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On αrω-Homeomorphisms in Topological Spaces

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Abstract - A bijection $f:(X, \tau)\rightarrow(Y, \sigma)$ is called αrω-homeomorphism if f and f^{-1} are αrω-continuous. Also we introduce new class of maps, namely αrω*c*-homeomorphisms which form a subclass of αrω-homeomorphisms. This class of maps is closed under composition of maps. We prove that the set of all αrω*c*-homeomorphisms forms a group under the operation composition of maps.

Keywords - *αrω-closed maps, αrω*-closed maps and αrω-open maps, αrω*-open maps, αrω-homeomorphism, αrωc-homeomorphism.*

1 introduction

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional Analysis. Closed and open mappings are one such mappings which are studied for different types of closed sets by various mathematicians for the past many years. Levine [16] introduced the notion of generalized closed sets. After him different mathematicians worked and studied on different versions of generalized closed sets and related topological properties.

Generalized Homeomorphisms, wgrα-homeomorphisms, rgα-homeomorphisms, rps-homeomorphisms and gs and sg homeomorphisms have been introduced and studied by Maki et al. [19], Sakthivel and Uma [25], Vadivel and Vairamanickam [30], Mary and Thangavelu [27], Devi et al. [9] respectively.

We give the definitions of some of them which are used in our present study. In this paper, we introduce the concept of αrω-homeomorphism and study the relationship between homeomorphisms, wgrα-homeomorphisms, rgα-homeomorphisms, rps-homeomorphisms, w-homeomorphisms, g-homeomorphisms and rwg-homeomorphisms. Also we introduce

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new class of maps αωc-homeomorphisms which form a subclass of αrω-homeomorphisms. This class of maps is closed under composition of maps. We prove that the set of all αrω*c*-homeomorphisms forms a group under the operation composition of maps.

2. Preliminaries

Throughout this paper (X,τ) and (Y,σ) (or simply X and Y) always denote topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. X\A or A^c denotes the complement of A in X.

We recall the following definitions and results.

Definition 2.1 A subset A of a topological space (X, τ) is called

(i) semi-open set [17] if A ⊆ cl(int(A)) and semi-closed set if int(cl(A)) ⊆ A.
(ii) pre-open set [21] if A ⊆ int(cl(A)) and pre-closed set if cl(int(A)) ⊆ A.
(iii) α-open set [13] if A ⊆ int(cl(int(A))) and α -closed set if cl(int(cl(A)))⊆ A.
(iv) semi-pre open set [2] (=β-open[1] if A⊆cl(int(cl(A)))) and a semi-pre closed set (=β-closed) if int(cl(int(A)))⊆A.
(v) regular open set [28] if A = int(clA)) and a regular closed set if A = cl(int(A)).
(vi) Regular semi open set [8] if there is a regular open set U such that U ⊆ A ⊆ cl(U).
(vii) Regular α-open set[31] (briefly,rα-open) if there is a regular open set U such that U ⊆A ⊆αcl(U).

Definition 2.2 A subset A of a topological space (X, τ) is called

(i) regular generalized α-closed set (briefly, rgα-closed)[31] if αcl (A)⊆U whenever A⊆ U and U is regular α-open in X.
(ii) generalized closed set(briefly g-closed) [16] if cl(A)⊆U whenever A⊆U and U is open in X.
(iii) generalized semi-closed set(briefly gs-closed)[4] if scl(A) ⊆ U whenever A⊆U and U is open in X.
(iv) generalized semi pre regular closed (briefly gspr-closed) set[24] if spcl(A)⊆U whenever A⊆U and U is regular open in X.
(v) strongly generalized closed set [24]](briefly,g*-closed) if cl(A)⊆U whenever A⊆U and U is g-open in X.
(vi) α-generalized closed set(briefly αg-closed)[20] if αcl(A)⊆U whenever A⊆U and U is open in X.
(vii) ω-closed set[29] if cl(A)⊆U whenever A⊆U and U is semi-open in X.
(viii) weakely generalized closed set(briefly, wg-closed)[23] if cl(int(A))⊆U whenever A⊆U and U is open in X.
(ix) regular weakly generalized closed set (briefly, rwg-closed)[23] if cl(int(A))⊆U whenever A⊆U and U is regular open in X.
(x) semi weakly generalized closed set (briefly, swg-closed)[23] if cl(int(A))⊆U whenever A⊆U and U is semi open in X.
(xi) generalized pre closed (briefly gp-closed) set [18] if pcl(A) ⊆U whenever A⊆U and U is open in X.

