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Soft Topological Space in Virtue of Semi* Open Sets

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Abstract

The ultimate purpose of this research article is to originate and examine some new kind of open sets in soft topological spaces such as soft semi* - open and soft semi* - closed sets using generalized closure operator with illustrating counter examples.

1. Introduction

In our day-to-day life, we look out problems with unreliabilities. To handle the lack of unreliability and to solve the problems related to uncertainity, a short time ago numberless theories have been developed like Rough Sets, Fuzzy Sets and Vague Sets. However, these methodologies have their own risks. To circumvent these difficulties, Molodtsov [5] developed Soft set theory to deal with unreliability. The development of Soft Set theory is whistle stop now-a-days. Soft set theory has a wider application and its progress is very rapid in different fields [see [19], [20] and [10]]. The approach of Soft topological spaces was codified by Shabir et al. [12]. Many researchers defined some basic notions on soft topology and studied many properties see [[4], [13], [17], [16], [22], [7], [8] and [9]]. In this milieu, we define penetration of soft semi*-open and soft semi*-closed sets in soft topological spaces and then these are used to study properties of semi* - interior, semi* - closure of soft sets in soft topological spaces. Further the behavior of these concepts under various soft functions has obtained. Also we introduce and study soft semi*-connectedness and soft semi* - compactness using soft semi* - open sets.

2. Preliminaries

We roll call the following definitions with illustrated examples for the outpouring of this article.

Let $\mathcal{U}$ indicates initial universe set and let $\mathcal{E}$ be parameters proportionate to $\mathcal{U}$. Let $\mathcal{P}(\mathcal{U})$ denote the power set of $\mathcal{U}$, and let $\mathcal{A} \subseteq \mathcal{E}$. A subset A of a space (X,τ) is said to be generalized closed [15] (briefly g-closed), if $cl(\mathsf{A}) \subseteq \mathcal{U}$ whenever $\mathsf{A} \subseteq \mathcal{U}$ and $\mathcal{U}$ is open. The intersection of all g-closed sets containing A is called the g - closure of A and denoted by $cl^*(\mathsf{A})$ [21]. A subset A of a space (X,τ) is said to be generalized open if its complement is generalized closed and union of all g - open sets contained in A is called the g - interior of A and is denoted by $int^*(\mathsf{A})$. A subset S of a topological space (S,τ) is said to semi*-open if $\mathsf{S} \subseteq (cl^*(int(\mathsf{S}))$ [18]. The complement of a semi*-open set is semi*-closed. It is well known that a subset S is semi*-closed if and only if $int^*(cl(\mathsf{S})) \subseteq \mathsf{S}$ [3].

Definition 2.1. *[5] A soft set $\mathscr{F}_{\mathsf{A}}$ on the universe $\mathcal{U}$ is defined by the set of ordered pairs $\mathscr{F}_{\mathsf{A}} = \{(x, f_{\mathsf{A}}(x)) | x \in \mathcal{E}, f_{\mathsf{A}}(x) \in \mathcal{P}(\mathcal{U})\}$ where $\mathcal{E}$ is a set of parameters, $\mathsf{A} \subseteq \mathcal{E}$, $\mathcal{P}(\mathcal{U})$ is the power set of $\mathcal{U}$ and $f_{\mathsf{A}} : \mathsf{A} \to \mathcal{P}(\mathcal{U})$ such that $f_{\mathsf{A}}(x) = \emptyset$ if $x \notin \mathsf{A}$. Here f_{A} is called an approximate function of the soft set $\mathscr{F}_{\mathsf{A}}$. The value of $f_{\mathsf{A}}(x)$ may be arbitrary, some of them may be empty and some may have non-empty intersection. Note that the set of all soft sets over $\mathcal{U}$ is denoted by $\mathcal{SS}(\mathcal{U})_{\mathcal{E}}$.*
For illustration, we consider an example which we present below:

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Example 2.2. *Suppose $\mathcal{U}$=set of all real numbers on the closed interval $[a,b]$.
$\mathcal{E}$=set of parameters. Each parameter is a word or a sentence.
$\mathcal{E}$={Compact, Closed, Connected, Open}
In this case, to define a soft set means to point out closed set, connected set and so on. Let we consider below the same example in more detail. $\mathcal{U}=\{x : a \leq x \leq b\}$ and $\mathcal{E}=\{e_1,e_2,e_3,e_4\}$ where*

$e_1 \rightarrow$ *'compact',*
$e_2 \rightarrow$ *'closed',*
$e_3 \rightarrow$ *'connected',*
$e_4 \rightarrow$ *'open'.*

Suppose that

$f(e_1)=\{\mathsf{A}\subseteq[a,b] :$ *Every open cover for* A *in* $[a,b]$ *has finite subcover* $\}$.
$f(e_2)=\{[\alpha,\beta]\subseteq[a,b] : \alpha,\beta\in R\}$
$f(e_3)=\{A\subseteq[a,b] :$ *Separation does not exists for* A *in* $[a,b]\}$
$f(e_4)=\{(\alpha,\beta)\subseteq[a,b] : \alpha,\beta\in R\}$

$\mathscr{F}_{\mathsf{A}} \rightarrow$ parametrized family of subsets of the set $\mathcal{U}$. Consider the mapping f in which $f(e_1)\rightarrow$ subsets of $\mathcal{U}$ which are compact whose functional value is the set $\{\mathsf{A}\subseteq[a,b]$: Every open cover for A in $[a,b]$ has finite subcover $\}$. Hence the soft set $\mathscr{F}_{\mathsf{A}}$ is the collection of approximations given below:

$\{(compact,\{\mathsf{A}\subseteq[a,b]$: *Every open cover for* A *in* $[a,b]$ *has finite subcover* $\}),(Closed,\{[\alpha,\beta]\subseteq[a,b]:\alpha,\beta\in R\}),(Connected,\{\mathsf{A}\subseteq[a,b]$:*separation does not exist for* A *in* $R\}),(Open,\{(\alpha,\beta)\subseteq[a,b]:\alpha,\beta\in R\})\}=\mathscr{F}_{\mathsf{A}}$

Definition 2.3. *[12] Let $\tilde{\tau}$ be a collection of soft sets over a universe $\mathcal{U}$ with a fixed set $\mathcal{E}$ of parameters, then $\tilde{\tau}\subseteq SS(\mathcal{U})_{\mathcal{E}}$ is called a soft topology on $\mathcal{U}$ with a fixed set $\mathcal{E}$ if*

i. *$\phi_{\mathcal{E}}$, $\mathcal{U}_{\mathcal{E}}$ belong to $\tilde{\tau}$.*
ii. *The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.*
iii. *The intersection of any finite number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.*

The pair $(\mathcal{U}_{\mathcal{E}},\tilde{\tau})$ is called a soft topological space.

Definition 2.4. *[1] Let $\mathcal{U}$ be a universe and $\mathcal{E}$ a set of parameters. Then the collection $\mathcal{SS}(\mathcal{U})_{\mathcal{E}}$ of all soft sets over $\mathcal{U}$ with parameters from $\mathcal{E}$ is called a soft class.*

Definition 2.5. *[1] Let $\mathcal{S}(\mathcal{U})_{\mathcal{E}}$ and $\mathcal{S}(\mathcal{V})_{\mathcal{E}'}$ be two soft classes. Then $u:\mathcal{U}\mapsto\mathcal{V}$ and $p:\mathcal{E}\mapsto\mathcal{E}^{'}$ be two functions. Then a function $f:\mathcal{S}(\mathcal{U})_{\mathcal{E}}\mapsto\mathcal{S}(\mathcal{V})_{\mathcal{E}'}$ and its inverse are defined as*

(i) Let $\mathcal{L}_{\mathcal{A}}$ be a soft set in $\mathcal{SS}(\mathcal{U})_{\mathcal{E}}$ where $\mathcal{A}\tilde{\subseteq}\mathcal{E}$. The image of $\mathcal{L}_{\mathcal{A}}$ under a function f is a soft set in $\mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ such that $f(\mathcal{L}_{\mathcal{A}})(\beta)=\left(\cup_{\alpha\in p^{-1}(\beta)\cap\mathcal{A}}\mathcal{L}(\alpha)\right)$ for $\beta\in\mathcal{B}=p(\mathcal{A})\tilde{\subseteq}\mathcal{E}^{'}$.
(ii) Let $\mathcal{G}$ be the soft set in $\mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ where $\mathcal{C}\tilde{\subseteq}E^{'}$. Then the inverse image of $\mathcal{G}_{\mathcal{C}}$ under f is a soft set in $\mathcal{SS}(\mathcal{U})_{\mathcal{E}}$ such that $f^{-1}(\mathcal{G}_{\mathcal{C}})(\alpha)=u^{-1}(\mathcal{G}(p(\alpha)))$ for $\alpha\in p^{-1}(\mathcal{C})\tilde{\subseteq}\mathcal{E}$.

3. Semi*-open and semi*-closed soft sets

In this chunk, we expound soft semi*-closure and soft semi*-interior of a soft set are defined in terms of soft semi*-closed and soft semi*-open sets.

