**TURKISH JOURNAL OF SCIENCE**
**VOLUME 6, ISSUE 3, 162–166**
**ISSN: 2587–0971**

http://dergipark.gov.tr/tjos

# Mathematical Modeling of the Shape of Cavity Created with Laser Using Melting, Boiling Temperature

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**Abstract.** In this study, a groove was formed on the Ti-6Al-4V plate with Nd:YAG laser. The structure of the resulting grooves was examined. Mathematical modeling of the heat dissipation was made by making use of the melting and boiling temperatures observed on the plate. Later, to prove the validity of the obtained mathematical model, grooves with different geometries were obtained with different laser energies. The results obtained with the proposed mathematical model are quite compatible with the experimental results.

## 1. Introduction

When it is desired to modify the mechanical properties of materials such as friction and adhesion, one of the most used methods is the surface texturing process. For different materials, different texturing methods can be applied according to the material properties and the intended pattern properties. Surface texturing processes can be divided into three main groups as chemical, mechanical and thermal.

Ti-6Al-4V alloy is widely used in industry and especially in the healthcare industry due to its low density and high toughness. Titanium and titanium alloys are used in the production of many parts, especially in aviation, health and space technology, because they are more durable than steel but much lighter [1]. Ti-6Al-4V is a titanium alloy with high specific strength and excellent corrosion resistance. It is one of the most commonly used titanium alloys and is applied in a wide range of applications where low density and excellent corrosion resistance are necessary such as e.g. biomechanical applications (implants and prostheses) [2]. Additive Manufacturing [3], racing and aerospace industry [4], marine applications and chemical industry [5], etc..

Although Ti-6Al-4V alloy has superior properties and is preferred in many applications, tribological performance is inadequate. The tribologic properties can be improved by surface texturing. The process of creating regular patterns on the material surface by various methods can be called surface texturing. The sizes, shapes and proportions of these patterns on the surface greatly affect the adhesion and friction properties of the surface. Due to the different physical and chemical properties of materials, it can be processed with different methods for different materials. In addition to the many advantages of these methods, they also have some disadvantages such as environmental pollution, increased burrs from the material and wear of the parts.

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Received: 11 October 2021; Accepted:27 December 2021; Published: 30 December 2021
*Keywords*. (Parabolic equation, Boundry condition, Fourier methods, Lazer parameters)
2010 *Mathematics Subject Classification*. 35K05, 35K29, 65M06, 65M12
*Cited this article as:* Canel T, Bağlan İ. Mathematical Modeling of the Shape of Cavity Created with Laser Using Melting, Boiling Temperature, Turkish Journal of Science. 2021, 6(3), 162-166.

Lasers have many advantages in material processing. Lasers are preferred in many areas, especially thanks to their superior qualities such as the absence of wear on parts, precise processing and the preservation of this sensitivity in almost all products. Although many metals can be processed very easily by laser, there are difficulties in laser processing of Al and its alloys due to their high reflectivity.

Many studies have been carried out to control the dimensions and geometries of the patterns created on the material with the laser and to determine the appropriate laser parameters [6, 7]. Many parameters, including the properties of the laser used in laser material processing and the ambient conditions, affect the properties of the patterns obtained on the material. Numerous experimental studies have been carried out to obtain suitable parameters [8–10]. Since there are many parameters affecting the result in laser material processing, classical experimental methods take a lot of time and have high costs. For this, successful results are obtained with mathematical modeling as well as optimization studies [11–13].

In this study, a groove was created on the Ti-6Al-4V plate and mathematical modeling of the heat distribution was made with the measurements taken from the geometries of these grooves and the data obtained. In the mathematical modeling using the Fourier method, Melting, Boiling and melting Temperature were used as boundary conditions.

The effects of the laser beam energy on the groove width of Ti-6Al-4V plate were investigated in the mathematical model. Physical properties of Ti-6Al-4V and laser parameters were used to conduct model.

The heat distribution equation on surface can be written as below;

$$\frac{\partial T(x,t)}{\partial t} = \alpha^2 \frac{\partial^2 T(x,t)}{\partial x^2} \tag{1}$$

where, T is the temperature as a function of time "$t$" and distance "$x$", $\alpha$ is the thermal diffusivity of the material that can be obtained as below;

$\alpha^2 = \frac{\lambda}{c\rho}$

where, $\lambda$ is the thermal conductivity, c specific heat and $\rho$ density of material.

