Journal of Universal Mathematics
Vol.8 No.1 pp.33-39 (2025)
ISSN-2618-5660
DOI: 10.33773/jum.1446824

FIXED POINT THEOREMS IN SOME FUZZY METRIC SPACES VIA INTERPOLATIVE CONTRACTIONS

MERYEM BOZDEMİR

0000-0002-2988-9419

Abstract. In this article, an interpolative contraction existing in the literature is adapted to different fuzzy metric spaces. Using this contraction, a fixed point theorem in two fuzzy metric spaces is proven and an example is presented. Thus, a more general form of some concepts and theorems existing in the literature has been obtained.

1. Introduction

In our daily lives, situations that are uncertain are often faced. For each scenario encountered,determining what is "right" or "wrong" using the logic-based approach relied upon by modern computers is difficult. Many events in nature involve uncertainty, and the concept of "fuzziness" provides the flexibility needed to accurately describe such situations. This idea was introduced by Lotfi Zadeh [12], allowing phenomena that were once considered unknowable to be explained.

In recent years, various generalizations of the metric concept, which is key in fixed point theory, have been developed. One such generalization was initially introduced in [9] and later modified in [2], leading to the development of the fuzzy metric space.

Following the work of Stefan Banach [1], who laid the foundation for the fixed point theorem, adaptations of this theorem to different types of spaces have been made, contributing to research in many scientific fields. It has become a crucial tool, not only in functional analysis but also in general topology and other disciplines.

After the contributions of Grabiec [3], significant progress has been made on this theorem in the context of two spaces ([6], [10]). The type of space being studied and the contraction mapping used are the two main aspects that need to be considered.

2. Preliminaries

After defining the $t-norm$, which is considered the basic operator of fuzzy logic, some concepts to be used in this article will be presented.

Date: **Received:** 2024-07-30; **Accepted:** 2024-12-02.
Key words and phrases. Fuzzy metric space, Fixed-point, Interpolative contraction.
Mathematics Subject Classification: 03E72, 47H10, 54H25, 54E50.

Definition 1. [11] *Let* $*: [0,1] \times [0,1] \to [0,1]$ *be a binary operation that called a continuous t-norm if the conditions hold; for all* $\acute{y}, \breve{u}, \check{z}, \dot{g} \in [0,1]$ $\acute{y} * 1 = \acute{y}$ *and* $\acute{y} * \breve{u} < \check{z} * \dot{g}$, *whenever* $\acute{y} < \check{z}$ *and* $\breve{u} < \dot{g}$ *and in addition associative, commutative and continuous.*

After KM [9] and GV [2], a lot of definitions and theorems were created for fuzzy metric space (FMS). So these important discoveries attracted the attention of many writers.

Definition 2. [2] $(\hat{W}, \hat{Y}, *)$, $\hat{W}$ $(\neq \varnothing)$, *is called a FMS; provided that* $*$ *is a continuous* $t-norm$, $\hat{Y}$ *is a fuzzy set on* $\hat{W}^2 \times (0, \infty)$ *satisfying the conditions* $\forall$ $\gamma, \rho, \eta \in \hat{W}$ *and* $\acute{s}, \acute{r} > 0$;

(FM₁) $\hat{Y}(\gamma, \rho, \acute{s}) > 0$;
(FM₂) $\hat{Y}(\gamma, \rho, \acute{s}) = 1 \iff \gamma = \rho$;
(FM₃) $\hat{Y}(\gamma, \rho, \acute{s}) = \hat{Y}(\rho, \gamma, \acute{s})$;
(FM₄) $\hat{Y}(\gamma, \rho, \acute{s}) * \hat{Y}(\rho, \eta, \acute{r}) \leq \hat{Y}(\gamma, \eta, \acute{s} + \acute{r})$;
(FM₅) $\hat{Y}(\gamma, \rho, \cdot) : (0, \infty) \to [0,1]$ *is continuous*,

When (FM₄) is replaced by (NA),

$$(NA) = \hat{Y}(\gamma, \rho, \acute{s}) * \hat{Y}(\rho, \eta, \acute{r}) \leq \hat{Y}(\gamma, \eta, \max\{\acute{s}, \acute{r}\})$$

or

$$\hat{Y}(\gamma, \rho, \acute{s}) * \hat{Y}(\rho, \eta, \acute{s}) \leq \hat{Y}(\gamma, \eta, \acute{s})$$

then $(\hat{W}, \hat{Y}, *)$ *is named Non-Archimedean* (NA) *FMS* [7] .