Definition 3.1. *In a soft topological space $(\mathcal{U}_{\mathcal{E}},\tilde{\tau})$ a soft set*

(i) $\mathcal{G}_{\mathsf{C}}$ is termed as semi-open soft set if there exists an open soft set $\mathcal{H}_{\mathsf{B}}$ such that $\mathcal{H}_{\mathsf{B}}\tilde{\subseteq}\mathcal{G}_{\mathsf{C}}\tilde{\subseteq}cl^*(\mathcal{H}_{\mathsf{B}})$.*
(ii) $\mathcal{L}_{\mathsf{A}}$ is termed as semi-closed soft set if there exists an closed soft set $\mathcal{K}_{\mathsf{D}}$ such that $int^*(\mathcal{K}_{\mathsf{D}})\tilde{\subseteq}\mathcal{L}_{\mathsf{A}}\tilde{\subseteq}\mathcal{K}_{\mathsf{D}}$.*

We denote the set of all semi - closed Soft sets (respectively, Semi* - open Soft sets) over $\mathcal{U}$ by $\mathcal{S}^*\mathcal{CSS}(\mathcal{U})_{\mathcal{E}}$ (respectively, $\mathcal{S}^*\mathcal{OSS}(\mathcal{U})_{\mathcal{E}}$)*

Theorem 3.2. *Let $\mathcal{G}_{\mathsf{C}}$ be a soft set in a soft topological space $(\mathcal{U}_{\mathcal{E}},\tilde{\tau})$. Then the subsequent are equivalent:*

1. *$\mathcal{G}_{\mathsf{C}}$ is a semi*-closed soft set.*
2. *$int^*(cl(\mathcal{G}_{\mathsf{C}}))\tilde{\subseteq}\mathcal{G}_{\mathsf{C}}$.*
3. *$cl^*(int(\mathcal{G}^c_{\mathsf{C}}))\tilde{\supseteq}\mathcal{G}^c_{\mathsf{C}}$.*
4. *$\mathcal{G}^c_{\mathsf{C}}$ is a semi*-open soft set.*

Proof. (1) ⇒(2): If $\mathcal{G}_{\mathsf{C}}$ is a semi-closed soft set, then there exists a closed soft set $\mathcal{H}_{\mathsf{B}}$ such that $int^*(\mathcal{H}_{\mathsf{B}})\tilde{\subseteq}\mathcal{G}_{\mathsf{C}}\tilde{\subseteq}\mathcal{H}_{\mathsf{B}}$. Also $cl(\mathcal{G}_{\mathsf{C}})$ is a smallest closed soft set that contains $\mathcal{G}_{\mathsf{C}}$. Therefore, $\mathcal{G}_{\mathsf{C}}\tilde{\subseteq}cl(\mathcal{G}_{\mathsf{C}})\tilde{\subseteq}\mathcal{H}_{\mathsf{B}}$ which implies $int^*(cl(\mathcal{G}_{\mathsf{C}}))\tilde{\subseteq}int^*(\mathcal{H}_{\mathsf{B}})\tilde{\subseteq}\mathcal{G}_{\mathsf{C}}$.*

(2)⇒(3): Assume that $int^*(cl(\mathcal{G}_{\mathsf{C}}))\tilde{\subseteq}\mathcal{G}_{\mathsf{C}}$. Now $\mathcal{G}^c_{\mathsf{C}}\tilde{\subseteq}(int^*(cl(\mathcal{G}_{\mathsf{C}})))^c$. This implies $\mathcal{G}^c_{\mathsf{C}}\tilde{\subseteq}cl^*(cl(\mathcal{G}_{\mathsf{C}}))^c$. This implies $\mathcal{G}^c_{\mathsf{C}}\tilde{\subseteq}cl^*(int(\mathcal{G}^c_{\mathsf{C}}))$. Hence $cl^*(int(\mathcal{G}^c_{\mathsf{C}}))\tilde{\supseteq}\mathcal{G}^c_{\mathsf{C}}$.

(3)⇒(4):Take $\mathcal{H}_{\mathsf{B}}=int(\mathcal{G}^c_{\mathsf{C}})$. Then $\mathcal{H}_{\mathsf{B}}$ is an open soft set such that $int(\mathcal{G}^c_{\mathsf{C}})\tilde{\subseteq}(\mathcal{G}^c_{\mathsf{C}})\tilde{\subseteq}cl^*(int(\mathcal{G}^c_{\mathsf{C}}))$ and hence $\mathcal{H}_{\mathsf{B}}\tilde{\subseteq}\mathcal{G}^c_{\mathsf{C}}\tilde{\subseteq}cl^*(\mathcal{H}_{\mathsf{B}})$, where $\mathcal{H}_{\mathsf{B}}$ is an open soft set. Therefore $\mathcal{G}^c_{\mathsf{C}}$ is a semi-open soft set.*

(4)⇒(1): Suppose $\mathcal{G}_C^c$ is a semi-open soft set. Then there exists an open soft set $\mathcal{H}_B$ such that $\mathcal{H}_B \tilde{\subseteq} \mathcal{G}_C^c \tilde{\subseteq} cl^*(\mathcal{H}_B)$. Hence $(cl^*(\mathcal{H}_B))^c \tilde{\subseteq} \mathcal{G}_C \tilde{\subseteq} (\mathcal{H}_B)^c$ and hence $(int^*(\mathcal{H}_B)^c) \tilde{\subseteq} \mathcal{G}_C \tilde{\subseteq} (\mathcal{H}_B)^c$. As $\mathcal{H}_B$ is an open soft set,$(\mathcal{H}_B)^c$ is a closed soft set. Therefore, there exists a closed soft set $(\mathcal{H}_B)^c$ such that $(int^*(\mathcal{H}_B)^c) \tilde{\subseteq} \mathcal{G}_C \tilde{\subseteq} (\mathcal{H}_B)^c$. Hence $\mathcal{G}_C$ is a semi*-closed soft set.*

Theorem 3.3. *In a soft topological space $(\mathcal{U}_\mathcal{E}, \tilde{\tau})$, every open soft set in a soft topological space is a semi*-open soft set.*
Proof. *Let $\mathcal{G}_A$ be an open soft set. Since $\mathcal{G}_A$ is an open set $int(\mathcal{G}_A) = \mathcal{G}_A$. Now $\mathcal{G}_A = int(\mathcal{G}_A) \tilde{\subseteq} cl^*(int(\mathcal{G}_A))$ and hence $\mathcal{G}_A \tilde{\subseteq} cl^*(int(\mathcal{G}_A))$. Then $\mathcal{G}_A$ is a semi*-open soft set.*

Theorem 3.4. *Every closed soft set in a soft topological space $(\mathcal{U}_\mathcal{E}, \tilde{\tau})$ is a semi*-closed soft set.*
Proof. *Let $\mathcal{G}_A$ be a closed soft set. Since $\mathcal{G}_A$ is closed $\mathcal{G}_A = cl(\mathcal{G}_A)$. Now $int^*(cl(\mathcal{G}_A)) = int^*(\mathcal{G}_A) \tilde{\subseteq} \mathcal{G}_A$.Then $\mathcal{G}_A$ is a semi*-closed soft set.*

Theorem 3.5. *Every semi*-open soft set is a semi-open soft set.*
Proof. *Let $\mathcal{G}_A$ be a semi*-open soft set. Then $\mathcal{G}_A \tilde{\subseteq} cl^*(int(\mathcal{G}_A))$. Also we see that, $cl^*(int(\mathcal{G}_A)) \tilde{\subseteq} cl(int(\mathcal{G}_A))$. That is $\mathcal{G}_A \tilde{\subseteq} cl(int(\mathcal{G}_A))$. Hence $\mathcal{G}_A$ is a soft semi-open set.*

Corollary 3.6. *Every semi*-closed soft set is a semi-closed soft set.*

Theorem 3.7. *The arbitrary union of semi*-open soft sets is a semi*-open soft set.*
Proof. *Let $\{(\mathcal{G}_{C_{\lambda:\in\Lambda}})\}$ be a collection of semi*-open soft sets of a soft topological space. Then there exist open soft sets $(\mathcal{H}_B)_\lambda$ such that $(\mathcal{H}_B)_\lambda \tilde{\subseteq} (\mathcal{G}_C)_\lambda \tilde{\subseteq} cl^*(\mathcal{H}_B)_\lambda$ for each λ. Hence $\tilde{\cup}(\mathcal{H}_B)_\lambda \tilde{\subseteq} \tilde{\cup}(\mathcal{G}_C)_\lambda \tilde{\subseteq} \tilde{\cup}(cl^*(\mathcal{H}_B)_\lambda = cl^*(\tilde{\cup}(\mathcal{H}_B)_\lambda$. Therefore $\cup(\mathcal{G}_C)_\lambda$ is a semi*-open soft set.*

Corollary 3.8. *The arbitrary intersection of semi*-closed soft sets is a semi*-closed soft set.*

Theorem 3.9. *Let $\mathcal{G}_C$ be a semi* - open soft set and $\mathcal{G}_C \tilde{\subseteq} \mathcal{K}_D \tilde{\subseteq} cl^*(\mathcal{G}_C)$, then $\mathcal{K}_D$ is also a semi*-open soft set.*
Proof. *Let $\mathcal{G}_C$ be a semi*-open soft set. Then there exists an soft open set $\mathcal{H}_B$ such that $\mathcal{H}_B \tilde{\subseteq} \mathcal{G}_C \tilde{\subseteq} cl^*(\mathcal{H}_B)$. By our assumption $\mathcal{H}_B \tilde{\subseteq} \mathcal{K}_D$ and $cl^*(\mathcal{G}_C) \tilde{\subseteq} cl^*(\mathcal{H}_B)$ which implies $\mathcal{K}_D \tilde{\subseteq} cl^*(\mathcal{G}_C) \tilde{\subseteq} cl^*(\mathcal{H}_B)$. That is $\mathcal{H}_B \tilde{\subseteq} \mathcal{K}_D \tilde{\subseteq} cl^*(\mathcal{H}_B)$. Therefore $\mathcal{K}_D$ is a semi*-open soft set.*

Theorem 3.10. *If a semi*-closed soft set $\mathcal{L}_A$ is such that $int^*\mathcal{L}_A \tilde{\subseteq} \mathcal{K}_D \tilde{\subseteq} \mathcal{L}_A$, then $\mathcal{K}_D$ is also semi*-closed.*
Proof. *Similar to the above theorem.*

Definition 3.11. *Let $\mathcal{G}_C$ be a soft set in a soft topological space.*

(i) The soft semi-closure of $\mathcal{G}_C$ is*
$ss^*cl(\mathcal{G}_C) = \tilde{\cap}\{\mathcal{S}_F / \mathcal{G}_C \tilde{\subseteq} \mathcal{S}_F$ *and* $\mathcal{S}_F \in S^*C\mathcal{S}\mathcal{S}(\mathcal{U})_\mathcal{E}\}$ *is a soft set.*

(ii) The soft semi-interior of $\mathcal{G}_C$ is*
$ss^*int(\mathcal{G}_C) = \tilde{\cup}\{\mathcal{S}_F / \mathcal{S}_F \tilde{\subseteq} \mathcal{G}_C$ *and* $\mathcal{S}_F \in S^*O\mathcal{S}\mathcal{S}(\mathcal{U})_\mathcal{E}\}$ *is a soft set.*

