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Research Article

MULTI-PERIOD MIXED INTEGER PROGRAMMING MODEL FOR SUPPLY CHAIN PLANNING UNDER SAFETY STOCK

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ABSTRACT
Supply chain management philosophy has been adopted by enterprises due to the requirement of customer demand satisfaction in reasonable times under market competition. In case of rapid increase in product demands and/or occurrence of supply problems in materials, enterprises choose holding some amount of safety stock of several materials and products. In this study, a multi-period, multi-product supply chain with different suppliers, material storages, production plants, distribution centers and customers is modeled. To determine the optimal production, supply and storage plans at minimum cost, a mixed-integer programming model is proposed. Capacity, bill-of-materials structure of products and placement of safety stocks are taken into account within the proposed model. Solutions of a set of examples are also presented in order to test the model.

Keywords: *Supply chain, Safety stock, Integer programming, Bill-of-materials*

1. INTRODUCTION

As a result of changing economic conditions, enterprises need to develop new relations with their customers and suppliers. Furthermore, recently developed customer oriented marketing strategies force enterprises to communicate with their customers in a continuous and dynamic way.

Enterprises, which do not desire to be behind their rivals in market, have to manage their supply chain. Supply chain is defined as the aggregation of whole processes and organizations related to material supply, transformation of materials to products and distribution of products to customers, effectively. Effective management of supply chains reduce costs and increase profit of the enterprise significantly.

One of the popular subjects related to supply chain management and inventory management is to save money and storage area by communication and coordination over supply network. The most important problems being faced at this point are the uncertainty and variability. To overcome the uncertainty and variability problems faced here, companies may hold some amount of safety stock of materials and products.

In this study, a multi-period, multi-product supply chain with several suppliers, material storages, production plants, distribution centers and customers is modelled. To determine the optimal supply, distribution and storage plan at minimum cost, a mixed-integer programming model is proposed. A schematic representation of the supply chain is given in Figure 1.

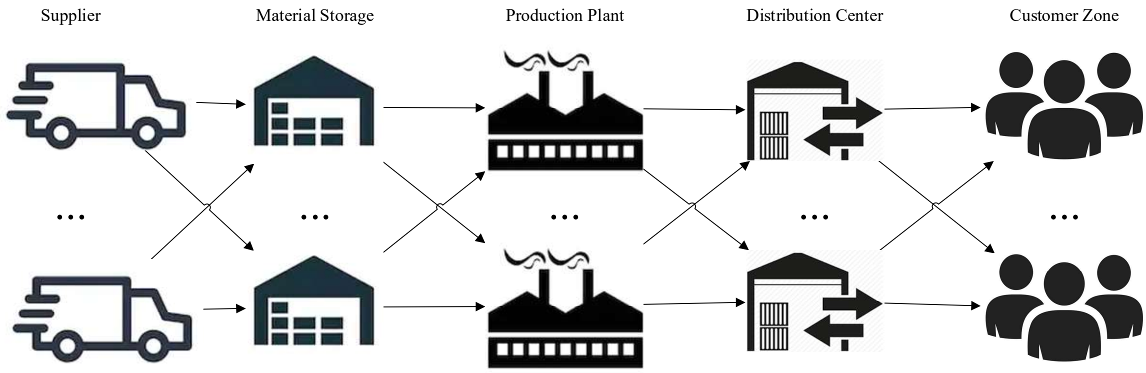

Figure 1. Schematic representation of supply chain network

Rest of the paper is organized as follows: In the second part, a literature review related to optimization of supply chains is presented. The proposed integer programming model is formulated in the third part by presenting the definitions of notations, parameters and decision variables of the model. Solution to a numerical example of the proposed model is given in the fourth part. The paper is concluded in the fifth part by giving further research suggestions.

