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Topological Mappings via $B\delta g$-Closed Sets

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Abstaract − In this paper we introduce a new class of functions called $B\delta g$-continuous functions. We obtain several characterizations and some their properties. Also we investigate its relationship with other types of functions. Further we introduce and study a new class of functions namely $B\delta g$-irresolute.

Keywords − *$B\delta g$-closed set, δ-continuous function, $B\delta g$-continuous function, $B\delta g$-irresolute function.*

1 Introduction

Levine [6], Noiri [10], Balachandran et al [2] and Dontchev and Ganster [3] introduced generalized closed sets, δ-continuity, generalized continuity and δ-generalized continuity (beiefly δg - continuity) & δ-generalized irresolute functions respectively. Devi et al [2] and Veerakumar [12] introduced semi-generalized continuity and $\hat{g}$-continuity in topological spaces. The purpose of this present paper is to define a new class of generalized continuous functions called $B\delta g$-continuous functions and investigate their relationships to other generalized continuous functions. We further introduce and study a new class of functions namely $B\delta g$-irresolute.

2 Preliminaries

Throughout this paper (X, τ) and, (Y, σ) and (Z, η) represent non-empty topological spaces on which no separation axioms are assumed unless or otherwise mentioned.

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For a subset A of X, $cl(A)$, $int(A)$ and A^c denote the closure of A, the interior of A and the complement of A respectively.

Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. A subset A of a space (X,τ) is called a

(i) semi-open set [5] if A $\subseteq$ cl(int(A)).

(ii) pre-open set [7] if A$\subseteq$ int(cl(A)).

(iii) α-open set [9] if A$\subseteq$ int(cl(int(A))).

The complement of a semi-open (resp. pre-open, α-open) set is called semi-closed (resp. semi-closed, α-closed).

Definition 2.2. The δ-interior [11] of a subset A of X is the union of all regular open sets of X contained in A and is denoted by $int_\delta(A)$. The subset A is called δ-open [11] if $A = int_\delta(A)$, i.e. a set is δ-open if it is the union of regular open sets. The complement of a δ-open set is called δ-closed. Alternatively, a set A$\subseteq$ (X,τ) is called δ-closed [11] if $A = cl_\delta(A)$, where $cl_\delta(A) = \{x \in X : int(cl(U)) \cap A \neq \phi, U \in \tau$ and $x \in U\}$.

Definition 2.3. [11] A subset A of a space (X, τ) is called a

(i) t-set if $int(A) = int(cl(A))$.

(ii) B-set if $A = G \cap F$ where G is open and F is a t-set in X .

Definition 2.4. A subset A of (X,τ) is called

(i) generalized closed (briefly g-closed) set [6] if cl(A)$\subseteq$U whenever A $\subseteq$U and U is open in (X,τ).

(ii) generalized semi-closed (briefly gs-closed) set [1] if scl(A)$\subseteq$U whenever A $\subseteq$U and U is open in (X,τ).

(iii) α- generalized closed (briefly αg-closed) set [2] if αcl(A)$\subseteq$U whenever A $\subseteq$U and U is open in (X,τ).

(iv) δ-generalized closed (briefly δg-closed) set [3] if $cl_\delta(A)$ $\subseteq$U whenever A $\subseteq$U and U is open in (X,τ).

(v) $\hat{g}$-closed set [12] if cl(A)$\subseteq$U whenever A $\subseteq$U and U is semi-open in (X, τ).

(vi) δ-$\hat{g}$-closed (briefly $\delta\hat{g}$-closed) set [4] if $cl_\delta(A) \subseteq U$ whenever A $\subseteq$U and U is $\hat{g}$-open in (X,τ).

(vii) $B\delta$-generalized closed (briefly $B\delta g$-closed) set [8] if $cl_\delta(A) \subseteq U$ whenever A $\subseteq$U and U is B-set in (X,τ).

