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NEW APPROACHES ON SYMMETRIC GENERALIZED INTUITIONISTIC FUZZY METRIC SPACES

M. JEYARAMAN AND M. SUGANTHI

ABSTRACT. In this paper, we prove the existence and uniqueness of a common fixed point in symmetric generalized intuitionistic fuzzy metric spaces using property (E.A.) or CLRg property. We introduce the new notion for a pair of mappings (f, g) on a generalized intuitionistic fuzzy metric space called weakly commuting of type (J_f) and *R*-weakly commuting of type (J_f) .

1. INTRODUCTION

In 2006, Mustafa and Sims [9] presented a definition of G-metric space. After that, several fixed point results were proved in G-metric spaces. On the other hand, Atanassov [3] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Park [12] has introduced and studied the notion of intuitionistic fuzzy metric spaces. In 2010, Sun and Yang [14] introduced the concept of generalized fuzzy metric space using the concept of continuous t-norm. In 2017, Muthuraj, Jeyaraman et al. [11], proved two unique common coupled fixed point theorems for Junck type and for three mappings in symmetric generalized intuitionistic fuzzy metric spaces. We prove the existence and uniqueness of a common fixed point in symmetric generalized intuitionistic fuzzy metric spaces using property (E.A.) or CLRg property. We introduce the new notion for a pair of mappings (f,g) on a generalized intuitionistic fuzzy metric space called weakly commuting of type (J_f) and R-weakly commuting of type (J_f) .

2. Preliminaries

Definition 2.1. A 5-tuple $(X, G, H, *, \diamond)$ is said to be a generalized intuitionistic fuzzy metric space, if X is an arbitrary nonempty set, * is a continuous t-norm, \diamond is a continuous t-conorm, G and H are fuzzy sets on $X^3 \times (0, \infty)$ satisfying the following conditions:

For every $x, y, z, a \in X$ and t, s > 0. (i) $G(x, y, z, t) + H(x, y, z, t) \le 1$, (ii) G(x, x, y, t) > 0 for $x \ne y$,

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 $\begin{array}{ll} (\mathrm{iii}) \ G(x,x,y,t) \geq G(x,y,z,t) \quad \mathrm{for} \quad y \neq z, \\ (\mathrm{iv}) \ G(x,y,z,t) = 1 \quad \mathrm{if} \ \mathrm{and} \ \mathrm{only} \ \mathrm{if} \quad x = y = z, \\ (\mathrm{v}) \ G(x,y,z,t) = G(p(x,y,z),t), \quad \mathrm{where} \ p \ \mathrm{is} \ \mathrm{a} \ \mathrm{permutation} \ \mathrm{function}, \\ (\mathrm{vi}) \ G(x,a,a,t) \ast G(a,y,z,s) \leq G(x,y,z,t+s), \\ (\mathrm{vii}) \ G(x,y,z,.) : (0,\infty) \rightarrow [0,1] \quad \mathrm{is} \ \mathrm{continuous}, \\ (\mathrm{viii}) \ G \ \mathrm{is} \ \mathrm{a} \ \mathrm{non-decreasing} \ \mathrm{function} \ \mathrm{n} \ R^+, \\ & \lim_{t \rightarrow \infty} G(x,y,z,t) = 1 \quad \mathrm{and} \ \lim_{t \rightarrow 0} H(x,y,z,t) = 0 \end{array}$

(ix)
$$H(x, x, y, t) < 1$$
 for $x \neq y$,
(x) $H(x, x, y, t) \leq H(x, y, z, t)$ for $y \neq z$,
(xi) $H(x, y, z, t) = 0$ if and only if $x = y = z$,
(xii) $H(x, y, z, t) = H(p(x, y, z), t)$, where p is a permutation function,
(xiii) $H(x, a, a, t) \diamond H(a, y, z, s) \geq H(x, y, z, t + s)$,
(xiv) $H(x, y, z, .) : (0, \infty) \rightarrow [0, 1]$ is continuous,
(xv) H is a non-increasing function on R^+ ,

$$\lim_{t \to \infty} G(x, y, z, t) = 0 \quad \text{and} \quad \lim_{n \to \infty} H(x, y, z, t) = 1$$

In this case, the pair (G, H) is called a generalized intuitionistic fuzzy metric on X.

