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Different Computational Approach for Sumudu Integral Transform by Using Differential Transform Method

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Abstract

In this work, we present a different technique for calculation of Sumudu Integral Transform (SIT) by considering differential transform method (DTM). By means of our technique, Sumudu Transform of functions is obtained easily without complicated integration procedures.

1. Introduction

In mathematical calculus, integral transforms are a specific branch that has used various applied area. In 1993, Watugala introduced a new integral transform called Sumudu Integral Transform (SIT) to solve differential equations and engineering problems [1]. The Sumudu Transform of the function $f(t)$ is defined over the set of A (seen [1], [2], [3], [4])

$$A = \{f(t) : \exists M, \tau_1, \tau_2 > 0, |f(t)| < Me^{\frac{|t|}{\tau_i}}, if \quad t \in (-1)^i \times [0,\infty)\}$$

as below formula

$$F(u) = S[f(t) : u] = \int_0^\infty f(ut)e^{-t}dt \quad u \in (-\tau_1, \tau_2) \tag{1.1}$$

Also, modified version of (1.1) is presented as

$$F(u) = \int_0^\infty \frac{f(t)e^{-\frac{t}{u}}}{u}dt \quad u \in (-\tau_1, \tau_2) \tag{1.2}$$

by Watugala [1], [4] and Belgacem [2],[3]. Hereafter, many authors consider the Sumudu Integral Transform to investigate properties, applications and relations with other transforms [1]-[10].

In recent time, homotopy perturbation, differential transform and adomian decomposition methods are applied to find Laplace transform as seen [11],[12],[13] respectively. Furthermore, homotopy perturbation method is also applied to Sumudu transform [8].

The goal of this paper is to present a different approach to obtain Sumudu transform of functions. In order to do this, we use the differential transform method (DTM) and first order initial value problem which has a solution that corresponds to Sumudu transform of desired functions. DTM is very famous and powerful analytic technique and it does not required complex integration process. So, very accurate and efficient results are obtained easily.

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Functions	Transformed form of functions
$v(t)=e^{at}$	$V(k)=\frac{a^k}{k!}$
$v(t)=\cos(at)$	$V(k)=\begin{cases}\frac{a^k(-1)^{\frac{k}{2}}}{k!}, & if \quad k \quad even\\ 0, & if \quad k \quad odd\end{cases}$
$v(t)=\sin(at)$	$V(k)=\begin{cases}\frac{a^k(-1)^{\frac{k-1}{2}}}{k!}, & if \quad k \quad odd\\ 0, & if \quad k \quad even\end{cases}$
$v(t)=\cosh(at)$	$V(k)=\begin{cases}\frac{a^k}{k!}, & if \quad k \quad even\\ 0, & if \quad k \quad odd\end{cases}$
$v(t)=\sinh(at)$	$V(k)=\begin{cases}\frac{a^k}{k!}, & if \quad k \quad odd\\ 0, & if \quad k \quad even\end{cases}$

Table 1: Basic transformations of DTM for some functions.

2. Basic idea of DTM

The differential transform of the analytical $v(t)$ function is defined (seen [14], [15], [16], [17]) as

$$V(k)=\frac{1}{k!}\left[\frac{d^k}{dt^k}v(t)\right]_{t=0} \tag{2.1}$$

where $V(k)$ is the transformed function of $v(t)$ which is called spectrum function. And the inverse transform of $V(k)$ is defined (seen [14], [15], [16], [17]) as

$$v(t)=\sum_{k=0}^{\infty}V(k)t^k \tag{2.2}$$

Combining (2.1) and (2.2), we obtain the DTM solution of $v(t)$ as follow

$$v(t)=\sum_{k=0}^{n}\frac{1}{k!}\left[\frac{d^k}{dt^k}v(t)\right]_{t=0}t^k+R_{n+1}(t) \tag{2.3}$$

Here $R_{n+1}(t)=\sum_{k=n+1}^{\infty}V(k)t^k$ are remaining terms of solution series. Some of the transformed functions are presented in Table 1.

