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Original Article

ON SOME IDEALS OF INTUITIONISTIC FUZZY POINTS SEMIGROUPS

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Abstract – In this paper, the minimal ideal A of a semigroup S is characterized by the intuitionistic characteristic function χ_A . The existence of an intuitionistic fuzzy kernel in a semigroup is explored. Finally, we consider the semigroup \underline{S} of the intuitionistic fuzzy points of a semigroup S and discuss some relations between some ideals A of S and the subset \underline{C}_A of the semigroup \underline{S} .

Keywords – Semigroups; Intuitionistic fuzzy points; Intuitionistic fuzzy ideals.

1 Introduction

Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis. Zadeh [16] introduced the concept of a fuzzy set for the first time and this concept was applied by Rosenfeld [14] to define fuzzy subgroups and fuzzy ideals. Based on this crucial work, Kuroki [9,10,11,12] defined a fuzzy semigroup and various kinds of fuzzy ideals in semigroups and characterized them. In [13], Kim considered the semigroup \underline{S} of the fuzzy points of a semigroup S , and discussed the relation between the fuzzy interior ideals and the subsets of \underline{S} , also see [6, 7]. Atanassov [4, 5] introduced the notion of intuitionistic fuzzy sets as a generalization of fuzzy sets. Many concepts in fuzzy set theory were also extended to intuitionistic fuzzy set theory, such as intuitionistic fuzzy relations, intuitionistic L- fuzzy sets, intuitionistic fuzzy implications, intuitionistic fuzzy logics, intuitionistic fuzzy semigroups etc. Jun and Song [8] introduced the notion of intuitionistic fuzzy points. In [15] Sardar et al., defined some relations between the intuitionistic fuzzy ideals of a semigroup S and the set of all intuitionistic fuzzy points of S . In [3] Akram characterized intuitionistic fuzzy ideals in ternary semigroups by intuitionistic fuzzy points. Also in [2] he analyzed some relations between the intuitionistic fuzzy Γ -ideals and the sets of intuitionistic fuzzy points of these Γ -ideals of a Γ -semigroup. In this paper, we consider the

semigroup \underline{S} of the intuitionistic fuzzy points of a semigroup S , and discuss some relations between some ideals A of S and the subset \underline{C}_A of the semigroup \underline{S} .

2 Basic Definitions and Results

Let S be a semigroup. A nonempty subset A of S is called a *left (resp., right) ideal* of S if $SA \subseteq A$ (resp., $AS \subseteq A$), and a *two-sided ideal* (or simply *ideal*) of S if A is both a left and a right ideal of S . A nonempty subset A of S is called an *interior ideal* of S if $A^2 \subseteq A$. An ideal A of S is called *minimal ideal* of S if A does not properly contains any other ideal of S . If the intersection K of all the ideals of a semigroup S is nonempty then we shall call K the *kernel* of S . A sub-semigroup A of S is called a *bi-ideal* of S if $ASA \subseteq A$. A function f from S to the closed interval $[0, 1]$ is called a *fuzzy set* in S . The semigroup S itself is a fuzzy set in S such that $S(x) = 1$ for all $x \in S$, denoted also by S . Let A and B be two fuzzy sets in S . Then the inclusion relation $A \subseteq B$ is defined $A(x) \leq B(x)$ for all $x \in S$. $A \cap B$ and $A \cup B$ are fuzzy sets in S defined by

$$(A \cap B)(x) = A(x) \wedge B(x) = \min\{A(x), B(x)\}$$

$$(A \cup B)(x) = A(x) \vee B(x) = \max\{A(x), B(x)\}$$

for all $x \in S$.

For any $\alpha \in (0, 1]$ and $x \in S$, a fuzzy set x_α in S is called a *fuzzy point* in S if

$$x_\alpha(y) = \begin{cases} \alpha & \text{if } x = y, \\ 0 & \text{otherwise,} \end{cases}$$

for all $x \in S$. The fuzzy point x_α is said to be contained in a fuzzy set A , denoted by $x_\alpha \in A$, iff $\alpha \leq A(x)$.

Definition 1. [4, 5] The intuitionistic fuzzy sets (IFS, for short) defined on a non-empty set X as objects having the form

$$A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \},$$

where the functions $\mu_A: X \rightarrow [0, 1]$ and $\gamma_A: X \rightarrow [0, 1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in X$.

