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LACUNARY STATISTICAL p-QUASI CAUCHY SEQUENCES

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Abstract. In this paper, we introduce a concept of lacunary statistically p-quasi-Cauchyness of a real sequence in the sense that a sequence (α_k) is lacunary statistically p-quasi-Cauchy if $\lim_{r\to\infty} \frac{1}{h_r} |\{k \in I_r : |\alpha_{k+p} - \alpha_k| \geq \varepsilon\}| = 0$ for each $\varepsilon > 0$. A function f is called lacunary statistically p-ward continuous on a subset A of the set of real numbers $\mathbb{R}$ if it preserves lacunary statistically p-quasi-Cauchy sequences, i.e. the sequence $(f(\alpha_n))$ is lacunary statistically p-quasi-Cauchy whenever $\boldsymbol{\alpha} = (\alpha_n)$ is a lacunary statistically p-quasi-Cauchy sequence of points in A. It turns out that a real valued function f is uniformly continuous on a bounded subset A of $\mathbb{R}$ if there exists a positive integer p such that f preserves lacunary statistically p-quasi-Cauchy sequences of points in A.

1. Introduction

Throughout this paper, $\mathbb{N}$, and $\mathbb{R}$ will denote the set of positive integers, and the set of real numbers, respectively. p will always be a fixed element of $\mathbb{N}$. The boldface letters such as $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\zeta}$ will be used for sequences $\boldsymbol{\alpha} = (\alpha_n)$, $\boldsymbol{\beta} = (\beta_n)$, $\boldsymbol{\zeta} = (\zeta_n)$, ... of points in $\mathbb{R}$. A function $f : \mathbb{R} \longrightarrow \mathbb{R}$ is continuous if and only if it preserves convergent sequences. Using the idea of continuity of a real function in this manner, many kinds of continuities were introduced and investigated, not all but some of them we recall in the following: ward continuity ([15], [4]), p-ward continuity ([23]), δ-ward continuity ([18]), δ^2-ward continuity ([3]), statistical ward continuity, ([19]), λ-statistical ward continuity ([36]), ρ-statistical ward continuity ([6], [25]), slowly oscillating continuity ([12, 59, 35]), quasi-slowly oscillating continuity ([42]), Δ-quasi-slowly oscillating continuity ([16]), arithmetic continuity ([60], [5]), upward and downward statistical continuities ([24]), lacunary statistical ward continuity ([7], [66]), lacunary statistical δ ward continuity ([31]), lacunary statistical δ^2 ward continuity ([64]), N_θ-ward continuity ([22], [30], [48], [8], [48], [47]), N_θ-δ-ward continuity, and ([8]), which enabled some authors to obtain interesting results.

In [45] Fridy and Orhan introduced the concept of lacunary statistically convergence in the sense that a sequence (α_k) of points in $\mathbb{R}$ is called lacunary statistically convergent, or S_θ-convergent, to an element L of $\mathbb{R}$ if $\lim_{r\to\infty} \frac{1}{h_r} |\{k \in I_r : |\alpha_k - L| \geq \varepsilon\}| = 0$ for every positive real number ε where $I_r = (k_{r-1}, k_r]$ and $k_0 = 0$,

$h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$ and $\theta = (k_r)$ is an increasing sequence of positive integers (see also [46], [56] [11], [57], and [44]). In this case we write $S_\theta - lim\alpha_k = L$. The set of lacunary statistically convergent sequences of points in R is denoted by S_θ. In the sequel, we will always assume that $\lim inf_r q_r > 1$. A sequence (α_k) of points in R is called lacunary statistically quasi-Cauchy if $S_\theta - lim\Delta\alpha_k = 0$, where $\Delta\alpha_k = \alpha_{k+1} - \alpha_k$ for each positive integer k. The set of lacunary statistically quasi-Cauchy sequences will be denoted by ΔS_θ.

The purpose of this paper is to introduce lacunary statistically p-quasi-Cauchy sequences, and prove interesting theorems.