Example 2.2. Let X be a nonempty set and G and H be metrics on X^3 where t-norm is defined by $a * b = \min\{a, b\}$ and t-conorm is defined by $a \diamond b = \max\{a, b\}$. For all $x, y, z \in X$ and t > 0,

$$G(x, y, z, t) = \frac{t}{t + G(x, y, z)}$$
 and $H(x, y, z, t) = \frac{G(x, y, z)}{t + G(x, y, z)}$.

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Then $(X, G, H, *, \diamond)$ is a generalized intuitionistic fuzzy metric space.

Definition 2.3. Let $(X, G, H, *, \diamond)$ be a generalized intuitionistic fuzzy metric space.

(i) A sequence $\{x_n\}$ in X is said to be convergent to x if

$$\lim_{n \to \infty} G(x_n, x_n, x, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} H(x_n, x_n, x, t) = 0$$

(ii) A sequence $\{x_n\}$ in X is said to be a Cauchy sequence if

$$\lim_{n,m\to\infty} G(x_n, x_n, x_m, t) = 1 \quad \text{and} \quad \lim_{n,m\to\infty} H(x_n, x_n, x_m, t) = 0,$$

that is, for any $\epsilon > 0$ and for each t > 0, there exists $n_0 \in N$ such that

 $G(x_n, x_n, x_m, t) > 1 - \epsilon$ and $H(x_n, x_n, x_m, t) < \epsilon$ for $n, m \ge n_0$.

(iii) A generalized intuitionistic fuzzy metric space $(X, G, H, *, \diamond)$ is said to be complete if every Cauchy sequence in X is convergent.

Definition 2.4. Let $(X, G, H, *, \diamond)$ be a generalized intuitionistic fuzzy metric space. If the conditions

 $\lim_{n \to \infty} G(x_n, y_n, z_n, t_n) = G(x, y, z, t) \text{ and } \lim_{n \to \infty} H(x_n, y_n, z_n, t_n) = H(x, y, z, t)$ are satisfied whenever $\lim_{n \to \infty} x_n = x$, $\lim_{n \to \infty} y_n = y$, $\lim_{n \to \infty} z_n = z$ and

$$\lim_{n \to \infty} G(x, y, z, t_n) = G(x, y, z, t) \quad \text{and} \quad \lim_{n \to \infty} H(x, y, z, t_n) = H(x, y, z, t),$$

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then G and H are called convergent functions on $X^3 \times (0, \infty)$.

Lemma 2.5. Let $(X, G, H, *, \diamond)$ be a generalized intuitionistic fuzzy metric space. Then G and H are continuous function on $X^3 \times (0, \infty)$.

Proof.

Since
$$\lim_{n \to \infty} x_n = x$$
, $\lim_{n \to \infty} y_n = y$, $\lim_{n \to \infty} z_n = z$

and $\lim_{n \to \infty} G(x, y, z, t_n) = G(x, y, z, t), \lim_{n \to \infty} H(x, y, z, t_n) = H(x, y, z, t),$

there is $n_0 \in N$ such that $|t - t_n| < \epsilon$ and $|t - t_n| > \delta$ for $n \ge n_0$ and $\epsilon < \frac{t}{2}$ and $\delta > \frac{t}{2}.$

We know that G(x, y, z, t) is non-decreasing and H(x, y, z, t) is non-increasing with respect to t. Hence we have

$$\begin{aligned} G(x_n, y_n, z_n, t) &\geq G(x_n, y_n, z_n, t - \epsilon) \\ &\geq G(x_n, x, x, \frac{\epsilon}{3}) * G(x, y_n, z_n, t - \frac{4\epsilon}{3}) \\ &\geq G(x_n, x, x, \frac{\epsilon}{3}) * G(y_n, y, y, \frac{\epsilon}{3}) * G(y, x, z_n, t - \frac{5\epsilon}{3}) \\ &\geq G(x_n, x, x, \frac{\epsilon}{3}) * G(y_n, y, y, \frac{\epsilon}{3}) * G(z_n, z, z, \frac{\epsilon}{3}) * G(z, y, z, t - 2\epsilon) \\ &\text{and} \end{aligned}$$