For the sake of simplicity, we shall use $A = (\mu_A, \gamma_A)$ for intuitionistic fuzzy set $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \}$.

Definition 2. [15] Let $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point, written as $x_{(\alpha, \beta)}$ is defined to be an intuitionistic fuzzy subset of S , given by

$$x_{(\alpha, \beta)}(y) = \begin{cases} (\alpha, \beta) & \text{if } x = y \\ (0, 1) & \text{otherwise} \end{cases}$$

Definition 3. [15] A non-empty IFS $A = (\mu_A, \gamma_A)$ of a semigroup S is called an intuitionistic fuzzy subsemigroup of S if

- (i) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y), \forall x, y \in S,$
- (ii) $\gamma_A(xy) \leq \gamma_A(x) \vee \gamma_A(y), \forall x, y \in S.$

Definition 4. [15] An intuitionistic fuzzy subsemigroup $A = (\mu_A, \gamma_A)$ of a semigroup S is called an intuitionistic fuzzy interior ideal of S if

- (i) $\mu_A(xay) \geq \mu_A(x) \forall x, a, y \in S,$
- (ii) $\gamma_A(xay) \leq \gamma_A(a) \forall x, a, y \in S.$

Definition 5. [15] An intuitionistic fuzzy subsemigroup $A = (\mu_A, \gamma_A)$ of a semigroup S is called an intuitionistic fuzzy bi-ideal of S if

- (i) $\mu_A(xwy) \geq \mu_A(x) \wedge \mu_A(y), \forall x, w, y \in S,$
- (ii) $\gamma_A(xwy) \leq \gamma_A(x) \vee \gamma_A(y) \forall x, w, y \in S.$

Definition 6. [15] A non-empty IFS $A = (\mu_A, \gamma_A)$ of a semigroup S is called an intuitionistic fuzzy left (right) ideal of S if

- (i) $\mu_A(xy) \geq \mu_A(y)$ (resp. $\mu_A(xy) \geq \mu_A(x)$) $\forall x, y \in S,$
- (ii) $\gamma_A(xy) \leq \gamma_A(y)$ (resp. $\gamma_A(xy) \leq \gamma_A(x)$) $\forall x, y \in S.$

Definition 7. [15] A non-empty IFS $A = (\mu_A, \gamma_A)$ of a semigroup S is called an intuitionistic fuzzy two-sided ideal or an intuitionistic fuzzy ideal of S if it is both an intuitionistic fuzzy left and an intuitionistic fuzzy right ideal of S .

Let A be a subset of a semigroup S and A^c be the complement of A . $C_A = (C_A, C_{A^c})$ is defined as:

$$C_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{otherwise,} \end{cases} \quad C_{A^c}(x) = \begin{cases} 0 & \text{if } x \in A, \\ 1 & \text{otherwise,} \end{cases}$$

for all $x \in S$.

Let $\mathcal{IF}(S)$ be the set of all intuitionistic fuzzy sets in a semigroup S . For each $A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \mathcal{IF}(S)$, the product of A and B is an intuitionistic fuzzy set $A \circ B$ defined as follows:

$$A \circ B = \{ \langle x, \mu_{A \circ B}(x), \gamma_{A \circ B}(x) \rangle : x \in S \},$$

where

$$\mu_{A \circ B}(x) = \begin{cases} \bigvee_{x=uv} \mu_A(u) \wedge \mu_B(v) & \text{if } x = uv \\ 0 & \text{otherwise.} \end{cases}$$

$$\gamma_{A \circ B}(x) = \begin{cases} \bigwedge_{x=uv} \gamma_A(u) \vee \gamma_B(v) & \text{if } uv = x \\ 1 & \text{otherwise.} \end{cases}$$

Lemma 1. [1] For any nonempty subsets A and B of a semigroup S , we have $A \subseteq B$ if and only if $\chi_A \subseteq \chi_B$.

Lemma 2. [1] Let A be a nonempty subset of a semigroup S , then A is an ideal of S if and only if χ_A is an intuitionistic fuzzy ideal of S .