2. VARIATIONS ON LACUNARY STATISTICAL WARD COMPACTNESS

The concept of a Cauchy sequence involves far more than that the distance between successive terms is tending to 0 and lacunary statistically tending to zero, and more generally speaking, than that the distance between p-successive terms is lacunary statistically tending to zero, by p-successive terms we mean α_{k+p} and α_k. Nevertheless, sequences which satisfy this weaker property are interesting in their own right.

Before giving our main definition we recall basic concepts. A sequence (α_n) is called quasi Cauchy if $\lim_{n\to\infty} \Delta\alpha_n = 0$, where $\Delta\alpha_n = \alpha_{n+1} - \alpha_n$ for each $n \in \mathbb{N}$ ([4], [15]). The set of all bounded quasi-Cauchy sequences is a closed subspace of the space of all bounded sequences with respect to the norm defined for bounded sequences ([51]). A sequence (α_k) of points in $\mathbb{R}$ is slowly oscillating if $\lim_{\lambda\to 1+} \overline{\lim}_n \max_{n+1\leq k\leq[\lambda n]} |\alpha_k - \alpha_n| = 0$, where $[\lambda n]$ denotes the integer part of λn ([41]). A sequence (α_k) is quasi-slowly oscillating if $(\Delta\alpha_k)$ is slowly oscillating. A sequence (α_n) is called statistically convergent to a real number L if $\lim_{r\to\infty} \frac{1}{h_r}|\{k \in I_r : |\alpha_k - L| \geq \varepsilon\}| = 0$ for each $\varepsilon > 0$ ([43], [14], [32], and [9]). Recently in [23] it was proved that a real valued function is uniformly continuous whenever it is p-ward continuous on a bounded subset of $\mathbb{R}$. Now we introduce the concept of a lacunary statistically p-quasi-Cauchy sequence.

Definition 2.1. *A sequence (α_k) of points in $\mathbb{R}$ is called lacunary statistically p-quasi-Cauchy if $S_\theta - lim_{k\to\infty}\Delta_p\alpha_k = 0$, i.e. $\lim_{r\to\infty} \frac{1}{h_r}|\{k \in I_r : |\Delta_p\alpha_k| \geq \varepsilon\}| = 0$ for each $\varepsilon > 0$, where $\Delta_p\alpha_k = \alpha_{k+p} - \alpha_k$ for every $k \in \mathbb{N}$.*

We will denote the set of all lacunary statistically p-quasi-Cauchy sequences by Δ_p^θ. The sum of two lacunary statistically p-quasi-Cauchy sequences is lacunary statistically p-quasi-Cauchy, the product of a lacunary statistically p-quasi-Cauchy sequence and a constant real number is lacunary statistically p-quasi-Cauchy, so that the set of all lacunary statistically p-quasi-Cauchy sequences Δ_p^θ is a vector space. We note that a sequence is lacunary statistically quasi-Cauchy when $p = 1$, i.e. lacunary statistically 1-quasi-Cauchy sequences are lacunary statistical quasi-Cauchy sequences. It follows from the inclusion
$\{k \in I_r : |\alpha_{k+p} - \alpha_k| \geq \varepsilon\} \subseteq$
$\subseteq \{k \in I_r : |\alpha_{k+p} - \alpha_{k+p-1}| \geq \frac{\varepsilon}{p}\} \cup \{k \in I_r : |\alpha_{k+p-1} - \alpha_{k+p-2}| \geq \frac{\varepsilon}{p}\} \cup ...$
$\cup \{k \in I_r : |\alpha_{k+2} - \alpha_{k+1}| \geq \frac{\varepsilon}{p}\} \cup \{k \in I_r : |\alpha_{k+1} - \alpha_k| \geq \frac{\varepsilon}{p}\}$
that any lacunary statistically quasi-Cauchy sequence is also lacunary statistically p-quasi-Cauchy, but the converse is not always true as it can be seen by considering the the sequence (α_k) defined by $(\alpha_k) = (0, 1, 0, 1, ..., 0, 1, ...)$ is lacunary statistically 2-quasi Cauchy which is not lacunary statistically quasi Cauchy. More examples

can be seen in [51, Section 1.4]. It is clear that any Cauchy sequence is in $\bigcap_{\infty}^{p=1} \Delta_p^\theta$, so that each Δ_p^θ is a sequence space containing the space $\mathcal{C}$ of Cauchy sequences. It should also be noted that $\mathcal{C}$ is a proper subset of Δ_p^θ for each $p \in \mathbb{N}$.