 $H(x_n, y_n, z_n, t) \le H(x_n, y_n, z_n, t - \delta)$

$$\leq H(x_n, x, x, \frac{\delta}{3}) \diamond H(x, y_n, z_n, t - \frac{4\delta}{3})$$

$$\leq H(x_n, x, x, \frac{\delta}{3}) \diamond H(y_n, y, y, \frac{\delta}{3}) \diamond H(y, x, z_n, t - \frac{5\delta}{3})$$

$$\leq H(x_n, x, x, \frac{\delta}{3}) \diamond H(y_n, y, y, \frac{\delta}{3}) \diamond H(z_n, z, z, \frac{\delta}{3}) \diamond H(z, y, z, t - 2\delta).$$

$$G(x, y, z, t+2\epsilon) \ge G(x, y, z, t_n+\epsilon)$$

$$\geq G(x, x_n, x_n, \frac{\epsilon}{3}) * G(x_n, y, z, t_n + \frac{2\epsilon}{3})$$

$$\geq G(x, x_n, x_n, \frac{\epsilon}{3}) * G(y, y_n, y_n, \frac{\epsilon}{3}) * G(y_n, x_n, z, t_n + \frac{\epsilon}{3})$$

$$\geq G(x, x_n, x_n, \frac{\epsilon}{3}) * G(y, y_n, y_n, \frac{\epsilon}{3}) * G(z, z_n, z_n, \frac{\epsilon}{3}) * G(z, y, x, t_n)$$
and

$$\begin{split} H(x,y,z,t+2\delta) &\leq H(x,y,z,t_n+\delta) \\ &\leq H(x,x_n,x_n,\frac{\delta}{3}) \diamond H(x_n,y,z,t_n+\frac{2\delta}{3}) \\ &\leq H(x,x_n,x_n,\frac{\delta}{3}) \diamond H(y,y_n,y_n,\frac{\delta}{3}) \diamond H(y_n,x_n,z,t_n+\frac{\delta}{3}) \\ &\leq H(x,x_n,x_n,\frac{\delta}{3}) \diamond H(y,y_n,y_n,\frac{\delta}{3}) \diamond H(z,z_n,z_n,\frac{\delta}{3}) \diamond H(z,y,x,t_n) \end{split}$$

Let $n \to \infty$. By continuity of the functions G and H, with respect to t, we can get $G(x,y,z,t+2\epsilon) \geq G(z,y,x,t) \geq G(z,y,x,t-2\epsilon)$ and

 $H(x,y,z,t+2\delta)\leq H(z,y,x,t)\leq H(z,y,x,t-2\delta).$ Therefore G and H are continuous functions on $X^3\times(0,\infty).$

3. Weakly Commuting of Type (J_f)

Definition 3.1. A pair of self mapping (f, g) of generalized intuitionistic fuzzy metric space $(X, G, H, *, \diamond)$ is said to be weakly commuting of type (J_f) if

$$G(fgx, gfx, ffx, t) \ge G(fx, gx, fx, t)$$

and

$$H(fgx, gfx, ffx, t) \le H(fx, gx, fx, t)$$

for all $x \in X$ and t > 0.

Definition 3.2. A pair of self mapping (f, g) of generalized intuitionistic fuzzy metric space $(X, G, H, *, \diamond)$ is said to be *R*-weakly commuting of type (J_f) if there exists some positive real number *R* such that

$$G(fgx, gfx, ffx, t) \ge G(fx, gx, fx, t/R)$$

and

$$H(fgx, gfx, ffx, t) \le H(fx, gx, fx, t/R)$$

for all $x \in X$ and t > 0.