Theorem 1. A nonempty subset A of a semigroup S is a minimal ideal if and only if χ_A is a minimal intuitionistic fuzzy ideal of S .

Proof. Let A be a minimal ideal of S , then by lemma 2., χ_A is an intuitionistic fuzzy ideal of S . Suppose that χ_A is not minimal intuitionistic fuzzy ideal of S , then there exists some intuitionistic fuzzy ideal χ_B of S such that $\chi_B \subseteq \chi_A$. Hence, lemma 1 implies that $B \subseteq A$, where B is an ideal of S . This is a contradiction to the fact that A is minimal ideal of S . Thus χ_A is minimal intuitionistic fuzzy ideal of S . Conversely, let χ_A be a minimal intuitionistic fuzzy ideal of S , then A is an ideal of S . Suppose that A is not minimal ideal of S , then there exists some ideal B of S such that $B \subseteq A$. Now by lemma 1, $\chi_B \subseteq \chi_A$ where χ_B is an intuitionistic fuzzy ideal of S . This contradicts that χ_A is a minimal intuitionistic fuzzy ideal of S . Thus A is minimal ideal of S . ■

Lemma 3. If $A = (\mu_A, \gamma_A)$ is a minimal intuitionistic fuzzy ideal of a semigroup S , then A is the intuitionistic fuzzy kernel of S .

Proof. Let $B = (\mu_B, \gamma_B)$ be any intuitionistic fuzzy ideal of S , then $B \circ A \subseteq B \cap A$. Since $B \cap A$ is an intuitionistic fuzzy ideal of S and $B \cap A \subseteq A$, it follows that $B \cap A = A$. But then $A = B \cap A \subseteq B$, so A is contained in every intuitionistic fuzzy ideal of S and hence is an intuitionistic fuzzy kernel of S . ■

Lemma 4. If $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy kernel of a semigroup S , then A is a simple intuitionistic fuzzy subsemigroup of S .

Proof. Since A is an intuitionistic fuzzy ideal of S , so A is an intuitionistic fuzzy subsemigroup of S . To show that A is simple, let B be any intuitionistic fuzzy ideal of A , then $A \circ B \circ A$ is an intuitionistic fuzzy ideal of S , since

$$S \circ (A \circ B \circ A) = (S \circ A) \circ B \circ A \subseteq A \circ B \circ A$$

and

$$(A \circ B \circ A) \circ S = A \circ B \circ (A \circ S) \subseteq A \circ B \circ A$$

Also, $A \circ B \circ A \subseteq A \circ A \subseteq A$, but by lemma 3, A is minimal intuitionistic fuzzy ideal of S . Hence $A \circ B \circ A = A$. Also, $A \circ B \circ A \subseteq B \circ A \subseteq B$, which implies that $A \subseteq B$. Thus $A = B$, that is A is simple subsemigroup of S . ■

Lemma 5. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy left ideal of a semigroup S and $x_{(\alpha, \beta)}$ be any intuitionistic fuzzy point of S , then $A \circ x_{(\alpha, \beta)}$ is a minimal intuitionistic fuzzy left ideal of S .

Proof. $A \circ x_{(\alpha, \beta)}$ is an intuitionistic fuzzy left ideal of S , since $S \circ (A \circ x_{(\alpha, \beta)}) = (S \circ A) \circ x_{(\alpha, \beta)} \subseteq A \circ x_{(\alpha, \beta)}$. Suppose B is an intuitionistic fuzzy left ideal of $A \circ x_{(\alpha, \beta)}$ and let $D = \{y_{(\gamma, \delta)} \in A : y_{(\gamma, \delta)} \circ x_{(\alpha, \beta)} \subseteq B\}$. Let $z_{(\sigma, \tau)}$ be in S and $y_{(\gamma, \delta)}$ be in D , then $z_{(\sigma, \tau)} \circ y_{(\gamma, \delta)} \in A$ and so $z_{(\sigma, \tau)} \circ y_{(\gamma, \delta)} \circ x_{(\alpha, \beta)} \subseteq B$. Hence $z_{(\sigma, \tau)} \circ y_{(\gamma, \delta)} \in D$, which implies that $S \circ D \subseteq D$. Thus D is an intuitionistic fuzzy left ideal of S contained in A and because of minimality of A , we get $D = A$. Thus for all $y_{(\gamma, \delta)} \in A$, $y_{(\gamma, \delta)} \circ x_{(\alpha, \beta)} \in B$, which implies that $A \circ x_{(\alpha, \beta)} \subseteq B$. Hence $A \circ x_{(\alpha, \beta)} = B$ and therefore, $A \circ x_{(\alpha, \beta)}$ is a minimal intuitionistic fuzzy left ideal of S . ■

