Konuralp Journal of Mathematics
Volume 2 No. 2 pp. 102–108 (2014) ©KJM

.
.

NEIGHBOURHOODS OF A SUBCLASS OF UNIFORMLY CONVEX FUNCTIONS

T. RAM REDDY AND P. THIRUPATHI REDDY

Abstract. In this paper, we investigate the properties of neighbourhoods of functions for the classes $UCV(\alpha)$ and $Sp(\alpha)$. First we established an inclusion relationship between them and proved a necessary and sufficient condition interms of convolutions for a function f to be in $Sp(\alpha)$. Next we show that the class $Sp(\alpha)$ is closed under convolution with functions f(z) which are convex univalent. The results obtained in this which generalizes the results of Padmanabhan [8] and Ronning [9].

1. Introduction:

Let A denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the unit disk $E = \{z \colon |z| < 1\}$. Further, let S be the subclass of A consisting of those functions that are univalent in E. Let CV and ST denote the subclasses of S consisting of convex and starlike functions respectively.

If $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$ then the convolution or Hadamard product of $f(z)$ and $g(z)$ denoted by $f * g$ is defined by

$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n$. Clearly
$f(z) * \frac{z}{(1-z)^2} = zf'(z)$ and $f(z) * \frac{z}{(1-z)} = f(z)$

Goodman[3,4] defined the following subclasses of CV and ST.

Definition A: A function f is uniformly convex (Starlike) in E if f is in CV (ST) and has the property that for every circular arc γ contained in E with centre ξ also in E, the arc $f(\gamma)$ is convex (Starlike w.r.t $f(\xi)$.

Goodman [3,4] then gave the following two variable analytic characterizations of these classes, denoted by UCV and UST.

2000 *Mathematics Subject Classification.* 30C45.
Key words and phrases. Neighbourhood, Subordination, Hadamard Product.

Theorem A: A function f of the form (1.1) is in UCV if and only if

$$Re\left\{1+(z\ -\ \xi)\ \frac{f''(z)}{f'}\right\}\ \geq\ 0,\ (z,\ \xi)\ \in\ EXE \tag{1.2}$$

and is in UST if any only if

$$Re\ \left\{\frac{f(z)\ \ -\ f(\xi)}{(z\ -\ \xi)\,f'(z)}\right\}\ \geq\ 0,\ \ (z,\ \xi)\ \ \in\ EXE \tag{1.3}$$

The classical Alexander result that $f\ \in\ CV$ if and only if $zf\prime\ \in\ ST$ does not hold between the classes UCV and UST. Ronning [7] defined a subclass of starlike functions Sp with the property that a function $f\ \in\ UCV$ if and only if $zf\prime \in Sp$.

Definition B: Let $Sp=\{F\ \in\ ST/F(z)\ =\ zf'(z)\,,f\ \in\ UCV\}$

Ma and Minda [6] and Ronning [10] independently found a more applicable one variable characterization for UCV.

Theorem B: A function f is in UCV if and only if

$$Re\left\{1\ +\ \frac{zf''(z)}{f'(z)}\right\}\ \geq\ \left|\frac{zf''(z)}{f'(z)}\right|,\quad z\in E. \tag{1.4}$$

Ronning [10] proved a one variable characterization for Sp as follows:

Theorem C: A function f is in Sp if and only if

$$\left|\frac{zf'(z)}{f(z)}\ -\ 1\right|\ \leq\ Re\ \left\{\frac{zf'(z)}{f(z)}\right\},\ z\ \in\ E. \tag{1.5}$$

A function $f\ \in\ A$ is uniformly convex of order α for -1$\leq\ \alpha\ <\ 1$ if and only if $1\ +\ \frac{zf''(z)}{f'(z)}$ lies in the parabolic region

(??) Re $\{\omega$ - α $\}$ > $|\omega$ - 1 $|$

In otherwords, the function f is uniformly convex of order α if

$$1\ +\ \frac{zf''(z)}{f'(z)}\prec\ 1\ +\ \frac{2\,(1\ -\ \alpha)}{\pi^2}\ \left[\log\left(\frac{1\ +\ \sqrt{z}}{1\ -\ \sqrt{z}}\right)\right]^2,\ z\ \in\ E \tag{1.6}$$

where the symbol $\prec$ denotes subordination. This class was introduced by Ronning [9] and it is denoted by $UCV\ (\alpha)$. The class of all analytic functions $f(z)\in A$ for which $\frac{zf'(z)}{f(z)}$ lies in the parabolic region is denoted by $Sp(\alpha)$ and defined as follows.