Let $t_p > 0$ be a fixed number and denote by $D = \{(x.t) : 0 < x < l, 0 < t < t_p\}$

where $x$ is the investigated length that varies between zero and $l$. $t_p$ is the pulse duration that means laser beam start at "0" and laser is beam is cut of at $t_p$.

Therefore one of the initial condition can be written as;

$T(x,0) = T_0, 0 < x < l$

where $T_0$ is the initial temperature of the material. It was assumed that all the energy absorbed by the surface was transmitted to the material. Thus, in the absence of heat loss, the boundary condition ($x = 0$) on the surface can be written as follows:

$(\partial T(0,t))/\partial t = 0, (\partial T(l,t))/\partial t = 0 (t > 0)$

This is a parabolic problem. Classical solution of the problem (1)-(3) is $T(x,t) \in C^{2,1}(D) \cap C^{1,0}(D)$. The heat source problem has been investigated with parabolic equation in many studies.

By applying the standard procedure of the Fourier method, we obtain the following representation for the solution of (1)-(3).

$T(x,t) = Z(x)T(t)$

$(X''(x))/(X(x)) = (T'(t))/(\alpha^2 T(t)) = -\lambda^2$

where $\lambda$ is fix number.

The eigen values are

$\lambda_k = (2\pi k/l)^2, k = 1, ..., \infty$

The eigien functions are

$X_1(x) = cos\frac{2\pi k}{l}x, X_2 = sin\frac{2\pi k}{l}x,$

$X(x) = C_1 cos\frac{2\pi k}{l}x + C_2 sin\frac{2\pi k}{l}x.$

$T(t) = C_3 e^{-(\frac{2\pi\alpha k}{l})^2 t}.$

Then the following solution is obtained using Fourier method.

$T(x,t) = \sum_{k=1}^{\infty} \left(T_{ck} \cos\frac{2\pi\alpha k}{l} + T_{sk} \sin\frac{2\pi\alpha k}{l}\right) e^{-(\frac{2\pi\alpha k}{l})^2 t}.$

The laser intensity within the material can be found using the Beer-Lambert's Law:
$\frac{dI(x)}{dx} = -aI$
Where $I(x)$ is the laser intensity as a function of distance from laser spot and $a$ is the absorption coefficient of the material respectively. Although absorption coefficient is changed within the material but it was taken as constant in our study. Laser intensity as a function of distance within material can be written as;
$I = I_0 e^{-\int_b^z a dx}$.
Actually most of the beam intensities have Gaussian distribution. We made one more assumption that our laser beam is top-hat beam that means intensity is homogeneously distributed in spot area.
The heat generation from the laser beam absorbed by the material is defined as,
$S = \frac{-dI}{dx}$
Using Leibniz rule yields, the heat source can be written as;
$S = I_0 e^{-\int_b^z a dx}$
The temperature distribution as a function was obtained as given below;

$$
\begin{aligned}
T(x,t) \quad = \quad & \sum_{k=1}^{\infty}\left(\varphi_{ck} e^{-(\frac{2\pi\alpha k}{l})^2 t} + \int_0^t \int_0^\pi S(x,t)\cos\frac{2\pi\alpha k}{l} x e^{-(\frac{2\pi\alpha k}{l})^2(t-\tau)} dx d\tau\right)\cos\frac{2\pi\alpha k}{l} x \\
& + \sum_{k=1}^{\infty}\left(\varphi_{sk} e^{-(\frac{2\pi\alpha k}{l})^2 t} + \int_0^t \int_0^\pi S(x,t)\sin\frac{2\pi\alpha k}{l} x e^{-(\frac{2\pi\alpha k}{l})^2(t-\tau)} dx d\tau\right)\sin\frac{2\pi\alpha k}{l} x - -xH/l\lambda.
\end{aligned}
\tag{2}
$$

## 2. MATERIAL AND EXPERIMENTAL SETUP

The Ti-6Al-4V plates with 2.5 cm x 2.5 cm having an area of 3 mm thick were used for to surface machining process. Some physical and thermal properties of Ti-6Al-4V which were used in mathematical modeling have been listed in Table 1. In the ablation process commercial Nd:YAG laser was used with different energy at constant scan speed. The laser beams were focused 1 mm above the surface, the spot diameters were obtained as 580 $\mu$m.