2. LITERATURE REVIEW

Supply chain planning has taken attention of researchers in recent 20 years. Some of the studies related to this topic can be summarized as follows:

Value add of the activities and resources within the supply chain on overall performance is investigated under the demand and capacity considerations (Lakhal et al., 2001). An integrated solution approach based on Analytic Hierarchy Process and Integer Programming is developed in order to determine partner selection and production-distribution planning decisions in supply chains (Sha and Che, 2004). A two stage dynamic programming approach is used to find the optimal supply chain configuration for a new designed product (Graves and Willems, 2005). Uncertainty in supply chains is modelled by Santoso et al. (2005) by using stochastic programming approach. In another study taking uncertainty into consideration in supply chains (Aliev et al., 2007), fuzzy genetic programming approach is used to determine aggregate production-distribution planning.

A two-stage supply chain under stockout, capacity and safety stock considerations is modelled by Romeijn et al. (2007). Fuzzy multi-objective linear programming formulation is used by Liang (2008) to determine the multi-period production-distribution plan in a multi-product supply chain subject to demand, workforce and machine capacity constraints by minimizing cost and delivery time. A mixed-integer programming model considering quality constraints, tax and exchange rates is proposed to optimize production-distribution plans in supply chains (Tsiakis and Papageorgiou, 2008). A real case study of multi-stage supply chains with several production plants is solved by using fuzzy goal programming to determine aggregated plans of supply, production and distribution (Torabi and Hassini, 2009).

A mixed-integer programming model subject to supplier capacity constraints is proposed to optimize supply chain configuration in the agile manufacturing supply chains (Constantino et al., 2012). The trade-off between quality and profit is searched in a study that maximizes the profit on a supply chain (Paksoy et al., 2012).

A mixed-integer program is proposed to optimize cost and reliability objectives in a three echelon production distribution system (Khalifehzadeh et al., 2017). Miranda et al. (2018) propose an integrated production, distribution, routing and inventory planning model for the small furniture companies. A bi-objective production-distribution planning model is proposed by Rafiei et al. (2018) to optimize cost and service level objectives. A goal programming model taking lead times, bill-of-materials, capacity and demand issues into account is proposed by Aktas and Temiz (2020) to determine the trade-off point between profit and emission caused of transportation activities over the supply chain.

Production, inventory and distribution decisions in a supply chain are optimized by using a column generation and MILP based two-stage approach to obtain maximum profit (Cóccola et al., 2020). A two – stage stochastic mixed integer programming formulation is proposed to determine integrated production-distribution decisions in dairy products supply chain (Guarnaschelli et al., 2020). Optimal production capacity and safety stock levels in a multi-product serial production-distribution network are determined under guaranteed service approach (Ghadimi and Aouam, 2021). The problem of the study is formulated as a non-convex program subject to budget limitation and solution is obtained by developing a nested Lagrangian relaxation algorithm.

In the light of reviewed studies, a mixed-integer programming model for the optimization of a multi-product supply chain is proposed in this study by considering material supply, production, distribution and inventory planning decisions in an integrated manner.

3. PROPOSED MODEL

The main aim of this study is to propose a mathematical model to support multi-period planning decisions in multi-product multi-stage supply chains with safety stock consideration. The supply chain defined with the mathematical model consists several suppliers, customers, material storage, production and distribution center plants. Each material storage must have inventory of each item at least safety stock level and cannot have more than storage capacity. Similarly, distribution centers must have products more than safety stock level and can have at most storage capacity of products. Moreover, production capacity and bill-of-materials structure are also taken into consideration in the proposed model. Distribution centers send products to customer locations to satisfy customer demands.

Indices, parameters and decision variables of the model are defined and mathematical formulation of the problem is presented as follows:

Indices

m	Materials
s	Suppliers
p	Material storage plants
i	Products
w	Production plants
d	Distribution centers
c	Customer zones
t	Planning periods

Parameters

c_{msp}	unit variable transportation cost of material m from supplier s to material storage p
c_{mpw}	unit variable transportation cost of material m from material storage p to product plant w
c_{iw}	unit variable production cost of product i at production plant w
c_{iwd}	unit variable transportation cost of product i from production plant w to distribution center d
c_{idc}	unit variable transportation cost product i from distribution center d customer zone c
h_{mp}	unit variable inventory holding cost of material m at material storage p
h_{id}	unit variable inventory holding cost of product i at distribution center d
cap_{mp}	storage capacity of material storage p for material m
ss_m	safety stock level for material m
b_{im}	required number of material m for product i
a_i	unit production time for product i
cap_w	production capacity of production plant w
D_{ict}	demand of customer c for product i in period t
cap_{id}	storage capacity of distribution center d for product i
ss_i	safety stock level of product i