The complement of a g-closed (resp. gs-closed, αg-closed, δg-closed, $\hat{g}$-closed, $\delta\hat{g}$-closed, $B\delta g$-closed) set is called g-open (resp. gs-open, αg-open, δg-open, $\hat{g}$-open, $\delta\hat{g}$-open, $B\delta g$-open).

Definition 2.5. A function $f : (X, \tau) \to (Y, \sigma)$ is called

(i) semi-continuous [5] if $f^{-1}(V)$ is semi-closed in (X, τ) for every closed set V of (Y, σ).

(ii) g-continuous [2] if $f^{-1}(V)$ is g-closed in (X, τ) for every closed set V of (Y, σ).

(iii) gs-continuous [2] if $f^{-1}(V)$ is gs-closed in (X, τ) for every closed set V of (Y, σ).

(iv) αg-continuous [2] if $f^{-1}(V)$ is αg-closed in (X, τ) for every closed set V of (Y, σ).

(v) super continuous [10] if $f^{-1}(V)$ is δ-open in (X, τ) for every open set V of (Y, σ).

(vi) $\hat{g}$-continuous [12] if $f^{-1}(V)$ is $\hat{g}$-closed in (X, τ) for every $\hat{g}$-closed set V of (Y, σ).

(vii) δ-continuous [10] if $f^{-1}(V)$ is δ-open in (X, τ) for every δ-open set V of (Y, σ).

(viii) δ-closed [10] if $f(V)$ is δ-closed in (Y, σ) for every δ-closed set V of (X, τ).

(ix) δg-continuous [3] if $f^{-1}(V)$ is δg-closed in (X, τ) for every closed set V of (Y, σ).

(x) $\delta \hat{g}$-continuous [4] if $f^{-1}(V)$ is $\delta \hat{g}$-closed in (X, τ) for every closed set V of (Y, σ).

Proposition 2.6. [8] If A and B are $B\delta g$-closed sets, then $A \cup B$ is $B\delta g$-closed.

3 $B\delta g$-Continuous and $B\delta g$-Irresolute Functions

In this section we introduce the following definitions.

Definition 3.1. A function $f : (X, \tau) \to (Y, \sigma)$ is called $B\delta g$-continuous if $f^{-1}(V)$ is $B\delta g$-closed in (X, τ) for every closed set V of (Y, σ).

Example 3.2. Let $X = \{a, b, c\}$and $Y = \{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{q\}, \{p, q\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = p$, $f(b) = q$ and $f(c) = r$. Clearly f is $B\delta g$-continuous.

Definition 3.3. A function $f : (X, \tau) \to (Y, \sigma)$ is called $B\delta g$-irresolute if $f^{-1}(V)$ is $B\delta g$-closed in (X, τ) for every $B\delta g$-closed set V of (Y, σ).

Example 3.4. Let $X = \{a, b, c\} = Y = \{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{q\}, \{q, r\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = p$, $f(b) = r$ and $f(c) = q$. Clearly f is $B\delta g$-irresolute.

Proposition 3.5. If $f : (X, \tau) \to (Y, \sigma)$ is $B\delta g$-continuous then f is g-continuous, αg-continuous, gs-continuous and δg-continuous maps.

Proof. It is true that every $B\delta g$-closed set is g-closed, αg-closed, gs-closed and δg-closed. □

Remark 3.6. The converses of the above proposition are not true in general as seen from the following examples.

Example 3.7. Let $X = \{a, b, c\}$and $Y = \{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{p\}, \{p, r\}, Y\}$. Define the map $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = p$, $f(b) = q$ and $f(c) = r$. Clearly f is not $B\delta g$-continuous because $\{q, r\}$ is closed in (Y, σ) but $f^{-1}(\{q, r\}) = \{b, c\}$ is not $B\delta g$-closed in (X, τ). However f is g-continuous.

