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FUZZY ALMOST CONTRA θ-SEMIGENERALIZED-CONTINUOUS FUNCTIONS

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Abstract – The aim of this paper is to introduce new notion of the fuzzy almost contra θ-semigeneralized-continuous functions using fuzzy θ-semigeneralized-closed set and to investigate properties and relationships of fuzzy functions.

Keywords – *Fuzzy θsg-closed set, Fuzzy almost contra θsg-continuous, FTSGO-connected space, FTSGO-compact space, fuzzy θsg-normal space, fuzzy $\theta sg - T_1$, fuzzy $\theta sg - T_2$.*

1 Introduction

The concept of fuzzy sets due to Zadeh [10] naturally plays important role in the study of fuzzy topological space which has been introduced by Chang [2]. In 2013, Zabidin Salleh et al introduced and studied the notion of θ-semi-generalized-closed sets in fuzzy topological spaces. Ekici and Kerre [4] introduced the concept of fuzzy contra continuous functions. The purpose of this paper is to introduce the forms of fuzzy almost contra θsg-continuous functions and to investigate properties and relationships of fuzzy functions. We have also defined fuzzy θsg-compact and fuzzy θsg-connected spaces.

2 Preliminary

Throughout this paper X be a set and I the unit interval. A fuzzy set in X is an element of the set of all functions from X to I. The family of all fuzzy sets in X is denoted by I^X. A fuzzy singleton x_α is a fuzzy set in X define by $x_\alpha(x) = \alpha$, $x_\alpha(y) = 0$ for all $y \neq x, x \in (0, 1]$. The set of all fuzzy singletons in X is denoted by $S(X)$. For every $x_\alpha \in S(X)$ and $\mu \in I^X$, we define $x_\alpha \in \mu$ if and only if $x_\alpha \leq \mu(x)$. The members of τ are called fuzzy open sets and their complements are fuzzy closed sets. Spaces (X, τ) and (Y, σ)(or simply, X and Y) always mean fuzzy topological spaces in the sense of Chang [2]. By 1_X and 0_X, we mean fuzzy sets with constant function 1 (unit function) and 0 (zero function), respectively.

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For a fuzzy set μ of X, fuzzy closure and fuzzy interior of μ denoted by $cl(\mu)$ and $int(\mu)$, respectively. The operators fuzzy closure and fuzzy interior are defined by $cl(\mu) = \wedge\{\lambda : \lambda \geq \mu, 1-\mu \in \tau\}$ where λ is fuzzy closed set in X and $int\mu = \wedge\{\eta : \eta \leq \mu, \eta \in \tau$ [9] where η is fuzzy open set in X. Fuzzy semi-closure [9] of μ denoted by $scl(\mu) = \wedge\{\eta : \mu \leq \eta, \eta \in FSC(X)\}$ and fuzzy θ-closure of μ denoted by $cl_\theta = \wedge\{cl(\eta) : \mu \leq \eta, \eta \in \tau\}$[3]. θ-semi-generalized closed set in fuzzy topology is introduced by Z.Salleh et al [8].

Definition 2.1. A subset A of a space X is called
(1) Fuzzy semi-open(briefly, Fs-open) set [1] if $A \leq cl(int(A))$.
(2) Fuzzy semi-closed (briefly, Fs-closed)set[1] if $int(cl(A)) \leq A$.
(3) Fuzzy regular closed [1] if $cl(int(A)) = A$ and fuzzy regular open if $int(cl(A)) = A$. The family of all fuzzy semi open and fuzzy semi closed sets in X will be denoted by FSO(X)and FSC(X), respectively.

Definition 2.2. [8] Let X be a fuzzy topological space and μ be a fuzzy set of X. Then the operators semi-θ-closure of μ denoted by $scl_\theta(\mu)$ and fuzzy semi-θ-interior of μ is denoted by $sint_\theta(\mu)$ are defined as follows,
$scl_\theta(\mu) = \wedge\{scl(\eta) : \mu \leq \eta, \eta \in FSO(X)\}$,
$sint_\theta(\mu) = \vee\{sint(\eta) : \mu \geq \eta, \eta \in FSC(X)\}$.