3 Main Results

If S is a semigroup, then $\mathcal{IF}(S)$ is a semigroup with the product " \circ " [15]. Let \underline{S} be the set of all intuitionistic fuzzy points in a semigroup S . Then $x_{(\alpha, \beta)} \circ y_{(\gamma, \delta)} = (xy)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in \underline{S}$, for $x_{(\alpha, \beta)}, y_{(\gamma, \delta)} \in \underline{S}$ and $y_{(\gamma, \delta)} \circ (x_{(\sigma, \tau)} \circ z_{(\gamma, \delta)}) = (y_{(\alpha, \beta)} \circ x_{(\sigma, \tau)}) \circ z_{(\gamma, \delta)}$. Thus \underline{S} is a subsemigroup of $\mathcal{IF}(S)$ [15]. For any $A \in \mathcal{IF}(S)$, \underline{A} denotes the set of all intuitionistic fuzzy points contained in A , that is, $\underline{A} = \{x_{(\alpha, \beta)} \in \underline{S} : \mu_A(x) \geq \alpha, \gamma_A(x) \leq \beta\}$. For any $\underline{A}, \underline{B} \in \underline{S}$, we define the product of \underline{A} and \underline{B} as $\underline{A} \circ \underline{B} = \{x_{(\alpha, \beta)} \circ y_{(\gamma, \delta)} : x_{(\alpha, \beta)} \in \underline{A}, y_{(\gamma, \delta)} \in \underline{B}\}$.

Lemma 6. [15] Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be two intuitionistic fuzzy subsets of a semigroup S , then

- 1) $\underline{A} \cup \underline{B} = \underline{A \cup B}$
- 2) $\underline{A} \cap \underline{B} = \underline{A \cap B}$
- 3) $\underline{A} \circ \underline{B} \subseteq \underline{A \circ B}$.

Lemma 7. Let A be nonempty subset of a semigroup S , we have $x_{(\alpha, \beta)} \in \underline{X_A}$ if and only if $x \in A$.

Proof. Suppose that $x_{(\alpha, \beta)} \in \underline{X_A}$ for any $x \in S$, then $C_A(x) \geq \alpha$. Hence $C_A(x) = 1$ for any $\alpha > 0$, which implies that $x \in A$. Conversely, Let $x \in A$, then $C_A(x) = 1 \geq \alpha$ and $C_A(x) = 0 < \beta$ for any $\alpha, \beta > 0$. This means that $x_{(\alpha, \beta)} \in \underline{X_A}$. ■

Lemma 8. For any nonempty subsets A and B of a semigroup S , we have

- 1) $A \subseteq B$ if and only if $\underline{X_A} \subseteq \underline{X_B}$.
- 2) $\underline{X_A} \subseteq \underline{X_B}$ if and only if $\underline{X_A} \subseteq \underline{X_B}$.

Proof. (1) Assume that $A \subseteq B$ and let $x_{(\alpha, \beta)} \in \underline{X_A}$. By lemma 7, $x \in A \subseteq B$ and $x_{(\alpha, \beta)} \in \underline{X_B}$, this implies that $\underline{X_A} \subseteq \underline{X_B}$. Conversely, suppose that $\underline{X_A} \subseteq \underline{X_B}$. Let $x \in A$, then by lemma 7, for any $\alpha, \beta > 0$ $x_{(\alpha, \beta)} \in \underline{X_A}$ and $x_{(\alpha, \beta)} \in \underline{X_B}$ which implies that $x \in B$. (2) it is obvious that if $\underline{X_A} \subseteq \underline{X_B}$ then $\underline{X_A} \subseteq \underline{X_B}$. Now assume that $\underline{X_A} \subseteq \underline{X_B}$ and let $x_{(\alpha, \beta)} \in \underline{X_A} \subseteq \underline{X_B}$, then $A \subseteq B$ and consequently, we have $\underline{X_A} \subseteq \underline{X_B}$. This completes the proof. ■

Lemma 9. Let A be a nonempty subset of a semigroup S . Then A is an ideal of S if and only if $\underline{X_A}$ is an ideal of \underline{S} .