(xii) regular ω-closed (briefly rω -closed) set [5] if cl(A) ⊆ U whenever A⊆U and U is regular semi-open in X.
(xiii) g*-pre closed (briefly g*p-closed) [32] if pcl(A) ⊆ U whenever A⊆ U and U is g-open in X
(xiv) generalized regular closed (briefly gr–closed)set[7] if rcl(A)⊆ U whenever A⊆ U and U is open in X.
(xv) regular generalized weak (briefly rgw-closed) set[22] if cl(int(A))⊆U whenever A⊆U and U is regular semi open in X.
(xvi) weak generalized regular–α closed (briefly wgrα-closed) set[14]if cl(int(A)⊆U whenever A⊆ U and U is regular α-open in X.
(xvii) regular pre semi–closed (briefly rps-closed) set [26]if spcl(A)⊆U whenever A⊆U and U is rg- open in X.
(xviii) generalized pre regular weakly closed (briefly gprw-closed) set [15] if pcl(A)⊆U whenever A⊆ U and U is regular semi- open in X.
(xix) α-generalized regular closed (briefly αgr-closed) set [33] if αcl(A)⊆U whenever A⊆U and U is regular open in X.
(xx) R*-closed set [12] if rcl(A)⊆U whenever A⊆U and U is regular semi- open in X.
(xxi) generalized pre regular closed set(briefly gpr-closed)[11] if pcl(A)⊆U whenever A⊆U and U is regular open in X.
(xxii) ωα- closed set [6] if αcl(A)⊆U whenever A⊆ U and U is ω-open in X.
(xxiii) α regular ω- closed (briefly αrω -closed) set[37] if αcl(A) ⊆ U whenever A⊆U and U is rw-open in X.

The compliment of the above mentioned closed sets are their open sets respectively.

Definition 2.3 A map f: (X,τ)→(Y,σ) is said to be
(i) regular-continuous(r-continuous) [3] if $f^{-1}(V)$ is r-closed in X for every closed subset V of Y.
(ii) Completely-continuous [3]if $f^{-1}(V)$ is regular closed in X for every closed subset V of Y.
(iii) strongly α-continuous [38] if $f^{-1}(V)$ is α-closed in X for every semi-closed subset V of Y.
(iv) αrω-continuous [35] if $f^{-1}(V)$ is αrω –closed in X for every closed subset V of Y
(v) Strongly-continuous[28] if $f^{-1}(V)$ is Clopen (both open and closed) in X for every subset V of Y.
(vi) α-continuous[13] if $f^{-1}(V)$ is α–closed in X for every closed subset V of Y.
(vii) αg-continuous[20] if $f^{-1}(V)$ is αg–closed in X for every closed subset V of Y.
(viii) wg-continuous[23] if $f^{-1}(V)$ is wg–closed in X for every closed subset V of Y.
(ix) rwg-continuous[23] if $f^{-1}(V)$ is rwg–closed in X for every closed subset V of Y.
(x) gs-continuous [4] if $f^{-1}(V)$ is gs–closed in X for every closed subset V of Y.
(xi) gp-continuous [18] if $f^{-1}(V)$ is gp–closed in X for every closed subset V of Y.
(xii) gpr-continuous [11] if $f^{-1}(V)$ is gpr–closed in X for every closed subset V of Y.
(xiii) αgr-continuous [33] if $f^{-1}(V)$ is αgr–closed in X for every closed subset V of Y.
(xiv) ωα-continuous [6] if $f^{-1}(V)$ is ωα–closed in X for every closed subset V of Y.
(xv) gspr-continuous [24] if $f^{-1}(V)$ is gspr–closed in X for every closed subset V of Y.
(xvi) g-continuous [6] if $f^{-1}(V)$ is g–closed in X for every closed subset V of Y
(xvii) ω-continuous [29] if $f^{-1}(V)$ is ω–closed in X for every closed subset V of Y
(xviii) rgα-continuous [31] if $f^{-1}(V)$ is rgα–closed in X for every closed subset V of Y.
(xix) gr-continuous [7] if $f^{-1}(V)$ is gr–closed in X for every closed subset V of Y.