Definition 2.3. A fuzzy set μ in X is called
(1) fuzzy θ-generalized closed [3] (briefly, f-θg-closed set) if $cl_\theta(\mu) \leq \eta$ whenever $\mu \leq \eta$ and η is fuzzy open
(2)fuzzy θ-semigeneralized-closed set [8] (briefly, f-θsg-closed set) if $scl_\theta(\mu) \leq \eta$ whenever $\mu \leq \eta$ and η is fuzzy semiopen. The coplement of fuzzy θ-semi-generalized-closed set is fuzzy θ-semi-generalized-open set (briefly, f-θsg-open set). The family of all Fθsg-closed sets in X are denoted by $F\theta$SGC(X)and The family of all f-θsg-open sets in X are denoted by $F\theta$SGO(X)

Definition 2.4. [8] A function $f : X \to Y$ is said to be
(1) fuzzy θ-semi-generalized continuous(briefly, f-θsg-continuous) if $f^{-1}(\lambda)$ is f-θsg-closed in X for each fuzzy semi-closed set λ in Y.
(2)fuzzy θ-semi-generalized irresolute(briefly, f-θsg-irresolute) if $f^{-1}(\lambda)$ is f-θsg-closed in X for each f-θsg-closed set λ in Y.
(3) fuzzy θ-semi-generalized open(briefly, f-θsg-open) if $f(\lambda)$ of Y and for each f-θsg-open in Y for every fuzzy semi-open set λ in X.

3 Fuzzy Almost Contra θ-Semigeneralized-Continuous Functions

In this section, the notion of fuzzy almost contra θsg-continuous functions via f-θsg-closed set is introduced.

Definition 3.1. Let X and Y be fuzzy topological spaces. A fuzzy function $f : X \to Y$ is said to be fuzzy almost θ-semigeneralized-continuous (briefly, fuzzy almost contra θsg-continuous) if inverse image of each fuzzy regular open set in Y is f-θsg-closed in X.

Example 3.2. Let $X = Y = \{a, b, c\}$. A, B, C are fuzzy sets of X defined as A(a) =0, A(b) = 1, A(c) = 0, B(a) = 0, B(b)= 0, B(c)= 1 and C(a)=0, C(b)=1, C(c)=1 and D be a fuzzy set of Y defined as D(a)=1, D(b)=0, D(c)=0. Then $\tau = \{0, 1, A, B, C\}$and $\mu = \{0, 1, D\}$ be fuzzy toplogies on sets X and Y respectively. The identity function $f : X \to Y$ is fuzzy almost contra-θsg-continuous fucntion.

Theorem 3.3. For a function $f : X \to Y$, the following statements are equivalent:
(i) f is fuzzy almost contra θsg-continuous.
(ii) For every fuzzy regular closed set μ in Y, $f^{-1}(\mu)$ is f-θsg-open.
(iii) For each $x \in X$ and each fuzzy regular closed set λ in Y containing f(x), there exists a f-θsg-open set η in X containing x such that $f(\eta) \leq \lambda$.
(iv) For each $x \in X$ and fuzzy regular open set μ in Y containing f(x), there exists a f-θsg-open set ω in X containing x such that $f^{-1}(\mu) \leq \omega$.

Proof:$(i) \Rightarrow (ii)$. Let μ be a fuzzy regular closed set in Y, then Y-μ is fuzzy regular open set in Y. By (i) $f^{-1}(Y-\mu) = X - f^{-1}(\mu)$ is f-θsg-closed set in X. This implies that $f^{-1}(\mu)$ is f-θsg-open set in X. Therefore (ii) holds.
$(ii) \Rightarrow (i)$. Let G be a fuzzy regular open set of Y. Then Y-G be a fuzzy regular closed set in Y. By (ii) $f^{-1}(Y-G)$ is f-θsg-open set in X. This implies that $X - f^{-1}(G)$ is f-θsg-open in X, which implies $f^{-1}(G)$ is f-θsg-closed set in X. Therefore (i) holds.
$(ii) \Rightarrow (iii)$. Let λ be a fuzzy regular closed set of Y containing f(x). By (ii) $f^{-1}(\lambda)$ is f-θsg-open set in X and $x \in f^{-1}(\lambda)$. Take $\eta = f^{-1}(\lambda)$. Then $f(\eta) \leq \lambda$.
$(iii) \Rightarrow (ii)$. Let λ be a fuzzy regular closed set of Y and $x \in f^{-1}(\lambda)$. From (iii), there exists a f-θsg-open set η in X containing x suxh that $\eta \leq f^{-1}(\lambda)$. We have $f^{-1}(\lambda) = \vee_{x \in f^{-1}(\lambda)} \eta$. Thus $f^{-1}(\lambda)$ is f-θsg-open set in X.
$(iii) \Rightarrow (iv)$. Let μ be a fuzzy regular open set in Y not containing f(x). Then and $1-\mu$ is a fuzzy regular closed set containing f(x). By (iii), there exists a f-θsg-open set η in X containing x suxh that $f(\eta) \leq 1-\mu$. Hence $\eta \leq f^{-1}(1-\mu) \leq 1 - f^{-1}(\mu)$ and then $f^{-1}(\mu) \leq 1-\eta$. Take $\omega = 1-\eta$. Therefore we obtain that ω is a f-θsg-open set in X not containing x. The converse can be be shown easily.

