chen equation are fallowed up in the given order.

- When a comparison according to degree of the error is made, Goudar-Sonnad equation with small percentage error order exceeds 10^{-9} % is very close to the result obtained from Colebrook-White equation. Then the next best equation is Serghides equation with percentage error order 10^{-5} % which is also can be used practically.

- Because of these equations is precision enough, the need for use Colebrook-White iterative solution seems to be eliminated.

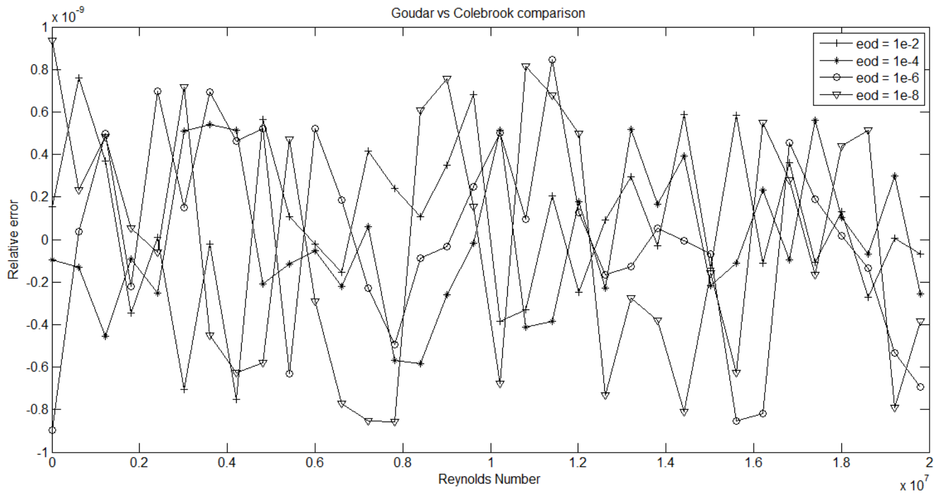

Figure 1. The amount of relative error comparison between Goudar and Colebrook-White equations

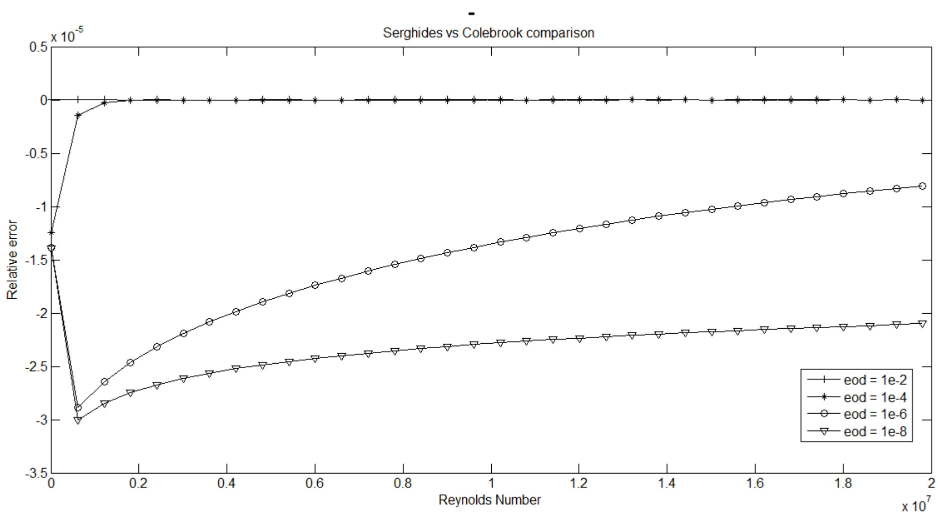

Figure 2. The amount of relative error comparison between Serghides and Colebrook-White equations

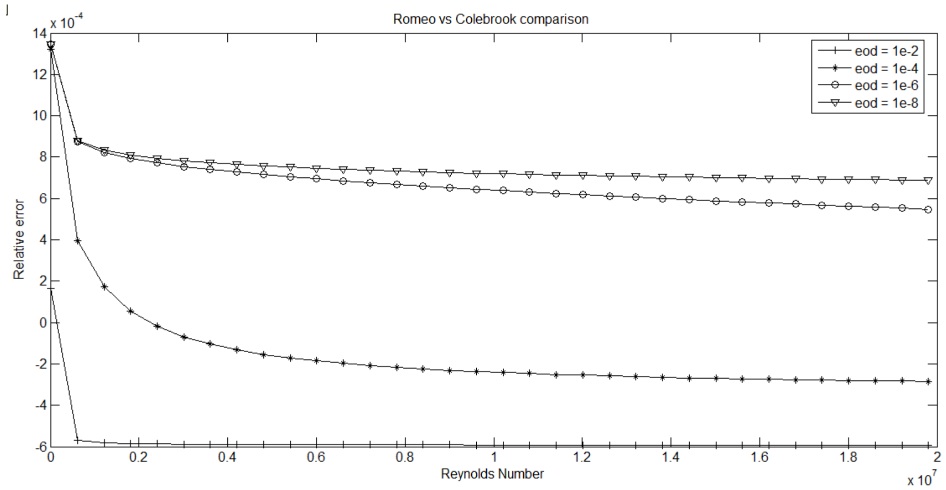

Figure 3. The amount of relative error comparison between Romeo and Colebrook-White equations