Metrics that do not depend on "t" are called stationary fuzzy metrics. When examined from this aspect; it is clearly seen that these fuzzy metrics are the most similar to classical ones.

Definition 3. [5] $(\hat{W}, \hat{Y}, *)$, $\hat{W}$ $(\neq \varnothing)$, *is called a stationary FMS (SFMS); If* $*$ *is a continuous t-norm,* $\hat{Y}$ *is a fuzzy set on* $\hat{W}^2$ *satisfying the conditions* $\forall$ $\gamma, \rho \in \hat{W}$;

(SF_1) $\hat{Y}(\gamma, \rho) > 0$;
(SF_2) $\hat{Y}(\gamma, \rho) = 1 \iff \gamma = \rho$;
(SF_3) $\hat{Y}(\gamma, \rho) = \hat{Y}(\rho, \gamma)$;
(SF_4) $\hat{Y}(\gamma, \rho) * \hat{Y}(\rho, \eta) \leq \hat{Y}(\gamma, \eta)$.

$(\gamma_i)_{i \in \mathbb{N}}$ *in this space* $(\hat{W}, \hat{Y})$ *is Cauchy if* $\lim_{i,j \to \infty} \hat{Y}(\gamma_i, \gamma_j) = 1$;

$(\gamma_i)_{i \in \mathbb{N}} \to \gamma \in \hat{W}$ *if* $\lim_{i \to \infty} \hat{Y}(\gamma_i, \gamma) = 1$.

Now a newly fuzzy metrics defined in [4] is presented below that in the study " $\wedge_{t>0} \hat{Y}(\gamma, \rho, t) > 0$ on $\hat{W}$" were examined.

Definition 4. [4] $(\hat{W}, \hat{Y}^0, *)$, $\hat{W}$ $(\neq \varnothing)$,*is called an extended FMS (EFMS);If* $*$ *is a continuous t-norm,* $\hat{Y}^0$ *is a fuzzy set on* $\hat{W}^2 \times$ $[0, \infty)$ *satisfying the conditions* $\forall$ $\gamma, \rho, \eta \in \hat{W}$ *and* $\acute{s}, \acute{r} \geq 0$;

(EF₁) $\hat{Y}^0(\gamma, \rho, \acute{s}) > 0$;
(EF₂) $\hat{Y}^0(\gamma, \rho, \acute{s}) = 1 \iff \gamma = \rho$;
(EF₃) $\hat{Y}^0(\gamma, \rho, \acute{s}) = \hat{Y}^0(\rho, \gamma, \acute{s})$;
(EF₄) $\hat{Y}^0(\gamma, \rho, \acute{s}) * \hat{Y}^0(\rho, \eta, \acute{r}) \leq \hat{Y}^0(\gamma, \eta, \acute{s} + \acute{r})$;
(EF₅) $\hat{Y}^0_{\gamma,\rho} : [0, \infty) \to (0, 1]$ *is continuous.*

Similarly replacing (EF$_4$) by $(NA)^* = \hat{Y}^0(\gamma,\rho,\acute{s}) * \hat{Y}^0(\rho,\eta,\acute{r}) \leq \hat{Y}^0(\gamma,\eta,\max\{\acute{s},\acute{r}\})$ *or* $\hat{Y}^0(\gamma,\rho,\acute{s}) * \hat{Y}^0(\rho,\eta,\acute{s}) \leq \hat{Y}^0(\gamma,\eta,\acute{s})$ *for* $\forall\ \gamma,\rho,\eta \in \hat{W}$ *and* $\acute{s},\acute{r} \geq 0$*; then* $(\hat{W},\hat{Y}^0,*)$ *is named* $NA\ EFMS$.