3. Results by using DTM

Theorem 3.1. *Let $v(t)$ is an analytic function and r is positive constant. Also, we consider the linear initial value problem as follow*

$$\begin{aligned} v(t)' &= \frac{1}{r}v(t)+\frac{1}{r}q(t) \\ v(0) &= 0 \end{aligned} \tag{3.1}$$

Then, the Sumudu transform of $q(t)$ is

$$S[q(t)]=\left[e^{-\frac{t}{r}}\sum_{i=0}^{\infty}V(i)t^i\right]_{t=0}^{t=\infty}$$

Here, $V(i)$ is differential transform of $v(t)$.

Proof. First of all, we can write the solution of (3.1) as

$$v(t)=e^{\frac{t}{r}}\left(\int\frac{q(t)e^{-\frac{t}{r}}}{r}dt\right) \tag{3.2}$$

and by rewriting two side of (3.2) from zero to infinity, we obtain the relation between (3.2) and Sumudu transform as follow

$$\left[v(t)e^{-\frac{t}{r}}\right]_{t=0}^{t=\infty}=\left(\int_0^{\infty}\frac{q(t)e^{-\frac{t}{r}}}{r}dt\right) \tag{3.3}$$

It is clearly seen that right hand side of (3.3) is the definition of Sumudu transform of $q(t)$ as seen in (1.2).
In order to find Sumudu transform of $q(t)$, we construct the differential transformed form of (3.1) as

$$V(i+1)=\frac{1}{r}\frac{V(i)}{i+1}+\frac{1}{r}\frac{Q(i)}{i+1} \qquad (3.4)$$
$$V(0)=0$$

where $V(i)$, $Q(i)$ are differential transformed functions of $v(t)$ and $q(t)$ respectively. Then, by using the inverse differential transform as in (2.2), (2.3), we obtain the DTM solution of $v(t)$ as

$$v(t)=\sum_{i=0}^{\infty}V(i)t^i \qquad (3.5)$$

Finally, put (3.5) into (3.3), we find the sumudu transform of $q(t)$ as

$$S[q(t)]=\left(\int_0^{\infty}\frac{q(t)e^{-\frac{t}{r}}}{r}dt\right)=\left[e^{-\frac{t}{r}}\sum_{i=0}^{\infty}V(i)t^i\right]_{t=0}^{t=\infty} \qquad (3.6)$$

This completes the proof.

□

The following illustrations are given to show accuracy, efficiency and easy applicability of our approach to find Sumudu transform of functions.

Case 1: In the Theorem 3.1, let $q(t)=e^{at}$. Then considering (3.4) and transformed form of $q(t)=e^{at}$, we can write

$$V(i+1)=\frac{1}{r}\frac{V(i)}{i+1}+\frac{1}{r(i+1)}\frac{a^i}{i!}$$
$$V(0)=0$$

Some of the $V(i)$ are obtained as

$$\begin{aligned} V(1)&=\frac{1}{r} \quad V(2)=\frac{1+ar}{2!r^2} \quad V(3)=\frac{1+ar+a^2r^2}{3!r^3}\\ V(4)&=\frac{1+ar+a^2r^2+a^3r^3}{4!r^4} \quad V(5)=\frac{1+ar+a^2r^2+a^3r^3+a^4r^4}{5!r^5}\\ V(6)&=\frac{1+ar+a^2r^2+a^3r^3+a^4r^4+a^5r^5}{6!r^6}\\ &\vdots \qquad \vdots \qquad \vdots \end{aligned} \qquad (3.7)$$

From (3.5) and from (3.7), we have

$$\begin{aligned}\sum_{i=0}^{\infty}V(i)t^i&=\left(\frac{t}{r}+\frac{t^2}{2!r^2}+\frac{t^3}{3!r^3}+\frac{t^4}{4!r^4}+\frac{t^5}{5!r^5}+\cdots\right)+ar\left(\frac{t^2}{2!r^2}+\frac{t^3}{3!r^3}+\frac{t^4}{4!r^4}+\frac{t^5}{5!r^5}+\frac{t^6}{6!r^6}+\cdots\right)\\ &+a^2r^2\left(\frac{t^3}{3!r^3}+\frac{t^4}{4!r^4}+\frac{t^5}{5!r^5}+\frac{t^6}{6!r^6}+\frac{t^7}{7!r^7}+\cdots\right)+a^3r^3\left(\frac{t^4}{4!r^4}+\frac{t^5}{5!r^5}+\frac{t^6}{6!r^6}+\frac{t^7}{7!r^7}+\frac{t^8}{8!r^8}+\cdots\right)+\cdots\end{aligned} \qquad (3.8)$$