Definition C: A function $f(z)$ is said to be in the class $Sp(\alpha)$ if for all $z\in E$,

$$\left|\frac{zf'(z)}{f(z)}\ -\ 1\right|\ \leq\ Re\ \left\{\frac{zf'(z)}{f(z)}\right\}\ -\ \alpha,\ \text{for}\ -\ 1\ <\ \alpha\ <\ 1. \tag{1.7}$$

This implies $f\ \in\ Sp(\alpha)$ for $z\ \in\ E$ if and only if $\frac{zf'(z)}{f(z)}$ lies in the region $\Omega\alpha$ bounded by a parabola with vertex at $\left(\frac{1+\alpha}{2},\ 0\right)$ and parameterized by $\frac{t^2+\ 1\ -\ \alpha^2\ +\ 2\ it\ (1\ -\ \alpha)}{2(1\ -\ \alpha)}$ for any real t.

It is known [9] that the function

$$P_\alpha(z)\ =\ 1\ +\ \frac{2\,(1\ -\ \alpha)}{\pi^2}\left[\log\frac{1\ +\ \sqrt{z}}{1-\sqrt{z}}\right]^2 \tag{1.8}$$

maps the unit disk E on to the parabolic region $\Omega\alpha$ (The branch $\sqrt{z}$ is choosen in such a way that Im $\sqrt{z} \geq 0$). Then from the above definition $f \in A$ is in the class $Sp(\alpha)$ if and only if $\frac{zf'(z)}{f(z)} \prec P_\alpha(z)$.
The notion of δ - neghbourhood was first introduced by St. Ruscheweyh [11].
Definition D: For $\delta \geq 0$, the δ - neighbourhood of $f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in A$ is defined by

$$N_\delta(f) = \left\{ g(z) = z + \sum_{n=2}^{\infty} b_n z^n : \sum_{n=2}^{\infty} n|a_n - b_n| \leq \delta \right\}. \tag{1.9}$$

Recently Padmanabhan [8] has introduced the neighbourhoods of functions in the calss Sp and studied various properties.

In this paper we studied some related work on the neighbourhood problems for k-uniformly convex functions of Kanas[5].The work of Ma and Minda [7] generalize many studies on subclasses of starlike and convex functions. we introduce a new class of functions and study the properties of neighbourhoods, of functions in this class which generalizes the recent results of Padmanabhan [8] and Ronning [9].

First let us state lammas which are needed to establish our results in the sequel.
Lemma A [2]: Let $\beta, \gamma \in C$, let $h(z)$ be analytic, univalent and convex in E with $h(0) = 1$ and Re $(\beta\ h\ (z) + \gamma) > 0$, $z \in E$ and let $p(z) = 1 + p1\ z + \ldots.\ z \in E$, then

$$p(z) + \frac{z\,p'(z)}{\beta\,p(z) + \gamma} \prec h(z) \Rightarrow p(z) \prec h(z). \tag{1.10}$$

Lemma B [12]: Let $f(z) = \sum_{n=2}^{\infty} a_n z^n$ and $g(z) = \sum_{n=2}^{\infty} a_n z^n$ be in $ST\left(\frac{1+\alpha}{2}\right)$ denote by $f * g$ the Hadamard product $(f * g)(z) = \sum_{n=2}^{\infty} a_n\ b_n\ z^n$. Then for any function $F(z)$ analytic in E, we have for $z \in E$ that

$$\frac{f(z) * g(z)\ F(z)}{f(z) * g(z)} \subset \overline{Co}\ (F(E))$$

$\overline{Co}$ denotes the closed convex hull.