(xx) g*p-continuous [32] if $f^{-1}(V)$ is g*p–closed in X for every closed subset V of Y.
(xxi) rps-continuous [26] if $f^{-1}(V)$ is rps–closed in X for every closed subset V of Y.
(xxii) R*-continuous [12] if $f^{-1}(V)$ is R*–closed in X for every closed subset V of Y.
(xxiii) gprw-continuous [15] if $f^{-1}(V)$ is gprw–closed in X for every closed subset V of Y.
(xxiv) wgrα-continuous [14] if $f^{-1}(V)$ is wgrα–closed in X for every closed subset V of Y.
(xxv) swg-continuous [23] if $f^{-1}(V)$ is swg–closed in X for every closed subset V of Y.
(xxvi) rw-continuous [5] if $f^{-1}(V)$ is rw–closed in X for every closed subset V of Y.
(xxvii) rgw-continuous [22] if $f^{-1}(V)$ is rgw–closed in X for every closed subset V of Y.

Definition 2.4 A map f: (X,τ)→(Y,σ) is said to be
(i) α-irresolute [13] if $f^{-1}(V)$ is α-closed in X for every α-closed subset V of Y.
(ii) irresolute [6] if $f^{-1}(V)$ is semi-closed in X for every semi-closed subset V of Y.
(iii) contra ω-irresolute [29] if $f^{-1}(V)$ is ω-open in X for every ω-closed subset V of Y.
(iv) contra irresolute [13] if $f^{-1}(V)$ is semi-open in X for every semi-closed subset V of Y.
(v) contra r-irresolute [3] if $f^{-1}(V)$ is regular-open in X for every regular-closed subset V of Y
(vi) rw*-open(resp rw*-closed) [5] map if f(U) is rw-open (resp. rw-closed) in Y for every rw-open (resp. rw-closed) subset U of X.
(vii) contra continuous [10] if $f^{-1}(V)$ is open in X for every closed subset V of Y .

Lemma 2.5 [37]
i) Every closed (resp. regular-closed, α-closed) set is αrω-closed set in X.
ii) Every αrω-closed set is αg-closed set
iii) Every αrω-closed set is αgr-closed (resp. ωα-closed, gs-closed, gspr-closed, wg-closed, rwg-closed, gp-closed, gpr-closed) set in X

Lemma 2.6 [37] If a subset A of a topological space X and
i) If A is regular open and αrω-closed then A is α-closed set in X
ii) If A is open and αg-closed then A is αrω-closed set in X
iii) If A is open and gp-closed then A is αrω-closed set in X
iv) If A is regular open and gpr-closed then A is αrω-closed set in X
v) If A is open and wg-closed then A is αrω-closed set in X
vi) If A is regular open and rwg-closed then A is αrω-closed set in X
vii) If A is regular open and αgr-closed then A is αrω-closed set in X
viii) If A is ω-open and ωα-closed then A is αrω-closed set in X

Lemma 2.7 [37] If a subset A of a topological space X and
i) If A is semi-open and sg-closed then it is αrω-closed.
ii) If A is semi-open and ω-closed then it is αrω-closed.
iii) A is αrω-open iff $U \subseteq \alpha int(A)$, whenever U is rw-closed and $U \subseteq A$.

Definition 2.8 A topological space (X,τ) is called an α-space if every α-closed subset of X is closed in X.

Definition 2.9 A map f : (X,τ)→(Y,σ) is said to be
(i) **g-closed** [29] if f(F) is g-closed in (Y, σ) for every closed set F of (X, τ),
(ii) **w-closed** [22] if f(F) is w-closed in (Y, σ) for every closed set F of (X, τ),
(iii) **wg-closed** [23] if f(F) is wg-closed in (Y, σ) for every closed set F of (X, τ),
(iv) **rwg-closed** [23] if f(F) is rwg-closed in (Y, σ) for every closed set F of (X, τ),
(v) **rg-closed** [19] if f(F) is rg-closed in (Y, σ) for every closed set F of (X, τ) ,

(vi) **gpr-closed** [11] if f(F) is gpr-closed in (Y, σ) for every closed set F of (X, τ),
(vii) **regular closed** [31] if f(F) is closed in (Y, σ) for every regular closed set F of (X, τ).