Proof. By lemma 2, A is an ideal of S if and only if χ_A is a fuzzy ideal of S . And from theorem 3.5[13], χ_A is a fuzzy ideal of S if and only if $\underline{\chi}_A$ is an ideal of \underline{S} . ■

Theorem 2. A nonempty subset A of a semigroup S is minimal ideal if and only if χ_A is a minimal intuitionistic fuzzy ideal of S .

Proof. Let A be a minimal ideal of S , then by lemma 2, χ_A is an intuitionistic fuzzy ideal of S . Suppose that χ_A is not minimal intuitionistic fuzzy ideal of S , then there exists some intuitionistic fuzzy ideal χ_B of S such that $\chi_B \subseteq \chi_A$. Hence, lemma 1 implies that $B \subseteq A$, where B is an ideal of S . This is a contradiction to the fact that A is minimal ideal of S . Thus χ_A is minimal intuitionistic fuzzy ideal of S . Conversely, let χ_A be a minimal intuitionistic fuzzy ideal of S , then by lemma, A is an ideal of S . Suppose that A is not minimal ideal of S , then there exists some ideal B of S such that $B \subseteq A$. Now by lemma, $\chi_B \subseteq \chi_A$ where χ_B is an intuitionistic fuzzy ideal of S . This contradicts that χ_A is a minimal intuitionistic fuzzy ideal of S . Thus A is minimal ideal of S . ■

Theorem 3. Let A be a nonempty subset of a semigroup S . Then A is a minimal ideal of S if and only if $\underline{\chi}_A$ is a minimal ideal of \underline{S} .

Proof. By theorem 1, A is a minimal ideal of S if and only if χ_A is an intuitionistic fuzzy minimal ideal of S . We only need to prove that, χ_A is a minimal intuitionistic fuzzy ideal of S if and only if $\underline{\chi}_A$ is a minimal ideal of \underline{S} . Let χ_A be a minimal intuitionistic fuzzy ideal of S , then $\underline{\chi}_A$ is an ideal of \underline{S} . Suppose that $\underline{\chi}_A$ is not minimal, then there exists some ideals $\underline{\chi}_B$ of \underline{S} such that $\underline{\chi}_B \subseteq \underline{\chi}_A$ which implies that $\chi_B \subseteq \chi_A$, where χ_B is an intuitionistic fuzzy ideal of S . This is a contradiction to χ_A is a minimal intuitionistic fuzzy ideal of S . Thus $\underline{\chi}_A$ is a minimal ideal of \underline{S} . Conversely, assume that $\underline{\chi}_A$ is a minimal ideal of \underline{S} and that χ_A is not a minimal intuitionistic fuzzy ideal of S . Then there exists an intuitionistic fuzzy ideal χ_B of S such that $\chi_B \subseteq \chi_A$. Hence $\underline{\chi}_B \subseteq \underline{\chi}_A$, where $\underline{\chi}_B$ is an ideal of \underline{S} . This contradicts that $\underline{\chi}_A$ is a minimal ideal of \underline{S} . This completes the proof of the theorem. ■

Theorem 4. Let A be a nonempty subset of a semigroup S . Then A is the kernel of S if and only if $\underline{\chi}_A$ is the kernel of \underline{S} .

Proof. Suppose that A is the kernel of S , then $A = \bigcap_i I_i$ where I_i is an ideal of S . Let $\underline{\chi}_B$ be an ideal of \underline{S} , then B is an ideal of S . Now we need to show that, $\underline{\chi}_A \subseteq \underline{\chi}_B$. Let $x_{(\alpha, \beta)} \in \underline{\chi}_A$, by lemma 7, $x \in A$ and also $x \in B$, since A is the kernel of S . This implies that $x_{(\alpha, \beta)} \in \underline{\chi}_B$ and hence, $\underline{\chi}_A$ is the kernel of \underline{S} . Conversely, Let $\underline{\chi}_A$ be the kernel of \underline{S} , then $\underline{\chi}_A \subseteq \underline{\chi}_B$, for every ideal $\underline{\chi}_B$ of \underline{S} . Thus $A \subseteq B$ and therefore, A is the kernel of S . ■

The following lemma weakens the condition of theorem 4.