Decision Variables

XS_{mspt}	transportation amount of material m from supplier s to material storage p in period t
IP_{mpt}	storage amount of material m in material storage p in period t
XW_{iwt}	production amount of product i at production plant w in period t
XP_{mpwt}	transportation amount of material m from material storage p to production plant w in period t
XD_{iwdt}	transportation amount of product i from production plant w to distribution center d in period t
XC_{idct}	transportation amount of product i from distribution center d to customer center c in period t
ID_{idt}	storage amount of product i at distribution center d in period t

Mathematical Formulation

$$\min TC = \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{s=1}^{S}\sum_{p=1}^{P} c_{msp} * XS_{mspt} + \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{w=1}^{W} c_{mpw} * XP_{mpwt} + \sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{w=1}^{W} c_{iw} * XW_{iwt} + \sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{w=1}^{W}\sum_{d=1}^{D} c_{iwd} * XD_{iwdt} + \sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{d=1}^{D}\sum_{c=1}^{C} c_{idc} * XC_{idct} + \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{p=1}^{P} h_{mp} * IP_{mpt} + \sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{d=1}^{D} h_{id} * ID_{idt} \tag{1}$$

Subject to

$$\sum_{s=1}^{S} XS_{mspt} - \sum_{w=1}^{W} XP_{mpwt} + IP_{mp,t-1} = I_{mpt} \quad \forall m,p,t \qquad (2)$$
$$IP_{mpt} \leq cap_{mp} \quad \forall m,p,t \qquad (3)$$
$$\sum_{p=1}^{P} IP_{mpt} \geq ss_{m} \quad \forall m,t \qquad (4)$$
$$\sum_{i=1}^{N} b_{im} * X_{iwt} - \sum_{p=1}^{P} XP_{mpwt} = 0 \quad \forall m,w,t \qquad (5)$$
$$\sum_{i=1}^{N} a_{i} * XW_{iwt} \leq cap_{w} \quad \forall w,t \qquad (6)$$
$$\sum_{d=1}^{D} XD_{iwdt} = XW_{iwt} \quad \forall i,w,t \qquad (7)$$
$$\sum_{w=1}^{W} XD_{iwdt} - \sum_{c=1}^{C} XC_{icdt} + ID_{id,t-1} = I_{idt} \quad \forall i,d,t \qquad (8)$$
$$ID_{idt} \leq cap_{id} \quad \forall i,d,t \qquad (9)$$
$$\sum_{d=1}^{D} ID_{idt} \geq ss_{i} \quad \forall i,t \qquad (10)$$
$$\sum_{d=1}^{D} XC_{icdt} = D_{ict} \quad \forall i,c,t \qquad (11)$$
$$XS_{mspt} \geq 0 \text{ and integer} \quad \forall m,s,p,t \qquad (12)$$
$$XW_{iwt} \geq 0 \text{ and integer} \quad \forall i,w,t \qquad (13)$$
$$XP_{mpwt} \geq 0 \text{ and integer} \quad \forall m,p,w,t \qquad (14)$$
$$XD_{iwdt} \geq 0 \text{ and integer} \quad \forall i,w,d,t \qquad (15)$$
$$XC_{idct} \geq 0 \text{ and integer} \quad \forall i,d,c,t \qquad (16)$$
$$IP_{mpt} \geq 0 \text{ and integer} \quad \forall m,p,t \qquad (17)$$
$$ID_{idt} \geq 0 \text{ and integer} \quad \forall i,d,t \qquad (18)$$