Definition 2.10 A map f : (X, τ) → (Y, σ) is said to be
(i) **g-open** [15] if f(U) is g-open in (Y, σ) for every open set U of (X, τ),
(ii) **w-open** [22] if f(U) is w-open in (Y, σ) for every open set U of (X, τ),
(iii) **wg-open** [23] if f(U) is wg-open in (Y, σ) for every open set U of (X, τ),
(iv) rwg-**open** [23] if f(U) is rwg-open in (Y, σ) for every open set U of (X, τ),
(v) rg-**open** [19] if f(U) is rg-open in (Y, σ) for every open set U of (X, τ),
(vi) gpr-**open** [11] if f(U) is gpr-open in (Y, σ) for every open set U of (X, τ),
(vii) **regular open** [31] if f(U) is open in (Y, σ) for every regular open set U of (X, τ).

Definition 2.11 A bijective function f : (X, τ) → (Y, σ) is called
(i) **generalized homeomorphism** (g-homeomorphism) [2]if both f and f^{-1} are g-continuous,
(ii) **gc-homeomorphism** [2] if both f and f^{-1} are gc-irresolute,
(iii) **rwg-homeomorphism** [16] if both f and f^{-1} are rwg-continuous,
(iv) **w∗-homeomorphism** [20] if both f and f^{-1} are w-irresolute,
(v) **w-homeomorphism** [20] if both f and f^{-1} are w-contiuous.
(vi) **rps-homeomorphism** [27] if both f and f^{-1} are rps-contiuous.
(vii) **rgα-homeomorphism** [30] if both f and f^{-1} are rgα-contiuous.
(viii) **wgrα-homeomorphism** [25] if both f and f^{-1} are wgrα-contiuous

3 αrω-Homeomorphisms in Topological Spaces

We introduce the following definition.

Definition 3.1 A bijection f : (X, τ) → (Y, σ) is called α **regular ω-homeomorphism** (briefly, αrω-homeomorphism) if f and f^{-1} are αrω-continuous.

We denote the family of all αrω-homeomorphisms of a topological space (X, τ) onto itself by αrω-h(X, τ).

Example 3.2 Consider X = Y = {a, b, c, d} with topologies τ = σ ={X, ϕ, {a},{b}, {a, b}, {a, b, c}} Let f:(X,τ)→(Y,σ) by f(a)=c, f(b)=a, f(c)= b, f(d)=d. Then f is αrω-continuous and f^{-1} is αrω-continuous. Therefore f is αrω-homeomorphisms.

Theorem 3.3 Every homeomorphism is an αrω-homeomorphism, but not conversely.
Proof: Let f :(X,τ)→(Y,σ) be a homeomorphism. Then f and f^{-1} are continuous and f is bijection. As every continuous function is αrω-continuous, we have f and f^{-1} are αrω-continuous. Therefore f is αrω-homeomorphism.

The converse of the above theorem need not be true, as seen from the following example.

Example 3.4 Consider X = Y ={a, b, c, d} with topologies τ = σ ={X, ϕ, {a},{b},{a, b}, {a, b, c}} and Define a map f :(X, τ)→(Y, σ) by f(a)=c, f(b)=a, f(c)=b, f(d)=d. Then f is αrω-homeomorphism but it is not homeomorphism, since the inverse image of closed F = {c, d} in Y then f^{-1}(F)= {a, d} which is not closed set in X .

Theorem 3.5 Every α-homeomorphism is an αrω-homeomorphism but not conversely.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a α-homeomorphism. Then f and f^{-1} are α-continuous and f is bijection. As every α-continuous function is αrω-continuous, we have f and f^{-1} are αrω-continuous. Therefore f is αrω-homeomorphism.

The converse of the above theorem is not true in general as seen from the following example.