Theorem 3.4. Let $f : X \to Y$ be a function and let $g : X \to X \times Y$ be the fuzzy graph function of f defined by $g(x_\in) = (x_\in, f(x_\in))$ for every $x_\in \in X$. If g is fuzzy almost contra θsg-continuous, then f is fuzzy almost contra θsg-continuous.

Proof: Let μ be a fuzzy regular closed set in Y, then $X \times \mu$ is fuzzy regular closed set in $X \times Y$. Since g is fuzzy almost contra θsg-continuous, then $f^{-1}(\mu) = g^{-1}(X \times \mu)$ is f-θsg-open in X. Thus, f is fuzzy almost contra θsg-continuous.

Definition 3.5. A fuzzy filter base Λ is said to be fuzzy θsg-convergent to a fuzzy singleton $x_\in$ in X if for any f-θsg-open set μ in X containing $x_\in$, there exists a fuzzy set $\eta \in \Lambda$ such that $\eta \leq \mu$.

Definition 3.6. A fuzzy filter base Λ is said to be fuzzy rc-convergent[5] to a fuzzy singleton $x_\in$ in X if for any fuzzy regular closed set μ in X containing $x_\in$, there exists a fuzzy set $\eta \in \Lambda$ such that $\eta \leq \mu$.

Theorem 3.7. If a function $f : X \to Y$ is fuzzy almost contra θsg-continuous, then for each fuzzy singleton $x_\in \in X$ and each filter base Λ in X fuzzy θsg-converging to $x_\in$, the fuzzy filter base $f(\Lambda)$ is fuzzy rc-convergent to $f(x_\in)$.

Proof: Let $x_\in \in X$ and Λ be any fuzzy filter base in fuzzy θsg-converging to $x_\in$. Since f is fuzzy almost contra θsg-continuous, then for any fuzzy regular closed set λ in Y containing $f(x_\in)$, there exists a Fθsg-open set $\mu \in X$ containing $x_\in$ such that $f(\mu) \leq \lambda$. Since Λ is fuzzy θsg-converging to $x_\in$, there exists a $A \in \Lambda$ such that $A \leq \mu$. This means that $f(A) \leq \mu$ and therefore the fuzzy filter base $f(\Lambda)$ is fuzzy rc-convergent to $f(x_\in)$.

4 Fuzzy θ-Semigeneralized-Connectedness

In this section we introduce fuzzy θ-semigeneralized-connected (briefly, FTSGO-connected) and fuzzy θ-semigeneralized-normal spaces.

Definition 4.1. A fuzzy topological space X is called Fuzzy θ-semigeneralized-connected(briefly,FTSGO-Connected) if X is not the union of two disjoint nonempty f-θsg-open sets.

Definition 4.2. A fuzzy topological space X is called fuzzy connected [7] if X is not the union of two disjoint nonempty fuzzy open sets.

Theorem 4.3. If $f : X \to Y$ is fuzzy almost contra θsg-continuous surjection and X is FTSGO-connected, then Y is fuzzy connected.

Proof: Suppose Y is not fuzzy connected. Then there exist nonempty disjoint fuzzy open sets μ_1 and μ_2 such that $Y = \mu_1 \wedge \mu_2$. Therefore,μ_1 and μ_2 are fuzzy clopen in Y. Since f is fuzzy almost contra θsg-continuous , $f^{-1}(\mu_1)$ and $f^{-1}(\mu_2)$ are f-θsg-open in in X. Moreover, $f^{-1}(\mu_1)$ and $f^{-1}(\mu_2)$ are nonempty disjoint and $X = f^{-1}(\mu_1) \wedge f^{-1}(\mu_2)$. This shows that X is not FTSGO-connected. This contradicts the fact that Y is not Fuzzy connected assumed. Hence Y is fuzzy connected.