In short, $ss^*cl(\mathcal{G}_C)$ is the smallest semi-closed soft set containing $\mathcal{G}_C$ and $ss^*int(\mathcal{G}_C)$ is the largest semi*-open soft set contained in $\mathcal{G}_C$.*

Theorem 3.12. *Let $\mathcal{G}_C$ be a soft set in a soft topological space $(\mathcal{U}_\mathcal{E}, \tilde{\tau})$. Then the soft point $\ell_\mathcal{F} \in ss^*cl(\mathcal{G}_C)$ if and only if every soft semi*-open set containing $\ell_\mathcal{F}$ intersects $\mathcal{G}_C$.*
Proof. *We transform each implication to its contrapositve by $\ell_\mathcal{F} \notin ss^*cl(\mathcal{G}_C)$ if and only if there exists a soft semi*-open set $\mathcal{H}_B$ containing $\ell_\mathcal{F}$ that does not intersect $\mathcal{G}_C$*
Suppose assume that $\ell_\mathcal{F} \tilde{\notin} ss^*cl(\mathcal{G}_C)$. Then $\ell_\mathcal{F} \tilde{\in} (ss^*cl(\mathcal{G}_C))^c$. Then $(ss^*cl(\mathcal{G}_C))^c$ is a soft semi-open set containing $\ell_\mathcal{F}$ that does not intersect $\mathcal{G}_C$. Conversely if there exists a soft semi*-open set $\mathcal{H}_B$ containing $\ell_\mathcal{F}$ which does not intersect $\mathcal{G}_C$. Then $(\mathcal{H}_B)^c$ is a soft semi*-open set containing $\mathcal{G}_C$. By the definition of soft semi*-closure, $ss^*cl(\mathcal{G}_C)$ is contained in $(\mathcal{H}_B)^c$. Hence $\ell_\mathcal{F}$ cannot be in $ss^*cl(\mathcal{G}_C)$.*

Theorem 3.13. *Let $\mathcal{G}_C$ and $\mathcal{K}_D$ be two soft sets in a soft topological space. Then*

(i) $\mathcal{G}_C \in S^*C\mathcal{S}\mathcal{S}(\mathcal{U})_\mathcal{E}$ *if and only if* $\mathcal{G}_C = ss^*cl(\mathcal{G}_C)$.

(ii) $\mathcal{G}_C \in S^*O\mathcal{S}\mathcal{S}(\mathcal{U})_\mathcal{E}$ *if and only if* $\mathcal{G}_C = ss^*int(\mathcal{G}_C)$.

(iii) $(ss^*cl(\mathcal{G}_C))^c = ss^*int(\mathcal{G}_C^c)$.

(iv) $(ss^*int(\mathcal{G}_C))^c = ss^*cl(\mathcal{G}_C^c)$.

(v) $\mathcal{G}_C \tilde{\subseteq} \mathcal{K}_D$ *implies* $ss^*int(\mathcal{G}_C) \tilde{\subseteq} ss^*int(\mathcal{K}_D)$.

(vi) $\mathcal{G}_C \tilde{\subseteq} \mathcal{K}_D$ *implies* $ss^*cl(\mathcal{G}_C) \tilde{\subseteq} ss^*cl(\mathcal{K}_D)$.

(vii) $ss^*cl(\phi_\mathcal{E}) = \phi_\mathcal{E}, ss^*cl(\mathcal{U}_\mathcal{E}) = \mathcal{U}_\mathcal{E}$.

(viii) $ss^*int(\phi_\mathcal{E}) = \phi_\mathcal{E}, ss^*int(\mathcal{U}_\mathcal{E}) = \mathcal{U}_\mathcal{E}$.

(ix) $ss^*int(\mathcal{G}_C \tilde{\cap} \mathcal{K}_D) = ss^*int(\mathcal{G}_C) \tilde{\cap} ss^*int(\mathcal{K}_D)$.

(x) $ss^*cl(\mathcal{G}_C \tilde{\cap} \mathcal{K}_D) \tilde{\subseteq} ss^*cl(\mathcal{G}_C) \tilde{\cap} ss^*cl(\mathcal{K}_D)$.

(xi) $ss^*int(\mathcal{G}_C \tilde{\cup} \mathcal{K}_D) \tilde{\supseteq} ss^*int(\mathcal{G}_C) \tilde{\cup} ss^*int(\mathcal{K}_D)$.

(xii) $ss^*cl(ss^*cl(\mathcal{G}_{\mathsf{C}})) = ss^*cl(\mathcal{G}_{\mathsf{C}})$.

(xiii) $ss^*int(ss^*int(\mathcal{G}_{\mathsf{C}})) = ss^*int(\mathcal{G}_{\mathsf{C}})$.

Proof.

(i) *Let* $\mathcal{G}_{\mathsf{C}}$ *be a semi*-closed soft set. Then it is a smallest semi*-closed soft set containing itself. Then by the definition of soft semi*-closure we have* $\mathcal{G}_{\mathsf{C}} = ss^*cl(\mathcal{G}_{\mathsf{C}})$.
Conversely let $\mathcal{G}_{\mathsf{C}} = ss^*cl(\mathcal{G}_{\mathsf{C}})$. *since* $ss^*cl(\mathcal{G}_{\mathsf{C}})$ *is the intersection of all soft semi*-closed sets and by using Corollary 3.8,* $ss^*cl(\mathcal{G}_{\mathsf{C}}) \in S^*C\mathcal{SS}(\mathcal{U})_{\mathcal{E}}$. *Hence* $\mathcal{G}_{\mathsf{C}} \in S^*C\mathcal{SS}(\mathcal{U})_{\mathcal{E}}$.

(ii) *Let* $\mathcal{G}_{\mathsf{C}}$ *be a semi*-open soft set. Then it is a largest semi*-open soft set contained in itself. Then by the definition of soft semi*-interior* $\mathcal{G}_{\mathsf{C}} = ss^*int(\mathcal{G}_{\mathsf{C}})$ *Conversely let* $\mathcal{G}_{\mathsf{C}} = ss^*int(\mathcal{G}_{\mathsf{C}})$ *As* $ss^*int(\mathcal{G}_{\mathsf{C}})$ *is the union of all soft semi*-open sets and by using Theorem 3.7,* $ss^*int(\mathcal{G}_{\mathsf{C}}) \in S^*O\mathcal{SS}(\mathcal{U})_{\mathcal{E}}$. *This implies* $\mathcal{G}_{\mathsf{C}} \in S^*O\mathcal{SS}(\mathcal{U})_{\mathcal{E}}$.

(iii) $ss^*int(\mathcal{G}_{\mathsf{C}}) = \tilde{\bigcup}\{(\mathcal{H}_{\mathsf{D}})^c : \mathcal{H}_{\mathsf{D}}$ *is a semi*-closed soft set and* $(\mathcal{G}_{\mathsf{C}})^c \tilde{\subseteq} \mathcal{H}_{\mathsf{D}}\}$ *That is* $ss^*int(\mathcal{G}_{\mathsf{C}}) = [\tilde{\bigcap}\{\mathcal{H}_{\mathsf{D}} : \mathcal{H}_{\mathsf{D}}$ *is a semi*-closed soft set and* $(\mathcal{G}_{\mathsf{C}})^c \tilde{\subseteq} \mathcal{H}_{\mathsf{D}}\}]^c$. *This implies* $ss^*int(\mathcal{G}_{\mathsf{C}}) = [ss^*cl(\mathcal{G}^c_{\mathsf{C}})]^c$. *Hence* $(ss^*int(\mathcal{G}_{\mathsf{C}}))^c = ss^*cl(\mathcal{G}^c_{\mathsf{C}})$

(iv) *Similar to (iii).*

(v) $ss^*int(\mathcal{G}_{\mathsf{C}}) \tilde{\subseteq} \mathcal{G}_{\mathsf{C}} \tilde{\subseteq} \mathcal{K}_{\mathsf{D}}$ *implies that* $ss^*int(\mathcal{G}_{\mathsf{C}}) \tilde{\subseteq} \mathcal{K}_{\mathsf{D}}$. *As* $ss^*int(\mathcal{K}_{\mathsf{D}})$ *is the largest semi*-open soft set contained in* $\mathcal{K}_{\mathsf{D}}$, $ss^*int(\mathcal{G}_{\mathsf{C}}) \tilde{\subseteq} ss^*int(\mathcal{K}_{\mathsf{D}})$.

(vi) $\mathcal{K}_{\mathsf{D}} \tilde{\subseteq} ss^*cl(\mathcal{K}_{\mathsf{D}})$. *This implies* $\mathcal{G}_{\mathsf{C}} \tilde{\subseteq} \mathcal{K}_{\mathsf{D}} \tilde{\subseteq} ss^*cl(\mathcal{K}_{\mathsf{D}})$. *Hence* $\mathcal{G}_{\mathsf{C}} \tilde{\subseteq} ss^*cl(\mathcal{K}_{\mathsf{D}})$. *As* $ss^*cl(\mathcal{G}_{\mathsf{C}})$ *is the smallest semi*-closed soft set contains* $\mathcal{G}_{\mathsf{C}}$, $ss^*cl(\mathcal{G}_{\mathsf{C}}) \tilde{\subseteq} ss^*cl(\mathcal{K}_{\mathsf{D}})$.

(vii) *Since* $\phi_{\mathcal{E}}$ *and* $\mathcal{U}_{\mathcal{E}}$ *are semi*-closed soft set by (i),* $ss^*cl(\phi_{\mathcal{E}}) = \phi_{\mathcal{E}}$ *and* $ss^*cl(\mathcal{U}_{\mathcal{E}}) = \mathcal{U}_{\mathcal{E}}$.

(viii) *Similar to (vii).*

(ix) $\mathcal{G}_{\mathsf{C}} \tilde{\bigcap} \mathcal{K}_{\mathsf{D}} \tilde{\subseteq} \mathcal{G}_{\mathsf{C}}$ *and* $\mathcal{G}_{\mathsf{C}} \tilde{\bigcap} \mathcal{K}_{\mathsf{D}} \tilde{\subseteq} \mathcal{K}_{\mathsf{D}}$.