2. Main Results

First let us establish an inclusion relation.
Theorem 2.1: Let $f \in UCV\ (\alpha)$. Then $f \in Sp(\alpha)$.
Proof: Let $p(z) = \frac{zf'(z)}{f(z)}$. Then since $f \in UCV(\alpha)$

$$p(z) + \frac{zp'(z)}{p(z)} = 1 + \frac{zf''(z)}{f'(z)} \subset \Omega_\alpha$$

Since $\Omega\alpha$ is a convex damain, an application of Lemma A gives $\frac{zp'(z)}{p(z)} = p(z) \subset \Omega_\alpha,\ z \in\ E$ which implies that $f \in Sp(\alpha)$.

Now we give a characterization of the class $Sp(\alpha)$ in terms of convolution.
Definition 2.1: Let $S'_p(\alpha)$ be the class of all functions $h\alpha(z)$ in A of the form

$$h_\alpha(z) = \frac{2(1-\alpha)}{(1-\alpha)^2 - t^2 - 2it(1-\alpha)} \left[\frac{2}{(1-z)^2} - \frac{t^2 + 1 - \alpha^2 + 2it(1-\alpha)}{2(1-\alpha)} \frac{z}{(1-z)} \right] \tag{1.11}$$

for -1 $\leq \alpha < 1$ and for all real t.

Theorem 2.2: A function f(z) in A is in *Sp(α)* if and only if for all z in E ($z \neq 0$) there exists a function *hα(z)* in $S'_p(\alpha)$ such that $\frac{(f * h_\alpha)(z)}{z} \neq 0$.

Proof: Let us assume that $\frac{(f * h_\alpha)(z)}{z} \neq 0$, then for all $h\alpha(z) \in S'_p(\alpha)$ and for $z \in E$ ($z \neq 0$). From the definition of $h\alpha(z)$ it follows that

$$\frac{f(z) * h_\alpha(z)}{z} = \frac{2(1-\alpha)}{z\left[(1-\alpha)^2 - t^2 - 2it(1-\alpha)\right]}\left[f(z) * \frac{z}{(1-z)^2} - \frac{t^2+1-\alpha^2+2it(1-\alpha)}{2(1-\alpha)} f * \frac{z}{1-z}\right]$$

$$= \frac{2(1-\alpha)}{z\left[(1-\alpha)^2 - t^2 - 2it(1-\alpha)\right]}\left[zf'(z) - \frac{t^2+1-\alpha^2+2it(1-\alpha)}{2(1-\alpha)} f(z)\right]$$

$$\neq 0.$$

Equivalently $\frac{zf'(z)}{f(z)} \neq \frac{t^2+1-\alpha^2+2it(1-\alpha)}{2(1-\alpha)}, t \in R$. This means that $\frac{zf'(z)}{f(z)}$ lies completely either inside $\Omega\alpha$ or complement of $\Omega\alpha$ for all z in E. At $z = 0$, $\frac{zf'(z)}{f(z)} = 1 \in \Omega\alpha$, so $\frac{zf'(z)}{f(z)} \subset \Omega\alpha$ which means $f \in Sp(\alpha)$.

Conversely let $f \in Sp(\alpha)$. Hence $\frac{zf'(z)}{f(z)}$ lies with in the parabola with vertex at the point $\left(\frac{1+\alpha}{2}, 0\right)$ and the boundary of this is given by $\frac{t^2+1-\alpha^2+2it(1-\alpha)}{2(1-\alpha)}$ for $t \in R$. So $f \in Sp(\alpha)$ only when

$$\frac{zf'(z)}{f(z)} \neq \frac{t^2+1-\alpha^2+2it(1-\alpha)}{2(1-\alpha)}$$

Equivalently

$f(z) * \left[\frac{z}{(1-z)^2} - \frac{t^2+1-\alpha^2+2it(1-\alpha)}{2(1-\alpha)}\frac{z}{(1-z)}\right] \neq 0$ for $z \neq 0$.

Normalizing the function within the brackets we get $\frac{(f * h_\alpha)(z)}{z} \neq 0$ in E where $h\alpha(z)$ is the function defined in (1.11).

To investigate the δ neighbourhoods of functions belonging to the class $Sp(\alpha)$, we need the following lemmas.