Example 3.6 Consider X = Y = {a, b, c, d} with topologies τ = σ = {X, ϕ, {a}, {b},{a, b}, {a, b, c}} and define a map $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a)=c , f(b)=a ,f(c)=b , f(d)=d. Then f is αrω-homeomorphism but it is not α-homeomorphism, since the inverse image of closed F={c, d} in Y then $f^{-1}(F)$= {a, d} which is not α-closed set in X .

Theorem 3.7 i) Every αrω-homeomorphism is an αg-homeomorphism.
ii) Every αrω-homeomorphism is an wg-homeomorphism (resp. gs-homeomorphism, rwg-homeomorphism, gp-homeomorphism, gspr-homeomorphism, gpr-homeomorphism, ωα-homeomorphism, αgr-homeomorphism)

Proof: i) Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a αrω-homeomorphism. Then f and f^{-1} are αrω-continuous and f is bijection. As every αrω-continuous function is αg-continuous, we have f and f^{-1} are αg-continuous. Therefore f is αg-homeomorphism.

Similarly we can prove **ii)**

The converse of the above theorem is not true in general as seen from the following example.

Example 3.8 Consider X = Y = {a, b, c} with topologies τ = {X, ϕ, {a} ,{b ,c}} and σ = {Y, ϕ ,{a}}. Let $f :(X,\tau)\rightarrow(Y,\sigma)$ by f(a)=b, f(b)=a, f(c)=c. Then this function is αg-homeomorphism, (resp. wg-homeomorphism, gs-homeomorphism, rwg-homeomorphism, gp-homeomorphism, gspr-homeomorphism, gpr-homeomorphism, ωα-homeomorphism, αgr–homeomorphism) but it is not αrω-homeomorphism, since the closed set F={b, c} in Y, $f^{-1}(F)$= {a, c} which is not αrω-closed set in X .

Remark 3.9 The following examples shows that αrω-homeomorphism are independent of pre-homeomorphism, β-homeomorphism, g-homeomorphism, ω-homeomorphism, rw-homeomorphism, swg-homeomorphism, rgw-homeomorphism, wgrα-homeomorphism, rgα-homeomorphism, gprw-homeomorphism, g*p-homeomorphism, gr-homeomorphism, R*- homeomorphism, rps-homeomorphism, semi-homeomorphism .

Example 3.10 Let X=Y={a, b, c}, τ = {X, ϕ, {a} , {b,c}} σ ={Y, ϕ, {a}, {b,c}}, Let map f: X→Y defined by f(a)=b f(b)=a f(c)=c. Then pre-homeomorphism, β-homeomorphism, g-homeomorphism, ω-homeomorphism, rw-homeomorphism, swg-homeomorphism, rgw-homeomorphism, wgrα-homeomorphism, rgα-homeomorphism, gprw-homeomorphism, g*p-homeomorphism, gr-homeomorphism, R*-homeomorphism , rps-homeomorphism but it is not αrω-homeomorphism, since the inverse image of the closed set {b,c} in Y is {a, c} which is not αrω-closed set in X .

Example 3.11 X=Y={a,b,c,d}, τ = {X, ϕ ,{a},{b},{a,b},{a,b,c}} σ ={Y, ϕ, {a},{b},{a,b},{a,b,c}} Let map f: X→Y defined by f(a)=b, f(b)=a , f(c)=d, f(d)=c then αrω-homeomorphism but not, g-homeomorphism, ω-homeomorphism, rw-homeomorphism, gprw-homeomorphism, g*p-homeomorphism, gr-homeomorphism, R*-homeomorphism as closed set F={d} in X, then f(F)={c} in Y, which is not gr-closed (resp. g-closed, g*p-closed, ω-closed, rw-closed, gprw-closed, gr-closed, R*-closed) set in Y.

Theorem 3.12 Let f:(X,τ)→(Y,σ) be a bijective αrω-continuous map. Then the following are equivalent.
(i) f is a αrω-open map,
(ii) f is αrω-homeomorphism,
(iii) f is a αrω-closed map.

Proof: Proof follows from theorem 3.39 in [36].

Remark 3.13 The composition of two αrω-homeomorphism need not be a αrω-homeomorphism in general as seen from the following example.