Example 3.8. Let $X = \{a, b, c\}$and $Y = \{p, q, r\}$ with the topologies $\tau = \{\phi, \{c\}, X\}$ and $\sigma = \{\phi, \{p\}, \{p, q\}, \{p, r\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = r$, $f(b) = q$ and $f(c) = p$. Then f is αg-continuous and sg-continuous. But f is not $B\delta g$-continuous, for the closed set $\{q\}$ of (Y, σ), $f^{-1}(\{q\}) = \{b\}$ is not $B\delta g$-closed in (X, τ).

Example 3.9. Let $X = \{a, b, c\}$and $Y = \{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{p\}, \{q\}, \{p, q\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = q$, $f(b) = r$ and $f(c) = p$. Then f is not $B\delta g$-continuous, for $\{r\}$ is closed in (Y, σ), $f^{-1}(\{r\}) = \{b\}$ is not $B\delta g$-closed in (X, τ). However f is δg-continuous function.

Theorem 3.10. Every super continuous function is $B\delta g$-continuous.

Proof. It is true that every δ-closed set is $B\delta g$-closed. □

Remark 3.11. The converse of Theorem 3.10 need not be true as shown in the following example.

Example 3.12. Let $X = \{a, b, c\}$ and $Y = \{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{r\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = p$, $f(b) = r$ and $f(c) = q$. Then f is $B\delta g$-continuous. But f is not super continuous, for $\{r\}$ is open in (Y, σ), $f^{-1}(\{r\}) = \{b\}$ is not δ-open in (X, τ).

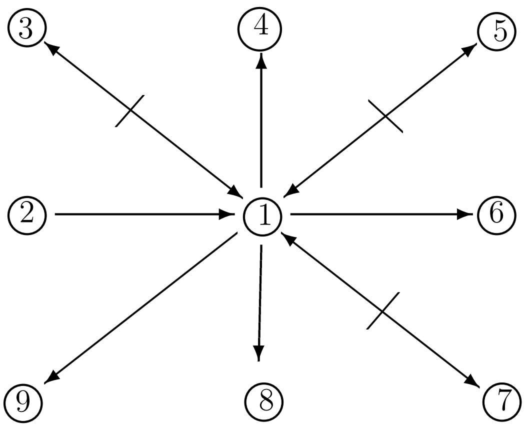
Remark 3.13. The following examples show that $B\delta g$-continuity is independent of semi-continuity, $\hat{g}$-continuity and $\delta\hat{g}$-continuity.

Example 3.14. Let $X = \{a, b, c\}$and $Y = \{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{p, q\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = q$, $f(b) = r$ and $f(c) = p$. Then f is semi-continuous but not $B\delta g$-continuous.

Example 3.15. Let $X = \{a, b, c\}$and $Y = \{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{q\}, Y\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = q$, $f(b) = p$ and $f(c) = r$. Then f is $\hat{g}$-continuous and $\delta\hat{g}$-continuous but not $B\delta g$-continuous.

Example 3.16. Let $X = \{a, b, c\}$and $Y = \{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{p\}, Y\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = q$, $f(b) = r$ and $f(c) = p$. Then f is neither semi-continuous nor $\hat{g}$-continuous. Moreover it is not $\delta\hat{g}$-continuous. However f is $B\delta g$-continuous function.

Remark 3.17. All the above discussions of this section can be represented by the following diagram. $A \to B (A \nleftrightarrow B)$ represents A implies B but not conversely (A and B are independent of each other).



1.$B\delta g$-continuous 2.δ-continuous 3.*semi*-continuous 4.αg-continuous 5.$\hat{g}$-continuous 6.δg-continuous 7.$\delta\hat{g}$-continuous 8.gs-continuous 9.g-continuous

4 Characterizations

Theorem 4.1. A function $f : (X, \tau) \to (Y, \sigma)$ is $B\delta g$-continuous iff $f^{-1}(U)$ is $B\delta g$-open in (X, τ) for every open set U in (Y, σ).