Remark 3.3. If we interchange f and g in above definitions, the pair of self mapping (g, f), of generalized intuitionistic fuzzy metric space $(X, G, H, *, \diamond)$, is said to be weekly commuting of type (J_g) and R-weakly commuting of type (J_g) , respectively.

Example 3.4. Let X = [0, 1] be endowed with a standard generalized intuitionistic fuzzy metric defined by

$$G(x, y, z, t) = \frac{t}{t + |x - y| + |y - z| + |z - x|}$$

and

$$H(x, y, z, t) = \frac{|x - y| + |y - z| + |z - x|}{t + |x - y| + |y - z| + |z - x|}$$

for all $x, y, z \in X$ and t > 0.

Define $f, g: X \to X$ by $fx = \frac{x^2}{4}$, $gx = x^2$, for all $x \in X$. Clearly, x = 0 is the only coincidence point of f and g.

So f and g are weakly compatible. It should be noted that

$$G(fgx,gfx,ffx,t) \geq G(fx,gx,fx,t), H(fgx,gfx,ffx,t) \leq H(fx,gx,fx,t)$$

for all $x \in X$ and t > 0.

Then the pair (f,g) is weakly commuting of type (J_f) but not weakly commuting of type (J_g) .

Example 3.5. Let X = [0, 2] be endowed with a standard generalized intuitionistic fuzzy metric. Define fx = 2 - x, gx = x, then by an easy calculation, one can show that the pair (f, g) is weakly commuting of type (J_f) and *R*-weakly commuting of type (J_f) .

Lemma 3.6. If f and g are weakly commuting of type (J_f) or R-weakly commuting of type (J_f) , then f and g are weakly compatible.

Proof. Let x be a coincidence point of f and g, i.e., fx = gx. If the pair (f, g) of generalized intuitionistic fuzzy metric space $(X, G, H, *, \diamond)$ is weakly commuting of type (J_f) , we have

$$G(fgx, gfx, fgx, t) = G(fgx, gfx, ffx, t) \ge G(fx, gx, gx, t) \ge 1,$$

and

$$H(fgx, gfx, fgx, t) = H(fgx, gfx, ffx, t) \le H(fx, gx, gx, t) \le 0.$$

It follows that fgx = gfx, and then they commute at their coincidence point. Similarly, if the pair (f,g) of generalized intuitionistic fuzzy metric space $(X, G, H, *, \diamond)$ is *R*-weakly commuting of type (J_f) , we have, for all $x \in X$,

$$G(fgx, gfx, fgx, t) \ge G(fgx, gfx, ffx, t) \ge G(fx, gx, fx, t/R) = 1,$$

$$H(fgx, gfx, fgx, t) \le H(fgx, gfx, ffx, t) \le H(fx, gx, fx, t/R) = 0.$$

Thus fgx = gfx, which implies that f and g are weakly compatible.

Definition 3.7. [1] A pair of self mapping (f, g) on X is said to satisfy the property (E.A.) if there exists a sequence $\{x_n\}$ such that

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = z \text{ for all } z \in X$$

Definition 3.8. A pair of self mapping (f, g) on X is said to satisfy CLRg property if there exists a sequence $\{x_n\}$ such that

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = gz \text{ for all } z \in X.$$

For proving our main results, we use the following relation: Define $\Phi = \{\phi : \mathbb{R}^+ \to \mathbb{R}^+\}$ where $\mathbb{R}^+ = [0, \infty)$ and each $\phi \in \Phi$ satisfies the following conditions:

 $(\Phi - 1)$: ϕ is strictly increasing.

 $(\Phi - 2)$: ϕ is upper semi-continuous from the right.

 $(\Phi - 3): \sum_{n=0}^{\infty} \phi^n(t) < \infty \text{ for all } t > 0.$

Lemma 3.9. Let $(X, G, H, *, \diamond)$ be a generalized intuitionistic fuzzy metric space. If there exists $\phi \in \Phi$ such that

$$G(x, y, z, \phi(t)) \ge G(x, y, z, t) \text{ and } H(x, y, z, \phi(t)) \le H(x, y, z, t) \text{ for all } t > 0,$$

then x = y = z.