Definition 4.4. A fuzzy space X is said to be fuzzy θsg-normal (briefly,f-θsg-normal)if every pair of nonempty disjoint fuzzy closed sets can be separated by disjoint f-θsg-open sets.

Definition 4.5. A fuzzy space X is said to be fuzzy strongly θsg-normal if every pair of nonempty disjoint fuzzy closed sets A and B there exist disjoint f-θsg-open sets U and V such that $A \leq U$, $B \leq V$ and $cl(A) \wedge cl(B) = \phi$.

Theorem 4.6. If Y is fuzzy strongly θsg-normal and $f : X \to Y$ is fuzzy almost contra θsg-continuous closed surjection, then X is f-θsg-normal.

Proof: Let A and B be disjoint nonempty fuzzy closed sets of X. Since f is injective and closed, f(A) and f(B) are disjoint fuzzy closed sets. Since Y is fuzzy strongly θsg-normal, then there exist f-θsg-open sets U and V such that $f(A) \leq U$ and $f(B) \leq V$ and $cl(A) \wedge cl(B) = \phi$. Then, since cl(A) and cl(B) are regular closed and f is fuzzy almost contra θsg-continuous, $f^{-1}(cl(A))$ and $f^{-1}(cl(B))$ are f-θsg-open sets. Since, $U \leq f^{-1}(cl(A))$, $V \leq f^{-1}(cl(B))$ and $f^{-1}(cl(A))$ and $f^{-1}(cl(B))$ are disjoint, X is f-θsg-normal.

Definition 4.7. A fuzzy space X is said to be fuzzy $\theta sg - T_1$ if for each pair of distinct fuzzy singletons x and y in X, there exist f-θsg-open sets U and V containing x and y respectively, such that $y \notin U$ and $x \notin V$.

Definition 4.8. A fuzzy space X is said to be fuzzy $\theta sg - T_2$ if for each pair of distinct fuzzy points x and y in X, there exist f-θsg-open set U containing x and f-θsg-open set V containinig y such that $U \wedge V = \phi$.

Theorem 4.9. If $f : X \to Y$ is a fuzzy almost contra θsg-continuous injection and Y is fuzzy Urysohn, then X is fuzzy $\theta sg - T_2$.

Proof: Let Y is fuzzy Urysohn. By the injectivity of f, it follows that $f(x) \neq f(y)$ for any distinct fuzzy singletons x and y in X. Since Y is fuzzy Urysohn, then there exist fuzzy open sets U and V such that $f(x) \in U$ and $f(y) \in V$ and $cl(U) \wedge cl(V) = \phi$. Since f is fuzzy almost contra θsg-continuous, then there exist fuzzy open sets W and Z in X containing x and y, respectively, such that $f(W) \leq cl(U)$ and $f(Z) \leq cl(V)$. Hence $W \wedge Z = \phi$. This shows that X is fuzzy $\theta sg - T_2$.

Definition 4.10. A fuzzy space X is said to be fuzzy weakly T_2 [5] if each element of X is an intersection of fuzzy regular closed sets.

Theorem 4.11. If $f : X \to Y$ is a fuzzy almost contra θsg-continuous injection and Y is fuzzy weakly T_2, then X is fuzzy $\theta sg - T_1$.

Proof: Suppose that Y is fuzzy weakly T_2. For any distinct points x and y in X, there exist fuzzy regular closed sets U , V in Y such that $f(x) \in U, f(y) \notin U$, $f(x) \notin V$ and $f(y) \in V$. Since f is fuzzy almost contra θsg-continuous , by Theorem 3.2(ii), $f^{-1}(U)$ and $f^{-1}(V)$ are f-θsg-open subsets of X such that $x \in f^{-1}(U), y \notin f^{-1}(U)$ and $x \notin f^{-1}(V), y \in f^{-1}(V)$. This shows that X is fuzzy $\theta sg - T_1$.

Definition 4.12. The fuzzy graph G(f) of a fuzzy function $f : X \to Y$ is said to be fuzzy strongly contra-θsg-closed if for each $(x, y) \in (X \times Y)$ - G(f), there exist a f-θsg-open set U in x containing x and a fuzzy regular closed set V in Y containing y, such that $(U \times V) \wedge G(f) = \phi$.

Lemma 4.13. The following properties are equivalent for the fuzzy graph G(f) of a fuzzy fuction f:
(i) G(f) is fuzzy strongly contra-θsg-closed.
(ii) For each $(x, y) \in (X \times Y)$-G(f), there exist a f-θsg-open set U in X containing x and a fuzzy regular closed set V containing y such that $f(U) \wedge V = \phi$.