Theorem 1. [4] *Let* $\hat{Y}$ *and its extension set* $\hat{Y}^0$ *be defined on* $\hat{W}^2 \times (0,\infty)$*, and* $\hat{W}^2 \times\ [0,\infty)$ *respectively.*

$$\hat{Y}^0(\gamma,\rho,\acute{s}) = \hat{Y}(\gamma,\rho,\acute{s}) \text{ for all } \gamma,\rho \in \hat{W},\ \acute{s} > 0 \quad \text{and}$$
$$\hat{Y}^0(\gamma,\rho,0) = \wedge_{t>0}\hat{Y}(\gamma,\rho,\acute{s}).$$

So, $(\hat{W},\hat{Y}^0,*)$ *is an EFMS if and only if* $(\hat{W}.\hat{Y},*)$ *is a FMS satisfying* $\forall\ \gamma,\rho \in \hat{W}$ *the condition* $\wedge_{\acute{s}>0}\hat{Y}(\gamma,\rho,\acute{s}) > 0$.

Proposition 1. [4] $(\hat{W},N_{\hat{Y}},*)$ *is a SFMS on X if and only if* $\wedge_{\acute{s}>0}\hat{Y}(\gamma,\rho,\acute{s}) > 0$ $\forall\ \gamma,\rho \in \hat{W}$*. That is,*

$$\hat{Y}^0(\gamma,\rho,0) = \wedge_{\acute{s}>0}\hat{Y}(\gamma,\rho,\acute{s}) = N_{\hat{Y}}(\gamma,\rho) \qquad (2.1)$$

Proposition 2. [4] $(\hat{W},\hat{Y}^0,*)$ *is complete if and only if* $(\hat{W},N_{\hat{Y}},*)$ *is complete.*

In the literature, the concepts of completeness and Caucy have been defined in various ways and used in fuzzy metric spaces ([2], [3]). One of them is adapted to EFMS in [4]. It is presented below;

Definition 5. [4] $\{\gamma_n\}$ *in* $\hat{W}$ *is named Cauchy sequence if given* $\delta \in (0,1)$*, it can be find* $n_\delta \in \mathbb{N}$ *such that* $\hat{Y}^0(\gamma_n,\gamma_m,0) > 1-\delta$ *for all* $n,m \geq n_\delta$ *.*$\{\gamma_n\}$ *is Cauchy sequence*$\Longleftrightarrow \lim_{m,n}\hat{Y}^0(\gamma_n,\gamma_m,0) = 1$.

Since the spaces to which every Cauchy sequence converges are complete, the same situation is valid in EFMS.

An interpolative type contraction was studied in [8] in partial metric space *(PMS)*;

Definition 6. [8] *Let* $(\hat{W},p)$ *be a PMS,* $\Im : X \longrightarrow X$ *is named an interpolative Reich-Rus-Ciric type contraction, if there exist constants* $\lambda \in [0,1)$ *and* $\alpha,\beta \in (0,1)$ *such that*

$$p(\Im\gamma,\Im\rho) \leq \lambda\,[p(\gamma,\rho)]^\beta\,[p(\gamma,\Im\rho)]^\alpha \,.\, [d(\rho,\Im\rho)]^{1-\alpha-\beta}$$

for all $\gamma,\rho \in X/Fix(\Im)$.

Theorem 2. [8] *In the framework of a PMS* $(\hat{W},p)$*, if* $\Im : \hat{W} \longrightarrow \hat{W}$ *is an interpolative Reich-Rus-Ciric type contraction, then* $\Im$ *posseses a fixed point in* $\hat{W}$.

In this article, it is intended to obtain generalized versions inspired the contraction obtained by interpolative approach and to adapt this contraction first to fuzzy metrics and then to extended ones.

3. Main Result

Definition 7. $\Omega : \hat{W} \longrightarrow \hat{W}$ *is called a fuzzy-interpolative Reich-Rus-Ciric type contraction; If* $(\hat{W},\hat{Y},*)$ *is a FMS and there exist constants* $\lambda \in [0,1)$ *and* $\alpha,\beta \in (0,1)$ *;*

$$\left[1-\hat{Y}(\Omega\gamma,\Omega\rho,\acute{s})\right] \geq \lambda\left[1-\hat{Y}(\gamma,\rho,\acute{s})\right]^\beta\left[1-\hat{Y}(\gamma,\Omega\gamma,\acute{s})\right]^\alpha.\left[1-\hat{Y}(\rho,\Omega\rho,\acute{s})\right]^{1-\alpha-\beta} \qquad (3.1)$$

for all $\gamma, \rho \in \hat{W}/Fix(\Omega)$.

Theorem 3. *Let* $(\hat{W}, \hat{Y}, *)$ *be a complete NA FMS. Provided that* $\Omega : \hat{W} \longrightarrow \hat{W}$ *is a fuzzy-interpolative Reich-Rus-Ciric type contraction, then* Ω *has a fixed point in* $\hat{W}$.