Lemma 3.10. Let $(X, G, H, *, \diamond)$ be a generalized intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X. If there exists $\phi \in \Phi$ such that

$$G(y_n, y_n, y_{n+1}, \phi(t)) \ge G(y_{n-1}, y_{n-1}, y_n, t) * G(y_n, y_n, y_{n+1}, t)$$

and

$$H(y_n, y_n, y_{n+1}, \phi(t)) \le H(y_{n-1}, y_{n-1}, y_n, t) \diamond H(y_n, y_n, y_{n+1}, t)$$

for all t > 0 and n = 1, 2, ...,

then $\{y_n\}$ is a Cauchy sequence in X.

4. Main Results

Theorem 4.1. Let $(X, G, H, *, \diamond)$ be a symmetric generalized intuitionistic fuzzy metric space and the mappings $f, g : X \to X$ satisfy the following conditions: (4.1.1) f and g are weakly commuting of type (J_f) , (4.1.2) $f(X) \subseteq g(X)$, (4.1.3) g(X) is a complete subspace of X,

(4.1.4) there exists a $\phi \in \Phi$ such that for all $x, y, z \in X$ and t > 0,

$$G(fx, fy, fz, \phi(t)) \ge G(gx, gx, fx, t) * G(gy, gy, fy, t) * G(gz, gz, fz, t),$$

$$H(fx,fy,fz,\phi(t)) \leq H(gx,gx,fx,t) \diamond H(gy,gy,fy,t) \diamond H(gz,gz,fz,t).$$

Then f and g have common fixed point.

Proof. Let $x_0, x_1, x_2 \in X$ be such that $fx_0 = gx_1$ and $fx_1 = gx_2$. Then by induction, we can define a sequence $\{y_n\} \in X$ as follows:

$$y_n = fx_n = gx_{n+1}, \ n \in N.$$

We will prove $\{y_n\}$ is a Cauchy sequence in X.

$$\begin{split} &G(y_n, y_n, y_{n+1}, \phi(t)) = G(fx_n, fx_n, fx_{n+1}, \phi(t)) \\ &\geq G(gx_n, gx_n, fx_n, t) * G(gx_n, gx_n, fx_n, t) * G(gx_{n+1}, gx_{n+1}, fx_{n+1}, t) \\ &\geq G(gx_n, gx_n, gx_{n+1}, t) * G(gx_n, gx_n, gx_{n+1}, t) * G(gx_{n+1}, gx_{n+2}, t) \\ &= G(gx_n, gx_n, gx_{n+1}, t) * G(gx_{n+1}, gx_{n+1}, gx_{n+2}, t) \quad \text{and} \\ &H(y_n, y_n, y_{n+1}, \phi(t)) = H(fx_n, fx_n, fx_{n+1}, \phi(t)) \\ &\leq H(gx_n, gx_n, fx_n, t) \diamond H(gx_n, gx_n, fx_n, t) \diamond H(gx_{n+1}, gx_{n+1}, fx_{n+1}, t) \\ &\leq H(gx_n, gx_n, gx_{n+1}, t) \diamond H(gx_n, gx_n, gx_{n+1}, t) \diamond H(gx_{n+1}, gx_{n+1}, gx_{n+2}, t) \\ &= H(gx_n, gx_n, gx_{n+1}, t) \diamond H(gx_n, gx_n, gx_{n+1}, t) \diamond H(gx_{n+1}, gx_{n+2}, t). \end{split}$$

This gives

$$G(y_n, y_n, y_{n+1}, \phi(t)) \ge G(y_{n-1}, y_{n-1}, y_n, t) * G(y_n, y_n, y_{n+1}, t)$$

and

$$H(y_n, y_n, y_{n+1}, \phi(t)) \le H(y_{n-1}, y_{n-1}, y_n, t) \diamond H(y_n, y_n, y_{n+1}, t).$$