And the equation (3.8) can be written as equivalently following

$$\begin{aligned}\sum_{i=0}^{\infty}V(i)t^i&=\left(e^{\frac{t}{r}}-1\right)+ar\left(e^{\frac{t}{r}}-1-\frac{t}{r}\right)+a^2r^2\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}\right)\\ &+a^3r^3\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}-\frac{t^3}{3!r^3}\right)+a^4r^4\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}-\frac{t^3}{3!r^3}-\frac{t^4}{4!r^4}\right)+\cdots\end{aligned} \qquad (3.9)$$

Finally, by using (3.6) and (3.9) we find the Sumudu transform of e^{at}

$$\begin{aligned}S\left[e^{at}\right]&=\left\{e^{-\frac{t}{r}}\left(e^{\frac{t}{r}}-1\right)+ar\times e^{-\frac{t}{r}}\left(e^{\frac{t}{r}}-1-\frac{t}{r}\right)+a^2r^2\times e^{-\frac{t}{r}}\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}\right)+\cdots\right\}_{t=0}^{t=\infty}\\ &=1+ar+a^2r^2+a^3r^3+a^4r^4+a^5r^5+a^6r^6+\cdots\\ &=\frac{1}{1-ar}\end{aligned}$$

Case 2: In the Theorem 3.1, let $q(t)=\cos(at)$. Then, from (3.4) and Table 1 we can write

$$V(i+1)=\frac{1}{r}\frac{V(i)}{i+1}+\frac{1}{r(i+1)}\begin{cases}\frac{a^i(-1)^{\frac{i}{2}}}{i!}, & i \quad even\\ 0, & i \quad odd\end{cases}$$
$$V(0)=0$$

Thus, $V(i)$ are obtained as

$$\begin{gathered}V(1)=\frac{1}{r}\quad V(2)=\frac{1+ar}{2!r^2}\quad V(3)=\frac{1-a^2r^2}{3!r^3}\quad V(4)=\frac{1-a^2r^2}{4!r^4}\\ V(5)=\frac{1-a^2r^2+a^4r^4}{5!r^5}\quad V(6)=\frac{1-a^2r^2+a^4r^4}{6!r^6}\\ V(7)=\frac{1-a^2r^2+a^4r^4-a^6r^6}{7!r^7}\quad V(8)=\frac{1-a^2r^2+a^4r^4-a^6r^6}{8!r^8}\\ \vdots\qquad\qquad\vdots\qquad\qquad\vdots\end{gathered}\tag{3.10}$$

Using the (3.5) and (3.10), we have

$$\begin{aligned}\sum_{i=0}^{\infty}V(i)t^i=&\left(\frac{t}{r}+\frac{t^2}{2!r^2}+\frac{t^3}{3!r^3}+\frac{t^4}{4!r^4}+\frac{t^5}{5!r^5}+\cdots\right)-a^2r^2\left(\frac{t^3}{3!r^3}+\frac{t^4}{4!r^4}+\frac{t^5}{5!r^5}+\frac{t^6}{6!r^6}+\cdots\right)\\ &+a^4r^4\left(\frac{t^5}{5!r^5}+\frac{t^6}{6!r^6}+\frac{t^7}{7!r^7}+\frac{t^8}{8!r^8}+\cdots\right)-a^6r^6\left(\frac{t^7}{7!r^7}+\frac{t^8}{8!r^8}+\frac{t^9}{9!r^9}+\frac{t^{10}}{10!r^{10}}+\cdots\right)\pm\cdots\end{aligned}\tag{3.11}$$