Proof. Let $\rho_0 \in \hat{W}$. $(\rho_n)_{n\in\mathbb{N}} \in \hat{W}$ is a sequence with $\rho_{n+1} = \Omega\rho_n$.

Here, by examining the cases where $\gamma_{n+1} = \gamma_n$ and $\gamma_n \neq \gamma_{n+1}$; it will be obtain that γ^* is the fixed point in the both cases.

Let be $\rho_{n+1} = \rho_n$ (for some $n \in \mathbb{N}$), $\gamma^* = \gamma_n$.

Let be $\rho_n \neq \rho_{n+1}$ ($\forall\ n \in \mathbb{N}$);

By replacing the values such as $\gamma = \rho_{n-1}$, $\rho = \rho_n$,

$$\begin{aligned}\left[1-\hat{Y}(\Omega\rho_{n-1},\Omega\rho_n,\acute{s})\right] &\geq \lambda\left[1-\hat{Y}(\rho_{n-1},\rho_n,\acute{s})\right]^{\beta}\left[1-\hat{Y}(\rho_{n-1},\Omega\rho_{n-1},\acute{s})\right]^{\alpha}.\left[1-\hat{Y}(\rho_n,\Omega\rho_n,\acute{s})\right]^{1-\alpha-\beta}\\ \left[1-\hat{Y}(\rho_n,\Omega\rho_n,\acute{s})\right]^{\alpha+\beta} &\geq \lambda\left[1-\hat{Y}(\rho_n,\rho_{n-1},\acute{s})\right]^{\beta}.\left[1-\hat{Y}(\rho_{n-1},\Omega\rho_{n-1},\acute{s})\right]^{\alpha}\\ &= \lambda.\left[1-\hat{Y}(\rho_{n-1},\Omega\rho_{n-1},\acute{s})\right]^{\alpha+\beta}\end{aligned}$$

and

$$\left[1-\hat{Y}(\rho_n,\Omega\rho_n,\acute{s})\right]^{\alpha+\beta} \geq \lambda.\left[1-\hat{Y}(\rho_{n-1},\Omega\rho_{n-1},\acute{s})\right]^{\alpha+\beta}$$

so $\left\{\hat{Y}(\rho_{n-1},\Omega\rho_{n-1},\acute{s})\right\}$ is non-increasing;

$$\left[1-\hat{Y}(\rho_n,\Omega\rho_n,\acute{s})\right] \geq \lambda.\left[1-\hat{Y}(\rho_{n-1},\Omega\rho_{n-1},\acute{s})\right]$$

this implies that,

$$\left[1-\hat{Y}(\rho_n,\Omega\rho_n,\acute{s})\right] \geq \lambda^n.\left[1-\hat{Y}(\rho_0,\rho_1,\acute{s})\right]$$

as $n \to \infty$,

$$\lim_{n\to\infty}\left[1-\hat{Y}(\rho_n,\Omega\rho_n,\acute{s})\right] \geq \lambda^n.\lim_{n\to\infty}\left[1-\hat{Y}(\rho_0,\rho_1,\acute{s}\right]$$

$\lambda^n \to 0$ we obtain,

$$\lim_{n\to\infty}\left[1-\hat{Y}(\rho_n,\Omega\rho_n,\acute{s})\right] = 0 \Longrightarrow \hat{Y}(\rho_n,\Omega\rho_n,\acute{s}) = 1.$$

Using Def.4 with (NA), for $n < m$;

$$\hat{Y}(\rho_n,\rho_m,\acute{s}) \geq \hat{Y}(\rho_n,\rho_{n+1},\acute{s}) * \hat{Y}(\rho_{n+1},\rho_{n+2},\acute{s}) * ... * \hat{Y}(\rho_{m-1},\rho_m,\acute{s})$$

and as $n, m \to \infty$,

$$\begin{aligned}\lim_{n,m\to\infty}\hat{Y}(\rho_n,\rho_m,\acute{s}) &\geq \lim_{n\to\infty}\hat{Y}(\rho_n,\rho_{n+1},\acute{s}) * \lim_{n\to\infty}\hat{Y}(\rho_{n+1},\rho_{n+2},\acute{s}) * ... * \lim_{n\to\infty}\hat{Y}(\rho_{m-1},\rho_m,\acute{s})\\ &\geq 1 * 1 * ... * 1\\ &\geq 1\end{aligned}$$

and

$$\lim_{n\to\infty}\hat{Y}(\rho_n,\rho_m,\acute{s}) = 1.$$

Because $\hat{Y}$ is complete and $\{\rho_n\}$ is a Cauchy, $\exists\ \rho^* \in \hat{Y}$: as $n \to \infty$ and $\rho_n \to \rho^*$.