Definition 2.2. *A subset A of $\mathbb{R}$ is called lacunary statistically p-ward compact if any sequence of points in A has a lacunary statistically p-quasi-Cauchy subsequence.*

We note that this definition of lacunary statistically p-ward compactness cannot be obtained by any summability matrix in the sense of [13] (see also [10], and [20]). Since any lacunary statistically quasi-Cauchy sequence is lacunary statistically p-quasi-Cauchy we see that any lacunary statistically ward compact subset of $\mathbb{R}$ is lacunary statistically p-ward compact for any $p \in \mathbb{N}$. A finite subset of $\mathbb{R}$ is lacunary statistically p-ward compact, the union of finite number of lacunary statistically p-ward compact subsets of $\mathbb{R}$ is lacunary statistically p-ward compact, and the intersection of any family of lacunary statistically p-ward compact subsets of $\mathbb{R}$ is lacunary statistically p-ward compact. Furthermore any subset of a lacunary statistically p-ward compact set of $\mathbb{R}$ is lacunary statistically p-ward compact and any bounded subset of $\mathbb{R}$ is lacunary statistically p-ward compact. These observations above suggest to us the following.

Theorem 2.1. *A subset A of $\mathbb{R}$ is bounded if and only if there exists a $p \in \mathbb{N}$ such that A is lacunary statistically p-ward compact.*

Proof. The bounded subsets of $\mathbb{R}$ are lacunary statistically p-ward compact, since any bounded sequence of points in a bounded subset of $\mathbb{R}$ is bounded and any bounded sequence has a convergent subsequence which is lacunary statistically p-quasi-Cauchy for any $p \in \mathbb{N}$. To prove the converse, suppose that A is not bounded. If it is unbounded above, pick an element α_1 of A greater than p. Then we can find an element α_2 of A such that $\alpha_2 > 2p + \alpha_1$. Similarly, choose an element α_3 of A such that $\alpha_3 > 3p + \alpha_2$. So we can construct a sequence (α_j) of numbers in A such that $\alpha_{j+1} > (j+1)p + \alpha_j$ for each $j \in \mathbb{N}$. Then the sequence (α_j) does not have any lacunary statistically p-quasi-Cauchy subsequence. If A is bounded above and unbounded below, then pick an element β_1 of A less than $-p$. Then we can find an element β_2 of A such that $\beta_2 < -2p + \beta_1$. Similarly, choose an element β_3 of A such that $\beta_3 < -3p + \beta_2$. Thus one can construct a sequence (β_i) of points in A such that $\beta_{i+1} < -(i+1)p + \beta_i$ for each $i \in \mathbb{N}$. Then the sequence (α_i) does not have any lacunary statistically p-quasi-Cauchy subsequence. Thus this contradiction completes the proof of the theorem. $\square$

It follows from Theorem 2.1 that lacunary statistically p-ward compactness of a subset of A of $\mathbb{R}$ coincides with either of the following kinds of compactness: p-ward compactness ([23, Theorem 2.3]), statistical ward compactness ([19, Lemma 2]), λ-statistical ward compactness ([36, Theorem 1]), ρ-statistical ward compactness ([6, Theorem 1]), strongly lacunary ward compactness ([22, Theorem 3.3]), slowly oscillating compactness ([17, Theorem 3]), lacunary statistical ward compactness (see [7]), and [17, Theorem 3]), ideal ward compactness ([29, Theorem 8]), Abel ward compactness ([?, Theorem 5]).

If a closed subset of $\mathbb{R}$ is lacunary statistically p-ward compact for a positive integer p, then any sequence of points in A has a (P_n, s)-absolutely almost convergent subsequence (see [27], [37], [52], [62], [2], [65], and [63]).