Eq. (1) shows the objective function of the model and minimizes the total cost of production, distribution and storage within the supply chain. The objective function consist costs of the material supply cost, the transportation costs of materials to production plants, production costs, transportation costs of products to distribution centers and customer locations and storage cost of materials and products, respectively. Eq. (2) expresses the balance constraint for materials. Eq. (3) and Eq. (4) states the lower (safety stock) and upper bounds (storage limitation) for material storage levels, respectively. Eq. (5) represents the equity of material amount from material storage plants to each production plant and material amount used in production in that plant. Eq. (6) shows the production capacity of production plants. Eq. (7) states that the production amount must be equal to the product amount transported to distribution centers. In other words, production plants cannot store any products. Eq. (8) shows the balance equation for products in distribution centers. Eq. (9) and Eq. (10) states the safety stock and storage capacity amounts for products, respectively. Eq. (11) expresses that customer demands are satisfied by product transportation to customer zones. Nonnegativity and integrity restrictions for the decision variables are given by Eq. (12) – Eq. (18).

4. NUMERICAL STUDIES

Solution of the proposed model is obtained for a set of examples on a supply chain with 2 suppliers, 2 material storage plants, 2 production plants, 2 distribution centers and 3 customer zones. Production-distribution plan for 12 months planning period is obtained for problems with several combinations of 3, 4 and 5 products and 5, 10 and 15 materials. Number of products and materials in each example problem is presented in Table 1.

Safety stock level for each material at each material storage plant is assumed to be 250 and each material storage can store 5000 materials. Distribution centers can store up to 500 products and safety stock of each product is 30. Production capacity of each plant is determined as 12000 minutes per planning period. Unit production time of products are generated randomly from uniform distribution between 5 and 15 minutes. Product demands are also randomly generated from uniform distribution between 100 and 200 units. Range value of other random parameters are presented in Table 2.

Table 1. Number of materials and products in example

Problem	# of Products	# of Materials
P1	3	5
P2	3	10
P3	3	15
P4	4	5
P5	4	10
P6	4	15
P7	5	5
P8	5	10
P9	5	15

Table 2. Parameter ranges

Parameter	Value Range
c_{msp}	(5,35)
c_{mpw}	(10,25)
c_{iw}	(7,32)
c_{iwd}	(9,40)
c_{idc}	(5,35)
c_{iwr}	(10,25)
h_{mp}	(10,30)
h_{id}	(8,30)

Proposed mathematical model is coded on GAMS software and randomly generated nine examples are solved by CPLEX 24.1.3 solver with a personnel computer with Intel i7 2.40 GHz processor and 8 GB RAM. Obtained solution results are given in Table 3.

Solution results show that the model can easily be solved by a commercial solver. Each of the example problems reached to the optimal solution in less than 1 second solution time. So, the model can be used as a multi-period production – distribution planning tool for multi – product supply chains.

Table 3. Solution results

Problem	Objective Value	Solution Time
P1	2392226	0.047 sec
P2	4675950	0.062 sec
P3	8166604	0.062 sec
P4	5347514	0.031 sec
P5	5606356	0.047 sec
P6	9576919	0.172 sec
P7	4106332	0.282 sec
P8	8135839	0.047 sec
P9	11369679	0.187 sec

According to the solution results, the increase of material and product numbers caused greater values of system cost. It seems sensible, because the more elements the supply chain contain brings extra cost of material purchase, transportation, also production and product transportation.

5. CONCLUSION

Effective management of the supply chain requires consideration of whole system entirely and making decisions according to this consideration. Besides, its negative effects on system costs, placement of safety stock in supply chains may be an appropriate solution to increase customer service level.

In this study a mixed-integer programming model for a supply chain with safety stocks is proposed. Within the proposed model, capacity constraints and bill-of-materials structure are considered. A set of numerical examples of the model is solved by a commercial solver software.

The novelty of the model consideration of bill-of-materials and safety stock placement in a supply chain with suppliers, material storages, production plants, distribution centers and customers. Researchers can extend this study by insertion of different aspects of supply chains. For practitioners, this model can be used to determine supply, production, distribution and storage decisions, since it reaches optimal solution in a very short time.

The main limitation of the study is that the decisions can be expressed by binary variables are ignored in this model. Operation decisions for plants, supplier selection decisions, linkage of products with production plants can be considered as extension paths of the study. Also, capacity and demand uncertainties can be taken into account by fuzzy or stochastic modelling.

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