Hence by (v)

$ss^*int(\mathcal{G}_{\mathsf{C}} \tilde{\bigcap} \mathcal{K}_{\mathsf{D}}) \tilde{\subseteq} ss^*int(\mathcal{G}_{\mathsf{C}})$ *and* $ss^*int(\mathcal{G}_{\mathsf{C}} \tilde{\bigcap} \mathcal{K}_{\mathsf{D}}) \tilde{\subseteq} ss^*int(\mathcal{K}_{\mathsf{D}})$. *This implies*

$ss^*int(\mathcal{G}_{\mathsf{C}} \tilde{\bigcap} \mathcal{K}_{\mathsf{D}}) \tilde{\subseteq} ss^*int(\mathcal{G}_{\mathsf{C}}) \tilde{\bigcap} ss^*int(\mathcal{K}_{\mathsf{D}})$. *Let* $\ell_{\mathscr{F}} \tilde{\notin} ss^*int(\mathcal{G}_{\mathsf{C}} \tilde{\bigcap} \mathcal{K}_{\mathsf{D}})$. *Then* $\ell_{\mathscr{F}} \tilde{\notin} \tilde{\bigcup}_{\lambda \in \Lambda} (\mathcal{H}_{\mathsf{B}})_\lambda$ *where* $(\mathcal{H}_{\mathsf{B}})_\lambda \in S^*O\mathcal{SS}(\mathcal{U})_{\mathcal{E}}$ *such that* $(\mathcal{H}_{\mathsf{B}})_\lambda \tilde{\subseteq} \mathcal{G}_{\mathsf{C}} \tilde{\bigcap} \mathcal{K}_{\mathsf{D}}$ *for all* $\lambda \in \Lambda$. *This implies* $\ell_{\mathscr{F}} \tilde{\notin} \tilde{\bigcup}_{\lambda \in \Lambda} (\mathcal{H}_{\mathsf{B}})_\lambda$ *where* $(\mathcal{H}_{\mathsf{B}})_\lambda \in S^*O\mathcal{SS}(\mathcal{U})_{\mathcal{E}}$ *such that* $(\mathcal{H}_{\mathsf{B}})_\lambda \tilde{\subseteq} \mathcal{G}_{\mathsf{C}}$ *and* $(\mathcal{H}_{\mathsf{B}})_\lambda \tilde{\subseteq} \mathcal{K}_{\mathsf{D}}$ *for all* $\lambda \in \Lambda$ *Hence* $\ell_{\mathscr{F}} \tilde{\notin} \tilde{\bigcup}_{\lambda \in \Lambda} (\mathcal{H}_{\mathsf{B}})_\lambda$ *where* $(\mathcal{H}_{\mathsf{B}})_\lambda \in S^*O\mathcal{SS}(\mathcal{U})_{\mathcal{E}}$ *such that* $(\mathcal{H}_{\mathsf{B}})_\lambda \tilde{\subseteq} \mathcal{G}_{\mathsf{C}}$ *and* $\ell_{\mathscr{F}} \tilde{\notin} \tilde{\bigcup}_{\lambda \in \Lambda} (\mathcal{H}_{\mathsf{B}})_\lambda$ *where* $(\mathcal{H}_{\mathsf{B}})_\lambda \in S^*O\mathcal{SS}(\mathcal{U})_{\mathcal{E}}$ *such that* $(\mathcal{H}_{\mathsf{B}})_\lambda \tilde{\subseteq} \mathcal{K}_{\mathsf{D}}$ *for all* $\lambda \in \Lambda$. *This implies*

$\ell_{\mathscr{F}} \tilde{\notin} ss^*int(\mathcal{G}_{\mathsf{C}})$ *and* $\ell_{\mathscr{F}} \notin ss^*int(\mathcal{K}_{\mathsf{D}})$. *Then* $\ell_{\mathscr{F}} \tilde{\notin} ss^*int(\mathcal{G}_{\mathsf{C}}) \tilde{\bigcap} ss^*int(\mathcal{K}_{\mathsf{D}})$.

Hence $ss^*int(\mathcal{G}_{\mathsf{C}}) \tilde{\bigcap} ss^*int(\mathcal{K}_{\mathsf{D}}) \tilde{\subseteq} ss^*int(\mathcal{G}_{\mathsf{C}} \tilde{\bigcap} \mathcal{K}_{\mathsf{D}})$. *Therefore* $ss^*int(\mathcal{G}_{\mathsf{C}} \tilde{\bigcap} \mathcal{K}_{\mathsf{D}}) = ss^*int(\mathcal{G}_{\mathsf{C}}) \tilde{\bigcap} ss^*int(\mathcal{K}_{\mathsf{D}})$.

(x) $\mathcal{G}_{\mathsf{C}} \tilde{\bigcap} \mathcal{K}_{\mathsf{D}} \tilde{\subseteq} \mathcal{G}_{\mathsf{C}}$ *and* $\mathcal{G}_{\mathsf{C}} \tilde{\bigcap} \mathcal{K}_{\mathsf{D}} \tilde{\subseteq} \mathcal{K}_{\mathsf{D}}$.

Hence by (vi), $ss^*cl(\mathcal{G}_{\mathsf{C}} \tilde{\bigcap} \mathcal{K}_{\mathsf{D}}) \tilde{\subseteq} ss^*cl(\mathcal{G}_{\mathsf{C}})$ *and* $ss^*cl(\mathcal{G}_{\mathsf{C}} \tilde{\bigcap} \mathcal{K}_{\mathsf{D}}) \tilde{\subseteq} ss^*cl(\mathcal{K}_{\mathsf{D}})$.

This implies

$ss^*cl(\mathcal{G}_{\mathsf{C}} \tilde{\bigcap} \mathcal{K}_{\mathsf{D}}) \tilde{\subseteq} ss^*cl(\mathcal{G}_{\mathsf{C}}) \tilde{\bigcap} ss^*cl(\mathcal{K}_{\mathsf{D}})$.

(xi) $\mathcal{G}_{\mathsf{C}} \tilde{\subseteq} \mathcal{G}_{\mathsf{C}} \tilde{\bigcup} \mathcal{K}_{\mathsf{D}}$ *and* $\mathcal{K}_{\mathsf{D}} \tilde{\subseteq} \mathcal{G}_{\mathsf{C}} \tilde{\bigcup} \mathcal{K}_{\mathsf{D}}$.*Then by (v),*$ss^*int(\mathcal{G}_{\mathsf{C}}) \tilde{\subseteq} ss^*int(\mathcal{G}_{\mathsf{C}} \tilde{\bigcup} \mathcal{K}_{\mathsf{D}})$ *and* $ss^*int(\mathcal{K}_{\mathsf{D}}) \tilde{\subseteq} ss^*int(\mathcal{G}_{\mathsf{C}} \tilde{\bigcup} \mathcal{K}_{\mathsf{D}})$.

Therefore

$ss^*int(\mathcal{G}_{\mathsf{C}} \tilde{\bigcup} \mathcal{K}_{\mathsf{D}}) \tilde{\supseteq} ss^*int(\mathcal{G}_{\mathsf{C}}) \tilde{\bigcup} ss^*int(\mathcal{K}_{\mathsf{D}})$.

(xii) *Since* $ss^*cl(\mathcal{G}_{\mathsf{C}}) \in S^*C\mathcal{SS}(\mathcal{U})_{\mathcal{E}}$, *by (i)* $ss^*cl(ss^*cl(\mathcal{G}_{\mathsf{C}})) = ss^*cl(\mathcal{G}_{\mathsf{C}})$.

(xiii) *Since* $ss^*int(\mathcal{G}_{\mathsf{C}}) \in S^*O\mathcal{SS}(\mathcal{U})_{\mathcal{E}}$, *by (ii)* $ss^*int(ss^*int(\mathcal{G}_{\mathsf{C}})) = ss^*int(\mathcal{G}_{\mathsf{C}})$.

4. Functions using soft semi*-open sets

On this spot, we elucidate generalizations of soft functions in soft topological spaces and investigate their properties.

Definition 4.1. *A soft function $f : \mathcal{SS}(\mathcal{U})_{\mathcal{E}} \to \mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ is said to be*

(i) *soft semi*-continuous if for each soft open set $\mathcal{G}_{\mathsf{C}}$ of $\mathcal{V}_{\mathcal{E}'}$, the inverse image $f^{-1}(\mathcal{G}_{\mathsf{C}})$ is a semi*-open soft set of $\mathcal{U}_{\mathcal{E}}$.*

(ii) *soft semi*-open function if for each open soft set $\mathcal{L}_{\mathsf{A}}$ of $\mathcal{U}_{\mathcal{E}}$, the image is a semi*-open soft set of $\mathcal{V}'_{\mathcal{E}}$.*

(iii) *soft semi*-closed function if for each closed soft set $\mathcal{K}_{\mathsf{D}}$ of $\mathcal{U}_{\mathcal{E}}$, the image $f(\mathcal{K}_{\mathsf{D}})$ is a semi*-closed soft set of $\mathcal{V}'_{\mathcal{E}}$.*

(iv) *soft semi*-irresolute if for each soft open set $\mathcal{G}_{\mathsf{C}}$ of $\mathcal{V}_{\mathcal{E}'}$, the inverse image $f^{-1}(\mathcal{G}_{\mathsf{C}})$ is a semi*-open soft set of $\mathcal{U}_{\mathcal{E}}$.*

Definition 4.2. *A soft function $f : \mathcal{SS}(\mathcal{U})_{\mathcal{E}} \to \mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ is soft semi* -continuous if for each closed soft set $K_{D'}$ of $\mathcal{V}_{\mathcal{E}'}$,the inverse image $f^{-1}(K_{D'})$ is a semi*-closed soft set of $\mathcal{U}_{\mathcal{E}}$.*

Theorem 4.3. *A soft function $f : \mathcal{SS}(\mathcal{U})_{\mathcal{E}} \to \mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ is soft semi* -continuous if and only if $f(ss^*cl(\mathcal{L}_{\mathsf{A}}))\tilde{\subseteq}cl(f(\mathcal{L}_{\mathsf{A}}))$, for every soft set $\mathcal{L}_{\mathsf{A}}$ of $\mathcal{U}_{\mathcal{E}}$.*