Corollary 2.2. A subset of R is statistically p-ward compact if and only if it is statistically q-ward compact for any $p, q \in \mathbb{N}$.

Corollary 2.3. A subset of R is statistically p-ward compact if and only if it is both statistically upward half compact and statistically downward half compact.

Proof. The proof follows from [24, Corollary 3.9], so is omitted.   □

Corollary 2.4. A subset of $\mathbb{R}$ is lacunary statistically p ward compact for a $p \in \mathbb{N}$ if and only if it is both lacunary statistically upward half compact and lacunary statistically downward half compact.

Proof. The proof follows from [33, Theorem 1.3 and Theorem 1.9], so is omitted.   □

3. Variations on lacunary statistical ward continuity

In this section, we investigate connections between uniformly continuous functions and lacunary statistically p-ward continuous functions. A function $f : \mathbb{R} \longrightarrow \mathbb{R}$ is continuous if and only if it preserves lacunary statistically convergent sequences. Using this idea, we introduce lacunary statistical p-ward continuity.

Definition 3.1. *A function f is called lacunary statistically p-ward continuous on a subset A of $\mathbb{R}$ if it preserves lacunary statistically p-quasi-Cauchy sequences, i.e. the sequence $(f(\alpha_n))$ is lacunary statistically p-quasi-Cauchy whenever (α_n) is lacunary statistically p-quasi-Cauchy of points in A.*

We see that this definition of lacunary statistically p-ward continuity can not be obtained by any summability matrix A (see [10]).

We note that the sum of two lacunary statistically p-ward continuous functions is lacunary statistically p-ward continuous, and for any constant $c \in \mathbb{R}$, cf is lacunary statistically p-ward continuous whenever f is a lacunary statistically p-ward continuous function, so that the set of all lacunary statistically p ward continuous functions is a vector space. The composite of two lacunary statistically p-ward continuous functions is lacunary statistically p-ward continuous, but the product of two lacunary statistically p-ward continuous functions need not be lacunary statistically p-ward continuous as it can be seen by considering product of the lacunary statistically p-ward continuous function $f(x) = x$ with itself. If f is a lacunary statistically p-ward continuous function, then $|f|$ is also lacunary statistically p-ward continuous since

$$|\{k \in I_r : |f(\alpha_{k+p}) - f(\alpha_k)| \geq \varepsilon\}| \subseteq |\{k \in I_r : ||f(\alpha_{k+p})| - |f(\alpha_k)|| \geq \varepsilon\}|$$

which follows from the inequality $||f(\alpha_{k+p})| - |f(\alpha_k)|| \leq |f(\alpha_{k+p}) - f(\alpha_k)|$. If f, and g are lacunary statistically p-ward continuous, then $max\{f, g\}$ is also lacunary statistically p-ward continuous, which follows from the equality $max\{f, g\} = \frac{1}{2}\{|f - g| + |f + g|\}$.

Theorem 3.1. *If f is lacunary statistically p-ward continuous on a subset A of $\mathbb{R}$ for some $p \in \mathbb{N}$, then it is lacunary statistically ward continuous on A.*

Proof. If $p = 1$, then it is obvious. So we would suppose that $p > 1$. Take any lacunary statistically p-ward continuous function f on A. Let (α_k) be any lacunary statistical quasi-Cauchy sequence of points in A. Write

$$(\xi_i) = (\alpha_1, \alpha_1, ..., \alpha_1, \alpha_2, \alpha_2, ..., \alpha_2, ..., \alpha_n, \alpha_n, ..., \alpha_n, ...),$$

where the same term repeats p times. The sequence

$$(\alpha_1, \alpha_1, ..., \alpha_1, \alpha_2, \alpha_2, ..., \alpha_2, ..., \alpha_n, \alpha_n, ..., \alpha_n, ...)$$

is also lacunary statistically quasi-Cauchy so it is lacunary statistically p-quasi-Cauchy. By the lacunary statistically p-ward continuity of f, the sequence