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an $B\delta g$-continuous function and U be an open set in (Y, σ). Then $f^{-1}(U^c)$ is $B\delta g$-closed set in (X, τ). But $f^{-1}(U^c) = [f^{-1}(U)]^c$ and hence $f^{-1}(U)$ is $B\delta g$-open in (X, τ). Conversely $f^{-1}(U)$ is $B\delta g$-open in (X, τ) for every open set U in (Y, σ). Then U^c is closed set in (Y, σ) and $[f^{-1}(U)]^c$ is $B\delta g$-closed in (X, τ). But $[f^{-1}(U)]^c = f^{-1}(U^c)$, so $f^{-1}(U^c)$ is $B\delta g$-closed set in (X, τ). Hence f is $B\delta g$-continuous. □

Theorem 4.2. Let $f : (X, \tau) \to (Y, \sigma)$ be a $B\delta g$-irresolute and $g : (Y, \sigma) \to (Z, \eta)$ a $B\delta g$-irresolute. Then their composition is $g \circ f : (X, \tau) \to (Z, \eta)$ is $B\delta g$-irresolute.

Proof. Let F be $B\delta g$-closed set in (Z, η). Then $g^{-1}(F)$ is $B\delta g$-closed in (Y, σ). Since f is $B\delta g$-irresolute, $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $B\delta g$-closed set of (X, τ) and so $g \circ f$ is $B\delta g$-irresolute function. □

Remark 4.3. The composition of two $B\delta g$-continuous functions need not be $B\delta g$-continuous as the following examples shows.

Example 4.4. Let $X = \{a, b, c\} = Y = Z$ with the topologies $\tau = \{\phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{b\}, \{a, c\}, Y\}$ and $\eta = \{\phi, \{a\}, \{b\}, \{a, b\}, Z\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = c$ and $f(c) = b$ and let $g : (Y, \sigma) \to (Z, \eta)$ be the identity function. Clearly f and g are $B\delta g$-continuous. But $g \circ f : (X, \tau) \to (Z, \eta)$ is not an $B\delta g$-continuous function because $(g \circ f)^{-1}(\{c\}) = f^{-1}(g^{-1}(\{c\})) = f^{-1}(\{c\}) = \{b\}$ is not an $B\delta g$-closed in (X, τ) where as $\{c\}$ is a closed set of (Z, η).

Theorem 4.5. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be two functions. Then

(i) $g \circ f : (X, \tau) \to (Z, \eta)$ is $B\delta g$-continuous, if g is continuous and f is $B\delta g$-continuous.

(ii) $g \circ f : (X, \tau) \to (Z, \eta)$ is $B\delta g$-continuous, if g is $B\delta g$-continuous and f is $B\delta g$-irresolute.

Proof. (i) Let F be any closed set in (Z, η). Since g is continuous, $g^{-1}(F)$ is closed in (Y, σ). But f is $B\delta g$-continuous, $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $B\delta g$-closed of (X, τ) and hence $g \circ f$ is $B\delta g$-continuous function.
(ii) Let G be any closed set in (Z, η). Then $g^{-1}(G)$ is $B\delta g$-closed in (Y, σ). Since f is $B\delta g$-irresolute, $(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is $B\delta g$-closed of (X, τ) and so $g \circ f$ is $B\delta g$-continuous functions. □

Theorem 4.6. Let $f : (X, \tau) \to (Y, \sigma)$ be continuous and δ-closed map. Then for every $B\delta g$-closed subset A of (X, τ), $f(A)$ is $B\delta g$-closed in (Y, σ).

Proof. Let A be $B\delta g$-closed in (X, τ). Let $f(A) \subseteq$ O where O is open in (Y, σ). Since A$\subseteq f^{-1}(O)$ is open in (X, τ), $f^{-1}(O)$ is B-set in (X, τ). Since A is $B\delta g$-closed and since $f^{-1}(O)$ is B-set in (X, τ), then $cl_\delta(A) \subseteq f^{-1}(O)$. Thus $f(cl_\delta(A)) \subseteq O$. Hence $cl_\delta(f(A)) \subseteq cl_\delta(f(cl_\delta(A)) = f(cl_\delta(A)) \subseteq O$, since f is δ-closed. Hence $f(A)$ is $B\delta g$-closed in (Y, σ). □

Remark 4.7. $B\delta g$-continuity and $B\delta g$-irresoluteness are independent notions as seen in the following examples.