Lemma 2.1: Let $h_\alpha(z) = z + \sum_{k=2}^{\infty} c_k z^k \in S'_p(\alpha)$. Then

$$|c_k| \leq \frac{2k-(1+\alpha)}{(1-\alpha)}, \quad k = 2, 3 \ldots.$$

Proof: Let $h_\alpha(z) \in S'_p(\alpha)$. Then for $t \in R$

$$h_\alpha(z) = \frac{2(1-\alpha)}{(1-\alpha)^2 - t^2 - 2it(1-\alpha)}\left[\frac{z}{(1-z)^2} - \frac{t^2+1-\alpha^2+2it(1-\alpha)}{2(1-\alpha)}\frac{z}{(1-z)}\right]$$

$$= \frac{2(1-\alpha)}{(1-\alpha)^2 - t^2 - 2it(1-\alpha)}\left[\left(z+2z^2+......\right) - \frac{t^2+1-\alpha^2+2it(1-\alpha)}{2(1-\alpha)}\left(z + z^2 + ...\right)\right]$$

$$= z + \sum_{k=2}^{\infty} c_k z^k$$

Now comparing the coefficients on either side we get

$$c_k = \frac{2k(1-\alpha) - t^2 - 1 + \alpha^2 - 2it(1-\alpha)}{(1-\alpha)^2 - t^2 - 2it(1-\alpha)}$$

After simplication we get

$$|c_k| \leq T_k = \frac{2k-(1+\alpha)}{(1-\alpha)}, \text{ for } k = 2, 3 \ldots$$

Lemma 2.2: For $f \in A$ and or every $\epsilon \in C$ such that $|\epsilon| < \delta$ if $F\epsilon(z) = \frac{f(z) + \varepsilon z}{1+\varepsilon} \in Sp(\alpha)$ then for every $h\alpha(z) \in S_p'(\alpha)$.

$$\left|\frac{(f * h_\alpha)(z)}{z}\right| \geq \delta, \quad z \in E.$$

Proof: Let $F\epsilon(z) \in Sp(\alpha)$. Then by Theorem 2.2, $\frac{F_\varepsilon(z) * h_\alpha(\alpha)}{z} \neq 0$, for all $h\alpha(z) \in S_p'(\alpha)$ and $z \in E$.
Equivalently

$$\frac{(f * h_\alpha)(z) + \varepsilon z}{(1+\varepsilon)z} \neq 0 \ \ or \ \frac{(f * h_\alpha)(z)}{z} \neq -\varepsilon,$$

that is

$$\left|\frac{(f * h_\alpha)(z)}{z}\right| \geq \delta.$$

Theorem 2.3: Let $f \in A$, $\epsilon \in C$ and for $|\epsilon| < \delta < 1$, if $F\epsilon$(z) $\in Sp(\alpha)$. Then $N\delta(f) \subset Sp(\alpha)$
for the sequence

$$T = T_k = \frac{2k - (1+\alpha)}{(1+\alpha)}$$

Proof: Let $h\alpha(z) \in S_p'(\alpha)$ and $g(z) = z + \sum_{k=2}^{\infty} b_k z^k$ is in $N\delta(f)$
Then

$$\left|\frac{(g * h_\alpha)(z)}{z}\right| = \left|\frac{(f * h_\alpha)(z)}{z} + \frac{((g - f) * h_\alpha)(z)}{z}\right|$$

$$\geq \left|\frac{(f * h_\alpha)(z)}{z} - \frac{(g - f)(z) * h_\alpha(z)}{z}\right|$$

$\geq \delta - \left|\sum_{k=2}^{\infty} \frac{(b_k - a_k)\, c_k\, z^k}{z}\right|$, by lemma 2.2.
We have

$$\left|\frac{(g * h_\alpha)(z)}{z}\right| \geq \delta - |z| \sum_{k=2}^{\infty} |c_k|\ |b_k - a_k|$$

$> \delta - \sum_{k=2}^{\infty} T_k\ |b_k - a_k|$, by lemma 2.1
$> \delta$ - $\delta = 0$.
Thus $\left|\frac{(g * h_\alpha)(z)}{z}\right| \neq 0$ in E for all $h\alpha \in S_p'(\alpha)$ and then by Theorem 2.2, we have $g \in Sp(\alpha)$. Hence we have $N\delta\ (f) \subset Sp(\alpha)$.

Next we show that the class $Sp(\alpha)$ is closed under convolution with functions f which are convex univalent in E.