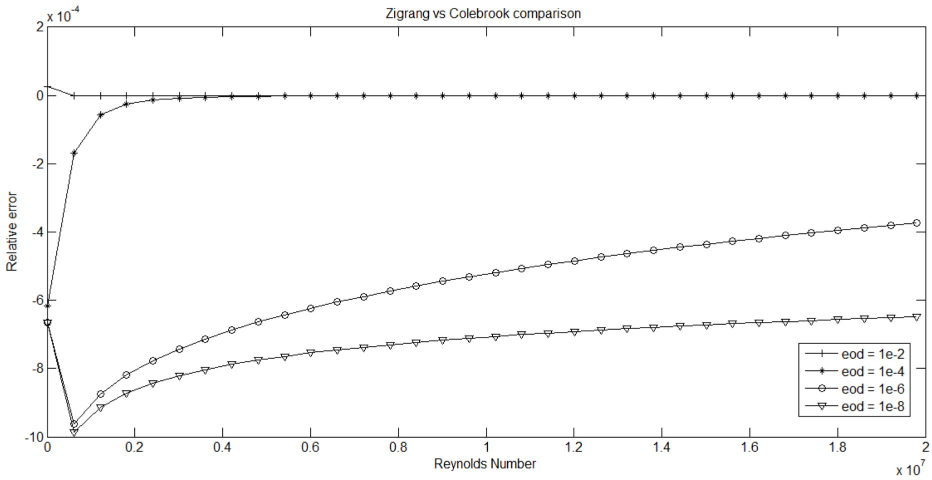

Figure 4. The amount of relative error comparison between Zigrang and Colebrook-White equations

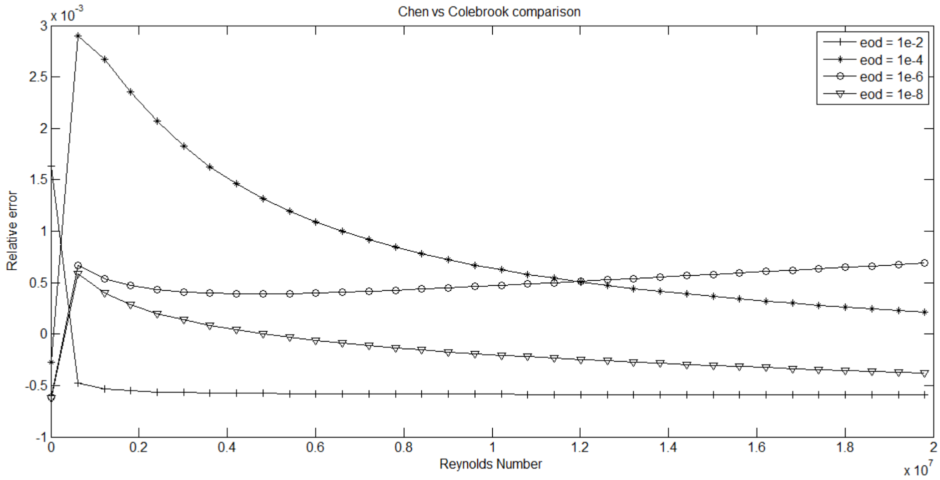

Figure 5. The amount of relative error comparison between Chen and Colebrook-White equations

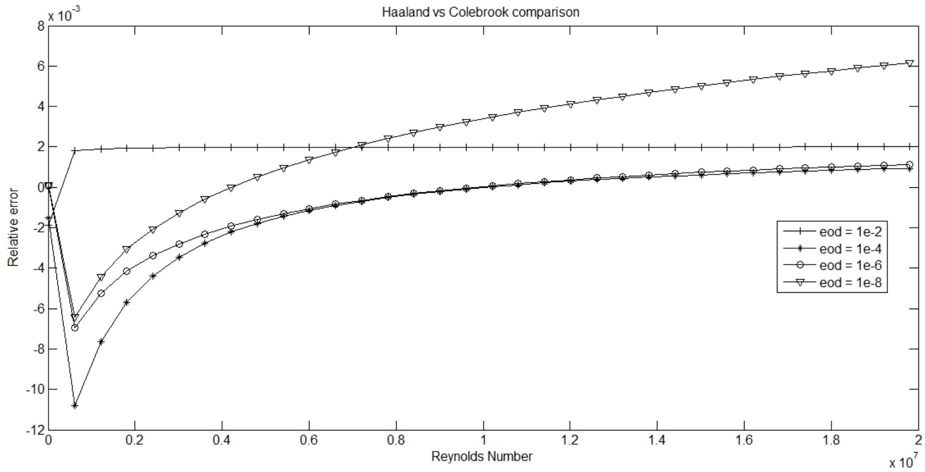

Figure 6. The amount of relative error comparison between Haaland and Colebrook-White equations

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