$$(f(\alpha_1), f(\alpha_1), ..., f(\alpha_1), f(\alpha_2), f(\alpha_2), ..., f(\alpha_2), ..., f(\alpha_n), f(\alpha_n), ..., f(\alpha_n), ...)$$

is lacunary statistically p-quasi-Cauchy, where the same term repeats p-times. Thus the sequence

$$(f(\alpha_1), f(\alpha_1), ..., f(\alpha_1), f(\alpha_2), f(\alpha_2), ..., f(\alpha_2), ..., f(\alpha_n), f(\alpha_n), ..., f(\alpha_n), ...)$$

is also lacunary statistically p quasi-Cauchy. It is easy to see that $S_\theta - lim(f(\alpha_{n+p}) - f(\alpha_n)) = 0$, which completes the proof of the theorem.

$\square$

Corollary 3.2. *If f is lacunary statistically p-ward continuous on a subset A of $\mathbb{R}$, then it is continuous on A in the ordinary case.*

Proof. The proof follows immediately from [19, Theorem 3] so is omitted. $\square$

Theorem 3.3. *Lacunary statistical p-ward continuous image of any lacunary statistically p-ward compact subset of $\mathbb{R}$ is lacunary statistically p-ward compact.*

Proof. Let f be a lacunary statistically p-ward continuous function, and A be a lacunary statistically p-ward compact subset of $\mathbb{R}$. Take any sequence $\boldsymbol{\beta} = (\beta_n)$ of terms in $f(E)$. Write $\beta_n = f(\alpha_n)$ where $\alpha_n \in E$ for each $n \in \mathbb{N}$, $\boldsymbol{\alpha} = (\alpha_n)$. Lacunary statistically p-ward compactness of A implies that there is a lacunary statistically p-quasi-Cauchy subsequence $\boldsymbol{\xi} = (\xi_k) = (\alpha_{n_k})$ of $\boldsymbol{\alpha}$. Since f is lacunary statistically p-ward continuous, $(t_k) = f(\boldsymbol{\xi}) = (f(\xi_k))$ is lacunary statistically p-quasi-Cauchy. Thus (t_k) is a lacunary statistically p-quasi-Cauchy subsequence of the sequence $f(\boldsymbol{\alpha})$. This completes the proof of the theorem. $\square$

Corollary 3.4. *Lacunary statistical p-ward continuous image of any G-sequentially connected subset of R is G-sequentially connected for a regular subsequential method G.*

Proof. The proof follows from the preceding theorem, so is omitted (see [21] and [50] for the definition of G-sequential connectedness and related concepts). $\square$

Theorem 3.5. *If f is uniformly continuous on a subset A of $\mathbb{R}$, then $(f(\alpha_n))$ is lacunary statistically p-quasi-Cauchy whenever (α_n) is a p-quasi-Cauchy sequence of points in A.*

Proof. Let (α_n) be any p-quasi-Cauchy sequence of points in A. Take any $\varepsilon > 0$. Uniform continuity of f on A implies that there exists a $\delta > 0$, depending on ε, such that $|f(x) - f(y)| < \varepsilon$ whenever $|x - y| < \delta$ and $x, y \in A$. For this $\delta > 0$, there exists an $N = N(\delta)$ such that $|\Delta_p \alpha_n| < \delta$ whenever $n > N$. Hence $|\Delta_p f(\alpha_n)| < \varepsilon$ if $n > N$. Thus $\{k \in I_r : |\Delta_p f(\alpha_k)| \geq \varepsilon\} \subseteq \{1, 2, ..., N\}$. Therefore $\lim_{r\to\infty} \frac{1}{h_r}|\{k \in I_r : |\Delta_p f(\alpha_k)| \geq \varepsilon\}| \leq \lim_{n\to\infty} \frac{1}{n}|\{k \leq N : k \in \mathbb{N}\}| = 0$. It follows from this that $(f(\alpha_n))$ is a lacunary statistically p-quasi-Cauchy sequence. This completes the proof of the theorem. $\square$