Theorem 2.4: Let $f \in CV$ the class of convex functions and $g(z) \in Sp(\alpha)$. Then $(f * g)(z) \in Sp(\alpha)$.
Proof: The proof of Theorem is similer result of T.N.Shanmugan [13], hence we omitted.
Theorem 2.5: Let $f \in ST\left(\frac{\alpha+1}{2}\right)$, $g \in Sp(\alpha)$. Then $(f * g)(z) \in Sp(\alpha)$.
Proof: Let $g \in Sp(\alpha)$. Assume $f \in ST\left(\frac{\alpha+1}{2}\right)$ and $\frac{zg'(z)}{g(z)}$ play in the role of F in Lemma B, and let $\Omega\alpha = \{|\omega\text{-}1|\text{Re }(\omega\text{-}\alpha)\}$. Using the Lemma B, we get for $z \in E$ that
$\frac{z(f * g)'(z)}{(f * g)(z)} = \frac{f(z) * zg'(z)}{(f * g)(z)} = \frac{f(z) * g(z)\frac{zg'(z)}{g(z)}}{(f * g)(z)} \subset \overline{Co}\,\frac{zg'(z)}{g(z)} \subset \Omega_\alpha$. Since $\Omega\alpha$ is convex and $g \in Sp(\alpha)$. This proves that $(f * g)(z) \in Sp(\alpha)$.

Setting $\alpha = 0$, the following result of Ronning [9] follows.
Corollary 2.1: Let $f \in ST\ (1/2)$, $g \in Sp(0) = Sp$, then $(f * g)(z) \in Sp$.
Theorem 2.6: Let $g \in UCV\ (\alpha)$ and $h(z) \in ST\left(\frac{\alpha+1}{2}\right)$. Then $(g * h)(z) \in UCV(\alpha)$.
Proof: If $g \in UCV(\alpha)$, then $z\,g'(z) \in Sp(\alpha)$. By Theorem 2.4 it follows that $h * zg' \in Sp(\alpha)$. So

$z(h * g)'(z) = h(z) * zg'(z) \in Sp(\alpha)$.

This proves that $(h * g)(z) \in UCV\ (\alpha)$.

Setting $\alpha = 0$, the following result of Padmanabhan [8] follows.
Corollary 2.2: Let $g \in UCV$ and $h(z) \in ST\ (1/2)$. Then $(g * h)(z) \in UCV(\alpha)$.
Theorem 2.7 : Let $f \in UCV(\alpha)$. Then $\frac{f(z) + \varepsilon z}{1+\varepsilon} \in S_p(\alpha)$ for $|\varepsilon| <$.
Proof: Let $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ then

$$\frac{f(z) + \varepsilon z}{1+\varepsilon} = \frac{z(1+\varepsilon) + \sum_{n=2}^{\infty} a_n z^n}{1+\varepsilon} = \frac{f(z) * [z(1+\varepsilon) + \sum_{n=2}^{\infty} z^n]}{1+\varepsilon}$$

$$= f(z) * \frac{\left(z - \frac{\varepsilon}{1+\varepsilon} z^2\right)}{(1-z)} = f(z) * h(z)$$

where $h(z) = \frac{\left[z - \frac{\varepsilon}{1+\varepsilon} z^2\right]}{(1-z)}$
Now

$$\frac{zh'(z)}{h(z)} = \frac{\left[z - \frac{2\varepsilon}{1+\varepsilon} z^2\right]}{\left[z - \frac{\varepsilon}{1+\varepsilon} z^2\right]} + \frac{z}{1-z} = \frac{-\rho z}{1-\rho z} + \frac{1}{1-z}$$

where $\rho = \frac{\varepsilon}{1+\varepsilon}$. Hence $|\rho| < \frac{\varepsilon}{1-|\varepsilon|} < 1/3$ gives $|\varepsilon| < 1/4$
Thus

$$Re\left\{\frac{zh'(z)}{h(z)}\right\} \geq \frac{1 - 2|\rho|\,|z| - |\rho|\,|z|^2}{(1-|\rho|\,|z|)\,(1 + |z|)} > 0$$

if $|\rho|(|z|2 + 2\,|z|) - 1 < 0$. This inequality holds for all $\rho < 1/3$ and $|z| < 1$, which is true for $|\varepsilon| < 1/4$. Therefore $h(z)$ is starlike in the unit disk and so $\int_0^z \frac{h(t)}{t}\,dt$ is convex.