Example 4.8. Let $X = \{a, b, c\}, Y = \{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{r\}, Y\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = p$, $f(b) = q$ and $f(c) = r$. Then f is $B\delta g$-continuous but it is not $B\delta g$-irresolute function because $f^{-1}(\{q, r\}) = \{b, c\}$ is not $B\delta g$-closed in (X, τ), where $\{q, r\}$ is $B\delta g$-closed in (Y, σ).

Example 4.9. Let $X = \{a, b, c\}$, $Y = \{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{r\}, \{q, r\}, Y\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = p$, $f(b) = q$ and $f(c) = r$. Then f is $B\delta g$-irresolute but it is not $B\delta g$-continuity function because $f^{-1}(\{p\}) = \{a\}$ is not $B\delta g$-closed in (X, τ), when $\{p\}$ is closed in (Y, σ).

Proposition 4.10. The product of two $B\delta g$-open sets of two spaces is $B\delta g$-open set in the product space.

Proof. Let A and B be two $B\delta g$-open sets of two spaces (X, τ) and (Y, σ) respectively and $V = A \times B \subseteq X \times Y$. Let F$\subseteq V$ be a complement of B-set in $X \times Y$, then there exists two complement of B-sets $F_1 \subseteq A$ and $F_2 \subseteq B$. So, $F_1 \subseteq int_\delta(A)$ and $F_2 \subseteq int_\delta(B)$. Hence $F_1 \times F_2 \subseteq A \times B$ and $F_1 \times F_2 \subseteq int_\delta(A) \times int_\delta(B) = int_\delta(A \times B)$. Therefore $A \times B$ is $B\delta g$-open subset of the space $X \times Y$. □

Theorem 4.11. Let $f_i : X_i \to Y_i$ be $B\delta g$-continuous functions for each $i \in \{1, 2\}$ and let $f : X_1 \times X_2 \to Y_1 \times Y_2$ be defined by $f((x_1, x_2)) = (f(x_1), f(x_2))$. Then f is $B\delta g$-continuous.

Proof. Let V_1 and V_2 be two open sets in Y_1 and Y_2 respectively. Since $f_i : X_i \to Y_i$ are $B\delta g$-continuous functions, for each i$\in$ {1,2}, $f_1^{-1}(V_1)$ and $f_2^{-1}(V_2)$ are $B\delta g$-open sets in X_1 and X_2 respectively. By the Proposition 4.10, $f_1^{-1}(V_1) \times f_2^{-1}(V_2)$ is $B\delta g$-open set in $X_1 \times X_2$. Therefore f is $B\delta g$-continuous. □

Theorem 4.12. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following statements are equivalent.

(i) f is $B\delta g$-irresolute.

(ii) For $x \in (X, \tau)$ and any $B\delta g$-closed set V of (Y, σ) containing $f(x)$ there exists an $B\delta g$-closed set U such that $x \in U$ and $f(U) \subset V$.

(iii) Inverse image of every $B\delta g$-open set of (Y, σ) is $B\delta g$-open in (X, τ).

Proof. $(i) \Rightarrow (ii)$. Let V be an $B\delta g$-closed set of (Y, σ) and $f(x) \in V$. Since f is $B\delta g$-irresolute, $f^{-1}(V)$ is $B\delta g$-closed in (X, τ) and $x \in f^{-1}(V)$. Put $U = f^{-1}(V)$. Then $x \in U$ and $f(U) \subset V$.
(ii) $\Rightarrow (i)$. Let V be an $B\delta g$-closed set of (Y, σ) and $x \in f^{-1}(V)$. Then $f(x) \in V$. Therefore by (ii) there exists an $B\delta g$-closed set U_x such that $x \in U_x$ and $f(U_x) \subset V$. Hence $x \in U_x \subset f^{-1}(V)$. This implies that $f^{-1}(V)$ is a union of $B\delta g$-closed sets of (X, τ). By Proposition 2.6, $f^{-1}(V)$ is $B\delta g$-closed set. This shows that f is $B\delta g$-irresolute.
$(i) \Leftrightarrow (iii)$. It is true that $f^{-1}(Y - V) =$ X$- f^{-1}(V)$. □