Example 3.14 Consider X = Y = Z= {a, b, c, d} with topologies τ = σ = μ = {X, ϕ, {a}, {b}, {a, b}, {a, b, c}} and define a map f : (X, τ)→(Y, σ) by f(a)=c, f(b)=a, f(c)=b, f(d)=d. and g :Y→Z defined by g(a) = b, g(b) = a, g(c) = d, g(d) = c then both f and g are αrω-homeomorphisms but their composition g ∘ f : (X, τ)→(Z, μ) is not αrω-homeomorphism because for the open set {a, b} of (X,τ), g ∘ f ({a,b}) =g(f({a, b}) = g(a,,c}) = {a, d}, which is not αrω-open in (z, μ). Therefore g ∘ f is not αrω-open and so g ∘ f is not αrω-homeomorphism.

Definition 3.15 A bijection f :(X,τ)→(Y,σ) is said to be **αrωc-homeomorphism** if both f and f^{-1} are αrω-irresolute. We say that spaces (X, τ) and (Y, σ) are αrωc-homeomorphic if there exists a αrωc-homeomorphism from (X, τ) onto (Y, σ).

We denote the family of all αrωc-homeomorphisms of a topological space (X,τ) onto itself by αrωc-h(X, τ).

Theorem 3.16: Every αrωc-homeomorphism is an αrω-homeomorphism but not conversely.

Proof: Let f :(X,τ)→(Y,σ) be an αrωc-homeomorphism. Then f and f^{-1} are αrω-irresolute and f is bijection. By theorem 3.20 in [35] f and f^{-1} are αrω-continuous. Therefore f is αrω-homeomorphism.

The converse of the above Theorem is not true in general as seen from the following example.

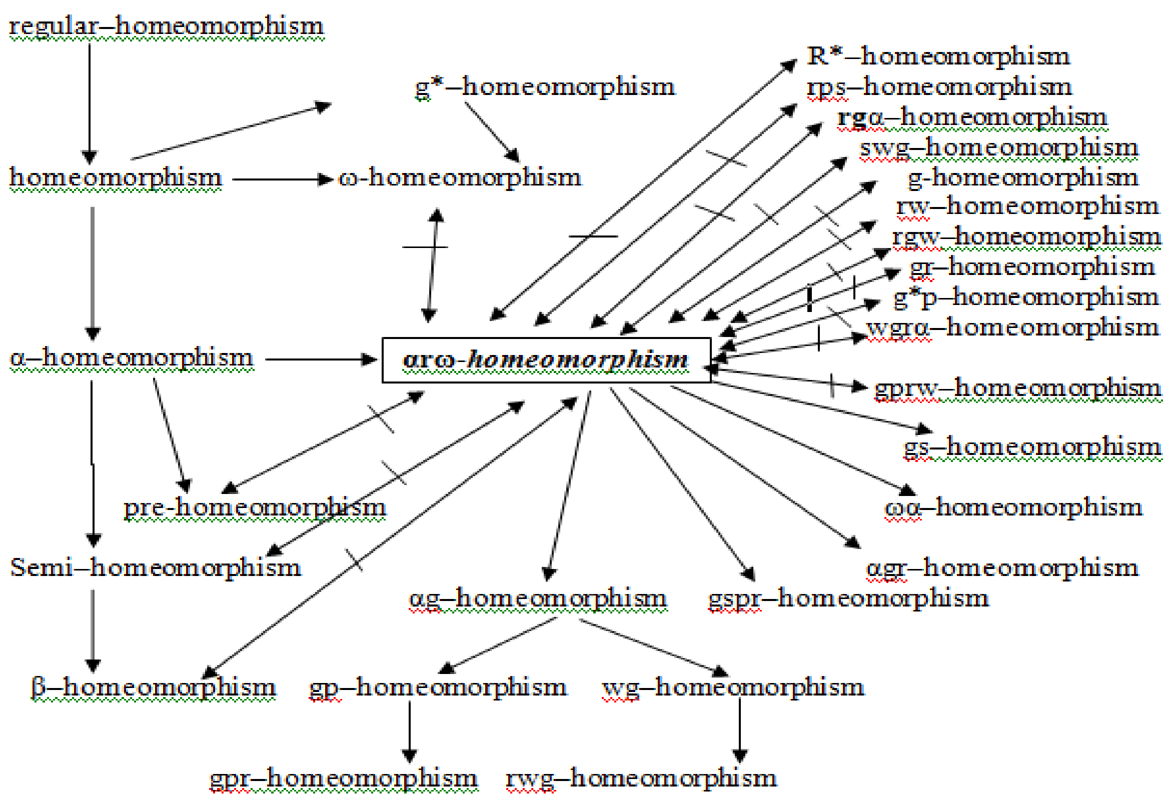
Example 3.17 Consider X = Y = Z= {a, b, c, d} with topologies τ = σ = μ = {X, ϕ, {a}, {b}, {a,b}, {a,b,c}} and define a map f : (X, τ) → (Y, σ) by f(a)=c, f(b)=a, f(c)=b, f(d)=d. Then f is αrω-homeomorphism but it is not αrωc-homeomorphism, since f is not αrω-irresolute.