Proof. *Let $f : \mathcal{SS}(\mathcal{U})_{\mathcal{E}} \to \mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ be a soft semi* -continuous function.Now $cl(f(\mathcal{L}_{\mathsf{A}}))$ is a soft closed set of $\mathcal{V}_{\mathcal{E}'}$. By using soft semi*-continuity of f, $f^{-1}(cl(f(\mathcal{L}_{\mathsf{A}})))$ is a semi*-closed soft set of $\mathcal{U}_{\mathcal{E}}$. Also $f(\mathcal{L}_{\mathsf{A}})\tilde{\subseteq}cl(f(\mathcal{L}_{\mathsf{A}}))$. This implies $\mathcal{L}_{\mathsf{A}}\tilde{\subseteq}f^{-1}(cl(f(\mathcal{L}_{\mathsf{A}})))$. Here $f^{-1}(cl(f(\mathcal{L}_{\mathsf{A}})))$ is a semi*-closed soft set containing $\mathcal{L}_{\mathsf{A}}$. But $ss^*cl(\mathcal{L}_{\mathsf{A}})$ is a smallest semi*-closed soft set containing $\mathcal{L}_{\mathsf{A}}$. Now $\mathcal{L}_{\mathsf{A}}\tilde{\subseteq}ss^*cl(\mathcal{L}_{\mathsf{A}})\tilde{\subseteq}f^{-1}(cl(f(\mathcal{L}_{\mathsf{A}})))$.$ss^*cl(\mathcal{L}_{\mathsf{A}})\tilde{\subseteq} f^{-1}(cl(f(\mathcal{L}_{\mathsf{A}})))$ which implies $f(ss^*cl(\mathcal{L}_{\mathsf{A}}))\tilde{\subseteq}cl(f(\mathcal{L}_{\mathsf{A}}))$.*

Conversely, assume that $f(ss^*cl(\mathcal{L}_{\mathsf{A}}))\tilde{\subseteq}cl(f(\mathcal{L}_{\mathsf{A}}))$.Let $\mathcal{G}_{\mathsf{C}}$ be any soft closed set of $\mathcal{V}_{\mathcal{E}'}$. Therefore $f^{-1}(\mathcal{G}_{\mathsf{C}}) \in \mathcal{U}_{\mathcal{E}}$ which implies $f(ss^*cl(f^{-1}(\mathcal{G}_{\mathsf{C}}))\tilde{\subseteq} cl(f(f^{-1}(\mathcal{G}_{\mathsf{C}})) = cl(\mathcal{G}_{\mathsf{C}}) = \mathcal{G}_{\mathsf{C}} \Rightarrow ss^*cl(f^{-1}(\mathcal{G}_{\mathsf{C}})\tilde{\subseteq}f^{-1}(\mathcal{G}_{\mathsf{C}})$. Always $f^{-1}(\mathcal{G}_{\mathsf{C}})\tilde{\subseteq}ss^*cl(f^{-1}(\mathcal{G}_{\mathsf{C}})$ and $f^{-1}(\mathcal{G}_{\mathsf{C}}) = ss^*cl(f^{-1}(\mathcal{G}_{\mathsf{C}}))$. Therefore,$f^{-1}(\mathcal{G}_{\mathsf{C}})$ is a semi-closed soft set. By using definition 4.2, f is a semi*-continuous soft function.*

Theorem 4.4. *A soft function $f : \mathcal{SS}(\mathcal{U})_{\mathcal{E}} \to \mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ is semi*-continuous if and only if $f^{-1}(int\mathcal{G}_{\mathsf{C}})\tilde{\subseteq}ss^*int(f^{-1}(\mathcal{G}_{\mathsf{C}}))$ for every soft set $\mathcal{G}_{\mathsf{C}}$ of $\mathcal{V}'_{\mathcal{E}}$.*

Proof. *Suppose $f : \mathcal{SS}(\mathcal{U})_{\mathcal{E}} \to \mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ is a soft semi*-continuous function. Now $int(\mathcal{G}_{\mathsf{C}})$ is a soft open set of $\mathcal{V}_{\mathcal{E}'}$. As f is a soft semi*-continuous function, $f^{-1}(int(\mathcal{G}_{\mathsf{C}}))$ is a soft semi*-open set of $\mathcal{U}_{\mathcal{E}}$. Also $int(\mathcal{G}_{\mathsf{C}})\tilde{\subseteq}\mathcal{G}_{\mathsf{C}}$ implies $f^{-1}(int(\mathcal{G}_{\mathsf{C}}))\tilde{\subseteq}f^{-1}(\mathcal{G}_{\mathsf{C}})$. As $ss^*int(f^{-1}(\mathcal{G}_{\mathsf{C}}))$ is a largest soft semi*-open set contained in $f^{-1}(\mathcal{G}_{\mathsf{C}})$, $f^{-1}(int(\mathcal{G}_{\mathsf{C}}))\tilde{\subseteq}ss^*int(f^{-1}(\mathcal{G}_{\mathsf{C}}))$.*

Conversely assume that $f^{-1}(int(\mathcal{G}_{\mathsf{C}}))\tilde{\subseteq}ss^*int(f^{-1}(\mathcal{G}_{\mathsf{C}}))$. Let $\mathcal{G}_{\mathsf{C}}$ be an soft open set of $\mathcal{V}_{\mathcal{E}'}$. Then $f^{-1}(\mathcal{G}_{\mathsf{C}}) = f^{-1}(int(\mathcal{G}_{\mathsf{C}}))\tilde{\subseteq}ss^*int(f^{-1}(\mathcal{G}_{\mathsf{C}}))$. This implies $f^{-1}(\mathcal{G}_{\mathsf{C}})\tilde{\subseteq}ss^*int(f^{-1}(\mathcal{G}_{\mathsf{C}}))$. Always $ss^*int(f^{-1}(\mathcal{G}_{\mathsf{C}}))\tilde{\subseteq}f^{-1}(\mathcal{G}_{\mathsf{C}})$. Hence $f^{-1}(\mathcal{G}_{\mathsf{C}}) = ss^*int(f^{-1}(\mathcal{G}_{\mathsf{C}}))$. That is $f^{-1}(\mathcal{G}_{\mathsf{C}})$ is a soft semi-open set. Hence f is a soft semi*-continuous function.*

Theorem 4.5. *A soft function $f : \mathcal{SS}(\mathcal{U})_{\mathcal{E}} \to \mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ is soft semi*-open if and only if $f(int\mathcal{L}_{\mathsf{A}})\tilde{\subseteq}ss^*int(f(\mathcal{L}_{\mathsf{A}}))$ for every soft set $\mathcal{L}_{\mathsf{A}}$ of $\mathcal{U}_{\mathcal{E}}$.*

Proof.Suppose $f : \mathcal{SS}(\mathcal{U})_{\mathcal{E}} \to \mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ is soft semi-open. Now $int(\mathcal{L}_{\mathsf{A}})$ is a soft open set in $\mathcal{U}_{\mathcal{E}}$ as f is soft semi*-open $f(int(\mathcal{L}_{\mathsf{A}}))$ is a soft semi*-open set. Also $int(\mathcal{L}_{\mathsf{A}})\tilde{\subseteq}\mathcal{L}_{\mathsf{A}}$. Hence $f(int(\mathcal{L}_{\mathsf{A}}))\tilde{\subseteq}f(\mathcal{L}_{\mathsf{A}})$. As $ss^*int(f(\mathcal{L}_{\mathsf{A}}))$ is the largest semi*-open soft set contained in $f(\mathcal{L}_{\mathsf{A}})$, $f(int(\mathcal{L}_{\mathsf{A}}))\tilde{\subseteq}ss^*int(f(\mathcal{L}_{\mathsf{A}}))$.*

Conversely assume that $f(int(\mathcal{L}_{\mathsf{A}}))\tilde{\subseteq}ss^*int(f(\mathcal{L}_{\mathsf{A}}))$. for every soft set $\mathcal{L}_{\mathsf{A}}$ of $\mathcal{U}_{\mathcal{E}}$. Let $\mathcal{G}_{\mathsf{C}}$ be a soft open set in $\mathcal{U}_{\mathcal{E}}$. Hence $f(\mathcal{G}_{\mathsf{C}}) = f(int(\mathcal{G}_{\mathsf{C}}))\tilde{\subseteq}ss^*int(f(\mathcal{G}_{\mathsf{C}}))$. Always $ss^*int(f(\mathcal{G}_{\mathsf{C}}))\tilde{\subseteq}f(\mathcal{G}_{\mathsf{C}})$. Therefore $f(\mathcal{G}_{\mathsf{C}})$ is a semi-open soft set in $\mathcal{V}_{\mathcal{E}'}$. Hence f is a semi*-open soft function.*

Theorem 4.6. *A soft function $f : \mathcal{SS}(\mathcal{U})_{\mathcal{E}} \to \mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ is soft semi*-closed if and only if $ss^*cl(f(\mathcal{L}_{\mathsf{A}}))\tilde{\subseteq}f(cl(\mathcal{L}_{\mathsf{A}}))$ for every soft set $\mathcal{L}_{\mathsf{A}}$ of $\mathcal{U}_{\mathcal{E}}$.*

Proof. *Let $f : \mathcal{SS}(\mathcal{U})_{\mathcal{E}} \to \mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ is a soft semi*-closed function. Since $cl(\mathcal{L}_{\mathsf{A}})$ is a soft closed set in $\mathcal{U}_{\mathcal{E}}$,$f(cl(\mathcal{L}_{\mathsf{A}}))$ is a soft semi*-closed set in $\mathcal{V}_{\mathcal{E}'}$. Also note that $\mathcal{L}_{\mathsf{A}}\tilde{\subseteq}cl(\mathcal{L}_{\mathsf{A}})$. This implies that $f(\mathcal{L}_{\mathsf{A}})\tilde{\subseteq}f(cl(\mathcal{L}_{\mathsf{A}}))$ .Since $ss^*cl(f(\mathcal{L}_{\mathsf{A}}))$ is the smallest semi*-closed soft set contains $f(\mathcal{L}_{\mathsf{A}})$, $ss^*cl(f(\mathcal{L}_{\mathsf{A}}))\tilde{\subseteq}f(cl(\mathcal{L}_{\mathsf{A}}))$. Conversely let $ss^*cl(f(\mathcal{L}_{\mathsf{A}}))\tilde{\subseteq}f(cl(\mathcal{L}_{\mathsf{A}}))$ for every soft set $\mathcal{L}_{\mathsf{A}}$ of $\mathcal{U}_{\mathcal{E}}$. Let $\mathcal{G}_{\mathsf{C}}$ be a soft closed set in $\mathcal{U}_{\mathcal{E}}$ .Then $\mathcal{G}_{\mathsf{C}} = cl(\mathcal{G}_{\mathsf{C}})$. This implies $f(\mathcal{G}_{\mathsf{C}}) = f(cl(\mathcal{G}_{\mathsf{C}}))$. Hence by our assumption $ss^*cl(f(\mathcal{G}_{\mathsf{C}}))\tilde{\subseteq}f(cl(\mathcal{G}_{\mathsf{C}})) = f(\mathcal{G}_{\mathsf{C}})$ Always $f(\mathcal{G}_{\mathsf{C}})\tilde{\subseteq}ss^*cl(f(\mathcal{G}_{\mathsf{C}}))$. Hence $f(\mathcal{G}_{\mathsf{C}}) = ss^*cl(f(\mathcal{G}_{\mathsf{C}}))$. This implies $f(\mathcal{G}_{\mathsf{C}})$ is a soft semi*-closed set. Hence f is a soft semi*-closed function.*

5. Soft semi*-compactness

In this tract, we define semi*-compactness in soft topological spaces and investigate some of its characteristics.