The (3.11) can be rewritten as follow

$$\begin{aligned}\sum_{i=0}^{\infty}V(i)t^i=&\left(e^{\frac{t}{r}}-1\right)-a^2r^2\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}\right)+a^4r^4\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}-\frac{t^3}{3!r^3}-\frac{t^4}{4!r^4}\right)\\ &-a^6r^6\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}-\frac{t^3}{3!r^3}-\frac{t^4}{4!r^4}-\frac{t^5}{5!r^5}-\frac{t^6}{6!r^6}\right)\pm\cdots\end{aligned}$$

At the end, the Sumudu transform of $\cos(at)$

$$\begin{aligned}S[\cos(at)]=&\left(1-\frac{1}{e^{\frac{t}{r}}}\right)_{t=0}^{t=\infty}-a^2r^2\left(1-\frac{1}{e^{\frac{t}{r}}}-\frac{t}{re^{\frac{t}{r}}}-\frac{t^2}{2r^2e^{\frac{t}{r}}}\right)_{t=0}^{t=\infty}+a^4r^4\left(1-\frac{1}{e^{\frac{t}{r}}}-\frac{t}{re^{\frac{t}{r}}}-\frac{t^2}{2r^2e^{\frac{t}{r}}}-\frac{t^3}{3!r^3e^{\frac{t}{r}}}-\frac{t^4}{4!r^4e^{\frac{t}{r}}}\right)_{t=0}^{t=\infty}\\ &-a^6r^6\left(1-\frac{1}{e^{\frac{t}{r}}}-\frac{t}{re^{\frac{t}{r}}}-\frac{t^2}{2r^2e^{\frac{t}{r}}}-\frac{t^3}{3!r^3e^{\frac{t}{r}}}-\frac{t^4}{4!r^4e^{\frac{t}{r}}}-\frac{t^5}{5!r^5e^{\frac{t}{r}}}-\frac{t^6}{6!r^6e^{\frac{t}{r}}}\right)_{t=0}^{t=\infty}\pm\cdots\\ &=1-a^2r^2+a^4r^4-a^6r^6+a^8r^8-a^{10}r^{10}\pm\cdots\\ &=\frac{1}{1+a^2r^2}\end{aligned}$$

Case 3: In the Theorem 3.1, let $q(t)=\frac{\sin(at)}{t}$. Again, by considering (3.4) and Table 1 we can write

$$\begin{aligned}&V(i+1)=\frac{1}{r}\frac{V(i)}{i+1}+\frac{1}{r(i+1)}\begin{cases}\frac{a^i(-1)^{\frac{i}{2}}}{(i+1)!}, & i \quad even\\ 0, & i \quad odd\end{cases}\\ &V(0)=0\end{aligned}\tag{3.12}$$

In that case, we can write some of $V(i)$ as

$$\begin{gathered}V(1)=\frac{1}{r}\quad V(2)=\frac{1}{2!r^2}\quad V(3)=\frac{3-a^2r^2}{3\times 3!r^3}\quad V(4)=\frac{3-a^2r^2}{3\times 4!r^4}\\ V(5)=\frac{15-5a^2r^2+3a^4r^4}{15\times 5!r^5}\quad V(6)=\frac{15-5a^2r^2+3a^4r^4}{15\times 6!r^6}\\ V(7)=\frac{105-35a^2r^2+21a^4r^4-15a^6r^6}{105\times 7!r^7}\\ \vdots\qquad\qquad\vdots\qquad\qquad\vdots\end{gathered}\tag{3.13}$$

By considering (3.5) in the Theorem 3.1 and using (3.13), we have

$$\sum_{i=0}^{\infty} V(i)t^i = a\left(\frac{t}{ar}+\frac{t^2}{2!ar^2}+\frac{t^3}{3!ar^3}+\frac{t^4}{4!ar^4}+\frac{t^5}{5!ar^5}+\cdots\right)-\frac{1}{3}a^3r^2\left(\frac{t^3}{3!ar^3}+\frac{t^4}{4!ar^4}+\frac{t^5}{5!ar^5}+\frac{t^6}{6!ar^6}+\cdots\right)$$
$$+\frac{1}{5}a^5r^4\left(\frac{t^5}{5!ar^5}+\frac{t^6}{6!ar^6}+\frac{t^7}{7!ar^7}+\frac{t^8}{8!ar^8}+\cdots\right)-\frac{1}{7}a^7r^6\left(\frac{t^7}{7!ar^7}+\frac{t^8}{8!ar^8}+\frac{t^9}{9!ar^9}+\frac{t^{10}}{10!ar^{10}}+\cdots\right)\pm\cdots \tag{3.14}$$