Theorem 4.14. If $f : X \to Y$ is fuzzy almost contra θsg-continuous and Y is fuzzy Urysohn, G(f) is fuzzy strongly contra-θsg-closed set in $X \times Y$.

Proof: Let Y is fuzzy Urysohn. Let $(x, y) \in (X \times Y)$ G(f). It follows that $f(x) \neq y$. Sinc Y is fuzzy Urysohn, then there exist fuzzy open sets U and V such that $f(x) \in U, y \in V$ and $cl(U) \wedge cl(V) = \phi$. Since f is fuzzy almost contra θsg-continuous, then there exists a f-θsg-open set μ in X containing x such that $f(\mu) \leq cl(U)$. Therefore, $f(\mu) \wedge cl(V) = \phi$ and G(f) is fuzzy strongly contra-θsg-closed in $X \times Y$.

Theorem 4.15. Let $f : X \to Y$ is fuzzy strongly contra-θsg-closed graph. If f is injective, then X is fuzzy $\theta sg - T_1$.

Proof: Let x and y be any two distinct points of X. Then, we have $(x, f(y)) \in (X \times Y)$-G(f). By Lemma 4.13, there exist a f-θsg-open set μ containing x and a fuzyy regular closed set η in Y containing f(y) such that $f(\mu) \wedge \eta = \phi$;hence $\mu \wedge f^{-1}(\eta) = \phi$. Therefore, we have $y \notin \mu$. This implies that X is fuzzy $\theta sg - T_1$.

5 Fuzzy Weakly Almost Contra-θ-Semigeralized-Continuous Functions

In this section, Fuzzy weakly almost contra-θ-semigeneralized-continuous function is introduced. The relationships between fuzzy almost contra-θsg-continuous functions and other forms are investigated. Also introduced the concept of Fuzzy θ-semigeneralized-compact (briefly, FTSGO-Compact)space.

Definition 5.1. A function $f : X \to Y$ is called fuzzy weakly almost contra-θsg-continuous if for each $x \in X$ and each fuzzy regular closed set η of Y containing f(x), there exists f-θsg-open set μ in X containing x, such that $int(f(\mu)) \leq \eta$.

Definition 5.2. A function $f : X \to Y$ is called fuzzy(θsg,s)-open if the image of each f-θsg-open set is Fs-open.

Theorem 5.3. If a function $f : X \to Y$ is fuzzy weakly almost contra-θsg-continuous and fuzzy (θsg,s)-open, then f is fuzzy almost contra-θsg-continuous.

Proof: Let $x \in X$ and η be a fuzzy regular closed set containing f(x). Since f is fuzzy weakly almost contra-θsg-continuous, there exists a f-θsg-open set μ in X containig x such that $int(f(\mu)) \leq \eta$. Since f is fuzzy (θsg, s)-open, $f(\mu)$ is a Fs-open set in Y and $f(\mu) \leq cl(int(f(\mu))) \leq \eta$. This shows that f is fuzzy almost contra-θsg-continuous.

Definition 5.4 (5)**.** A fuzzy space is said to be fuzzy P_Σ if for any fuzzy open set μ of X and each $x_\Sigma \in mu$, there exists fuzzy regular closed set ρ containing $x_\in$ such that $x_\in \in \rho \leq \mu$.

Theorem 5.5. Let $f : X \to Y$ ba a fuzzy function. Then, if f is fuzzy almost contra-θsg-continuous and Y is fuzzy P_Σ, then f is fuzzy almost contra-θsg-continuous.

Proof: Let μ be a fuzzy ope set in Y. Since Y is fuzzy P_Σ, there exists a family Ψ whose members are fuzzy regular closed set of Y such that $\mu = \wedge\{\rho : \rho \in \Psi\}$. Since f is fuzzy almost contra-θsg-continuous, $f^{-1}(\rho)$ is f-θsg-open in X for each $\rho \in \Psi$ and $f^{-1}(\mu)$ is f-θsg-open in X. Therefore, f is fuzzy almost contra-θsg-continuous.

Definition 5.6 (5)**.** A fuzzy space is said to be fuzzy weakly P_Σ if for any fuzzy regular open set μ of X and each $x_\Sigma \in mu$, there exists fuzzy regular closed set ρ containing $x_\in$ such that $x_\in \in \rho \leq \mu$.