Lemma 10. Let A be a minimal ideal of a semigroup S , then $\underline{\chi}_A$ is the kernel of \underline{S} .

Proof. Since A is a minimal ideal of S , then χ_A is a minimal intuitionistic fuzzy ideal of S . Also lemma 3 implies that χ_A is the fuzzy kernel of S . Now, let χ_B be an intuitionistic fuzzy

ideal of \underline{S} , then we have $\underline{X}_A \subseteq \underline{X}_B$ and hence $\underline{X}_A \subseteq \underline{X}_B$. So \underline{X}_A is a minimal ideal contained in every ideal of \underline{S} . Thus \underline{X}_A is the kernel of \underline{S} . ■

Theorem 5. Let A be a nonempty subset of a semigroup S . Then A is an interior ideal of S if and only if \underline{X}_A is an interior ideal of \underline{S} .

Proof. Let A be an interior ideal of S , and let $y_{(\alpha,\beta)}, z_{(\gamma,\delta)} \in \underline{S}$ and $x_{(\sigma,\tau)} \in \underline{X}_A$. Since $x \in A$, then $y_{(\alpha,\beta)} \circ x_{(\sigma,\tau)} \circ z_{(\gamma,\delta)} = (y_{\alpha\beta})_{(\alpha\wedge\sigma\wedge\gamma, \beta\vee\tau\vee\delta)} \in \underline{X}_A$. This implies that $\underline{S} \circ \underline{X}_A \circ \underline{S} \subseteq \underline{X}_A$, thus \underline{X}_A is an interior ideal of \underline{S} . Conversely, suppose that \underline{X}_A is an interior ideal of \underline{S} . Let $y, z \in S$ and $x \in A$, then $x_{(\sigma,\tau)} \in \underline{X}_A$. Assume that, $y_{(\alpha,\beta)} \circ x_{(\sigma,\tau)} \circ z_{(\gamma,\delta)} = (y_{\alpha\beta})_{(\alpha\wedge\sigma\wedge\gamma, \beta\vee\tau\vee\delta)} \in \underline{S} \circ \underline{X}_A \circ \underline{S} \subseteq \underline{X}_A$, then $yxz \in A$. This implies that $SAS \subseteq A$, and hence A is an interior ideal of S . ■

Theorem 6. Let A be a nonempty subset of a semigroup S . Then A is a bi-ideal of S if and only if \underline{X}_A is a bi-ideal of \underline{S} .

Proof. Let A be a bi-ideal of S , and let $y_{(\alpha,\beta)}, z_{(\gamma,\delta)} \in \underline{X}_A$ and $x_{(\sigma,\tau)} \in \underline{S}$. Since $y, z \in A$ and $yxz \in A$ then

$$y_{(\alpha,\beta)} \circ x_{(\sigma,\tau)} \circ z_{(\gamma,\delta)} = (y_{\alpha\beta})_{(\alpha\wedge\sigma\wedge\gamma, \beta\vee\tau\vee\delta)} \in \underline{X}_A.$$

This implies that $\underline{X}_A \circ \underline{S} \circ \underline{X}_A \subseteq \underline{X}_A$, thus \underline{X}_A is a bi-ideal of \underline{S} . Conversely, suppose that \underline{X}_A is a bi-ideal of \underline{S} . Let $y, z \in A$ and $x \in S$, then $y_{(\alpha,\beta)}, z_{(\gamma,\delta)} \in \underline{X}_A$ by lemma 7. Assume that, $y_{(\alpha,\beta)} \circ x_{(\sigma,\tau)} \circ z_{(\gamma,\delta)} = (y_{\alpha\beta})_{(\alpha\wedge\sigma\wedge\gamma, \beta\vee\tau\vee\delta)} \in \underline{X}_A \circ \underline{S} \circ \underline{X}_A \subseteq \underline{X}_A$, then $yxz \in A$. This implies that $ASA \subseteq A$, and hence A is a bi-ideal of S . ■

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