The (3.14) can be rewritten equally as below

$$\sum_{i=0}^{\infty} V(i)t^i = a\left[\frac{1}{a}\left(e^{\frac{t}{r}}-1\right)\right]-\frac{a^3r^2}{3}\left[\frac{1}{a}\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}\right)\right]+\frac{a^5r^4}{5}\left[\frac{1}{a}\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}-\frac{t^3}{3!r^3}-\frac{t^4}{4!r^4}\right)\right]$$
$$-\frac{a^7r^6}{7}\left[\frac{1}{a}\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}-\frac{t^3}{3!r^3}-\frac{t^4}{4!r^4}-\frac{t^5}{5!r^5}-\frac{t^6}{6!r^6}\right)\right]\pm\cdots$$

Finally, by using (3.6) we obtain the Sumudu transform of $\frac{\sin(at)}{t}$

$$S\left[\frac{\sin(at)}{t}\right]=\left[e^{-\frac{t}{r}}\sum_{i=0}^{\infty}V(i)t^i\right]_{t=0}^{t=\infty}=a-\frac{a^3r^2}{3}+\frac{a^5r^4}{5}-\frac{a^7r^6}{7}\pm\cdots=\frac{\tan^{-1}(ar)}{r}$$

Case 4: In the Theorem 3.1, let $q(t)=\sinh(at)$. Then, considering (3.4) and Table 1 we can write

$$V(i+1)=\frac{1}{r}\frac{V(i)}{i+1}+\frac{1}{r(i+1)}\begin{cases}\frac{a^i}{i!}, & i \quad odd\\ 0, & i \quad even\end{cases}$$
$$V(0)=0 \tag{3.15}$$

By means of (3.15), $V(i)$ are obtained following

$$\begin{gathered}V(1)=0 \quad V(2)=\frac{a}{2!r} \quad V(3)=\frac{a}{3!r^2} \quad V(4)=\frac{a(1+a^2r^2)}{4!r^3}\\ V(5)=\frac{a(1+a^2r^2)}{5!r^4} \quad V(6)=\frac{a(1+a^2r^2+a^4r^4)}{6!r^5}\\ V(7)=\frac{a(1+a^2r^2+a^4r^4)}{7!r^6} \quad V(8)=\frac{a(1+a^2r^2+a^4r^4+a^6r^6)}{8!r^7}\\ \vdots \qquad \vdots \qquad \vdots\end{gathered} \tag{3.16}$$

From (3.5) in the Theorem 3.1 and from (3.16), we have

$$\sum_{i=0}^{\infty} V(i)t^i = ar\left(\frac{t^2}{2!r^2}+\frac{t^3}{3!r^3}+\frac{t^4}{4!r^4}+\frac{t^5}{5!r^5}+\cdots\right)+a^3r^3\left(\frac{t^4}{4!r^4}+\frac{t^5}{5!r^5}+\frac{t^6}{6!r^6}+\cdots\right)$$
$$+a^5r^5\left(\frac{t^6}{6!r^6}+\frac{t^7}{7!r^7}+\frac{t^8}{8!r^8}+\cdots\right)+a^7r^7\left(\frac{t^8}{8!r^8}+\frac{t^9}{9!r^9}+\frac{t^{10}}{10!r^{10}}+\cdots\right)+\cdots \tag{3.17}$$