Definition 5.7. A function $f : X \to Y$ is said to be fuzzy almost θsg-continuous at $x_\in \in \mu$ if for each fuzzy open set η containing $f(x_\in)$, there exists a f-θsg-open set μ containing $x_\in$ such that $f(\mu) \leq int(cl(\eta))$.

Theorem 5.8. Let $f : X \to Y$ be a fuzzy almost contra-θsg-continuous function. If Y is fuzzy weakly P_Σ, then f is fuzzy almost θsg-continuous.

Proof: Let μ be any fuzzy regular open set of Y. Since Y is fuzzy weakly P_Σ, there exists a family Ψ whose members are fuzzy regular closed set of Y such that $\mu = \wedge\{\rho : \rho \in \Psi\}$. Since f is fuzzy almost contra-θsg-continuous, $f^{-1}(\mu)$ is f-θsg-open in X. Hence, f is fuzzy almost θsg-continuous.

Theorem 5.9. Let X, Y, Z be fuzzy topological spaces and let $f : X \to Y$ and $g : Y \to Z$ be fuzzy functions. If f is fuzzy θsg-irresolute and g is fuzzy almost contra-θsg-continuous, then $g \circ f : X \to Z$ is a fuzzy almost contra-θsg-continuous function.

Proof: Let $\mu \leq Z$ be any fuzzy regular closed set and let $(g \circ f)(x_\in) \in \mu$. Then $g(f(x_\in)) \in \mu$. Since g is fuzzy almost contra-θsg-continuous fuction, it follows that there exists a f-θsg-open set ρ containing $f(x_\in)$ such that $g(\rho) \leq \mu$. Since f is fuzzy θsg-irresolute fuction, it follows that there exists a f-θsg-open set η containiing $x_\in$ such that $f(\eta) \leq \rho$. From here we obtain that $(g \circ f)(\eta) = g(f(\eta)) \leq \ g(\rho) \leq \mu$. Thus we have shown that $g \circ f$ is fuzzy almost contra-θsg-continuous function.

Theorem 5.10. If $f : X \to Y$ is a surjective fuzzy θsg-open function and $g : Y \to Z$ is a fuzzy function such that $g \circ f : X \to Z$ is fuzzy almost contra-θsg-continuous, then g is fuzzy almost contra-θsg-continuous.

Proof: Suppose that $x_\in$ is a fuzzy singleton in X. Let η be regular closed set in Z containing $(g \circ f)(x_\in$. Then there exists a f-θsg-open set μ in X containing $x_\in$ such that $g(f(\mu)) \leq \eta$. Since f is f-θsg-open, $f(\mu)$ is a f-θsg-open set in Y containing $f(x_\in)$ such that $g(f(\mu)) \leq \eta$. This implies that g is fuzzy almost contra-θsg-continuous.

Corollary 5.11. If $f : X \to Y$ be a surjective f-θsg-irresolute and f-θsg-open function and let $g : Y \to Z$ is a fuzzy function. Then $g \circ f : X \to Z$ is fuzzy almost contra-θsg-continuous if and only if g is fuzzy almost contra-θsg-continuous.

Proof: It can be obtained from Theorem 5.9 and Theorem 5.10.

Definition 5.12. A space X is said to be fuzzy θsg-compact(briefly, FTSGO-Compact) if every f-θsg-open cover of X has a finite subcover.

Definition 5.13. A space X is said to be fuzzy θsg-closed-compact if every f-θsg-closed cover of X has a finite subcover.

Definition 5.14 (6)**.** A space X is said to be fuzzy nearly compact if every fuzzy regular open cover of X has a finite subcover.

Theorem 5.15. The fuzzy almost contra-θsg-continuous images of fuzzy θsg-closed-compact spaces are fuzzy nearly compact.

Proof: Suppose that $f : X \to Y$ is a fuzzy almost contra-θsg-continuous surjection. Let $\{\eta_i : i \in I\}$ be any fuzzy regular open cover of Y. Since f is fuzzy almost contra-θsg-continuous, then $\left\{f^{-1}(\eta_i) : i \in I\right\}$ uzzy is a f-θsg-closed cover of X. Since X is fuzzy θsg-closed-compact, there exists a finite subset I_o of I such that $X = \wedge \left\{f^{-1}(\eta_i) : i \in I_o\right\}$. Thus, we have $Y = \wedge \{\eta_i : i \in I_o\}$ and Y is nearly compact.

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