Table 1 Some physical and thermal properties of Ti-6Al-4V

| Properties | Value | Unit |
|---|---|---|
| Density | 4410 | kg/m3 |
| Specific Heat Capacity | 5263 | kJ / kg.K |
| Melting point | 1650 | K |
| Boiling Temperature | 3133 | K |
| Thermal Conductivity | 6.7 | W/mK |

## 3. RESULTS AND DISCUSSION

In this study, mathematical model has been proposed for the groove width on Ti-6Al-4V plate with 3 J of energy and 2 mm/s scan speed. An optical microscope was used to take the images of ablated surfaces of Ti-6Al-4V plate. Groove widths were measured from these images.

The Boiling and molten zone boundary distances were measured as 1310 $\mu$m and 1120 $\mu$m respectively. Temperatures at Boiling and molten zone boundary are 3133 K and 1650 K respectively. These temperatures are used in obtained mathematical model obtain the Fourier coefficients. These coefficients depend on the material properties. The coefficients in the temperature distribution equation (2) were calculated as $\varphi_{ck}$ (=701,68) and $\varphi_{sk}$ (-112.48). Then, in order to verify the validity of mathematical model, new grooves were created obtained using 2.5, 3.5,4, and 4.5 Joules of laser energies. The coefficients obtained with first experiment (conducted with 3 Joule of laser energy) were used to calculate temperature distribution for the Ti-6Al-4V plate and different laser beam energies.

Table 2 Laser Energies and groove widths measured from images.

| Laser Energy Joule | Boiled Zone width | Melt Zone width |
|---|---|---|
| 26 | 1252 | 1434 |
| 39 | 1319 | 1513 |
| 52 | 1367 | 1568 |
| 65 | 1404 | 1611 |
| 78 | 1434 | 1646 |

The calculated temperatures for boundaries are given in Table 3.

Table 3. Melting and Boiling Temperatures calculated with mathematical model, real values and percent error between them.

| Energy Joule | | T(x,t) (K) | T(x,t) (K) (calculated) | % error |
|---|---|---|---|---|
| 2.5 | Melting | 1650 | 1640 | 0.61 |
| 2.5 | Boiling | 3133 | 3121 | 0.38 |
| 3 | Melting | 1650 | (ref) | |
| 3 | Boiling | 3133 | (ref) | |
| 3.5 | Melting | 1650 | 1662 | 0.72 |
| 3.5 | Boiling | 3133 | 3178 | 1.44 |
| 4 | Melting | 1650 | 1674 | 1.45 |
| 4 | Boiling | 3133 | 3191 | 1.85 |
| 4.5 | Melting | 1650 | 1692 | 2.55 |
| 4.5 | Boiling | 3133 | 3209 | 2.43 |

## 4. CONCLUSION

Micro-scale patterns created on metal surfaces change the mechanical properties of the surfaces. In addition to the many advantages of laser surface treatment, it is very difficult to accurately predict the properties of the surface to be obtained due to the complexity of the laser-material interaction. Thanks to the mathematical modeling of the heat distribution of the surface to be obtained with the laser texture, the properties of the product to be obtained can be known in advance. In mathematical modeling, as in parameter optimizations, both time and material can be saved in experimental studies.

In this study, firstly, grooves were created on the Ti-6Al-4V plate with a 3 J laser. Measurements were made on the obtained through and the constants to be used in the temperature distribution equation were calculated. Then, grooves were obtained with 2.5, 3.5, 4 and 4.5 Joules energies to prove the validity of the mathematical model obtained. The measurements made on these grooves and the results obtained with the mathematical model were compared. The error rates of the results obtained vary between 0.38 and 2.55 %. The fact that the error rates are so low indicates that the proposed model is an acceptable one.

## 5. Acknowledgement

This work has been supported by Kocaeli University Scientific Research Projects Coordination Unit(FBA-2019-1586). The author is thankful to the referee for his/her valuable suggestions.

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