Definition 5.1. *A family ψ of soft sets is a cover of a soft set $\mathcal{F}_{\mathcal{A}}$ if $\mathcal{F}_{\mathcal{A}}\tilde{\subseteq}\tilde{\cup}\{(\mathcal{F}_i)_{\mathcal{A}} : (\mathcal{F}_i)_{\mathcal{A}} \in \psi, i \in I\}$. A subcover of ψ is a subfamily of ψ which is also a cover.*

Definition 5.2. *A soft topological space $(\mathcal{U}_{\mathcal{E}}, \tau)$ is said to be semi*-compact if each semi*-open soft cover of $\mathcal{U}_{\mathcal{E}}$ has a finite subcover.*

Theorem 5.3. *A soft topological space $(\mathcal{U}_{\mathcal{E}}, \tau)$ is semi*-compact if and only if each family of semi*-closed soft sets in $\mathcal{U}_{\mathcal{E}}$ with the finite intersection property has a non empty intersection.*

Proof. *Assume that $(\mathcal{U}_{\mathcal{E}}, \tilde{\tau})$ is a semi*-compact soft topological space. Let $\{(\mathcal{L}_{\mathsf{A}})_{\lambda} : \lambda \in \Lambda\}$ be a collection of semi*-closed soft sets with the finite intersection property. If possible, assume that $\tilde{\bigcap}_{\lambda\in\Lambda} (\mathcal{L}_{\mathsf{A}})_{\lambda} = \phi_{\mathcal{E}}$. This implies $\tilde{\bigcup}_{\lambda\in\Lambda} (\mathcal{L}_{\mathsf{A}})_{\lambda^c} = \mathcal{U}_{\mathcal{E}}$ So the collection $\{(\mathcal{L}_{\mathsf{A}})_{\lambda^c:\lambda\in\Lambda}\}$*

forms a soft semi-open cover of $\mathcal{U}_{\mathcal{E}}$, which is soft semi*-compact. So, there exists a finite sub collection Δ of Λ which also covers $\mathcal{U}_{\mathcal{E}}$. That is $\tilde{\bigcup}_{\lambda\in\Lambda}((\mathcal{L}_{\mathsf{A}})_{\lambda^c}=\mathcal{U}_{\mathcal{E}}$. This implies $\tilde{\bigcup}_{\lambda\in\Lambda}((L_A))^c=\phi_{\mathcal{E}}$. This is a contradiction to the finite intersection property. Hence $\tilde{\bigcap}_{\lambda\in\Lambda}(L_A)_\lambda\neq\phi_{\mathcal{E}}$. Conversely, assume that each family of semi*-closed soft sets in $\mathcal{U}_{\mathcal{E}}$ with the finite intersection property has a non empty intersection. If possible let us assume $(\mathcal{U}_{\mathcal{E}},\tau)$ is not semi*-compact. Then there exists a soft semi*-open cover $\{(\mathcal{G}_{\mathsf{C}})_{\lambda:\in\Lambda}\}$ of $\mathcal{U}_{\mathcal{E}}$ such that for every finite sub collection Δ of Λ we have $\tilde{\bigcup}_{\lambda\in\Delta}(\mathcal{G}_{\mathsf{C}})_\lambda\neq\mathcal{U}_{\mathcal{E}}$. Implies $\tilde{\bigcap}_{\lambda\in\Delta}((\mathcal{G}_{\mathsf{C}})_{\lambda^c}\neq\phi_{\mathcal{E}}$. Hence $\{((\mathcal{G}_{\mathsf{C}})_{\lambda^c:\in\Lambda}\}$ has a finite intersection property. So, by hypothesis $\tilde{\bigcap}_{\lambda\in\Lambda}((\mathcal{G}_{\mathsf{C}})_{\lambda^c}\neq\phi_{\mathcal{E}}$. Which implies $\tilde{\bigcup}_{\in\Lambda}(\mathcal{G}_{\mathsf{C}})_\lambda\neq\mathcal{U}_{\mathcal{E}}$. This is a contradiction to our assumption. Therefore $(\mathcal{U}_{\mathcal{E}},\tilde{\tau})$ is a semi*-compact soft topological space.*

Theorem 5.4. *A soft topological space $(\mathcal{U}_{\mathcal{E}},\tilde{\tau})$ is semi*-compact if and only if for every family ψ of soft sets with finite intersection property, $\tilde{\bigcap}_{\mathcal{G}_{\mathsf{C}}\in\psi} ss^*cl(\mathcal{G}_{\mathsf{C}})\neq\phi_{\mathcal{E}}$.*

Proof. *Let $(\mathcal{U}_{\mathcal{E}},\tilde{\tau})$ be a semi*-compact soft topological space. If possible let us assume that $\tilde{\bigcap}_{\mathcal{G}_{\mathsf{C}}\in\psi} ss^*cl(\mathcal{G}_{\mathsf{C}})=\phi_{\mathcal{E}}$ for some family ψ of soft sets with the finite intersection property. So $\tilde{\bigcup}_{\mathcal{G}_{\mathsf{C}}\in\psi}(ss^*cl(\mathcal{G}_{\mathsf{C}}))^c=\mathcal{U}_{\mathcal{E}}$. Hence $\Gamma=\{(ss^*cl(\mathcal{G}_{\mathsf{C}}))^c:\mathcal{G}_{\mathsf{C}}\in\psi\}$ forms an soft semi*-open cover for $\mathcal{U}_{\mathcal{E}}$. Then by semi*-compactness of $\mathcal{U}_{\mathcal{E}}$ there exists a finite subcover ω of ψ such that $\tilde{\bigcup}_{\mathcal{G}_{\mathsf{C}}\in\omega}(ss^*cl(\mathcal{G}_{\mathsf{C}}))^c=\mathcal{U}_{\mathcal{E}}$. We have $\mathcal{G}_{\mathsf{C}}\tilde{\subseteq}ss^*cl(\mathcal{G}_{\mathsf{C}})$. Then $\mathcal{U}_{\mathcal{E}}\tilde{\subseteq}\tilde{\bigcup}_{\mathcal{G}_{\mathsf{C}}\in\omega}(\mathcal{G}_{\mathsf{C}})^c$ and hence $\mathcal{U}_{\mathcal{E}}=\tilde{\bigcup}_{\mathcal{G}_{\mathsf{C}}\in\omega}(\mathcal{G}_{\mathsf{C}})^c$. Therefore $\tilde{\bigcap}_{\mathcal{G}_{\mathsf{C}}\in\omega}\mathcal{G}_{\mathsf{C}}=\phi_{\mathcal{E}}$. This is contradiction to the finite intersection property. Hence $\tilde{\bigcap}_{\mathcal{G}_{\mathsf{C}}\in\psi} ss^*cl(\mathcal{G}_{\mathsf{C}})\neq\phi_{\mathcal{E}}$.*

Conversely, assume that $\tilde{\bigcap}_{\mathcal{G}_{\mathsf{C}}\in\psi} ss^*cl(\mathcal{G}_{\mathsf{C}})\neq\phi_{\mathcal{E}}$ for every family ψ of soft sets with finite intersection property. Suppose assume that $(\mathcal{U}_{\mathcal{E}},\tau)$ is not soft semi-compact. Then there exists a family Γ of semi*-open soft sets covering $\mathcal{U}_{\mathcal{E}}$ without a finite subcover. So for every finite sub family ω of Γ we have $\tilde{\bigcup}_{\mathcal{G}_{\mathsf{C}}\in\omega}\mathcal{G}_{\mathsf{C}}\neq\mathcal{U}_{\mathcal{E}}$. This implies $\tilde{\bigcap}_{\mathcal{G}_{\mathsf{C}}\in\omega}(\mathcal{G}_{\mathsf{C}})^c\neq\phi_{\mathcal{E}}$.This implies $\{(\mathcal{G}_{\mathsf{C}})^c:\mathcal{G}_{\mathsf{C}}\in\Gamma\}$ is a family of soft sets with finite intersection property. Now $\tilde{\bigcup}_{\mathcal{G}_{\mathsf{C}}\in\Gamma}\mathcal{G}_{\mathsf{C}}=\mathcal{U}_{\mathcal{E}}$. This implies $\tilde{\bigcap}_{\mathcal{G}_{\mathsf{C}}\in\Gamma}(\mathcal{G}_{\mathsf{C}})^c=\phi_{\mathcal{E}}$. Since $\mathcal{G}_{\mathsf{C}}\tilde{\subseteq}ss^*cl(\mathcal{G}_{\mathsf{C}})$, $\tilde{\bigcap}_{\mathcal{G}_{\mathsf{C}}\in\Gamma} ss^*cl(\mathcal{G}_{\mathsf{C}})^c\tilde{\subseteq}\phi_{\mathcal{E}}$. Hence $\tilde{\bigcap}_{\mathcal{G}_{\mathsf{C}}\in\Gamma} ss^*cl(\mathcal{G}_{\mathsf{C}})^c=\phi_{\mathcal{E}}$. This is a contradiction. Therefore $(\mathcal{U}_{\mathcal{E}},\tilde{\tau})$ is semi*-compact soft topological space.*