Assuming $\Omega\rho^* \neq \rho^*$and implementing (3.1) with $\gamma = \rho_n$, $\rho = \rho^*$,

$$\left[1-\hat{Y}(\Omega\rho_n,\Omega\rho^*,\acute{s})\right] \geq \lambda\left[1-\hat{Y}(\rho_n,\rho^*,\acute{s})\right]^{\beta}\left[1-\hat{Y}(\rho_n,\Omega\rho_n,\acute{s})\right]^{\alpha}.\left[1-\hat{Y}(\rho^*,\Omega\rho^*,\acute{s})\right]^{1-\alpha-\beta}$$

and as $n \to \infty$,

$$\left[1 - \hat{Y}(\Omega\rho^*, \Omega\rho^*, \acute{s})\right] \geq \lambda \left[1 - \hat{Y}(\rho^*, \rho^*, \acute{s})\right]^{\beta} \left[1 - \hat{Y}(\rho^*, \Omega\rho^*, \acute{s})\right]^{\alpha} . \left[1 - \hat{Y}(\rho^*, \Omega\rho^*, \acute{s})\right]^{1-\alpha-\beta}$$

so $1 - \hat{Y}(\rho^*, \Omega\rho^*, \acute{s}) = 0 \Longrightarrow \Omega\rho^* = \rho^*$. It is a contradiction.
That is $\Omega\rho^* = \rho^*$ and ρ^* is a fixed point of Ω. □

Definition 8. *Let $(\hat{W}, \hat{Y}^0, *)$ be an EFMS. $\Omega : \hat{W} \longrightarrow \hat{W}$ is a fuzzy$-\hat{Y}^0-$interpolative Reich-Rus-Ciric type contraction, provided that (3.1) is satisfied for all $\acute{s} \geq 0$. Particularly, Ω is called $fuzzy - 0-$ interpolative Reich-Rus-Ciric type contraction, provided that (3.1) is satisfied for $\acute{s} = 0$.*

Theorem 4. *Let $(\hat{W}, \hat{Y}^0, *)$ be a complete NA EFMS. Provided that $\Omega : \hat{W} \longrightarrow \hat{W}$ is a $fuzzy - \hat{Y}^0-$ interpolative Reich-Rus-Ciric type contraction, then Ω has a fixed point in $\hat{W}$.*

Proof. It will be examine two cases.
I. $\acute{s} > 0$;
The situation where $\hat{Y}^0(\gamma, \rho, \acute{s}) = \hat{Y}(\gamma, \rho, \acute{s})\ \forall\ \gamma, \rho \in \hat{W}$ is actually the same as the case proven in Theorem3.1.
II. $\acute{s} = 0$;
Let $\gamma_0 \in \hat{W}$. $(\gamma_n)_{n\in\mathbb{N}} \in \hat{W}$ is a sequence with $\gamma_{n+1} = \Omega\gamma_n$
Here, by examining the cases where $\gamma_{n+1} = \gamma_n$ and $\gamma_n \neq \gamma_{n+1}$, it will be obtain that γ^* is a fixed point of Ω.
Let be $\gamma_{n+1} = \gamma_n$ (for some $n \in \mathbb{N}$), $\gamma^* = \gamma_n$.
Let be $\gamma_n \neq \gamma_{n+1}$ ($\forall\ n \in \mathbb{N}$)
Using (2.1) and (3.1) with $\gamma = \rho_{n-1}$, $\rho = \rho_n$, $\acute{s} = 0$

$$\left[1 - \hat{Y}^0(\Omega\rho_{n-1}, \Omega\rho_n, 0)\right] \geq \lambda \left[1 - N_{\hat{Y}}(\rho_{n-1}, \rho_n)\right]^{\beta} \left[1 - N_{\hat{Y}}(\rho_{n-1}, \Omega\rho_{n-1})\right]^{\alpha} . \left[1 - N_{\hat{Y}}(\rho_n, \Omega\rho_n)\right]^{1-\alpha-\beta}$$

and so

$$\left[1 - N_{\hat{Y}}(\rho_n, \Omega\rho_n)\right] \geq \lambda. \left[1 - N_{\hat{Y}}(\rho_{n-1}, \Omega\rho_{n-1})\right]$$