Corollary 3.6. *If f is slowly oscillating continuous on a bounded subset A of $\mathbb{R}$, then $(f(\alpha_n))$ is lacunary statistically p-quasi-Cauchy whenever (α_n) is a p quasi-Cauchy sequence of points in A.*

Proof. If f is a slowly oscillating continuous function on a bounded subset A of $\mathbb{R}$, then it is uniformly continuous on A by [38, Theorem 2.3]. Hence the proof follows from Theorem 3.5. $\square$

It is well-known that any continuous function on a compact subset A of $\mathbb{R}$ is uniformly continuous on A. We have an analogous theorem for a lacunary statistically p-ward continuous function defined on a lacunary statistically p-ward compact subset of $\mathbb{R}$.

Theorem 3.7. *If a function is lacunary statistically p-ward continuous on a lacunary statistically p-ward compact subset of $\mathbb{R}$, then it is uniformly continuous on A.*

Proof. Suppose that f is not uniformly continuous on A so that there exist an $\epsilon_0 > 0$ and sequences (α_n) and (β_n) of points in A such that $|\alpha_n - \beta_n| < 1/n$ and $|f(\alpha_n) - f(\beta_n)| \geq \epsilon_0$ for all $n \in \mathbb{N}$. Since A is lacunary statistically p-ward compact, there is a subsequence (α_{n_k}) of (α_n) that is lacunary statistically p-quasi-Cauchy. On the other hand, there is a subsequence $(\beta_{n_{k_j}})$ of (β_{n_k}) that is lacunary statistically p-quasi-Cauchy as well. It is clear that the corresponding sequence $(a_{n_{k_j}})$ is also lacunary statistically p-quasi-Cauchy, since

$\{j \in I_r : |\alpha_{n_{k_{j+p}}} - \alpha_{n_{k_j}}| \geq \varepsilon\} \subseteq \{j \in I_r : |\alpha_{n_{k_{j+p}}} - \beta_{n_{k_{j+p}}}| \geq \frac{\varepsilon}{3}\} \cup \{j \in I_r : |\beta_{n_{k_{j+p}}} - \beta_{n_{k_j}}| \geq \frac{\varepsilon}{3}\} \cup \{j \in I_r : |\beta_{n_{k_j}} - \alpha_{n_{k_j}}| \geq \frac{\varepsilon}{3}\}$

for every $n \in \mathbb{N}$, and for every $\varepsilon > 0$. Hence it is easy to establishe a contradiction. thus this completes the proof of the theorem. $\square$

Corollary 3.8. *If a function defined on a bounded subset of $\mathbb{R}$ is lacunary statistically p-ward continuous, then it is uniformly continuous.*

We note that when the domain of a function is restricted to a bounded subset of $\mathbb{R}$, lacunary statistically p-ward continuity implies not only ward continuity, but also slowly oscillating continuity.

4. Conclusion

In this paper, we introduce lacunary statistically p-quasi Cauchy sequences, and investigate conditions for a lacunary statistically p ward continuous real function to be uniformly continuous, and prove some other results related to these kinds of continuities and some other kinds of continuities. It turns out that lacunary statistically p-ward continuity implies uniform continuity on a bounded subset of $\mathbb{R}$. The results in this paper not only involves the related results in [7] as a special case for $p = 1$, but also some interesting results which are also new for the special case $p = 1$. The lacunary statistically p-quasi Cauchy concept for $p > 1$ might find more interesting applications than statistical quasi Cauchy sequences to the cases when statistically quasi Cauchy does not apply. For a further study, we suggest to investigate lacunary statistically p-quasi-Cauchy sequences of soft points and lacunary statistically p-quasi-Cauchy sequences of fuzzy points. However due to the change in settings, the definitions and methods of proofs will not always be analogous to those of the present work (for example see [1], [28], [40], and [49]). We

also suggest to investigate lacunary statistically p-quasi-Cauchy double sequences of points in $\mathbb{R}$ (see [55], [54], [39], and [34] for the related definitions in the double case). For another further study, we suggest to investigate lacunary statistically p-quasi-Cauchy sequences in abstract metric spaces (see [26], [53], [35], [58], and [61]).

5. Acknowledgments

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