Theorem 5.5. *Semi*-continuous image of a soft semi*-compact space is soft compact.*

Proof. *Let $f:\mathcal{SS}(\mathcal{U})_{\mathcal{E}}\to\mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ be a semi*-continuous function where $(\mathcal{U}_{\mathcal{E}},\tilde{\tau})$ is a semi*-compact soft topological space and $(\mathcal{V}_{\mathcal{E}'},\delta)$ is another soft topological space. Let $\{(\mathcal{G}_{\mathsf{C}})_{\lambda:\lambda\in\Lambda}\}$ be a soft open cover of $\mathcal{V}_{\mathcal{E}'}$. Since f is semi*-continuous,$\{f^{-1}(\mathcal{G}_{\mathsf{C}})_{\lambda:\in\Lambda}\}$ forms a soft semi*-open cover for $\mathcal{U}_{\mathcal{E}}$. This implies there exists a finite subset Δ of Λ such that $\{f^{-1}(\mathcal{G}_{\mathsf{C}})_{\lambda:\lambda\in\Delta}\}$ forms a soft semi*-open cover of $\mathcal{U}_{\mathcal{E}}$. Hence $\{(\mathcal{G}_{\mathsf{C}})_{\lambda:\lambda\in\Delta}\}$ forms a finite soft subcover of $\mathcal{V}_{\mathcal{E}'}$.*

Theorem 5.6. *Semi*-closed subspace of a semi*-compact soft topological space is soft semi*-compact.*

Proof. *Let $\mathcal{V}_{\mathsf{B}}$ be a semi*-closed subspace of a semi*-compact soft topological space $(\mathcal{U}_{\mathcal{E}},\tilde{\tau})$ and $\{(\mathcal{G}_{\mathsf{C}})_{\lambda:\lambda\in\Lambda}\}$ be a soft semi*-open cover for $\mathcal{V}_{\mathsf{B}}$. As $\mathcal{V}_{\mathsf{B}}$ is semi*-closed soft set $\mathcal{V}_{\mathsf{B}}^c$ is a semi*-open soft set. Hence $\Gamma=\{(\mathcal{G}_{\mathsf{C}})_{\lambda:\lambda\in\Lambda}\}\tilde{\bigcup}\mathcal{V}_{\mathsf{B}}^c$ forms a semi*-open soft cover for $\mathcal{U}_{\mathcal{E}}$. As $\mathcal{U}_{\mathcal{E}}$ is soft semi*-compact Λ has a finite sub family Δ such that $\mathcal{U}_{\mathcal{E}}=\mathcal{V}_{\mathsf{B}}^c\tilde{\bigcup}\{(\mathcal{G}_{\mathsf{C}})_{\lambda:\lambda\in\Delta}\}$. Then $\mathcal{V}_{\mathsf{B}}=\{(\mathcal{G}_{\mathsf{C}})_{\lambda:\lambda\in\Delta}\}$.*

Theorem 5.7. *Semi*-irresolute image of a semi*-compact soft topological space is semi*-compact.*

Proof. *Let $f:\mathcal{SS}(\mathcal{U})_{\mathcal{E}}\to\mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ be a semi*-irresolute soft function where $(\mathcal{U}_{\mathcal{E}},\tilde{\tau})$ is a semi*-compact soft topological space and $(\mathcal{V}'_{\mathcal{E}},\delta)$ be a soft topological space. Let $\{(\mathcal{G}_{\mathsf{C}})_{\lambda:\lambda\in\Lambda}\}$ be a soft semi*-open cover for $\mathcal{V}_{\mathcal{E}'}$. As f is a semi*-irresolute function $f^{-1}(\mathcal{G}_{\mathsf{C}})_\lambda$ is a soft semi*-open set for each $\lambda\in\Lambda$. Hence $\{f^{-1}(\mathcal{G}_{\mathsf{C}})_{\lambda:\lambda\in\Lambda}\}$ forms a semi*-open cover for $\mathcal{U}_{\mathcal{E}}$. Since $(\mathcal{U}_{\mathcal{E}},\tau)$ is a semi*-compact, there exists a finite subfamily Δ of Λ such that $\{f^{-1}(\mathcal{G}_{\mathsf{C}})_{\lambda:\lambda\in\Delta}\}$ covers $(\mathcal{U}_{\mathcal{E}},\tilde{\tau})$. Hence $\{(\mathcal{G}_{\mathsf{C}})_{\lambda:\lambda\in\Delta}\}$ forms a finite subcover of $f(\mathcal{U}_{\mathcal{E}})$. Hence $f(\mathcal{U}_{\mathcal{E}})$ is soft semi*-compact.*

6. Soft semi*-connectedness

Here, we come out with semi* - connectedness in soft topological spaces put into action with semi* - open soft sets and scrutinate its basic properties.

Definition 6.1. *[5] Two soft sets $\mathcal{L}_{\mathsf{A}}$ and $\mathcal{H}_{\mathsf{B}}$ are said to be disjoint if $\mathcal{L}_{\mathsf{A}}(a)\tilde{\cap}\mathcal{H}_{\mathsf{B}}(b)=\phi$ for all $a\in A,b\in B$*

Definition 6.2. *A soft semi*-separation of soft topological $(\mathcal{U}_{\mathcal{E}},\tau)$ is a pair $\mathcal{L}_{\mathsf{A}}$, $\mathcal{H}_{\mathsf{B}}$ of disjoint non null semi*-open sets whose union is $\mathcal{U}_{\mathcal{E}}$. If there does not exists a soft semi*-separation of $\mathcal{U}_{\mathcal{E}}$, then the soft topological space is said to be soft semi*-connected otherwise soft semi*-disconnected.*

Example 6.3. *Consider the soft topological space $(\mathcal{U}_{\mathcal{E}},\tau)$, where $U=\{h_1,h_2\},E=\{e_1,e_2\}$, and $\tau=\{\phi_{\mathcal{E}},\mathcal{U}_{\mathcal{E}},(e_1,\{h_1\}),(e_2,\{h_1,h_2\}),\{(e_1,\{h_1\}),(e_2.\{h_1,h_2\})\}$.The semi*-open soft sets are $\phi_{\mathcal{E}},\mathcal{U}_{\mathcal{E}},(e_1,\{h_1\}),(e_1,\{h_1,h_2\}),\{(e_1,\{h_1\}),(e_2,\{h_1,h_2\})\}$, $(e_2,\{h_1,h_2\}),\{(e_1,\{h_2\}),(e_2,\{h_1,h_2\})\}$. Here there does not exists a Soft semi* - separation of $\mathcal{U}_{\mathcal{E}}$. Therefore, $(\mathcal{U}_{\mathcal{E}},\tilde{\tau})$ is Soft semi*-connected.*

Theorem 6.4. *If the soft sets $\mathcal{L}_{\mathsf{A}}$ and $\mathcal{G}_{\mathsf{C}}$ form a soft semi*-separation of $\mathcal{U}_{\mathcal{E}}$ and if $\mathcal{V}_{\mathsf{B}}$ is a soft semi*-connected subspace of $\mathcal{U}_{\mathcal{E}}$ then $\mathcal{V}_{\mathsf{B}}\tilde{\subseteq}\mathcal{L}_{\mathsf{A}}$ or $\mathcal{V}_{\mathsf{B}}\tilde{\subseteq}\mathcal{G}_{\mathsf{C}}$.*

Proof. *Given $\mathcal{L}_{\mathsf{A}}$ and $\mathcal{G}_{\mathsf{C}}$ form a soft semi*-separation of $\mathcal{U}_{\mathcal{E}}$ Since $\mathcal{L}_{\mathsf{A}}$ and $\mathcal{G}_{\mathsf{C}}$ are disjoint semi*-open soft sets $\mathcal{L}_{\mathsf{A}}\tilde{\cap}\mathcal{V}_{\mathsf{B}}$ and $\mathcal{G}_{\mathsf{C}}\tilde{\cap}\mathcal{V}_{\mathsf{B}}$ are also semi*-open soft sets and their soft union gives $\mathcal{V}_{\mathsf{B}}$.That is they would constitute a soft semi*-separation of $\mathcal{V}_{\mathsf{B}}$. This is a contradiction. Hence one of $\mathcal{L}_{\mathsf{A}}\tilde{\cap}\mathcal{V}_{\mathsf{B}}$ and $\mathcal{G}_{\mathsf{C}}\tilde{\cap}\mathcal{V}_{\mathsf{B}}$ is empty. Therefore $\mathcal{V}_{\mathsf{B}}$ is entirely contained in one of them.*

Theorem 6.5. *Let $\mathcal{V}_\mathsf{B}$ be a soft semi*-connected subspace of $\mathcal{U}_\mathcal{E}$ and $\mathcal{K}_\mathsf{D}$ be a soft set in $\mathcal{U}_\mathcal{E}$ such that $\mathcal{V}_\mathsf{B}\tilde{\subseteq}\mathcal{K}_\mathsf{D}\tilde{\subseteq}cl(\mathcal{V}_\mathsf{B})$ then $\mathcal{K}_\mathsf{D}$ is also soft semi*-connected.*
Proof. Let the soft set $\mathcal{K}_\mathsf{D}$ satisfies the hypothesis. If possible, let $\mathscr{F}_\mathsf{A}$ and $\mathcal{G}_\mathsf{C}$ form a soft semi-separation of $\mathcal{K}_\mathsf{D}$. Then by the theorem 5.4, $\mathcal{V}_\mathsf{B}\tilde{\subseteq}\mathscr{F}_\mathsf{A}$ or $\mathcal{V}_\mathsf{B}\tilde{\subseteq}\mathcal{G}_\mathsf{C}$. Let $\mathcal{V}_\mathsf{B}\tilde{\subseteq}\mathscr{F}_\mathsf{A}$. This implies $ss^*cl(\mathcal{V}_\mathsf{B})\tilde{\subseteq}ss^*cl(\mathscr{F}_\mathsf{A})$. Since $ss^*cl(\mathscr{F}_\mathsf{A})$ and $\mathcal{G}_\mathsf{C}$ are disjoint, $\mathcal{V}_\mathsf{B}$ cannot intersects $\mathcal{G}_\mathsf{C}$. This is a contradiction. Hence $\mathcal{K}_\mathsf{D}$ is soft semi*-connected.*