By the lemma (3.10), the sequence $\{y_n\}$ is a Cauchy sequence. Since $y_n = gx_{n+1}$, $\{gx_{n+1}\}$ is a Cauchy sequence in g(X). By (4.1.3) hypotheses, we know that g(X) is complete and then there exists $u \in g(X)$ such that

$$\lim_{x \to 0} a_x = u = \lim_{x \to 0} f_x$$

$$\lim_{n \to \infty} gx_n = u = \lim_{n \to \infty} fx_n.$$

Now $u \in g(X)$, so there exists $p \in X$ such that u = gp. Therefore

$$\lim_{n \to \infty} gx_n = gp = \lim_{n \to \infty} fx_n.$$

We will prove that fp = gp.

$$\begin{split} G(fp, fp, fx_n, \phi(t)) &\geq G(gp, gp, fp, t) * G(gp, gp, fp, t) * G(gx_n, gx_n, fx_n, t) \text{ and} \\ H(fp, fp, fx_n, \phi(t)) &\leq H(gp, gp, fp, t) \diamond H(gp, gp, fp, t) \diamond H(gx_n, gx_n, fx_n, t). \end{split}$$
Taking limit as $n \to \infty$, we have

$$G(fp, fp, gp, \phi(t)) \ge G(gp, gp, fp, t) * G(gp, gp, fp, t) * G(gp, gp, fp, t)$$

or

$$\begin{split} G(fp,fp,gp,\phi(t)) &\geq G(gp,gp,fp,t) \text{ and} \\ H(fp,fp,gp,\phi(t)) &\leq H(gp,gp,fp,t) \diamond H(gp,gp,fp,t) \diamond H(gp,gp,gp,t) \end{split}$$

or

 $H(fp, fp, gp, \phi(t)) \le H(gp, gp, fp, t).$

Since generalized intuitionistic fuzzy metric space is symmetric, we have

 $G(fp, fp, gp, \phi(t)) \ge G(gp, gp, fp, t) = G(fp, fp, gp, t),$

 $H(fp, fp, gp, \phi(t)) \le H(gp, gp, fp, t) = H(fp, fp, gp, t)$

which implies fp = gp (by Lemma (3.9)).

Since the pair (f,g) is weakly commuting of type (J_f) , then

 $G(fgp, qfp, ffp, \phi(t)) \ge G(fp, qp, fp, t) = 1$

and

$$H(fgp, gfp, ffp, \phi(t)) \le H(fp, gp, fp, t) = 0,$$

implies that ffp = fgp = gfp = ggp. Hence fu = fgp = gfp = gu. Finally, we show that u = gp is common fixed point of f and g. Suppose $fu \neq u$, then

$$\begin{split} &G(fu, fp, fp, \phi(t)) \geq G(gu, gu, fu, t) * G(gp, gp, fp, t) * G(gp, gp, fp, t), \\ &G(fu, fp, fp, \phi(t)) \geq G(fu, fu, fu, t) * G(fp, fp, fp, t) * G(fp, fp, fp, t), \\ &G(fu, u, u, \phi(t)) \geq 1 * 1 * 1 = 1, \\ &H(fu, fp, fp, \phi(t)) \leq H(gu, gu, fu, t) \diamond H(gp, gp, fp, t) \diamond H(gp, gp, fp, t), \\ &H(fu, fp, fp, \phi(t)) \leq H(fu, fu, fu, t) \diamond H(fp, fp, fp, t) \diamond H(fp, fp, fp, t), \\ &H(fu, u, u, \phi(t)) \leq 0 \diamond 0 \diamond 0 = 0, \text{ a contradiction} \end{split}$$

Hence fu = gu = u.

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To prove the uniqueness, suppose we have u and v such that $u \neq v$ and

 $fu=gu=u \quad and \quad fv=gv=v$

Using condition (4.1.4), we have

$$\begin{split} G(u, v, v, \phi(t)) &= G(fu, fv, fv, \phi(t)) \\ &\geq G(gu, gu, fu, t) * G(gv, gv, fv, t) * G(gv, gv, fv, t) \\ &= 1 * 1 * 1 = 1, \\ H(u, v, v, \phi(t)) &= H(fu, fv, fv, \phi(t)) \\ &\leq H(gu, gu, fu, t) \diamond H(gv, gv, fv, t) \diamond H(gv, gv, fv, t) \\ &= 0 \diamond 0 \diamond 0 = 0. \end{split}$$