$\left\{N_{\hat{Y}}(\rho_n,\rho_{n+1})\right\}$ is non-increasing and by iterating

$$\left[1 - N_{\hat{Y}}(\rho_n,\rho_{n+1})\right] \geq \lambda^n \left[1 - N_{\hat{Y}}(\rho_0, \rho_1)\right].$$

Since, as $n \to \infty$ and $\lambda^n \to 0$,

$$N_{\hat{Y}}(\rho_n,\rho_{n+1}) \to 1.$$

Using (3.1) with $\gamma = \rho_n$, $\rho = \rho_m$, $\acute{s} = 0$ ($n < m$),

$$\begin{aligned} \lim_{n\to\infty} N_{\hat{Y}}(\rho_n, \rho_m) &\geq \lim_{n\to\infty} N_{\hat{Y}}(\rho_n, \rho_{n+1}) * \lim_{n\to\infty} N_{\hat{Y}}(\rho_{n+1}, \rho_{n+2}) * ... * \lim_{n\to\infty} N_{\hat{Y}}(\rho_{m-1}, \rho_m) \\ &\geq 1 * 1 * ... * 1 = 1 \end{aligned}$$

it is obtained that

$$\lim_{n\to\infty} N_{\hat{Y}}(\rho_n, \rho_m) = 1.$$

$\{\rho_n\}$ is a Cauchy and $\hat{W}$ is complete,then $\exists\ \rho^* \in \hat{W}$: as $n \to \infty$ and $\rho_n \to \rho^*$.
Because of Ω is continuous, $\Omega\rho_n \to \Omega\rho^*$ and by using (2.1),

$$\lim_{n\to\infty} N_{\hat{Y}}(\Omega\rho_n, \Omega\rho^*) = 1.$$

the limit is unique and so $\rho^* = \Omega\rho^*$.So the proof is completed. □

Example 1. *Let* $\hat{W} = \{1, 2, 3, 4\}$ *be a set,* $*$ *is product* $t - norm$, $\hat{Y}^0$ *is an EFMS on* $\hat{W}$ *and for* $\forall\ \gamma, \rho \in \hat{W}$;

$$\hat{Y}^0(\gamma, \rho, t) = e^{-\frac{|\gamma-\rho|}{t+1}}\ .$$

$(\hat{W}, \hat{Y}^0, *)$ *is a complete Non-Archimedean EFMS and we define a self mapping* $\Omega = \binom{1\ 2\ 3\ 4}{3\ 1\ 2\ 4}$ *on* $\hat{W}$.

Ω *is a* $fuzzy - \hat{Y}^0-$ *interpolative Reich-Rus-Ciric type contraction for all* $\zeta, \rho \in \hat{W}$ *and* $\lambda = \alpha = \beta = \frac{1}{2}$ *such that;*

I.*for* $\gamma = 1,\ \rho = 2$

$$\begin{aligned}
1 - \hat{Y}(3, 1.t) &= 1 - e^{-\frac{|\gamma-\rho|}{\acute{s}+1}} \\
&= 1 - e^{-\frac{2}{\acute{s}+1}} \\
&= \left(1 - e^{-\frac{1}{\acute{s}+1}}\right)\left(1 + e^{-\frac{1}{\acute{s}+1}}\right) \\
&> \left(1 - e^{-\frac{1}{\acute{s}+1}}\right) \\
&= \sqrt{1 - e^{-\frac{1}{\acute{s}+1}}}\sqrt{1 + e^{-\frac{1}{\acute{s}+1}}}\left(1 - e^{-\frac{1}{\acute{s}+1}}\right)^0 \\
&> \sqrt{1 - e^{-\frac{1}{\acute{s}+1}}}\sqrt{1 - e^{-\frac{2}{\acute{s}+1}}}\left(1 - e^{-\frac{1}{\acute{s}+1}}\right)^0 \\
&> \frac{1}{2}\left(1 - e^{-\frac{1}{\acute{s}+1}}\right)^{\frac{1}{2}}\left(1 - e^{-\frac{2}{\acute{s}+1}}\right)^{\frac{1}{2}}\left(1 - e^{-\frac{1}{\acute{s}+1}}\right)^0 \\
&= \lambda\left[1 - \hat{Y}(1, 2, t)\right]^{\beta}\left[1 - \hat{Y}(1, 3, t)\right]^{\alpha}.\left[1 - \hat{Y}(2, 1, t)\right]^{1-\alpha-\beta}
\end{aligned}$$