Theorem 6.6. *A soft topological space $(\mathcal{U}_\mathcal{E},\tilde{\tau})$ is soft semi*-disconnected if and only if there exists a non null proper soft subset of $\mathcal{U}_\mathcal{E}$ which is both soft semi*-open and soft semi*-closed.*
Let $\mathcal{U}_\mathcal{E}$ be soft semi-disconnected. Then there exist non null soft subsets $\mathcal{K}_\mathsf{D}$ and $\mathcal{H}_\mathsf{C}$ Such that $ss^*cl(\mathcal{K}_\mathsf{D})\tilde{\bigcap}\mathcal{H}_\mathsf{C}=\phi_\mathcal{E}$, $\mathcal{K}_\mathsf{D}\tilde{\bigcap}ss^*cl(\mathcal{H}_\mathsf{C})=\phi_\mathcal{E}$ and $\mathcal{K}_\mathsf{D}\tilde{\bigcup}\mathcal{H}_\mathsf{C}=\mathcal{U}_\mathcal{E}$ .Now $\mathcal{K}_\mathsf{D}\tilde{\subseteq}ss^*cl(\mathcal{K}_\mathsf{D})$ and $ss^*cl(\mathcal{K}_\mathsf{D})\tilde{\bigcap}\mathcal{H}_\mathsf{C}=\phi_\mathcal{E}$ .This implies $\mathcal{K}_\mathsf{D}\tilde{\bigcap}\mathcal{H}_\mathsf{C}=\phi_\mathcal{E}$, that is $\mathcal{H}_\mathsf{C}\tilde{\subseteq}(\mathcal{K}_\mathsf{D})^c$. Then $\mathcal{K}_\mathsf{D}\tilde{\bigcup}ss^*cl(\mathcal{H}_\mathsf{C})=\mathcal{U}_\mathcal{E}$ and $\mathcal{K}_\mathsf{D}\tilde{\bigcap}ss^*cl(\mathcal{H}_\mathsf{C})=\phi_\mathcal{E}$ this implies $\mathcal{K}_\mathsf{D}=(ss^*cl(\mathcal{H}_\mathsf{C}))^c$ similarly $\mathcal{H}_\mathsf{C}=(ss^*cl(\mathcal{K}_\mathsf{D}))^c$. Hence $\mathcal{K}_\mathsf{D}$ and $\mathcal{H}_\mathsf{C}$ are semi*-open soft sets being the complements of semi*-closed soft sets. Also $\mathcal{H}_\mathsf{C}\tilde{\subseteq}(\mathcal{K}_\mathsf{D})^c$. This implies $\mathcal{K}_\mathsf{D}$ and $\mathcal{H}_\mathsf{C}$ are also semi*-closed soft sets.*
Conversely, let $\mathcal{K}_\mathsf{D}$ be a non null proper soft subset of $\mathcal{U}_\mathcal{E}$ which is both semi-open and semi*-closed. Now let $\mathcal{H}_\mathsf{C}\tilde{\subseteq}(\mathcal{K}_\mathsf{D})^c$ is non null proper subset of $\mathcal{U}_\mathcal{E}$ which is also both semi*-open and semi*-closed. This implies $\mathcal{U}_\mathcal{E}$ can be expressed as the soft union of two semi*-separated soft sets $\mathcal{K}_\mathsf{D}$ and $\mathcal{H}_\mathsf{C}$. Hence $\mathcal{U}_\mathcal{E}$ is semi*-disconnected.*

Theorem 6.7. *Semi*-irresolute image of a soft semi*-connected soft topological space is soft semi*-connected.*
Let Let $f:\mathcal{SS}(\mathcal{U})_\mathcal{E}\to\mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ be a semi-irresolute soft function where $(\mathcal{U}_\mathcal{E},\tilde{\tau})$ is a semi*-connected soft topological space. Our aim is to prove is soft semi*-connected. Suppose assume that $f(\mathcal{U}_\mathcal{E})$ soft semi*-disconnected. Let $\mathcal{K}_\mathsf{D}$ and $\mathcal{H}_\mathsf{C}$ be non null disjoint semi*-open soft sets whose union is $f(\mathcal{U}_\mathcal{E})$. Since f is semi*-irresolute soft function $f^{-1}(\mathcal{K}_\mathsf{D})$ and $f^{-1}(\mathcal{H}_\mathsf{C})$ are semi*-open soft sets. Also they form a soft semi*-separation for $\mathcal{U}_\mathcal{E}$. This is a contradiction to the fact that $\mathcal{U}_\mathcal{E}$ is soft semi*-connected. Hence $f(\mathcal{U}_\mathcal{E})$ is soft semi*-connected.*

Theorem 6.8. *Semi*-continuous image of a soft semi*-connected soft topological space is soft connected.*
Let $f:\mathcal{SS}(\mathcal{U})_\mathcal{E}\to\mathcal{SS}(\mathcal{V})_{\mathcal{E}'}$ be a semi-continuous function where $(\mathcal{U}_\mathcal{E},\tilde{\tau})$ is a semi*-connected soft topological space and $(\mathcal{V}_{\mathcal{E}'},\delta)$ is a soft topological space. Our aim is to prove $f(\mathcal{U}_\mathcal{E})$ is soft connected. Suppose assume that $f(\mathcal{U}_\mathcal{E})$ is soft disconnected. Let $f(\mathcal{U}_\mathcal{E})=\mathcal{K}_\mathsf{D}\tilde{\bigcup}\mathcal{H}_\mathsf{C}$ be a soft separation that is $\mathcal{K}_\mathsf{D}$ and $\mathcal{H}_\mathsf{C}$ are disjoint soft open sets whose union is $f(\mathcal{U}_\mathcal{E})$. This implies $f^{-1}(\mathcal{K}_\mathsf{D})$ and $f^{-1}(\mathcal{H}_\mathsf{C})$ form a soft semi*-separation of $\mathcal{U}_\mathcal{E}$. This is a contradiction. Hence $f(\mathcal{U}_\mathcal{E})$ is soft connected.*

7. Conclusion

Topology and Soft sets are playing vital role in Pure and Applied Mathematics and gives more applications in real life using various Mathematical tools. Recently scientists have studied soft set theory, which is originated by a Mathematician Molodtsov and easily applied to the theory of uncertainties.In the present work, we have continued the study of soft sets and soft topological spaces.We investigate the behavior of Soft Semi*-open and Soft Semi*-closed sets, which is a step forward to further investigate the strong base of soft topological spaces. Further we planned to introduce and investigate soft semi*-separation Axioms using soft semi*-open and soft semi*-closed sets. We assure that the belongings in this paper will help researchers move into the new direction and promote the future work in soft topological spaces.

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References

[1] A. Kharal, B. Ahamad, *Mappings on soft classes,* New Math.Nat.Comput.**7**(3) (2011), 471-481.
[2] A. Robert, S.Pious Missier, *A new class of nearly open Sets*, Int. J. Math. Arch., **3**(7)(2012), 2575-2582.
[3] A. Robert, S.Pious Missier, *On semi*-closed sets ,* Asian J. Eng. Maths., **1**(4) (2012), 173-176.
[4] Ç. Gündüz Aras, S. Bayramov, *On the Tietze extension theorem in soft topological spaces,* Proc. Inst. Math. Mech., **43** (1) (2017), 105-115.
[5] D. Molodtsov, *Soft set theory-first results,* Comp. Math. Appl., **37** (1999), 19-31.
[6] E. Peyghan, B. Samadi, A. Tayebi, *On soft connectedness,* arXiv:1202.1668V1 [math.GN], 8 Feb 2012.
[7] İ. Demir, *An approach to the concept of soft vietoris topology,* Int. J. Anal. Appl., **12** (2016), 198-206.
[8] İ. Demir, O.B. Özbakir, İ. Yıldız, *A contribution to the study of soft proximity spaces,* Filomat, **31** (2017) 2023 - 2034.
[9] İ. Demir, O. B. Özbakir, *An extension of Lowen's uniformity to the fuzzy soft sets,* Konuralp J. Math., **6** (2018) 321 - 331.
[10] I. Zorlutuna, M. Akdağ, W.K. Min, S. Atmaca, *Remarks on soft topological spaces,* Ann. Fuzzy Math. Inform., **3** (2) (2012), 171-185.
[11] K. Kannan *Soft generalized closed sets in soft topological spaces,* J. Theoret. Appl. Tech., **37** (2012), 17-21.
[12] M. Shabir, M. Naz, *On Soft topological spaces,* Comp. Math. Appl., **61**(2011), 1786-1799.
[13] M.E. El-Shafei, M. Abo-Elhamayel, T.M. Al-Shami, *Two notes on "On soft Hausdorff spaces",* Ann. of Fuzzy Math. Inform., **16** (3) (2018), 333-336.
[14] N. Levine, *Semi-open sets and Semi- continuity in topological spaces,* Amer. Math. Monthly, **70**(1) (1963), 36-41.
[15] N. Levine, *Generalized closed sets in topology*, Rend. Circ. Mat. Palermo, **19** (1970),89–96.
[16] P.K.Maji, R.Biswas, R.Roy *Soft set theory*, Computer and Mathematics with Applications **45**(2003) 555-562.
[17] S.Hussain, B.Ahmed,*Some Properties of Soft Topological Spaces,* Comput. Math. Appl., **62** (2011), 4048-4067.
[18] S. Pious Missier, A. Robert *On semi*-open sets ,* International Journal of Mathematics and soft computing, **2**(2)(2012)95-105.
[19] S.M.Khalil, M.Ulrazaq, S.Abdul-Ghani, A.F. Al-Musawi, *σ-Algebra and σ-Baire in Fuzzy Soft Setting,* Advances in Fuzzy Systems , Volume 2018, Article ID 5731682, 10 pages. **7**(3),(2011),471-481.
[20] S.M.Khalil, *Decision making using algebraic operations on soft effect matrix as new category of similarity measures and study their application in medical diagnosis problems ,* Journal of Intelligent and Fuzzy Systems, **37**(2019) ,1865-1877.
[21] W. Dunham, *A New Closure Operator for Non-T1 Topologies,* Kyungpook Math. J. **22** (1982), 55–60.
[22] W.K.Min, *A Note on Soft Topological Spaces*, Comput. Math. Appl., **62**(2011),3524-3528.