Hence
$$G(u, v, v, \phi(t)) \ge 1$$
 and $H(u, v, v, \phi(t)) \le 0$,

which gives contradiction. Hence u = v. Therefore u is a unique common fixed point. \square **Example 4.2.** Let X = [0,1] be endowed with standard generalized intuitionistic fuzzy metric. Define $f, g: X \to X$ by $f(x) = \frac{x^2}{4}$, $g(x) = x^2$, $x \in X$. We see that x = 0 is the only coincidence point. So f and g are weakly compatible.

Also
$$G(fgx, gfx, ffx, t) \ge G(fx, gx, fx, t)$$

and $H(fgx, gfx, ffx, t) \leq H(fx, gx, fx, t)$.

Then the pair (f,g) is weakly commuting of type (J_f) but not weakly commuting of type (J_g) .

Example 4.3. Let X = [-1, 1] be endowed with standard generalized intuitionistic fuzzy metric. Let $\phi(t) = \frac{t}{2}$ and define $f, g: X \to X$ by $f(x) = \frac{x}{6}$, $g(x) = \frac{x}{2}(x+1)$, $x \in X$. We see that x = 0 is the only coincidence point and f and g are weakly compatible.

Let $\{x_n = \frac{1}{n}\}$ be a sequence such that

$$G(fp, fp, fx_n, \phi(t)) \ge G(fp, fp, gp, t)$$

and

$$H(fp, fp, fx_n, \phi(t)) \le H(fp, fp, gp, t)$$

where p is a coincidence point.

Then the pair (f,g) is weakly commuting of type (J_f) and f and g have a unique common fixed point.

Corollory 4.4. Theorem (4.1) remains true if we replace weakly commuting of type (J_f) by weakly commuting of type (J_g) or replace *R*-weakly commuting of type (J_q) .

Theorem 4.5. Let $(X, G, H, *, \diamond)$ be a symmetric generalized intuitionistic fuzzy metric space and suppose mappings $f, g : X \to X$ are weakly commuting of type (J_f) satisfying the following conditions:

(4.5.1) f and g satisfy the property (E.A.),

(4.5.2) g(X) is a closed subspace of X,

(4.5.3) there exists a $\phi \in \Phi$ such that for all $x, y, z \in X$ and t > 0,

$$G(fx, fy, fz, \phi(t)) \geq G(gx, gx, fx, t) * G(gy, gy, fy, t) * G(gz, gz, fz, t)$$

and

$$H(fx, fy, fz, \phi(t)) \le H(gx, gx, fx, t) \diamond H(gy, gy, fy, t) \diamond H(gz, gz, fz, t)$$

Then f and g have a unique common fixed point.

Proof. The mappings f and g satisfy the property (E.A.), then there exists a sequence $\{x_n\} \in X$ satisfying

$$\lim_{n \to \infty} f x_n = u = \lim_{n \to \infty} g x_n$$

for some $u \in X$.

Since g(X) is a closed subspace of X and

 $\lim_{n \to \infty} g x_n = u,$

then there exists $p \in X$ such that gp = u.

Also
$$\lim_{n \to \infty} gx_n = gp = \lim_{n \to \infty} fx_n$$

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We will prove, fp = gp.

 $G(fp,fp,fx_n,\phi(t)) \geq G(gp,gp,fp,t) * G(gp,gp,fp,t) * G(gx_n,gx_n,fx_n,t)$ and

 $H(fp, fp, fx_n, \phi(t)) \leq H(gp, gp, fp, t) \diamond H(gp, gp, fp, t) \diamond H(gx_n, gx_n, fx_n, t),$ taking limit as $n \to \infty$, we have

$$\begin{split} &G(fp, fp, gp, \phi(t)) \geq G(gp, gp, fp, t) * G(gp, gp, fp, t) * G(gp, gp, fp, t) \\ &G(fp, fp, gp, \phi(t)) \geq G(gp, gp, fp, t), \text{and} \\ &H(fp, fp, gp, \phi(t)) \leq H(gp, gp, fp, t) \diamond H(gp, gp, fp, t) \diamond H(gp, gp, fp, t) \\ &H(fp, fp, gp, \phi(t)) \leq H(gp, gp, fp, t). \end{split}$$