5 Applications

Definition 5.1. [8] A space (X, τ) is called a ${}_BT_{\delta g}$-space if every $B\delta g$-closed set in it is δ-closed.

Theorem 5.2. Let $f : (X, \tau) \to (Y, \sigma)$ be $B\delta g$-irresolute. Then f is δ-continuous if (X, τ) is ${}_BT_{\delta g}$-space.

Proof. Let V be a δ-closed subset of (Y, σ). Every δ-closed is $B\delta g$-closed and hence V is $B\delta g$-closed in (Y, σ). Since f is $B\delta g$-irresolute, $f^{-1}(V)$ is $B\delta g$-closed in (X, σ). Since X is ${}_BT_{\delta g}$-space, $f^{-1}(V)$ is δ-closed in (X, τ). Thus f is δ-continuous. □

Theorem 5.3. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be two functions. Let (Y, σ) be ${}_BT_{\delta g}$-space. Then $g \circ f$ is $B\delta g$-continuous if g is $B\delta g$-continuous and f is $B\delta g$-continuous.

Proof. Let G be any closed set in (Z, η). Then $g^{-1}(G)$ is $B\delta g$-closed in (Y, σ). Since (Y, σ) is ${}_BT_{\delta g}$-space, $g^{-1}(G)$ is closed in (Y, σ). Since f is $B\delta g$-continuous, $(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is $B\delta g$-closed in (X, τ). Hence $g \circ f$ is $B\delta g$-continuous function. □

Theorem 5.4. Let $f : (X, \tau) \to (Y, \sigma)$ be onto, $B\delta g$-irresolute and δ-closed. If (X, τ) is a ${}_BT_{\delta g}$-space, then (Y, σ) is also a ${}_BT_{\delta g}$-space.

Proof. Let B be a $B\delta g$-closed subset of (Y, σ). Since f is $B\delta g$-irresolute, then $f^{-1}(B)$ is $B\delta g$-closed set in (X, τ). Since (X, τ) is ${}_BT_{\delta g}$-space, $f^{-1}(B)$ is δ-closed in (X, τ). Also since f is surjective, B is δ-closed in (Y, σ). Hence (Y, σ) is ${}_BT_{\delta g}$-space. □

Theorem 5.5. If $f : (X, \tau) \to (Y, \sigma)$ is bijection, open and $B\delta g$-continuous, then f is $B\delta g$-irresolute.

Proof. Let V be $B\delta g$-closed in (Y,σ) and let $f^{-1}(V) \subseteq U$ where U is open in (X,τ). Since f is open, $f(U)$ is open in (Y,σ). Every open set is B-set and hence $f(U)$ is B-set. Clearly $V \subseteq f(U)$. Then $cl_\delta(V) \subseteq f(U)$ and hence $f^{-1}(cl_\delta(V)) \subseteq U$. Since f is $B\delta g$-continuous and since $cl_\delta(V)$ is a closed subset of (Y,σ), then $cl_\delta(f^{-1}(V)) \subseteq cl_\delta(f^{-1}(cl_\delta(V)) = f^{-1}(cl_\delta(V)) \subseteq U$. U is open and hence B-set in (X,τ). Thus we have $cl_\delta(f^{-1}(V)) \subseteq U$ whenever $f^{-1}(V) \subseteq U$ and U is B-set in (X,τ). This shows that $f^{-1}(V)$ is $B\delta g$-closed in (X,τ). Hence f is $B\delta g$-irresolute. □

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