Theorem 3.18 Every αrωc-homeomorphism is wg-homeomorphism (resp. αg-homeomorphism, gs-homeomorphism, rwg-homeomorphism, gp-homeomorphism, gspr-homeomorphism, gpr-homeomorphism, ωα-homeomorphism, αgr-homeomorphism) but not conversely.

Proof: Proof follows from lemma 2.5 and 2.6.

Remark 3.19 From the above discussions and known results we have the following implications.

A ←——→ B means A & B are independent of each other
A ——+——→ B means A implies B but not conversely



Theorem 3.20 Let f :(X,τ)→(Y,σ) and g :(Y,σ)→(Z,η) are αrωc-homeomorphisms, then their composition g ∘ f : (X, τ) → (Z, η) is also αrωc-homeomorphism.

Proof: Let U be a αrω-open set in (Z, η). Since g is αrω-irresolute, $g^{-1}(U)$ is αrω-open in (Y,σ). Since f is αrω-irresolute, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is αrω open set in (X, τ). Therefore g ∘ f is αrω-irresolute. Also for a αrω-open set G in (X,τ), we have (g ∘ f)(G) = g(f(G)) = g(W), where W =f(G). By hypothesis, f(G) is αrω-open in (Y, σ) and so again by hypothesis, g(f(G)) is a αrω-open set in (Z, η). That is (g ∘ f)(G) is a αrω-open set in (Z, η) and therefore $(g \circ f)^{-1}$ is αrω-irresolute. Also g ∘ f is a bijection. Hence g ∘ f is αrωc-homeomorphism.

Theorem 3.21: The set αrωc-h(X, τ) is a group under the composition of maps.

Proof: Define a binary operation $*$: αrωc-h(X,τ) × αrωc-h(X,τ)→αrωc-h(X,τ) by $f * g = g \circ f$ for all f,g∈αrωc-h(X,τ) and $\circ$ is the usual operation of composition of maps. Then by Lemma 2.8, $g \circ f$∈αrωc-h(X, τ). We know that the composition of maps is associative and the identity map I:(X,τ)→(X,τ) belonging to αrωc-h(X,τ) serves as the identity element. If f∈αrωc-h(X,τ), then f^{-1}∈αrωc-h(X,τ) such that $f \circ f^{-1} = f^{-1} \circ f = I$ and so inverse exists for each element of αrωc-h(X,τ). Therefore (αrωc-h(X,τ), ∘) is a group under the operation of composition of maps.

Theorem 3.22 Let f : (X, τ)→(Y, σ) be a αrωc-homeomorphism. Then f induces an isomorphism from the group αrωc-h(X, τ) onto the group αrωc-h(Y, σ).

Proof: Using the map f, we define a map Ψf : αrωc-h(X,τ)→αrωc-h(Y, σ) by $\Psi f(h) = f \circ h \circ f^{-1}$ for every h ∈αrωc-h(X,τ). Then Ψf is a bijection. Further, for all h_1, h_2 ∈ αrωc-h(X,τ), $\Psi f(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \Psi f(h_1) \circ \Psi f(h_2)$. Therefore Ψf is a homeomorphism and so it is an isomorphism induced by f.

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