Since generalized intuitionistic fuzzy metric space is symmetric, we have

 $G(fp, fp, gp, \phi(t)) \ge G(fp, fp, gp, t)$

and

$$H(fp, fp, gp, \phi(t)) \le H(fp, fp, gp, t),$$

which implies fp = gp = u.

Since the pair (f,g) is weakly commuting of type (J_f) , then

$$G(fgp, gfp, ffp, \phi(t)) \ge G(fp, gp, fp, t) = 1$$

and

$$H(fgp, gfp, ffp, \phi(t)) \le H(fp, gp, fp, t) = 0,$$

which implies ffp = fgp = gfp = ggp. Hence fu = fgp = gfp = gu. Now, we will show that fp = u is a common fixed point of f and g. Suppose $fu \neq u$, then

$$\begin{split} G(fu, u, u, \phi(t)) &= G(fu, fp, fp, \phi(t)) \\ &\geq G(gu, gu, fu, t) * G(gp, gp, fp, t) * G(gp, gp, fp, t) \\ &= G(fu, fu, fu, t) * 1 * 1 \\ &= 1 * 1 * 1 = 1, \\ H(fu, u, u, \phi(t)) &= H(fu, fp, fp, \phi(t)) \\ &\leq H(gu, gu, fu, t) \diamond H(gp, gp, fp, t) \diamond H(gp, gp, fp, t) \\ &= H(fu, fu, fu, t) \diamond 0 \diamond 0 \\ &= 0 \diamond 0 \diamond 0 = 0, \end{split}$$

a contradiction. Hence fu = u = gu.

To prove the uniqueness, suppose we have u and v such that $u \neq v$, fu = gu = uand fv = gv = v, then again using condition (4.5.3), we have,

$$\begin{split} G(u, v, v, \phi(t)) &= G(fu, fv, fv, \phi(t)) \\ &\geq G(gu, gu, fu, t) * G(gv, gv, fv, t) * G(gv, gv, fv, t) \\ &= 1 * 1 * 1 = 1, \\ H(u, v, v, \phi(t)) &= H(fu, fv, fv, \phi(t)) \\ &\leq H(gu, gu, fu, t) \diamond H(gv, gv, fv, t) \diamond H(gv, gv, fv, t) \\ &= 0 \diamond 0 \diamond 0 = 0, \end{split}$$

which is a contradiction. Hence u = v. Therefore u is a unique common fixed point of f and g.

Theorem 4.6. Let $(X, G, H, *, \diamond)$ be a symmetric generalized intuitionistic fuzzy metric space and suppose mappings $f, g : X \to X$ are weakly commuting of type (J_f) satisfying the following conditions: (4.6.1) f and g satisfy CLRg property,

(4.6.2) there exists a $\phi \in \Phi$ such that for all $x, y, z \in X$ and t > 0,

$$G(fx,fy,fz,\phi(t)) \geq G(gx,gx,fx,t) * G(gy,gy,fy,t) * G(gz,gz,fz,t)$$

$$H(fx, fy, fz, \phi(t)) \leq H(gx, gx, fx, t) \diamond H(gy, gy, fy, t) \diamond H(gz, gz, fz, t) \leq H(gz, gz$$

Then f and g have a unique common fixed point.

Proof. The proof follows on the same lines of Theorem (4.5) and by definition of CLRg property.

5. CONCLUSION

This paper introduced new kinds of pair of self mappings viz. weakly commuting of type (J_f) and R-weakly commuting of type (J_f) on generalized intuitionistic fuzzy metric spaces. Common fixed point theorems are proved based on these newly introduced pair of self mappings. This work provides a new approach to analyze intuitionistic fuzzy metric spaces and paves a path that leads to extensions of such conceptual work.

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