But $h(z) * \log\left(\frac{1}{1-z}\right) = \int_0^z \frac{h(t)}{t}\,dt$ and so $h(z) * \log\left(\frac{1}{1-z}\right)$ is convex in E and
$(f * h)(z) = (h * f)(z) = h(z) * \left[zf'(z) * \log\left(\frac{1}{1-z}\right)\right]$

$= zf\prime(z) * \left[h(z) * \log\left(\frac{1}{1-z}\right)\right]$

$f(z) \in UCV(\alpha)$ implies $zf\prime(z) \in Sp(\alpha)$ and $h(z) * \log\left(\frac{1}{1-z}\right) \in CV$. Now by Theorem 2.4 $h(z) * \left[zf'(z) * \log\left(\frac{1}{1-z}\right)\right]$ is in $Sp(\alpha)$. Thus $(f*h)(z) = \frac{f(z)+\varepsilon z}{1+\varepsilon} \in S_p(\alpha)$ for $|\varepsilon| < 1/4$.

Corollary 2. 3: If $f \in UCV(\alpha)$, then $f \in Sp(\alpha)$.

Proof: Choosing $\varepsilon = 0$ in the Theorem 2.7 we get the result.

Corollary 2. 4: If $f \in UCV(\alpha)$ then $\int_0^z \frac{f(t)}{t}\,dt \in UCV(\alpha)$.

Proof: $f \in UCV(\alpha)$ implies $f \in Sp(\alpha)$ by corollary2.3,so we can write $f(z) = zg\prime(z)$ for some $g \in UCV(\alpha)$ and $g\prime(z) = \frac{f(z)}{z}$ gives $g(z) = \int_0^z \frac{f(t)}{t}dt \in UCV(\alpha)$.

Acknowledgements The authors would like to thank the referee of the paper for his helpful comments.

References:

1. R.M.Ali and V.Ravichandran, Uniformly Convex and Uniformly Starlike functions Mathematics Newspaper, Vol. No 1, (2011),16-30.

2. P. Eenigenburg, S.S. Miller, P.T. Mocanu and M.O. Reade, On a Briot – Boquet Differential Subardination, General Inequalities 3, Birkhauser Verlag-Basel, (1983) 339-348.

3. A.W. Goodman. On uniformly convex functions. Ann.Polon. Math 56(1991), 87-92.

4. A.W. Goodman, On uniformly starlike functions, J. Math. Anal. Appl.155(1991), 364-370.

5. S.Kanas, Staility of convolution and dual sets for the class k- uniformly convex and k- starlike functions functions, Zeszyty Nauk.Politech. Rzeszowskiej Mat., (**??**) (1998) 51-64.

6. W.C. Ma and D. Minda, Uniformly convex functions, Ann. Polon, Math.

$$57(1992), 165 - 175.$$

7. W.C.Ma and D.Minda, A unified treatment of some special classes of univalent functions , in Proceedings of the Conference on Complex Analysis (Tianjin, 1992) , 157-169, Conf.Proc.Lecture Notes Anal.,I Int.Press ,Combridgema.MA.

8. K.S. Padmanabhan, On uniformly convex functions in the unit disk. J. Analysis, 2 (1994), 87-96.

9. F. Ronning, on starlike functions associated with parabolic regions, Ann. Univ. Mariae Curie-Sklodowska. Sect. A, 45 (1991), 117-122.

10. F. Ronning, Uniformly convex functions and a corresponding class of starlike functions, Proc. Amer. Math. Soc. 118 No.1 (1993), 189-196.

11. St. Ruscheweyh, Neighbourhoods of univalent functions, Proc. Amer Math. Soc. 81 (1981), 521-527.

12. St. Ruscheweyh, Convolutions in Geometric functions theory, Sem. Math. Sup. Vol.83, Presses Univ. de Montreal, placeCityMontreal 1982.

13. T.N.Shanmugam, Convolution and Differential subordination, Internat.J.Math & Math. Sci, 12 (**??**) (1989), 333-340.

E-mail address: reddypt2@yahoo.com

Department of Mathematics, Kakatiya University, Warangal - 506 009, INDIA.