And the equation (3.17) can be written as equivalently below

$$\sum_{i=0}^{\infty} V(i)t^i = ar\left(e^{\frac{t}{r}}-1-\frac{t}{r}\right)+a^3r^3\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}-\frac{t^3}{3!r^3}\right)+a^5r^5\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}-\frac{t^3}{3!r^3}-\frac{t^4}{4!r^4}-\frac{t^5}{5!r^5}\right)$$
$$+a^7r^7\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}-\frac{t^3}{3!r^3}-\frac{t^4}{4!r^4}-\frac{t^5}{5!r^5}-\frac{t^6}{6!r^6}-\frac{t^7}{7!r^7}\right)+\cdots$$

As a results, we find the Sumudu transform of $\sinh(at)$

$$S[\sinh(at)]=\left[e^{-\frac{t}{r}}\sum_{i=0}^{\infty}V(i)t^i\right]_{t=0}^{t=\infty}=ar+a^3r^3+a^5r^5+a^7r^7+a^9r^9+\cdots=\frac{ar}{1-a^2r^2}$$

Case 5: In the Theorem 3.1, let $q(t)=\cosh(at)$. Then, considering (3.4) and Table 1 we can write

$$V(i+1)=\frac{1}{r}\frac{V(i)}{i+1}+\frac{1}{r(i+1)}\begin{cases}\frac{a^i}{i!}, & i \quad even\\ 0, & i \quad odd\end{cases}$$
$$V(0)=0$$

Hence, some of the $V(i)$ are obtained as

$$\begin{gathered}
V(1)=\frac{1}{r} \quad V(2)=\frac{1}{2!r^2} \quad V(3)=\frac{1+a^2r^2}{3!r^3} \quad V(4)=\frac{1+a^2r^2}{4!r^4} \\
V(5)=\frac{1+a^2r^2+a^4r^4}{5!r^5} \quad V(6)=\frac{1+a^2r^2+a^4r^4}{6!r^6} \\
\vdots \qquad\qquad \vdots \qquad\qquad \vdots
\end{gathered} \tag{3.18}$$

Once again, by using (3.5) in the Theorem 3.1 and from (3.18), we have

$$\begin{aligned}
\sum_{i=0}^{\infty} V(i)t^i &= \left(\frac{t}{r}+\frac{t^2}{2!r^2}+\frac{t^3}{3!r^3}+\frac{t^4}{4!r^4}+\frac{t^5}{5!r^5}+\cdots\right)+a^2r^2\left(\frac{t^3}{3!r^3}+\frac{t^4}{4!r^4}+\frac{t^5}{5!r^5}+\frac{t^6}{6!r^6}+\cdots\right) \\
&+a^4r^4\left(\frac{t^5}{5!r^5}+\frac{t^6}{6!r^6}+\frac{t^7}{7!r^7}+\frac{t^8}{8!r^8}+\cdots\right)+a^6r^6\left(\frac{t^7}{7!r^7}+\frac{t^8}{8!r^8}+\frac{t^9}{9!r^9}+\frac{t^{10}}{10!r^{10}}+\cdots\right)+\cdots
\end{aligned} \tag{3.19}$$

The (3.19) can be rewritten equally as follow

$$\begin{aligned}
\sum_{i=0}^{\infty} V(i)t^i &= \left(e^{\frac{t}{r}}-1\right)+a^2r^2\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}\right) \\
&+a^4r^4\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}-\frac{t^3}{3!r^3}-\frac{t^4}{4!r^4}\right) \\
&+a^6r^6\left(e^{\frac{t}{r}}-1-\frac{t}{r}-\frac{t^2}{2r^2}-\frac{t^3}{3!r^3}-\frac{t^4}{4!r^4}-\frac{t^5}{5!r^5}-\frac{t^6}{6!r^6}\right)+\cdots
\end{aligned} \tag{3.20}$$

Finally, we find the Sumudu transform of $\cosh(at)$

$$S[\sinh(at)] = \left[e^{-\frac{t}{r}}\sum_{i=0}^{\infty} V(i)t^i\right]_{t=0}^{t=\infty} = 1+a^2r^2+a^4r^4+a^6r^6+a^8r^8+\cdots = \frac{1}{1-a^2r^2}$$

4. Conclusion

As a result, we use the differential transform method (DTM) to find Sumudu Transform of functions as a different way. Moreover, contrary to the literature we obtain the Sumudu transform of functions easily without complex integration and long calculations.

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