Similarly, it can be shown to be true for **II**.$(\gamma = 1, \rho = 3)$ *and for* **III**.$(\gamma = 2, \rho = 3)$.

So the conditions of Theo4. are satisfied. "4"*is unique fixed point of* Ω.

4. Conclusion

In the literature, many contraction mappings defined in metric spaces have been adapted to fuzzy metric spaces. However, the contraction used in this study is hybrid, that is, a contraction obtained by the interpolative approach. The contraction is first transferred to a fuzzy metric space and then adapted to an extended fuzzy metric space. In this way, many contraction mappings can be redefined by the interpolative approach and transferred to different fuzzy metric spaces.

5. ACKNOWLEDGMENTS

The author would like to thank the reviewers and editors of Journal of Universal Mathematics.

Funding
The author declared that has not received any financial support for the research, authorship or publication of this study.

The Declaration of Conflict of Interest/ Common Interest
The author declared that no conflict of interest or common interest

The Declaration of Ethics Committee Approval
This study does not be necessary ethical committee permission or any special permission.

The Declaration of Research and Publication Ethics
The author declared that they comply with the scientific, ethical, and citation rules of Journal of Universal Mathematics in all processes of the study and that they do not make any falsification on the data collected. Besides, the author declared that Journal of Universal Mathematics and its editorial board have no responsibility for any ethical violations that may be encountered and this study has not been evaluated in any academic publication environment other than Journal of Universal Mathematics.

References

[1] S. Banach, Sur les oprations dans les ensembles abstrails et leur application aux quations intgrales, Fund Math., Vol.3, pp.133-181 (1922).
[2] A. George and P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets and Systems., Vol.64, pp.395-399 (1994). Doi:10.1016/0165-0114(94)90162-7.
[3] M. Grabiec, Fixed points in fuzzy metric spaces, Fuzzy Sets and Systems., Vol.27, pp.385-389 (1988). Doi:10.1016/0165-0114(88)90064-4.
[4] V. Gregori, J. J. Minana, D. Miravet, Extended fuzzy metrics and fixed point theorems, Mathematics Journal, Vol.7, pp.303 (2019). Doi:10.3390/math7030303.
[5] V. Gregori, S. Romaguera, Characterizing completable fuzzy metric spaces, Fuzzy Sets and Systems, Vol.144, pp.411-420 (2014). Doi:10.1016/S0165-0114(03)00161-1.
[6] V. Gregori, A. Sapena, On fixed point theorems in fuzzy metric spaces, Fuzzy Sets and Systems, Vol.125, pp.245-252 (2002). Doi:10.1016/S0165-0114(00)00088-9.
[7] V. Istratescu, An introduction to theory of probabilistic metric spaces with applications, Ed Tehnica, Bucureşti-Romanian, (1974).
[8] E. Karapinar, R. P. Agarwal, H. Aydi, Interpolative Reich-Rus-Ciric type contractions on partial metric spaces, Mathematics, Vol.6, pp.256 (2018).
[9] I. Kramosil, J. Michalek, Fuzzy metrics and statistical metric spaces, Kybernetika, Vol.11, pp.336-344 (1975).
[10] D. Mihet, Fuzzy $\psi-$ contractive mappings in non-Archimedean fuzzy metric space, Fuzzy Sets and Systems, Vol.159, pp.736-744 (2008). Doi:10.1016/j.fss.2007.07.006.
[11] B. Schwizer, A. Sklar, Statistical metric spaces, Pacific Journal of Mathematics, Vol.10, pp.315-367 (1960).
[12] L. A. Zadeh, Fuzzy sets, Inform. Control., Vol.8, pp.338-353 (1965).

Department of Mathematics, Faculty of Sciences, Ankara University, 06100 Ankara, TURKEY
Email address: meryemsnck@gmail.com