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A NOTE ON DOMINATOR CHROMATIC NUMBER OF LINE GRAPH AND JUMP GRAPH OF SOME GRAPHS

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Abstract. A dominator coloring is a coloring of the vertices of a graph such that every vertex is either alone in its color class or adjacent to all vertices of at least one other color class. In this paper, we obtain the dominator chromatic number for the Line graph of some graphs, Central graph of Line graph of Star graph and Central graph of Line graph of Double Star graph. And also we obtain the dominator chromatic number for $J(S_n), J(C_n)$ and $J(K_{1,n,n,n})$ respectively.

1. Introduction and Preliminaries

All graphs considered here are finite, undirected and simple graphs. For graph theoretic terminology refer to D. B. West [10]. Let G be a graph, with vertex set $V(G)$ and edge set $E(G)$. A set $D \subseteq V(G)$ is a dominating set if every vertex of $V(G) \setminus D$ has a neighbor in D. The domination number $\gamma(G)$ is the minimum cardinality among all the dominating sets of G.

A proper coloring of a graph G is a function from the set of vertices of a graph to a set of colors such that any two adjacent vertices have different colors. A subset of vertices colored with the same color is called a color class. The chromatic number is the minimum number of colors needed in a proper coloring of a graph and is denoted by $\chi\ (G)$.

A dominator coloring of a graph G is a proper coloring of graph such that every vertex of V dominates all vertices of at least one color class (possibly its own class). i.e., it is coloring of the vertices of a graph such that every vertex is either alone in its color class or adjacent to all vertices of at least one other class and this concept was introduced by Ralucca Michelle Gera in 2006 [3] and studied further in [4]

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and [5] and recently in [2]. The dominator coloring of bipartite graph, central and middle graph of path and cycle graph were also studied in various paper [7, 8].

The central graph [9] $C(G)$ of a graph G is obtained from G by adding a new vertex on each edge of G, and then joining each pair of vertices of the original graph which were previously non-adjacent.

The line graph [6] of G, denoted by $L(G)$, is a graph whose vertices are the edges of G, and if $u, v \in E(G)$ then $uv \in E(L(G))$ if u and v share a vertex in G. The Line graph is defined as follows. The Line graph of G denoted by $L(G)$ is the intersection graph of the edges of G, representing each edge by the set of its two end vertices. Otherwise $L(G)$ is a graph such that

- Each vertex of $L(G)$ represents an edge of G .
- Two vertices of $L(G)$ are adjacent if their corresponding edges share a common end point in G.

The jump graph [1] $J(G)$ of a graph G is the graph defined on $E(G)$ and in which two vertices are adjacent if and only if they are not adjacent in G. Since both $L(G)$ and $J(G)$ are defined on the edge set of a graph G, it follows that isolated vertices of G (if G has) play no role in line graph and jump graph transformation. We assume that the graph G under consideration is nonempty and has no isolated vertices.

The line graph $L(G)$ of G has the edges of G as its vertices which are adjacent in $L(G)$ if and only if the corresponding edges are adjacent in G. We call the complement of line graph $L(G)$ as the jump graph $J(G)$ of G, found in [1].

2. Dominator Chromatic Number of Line Graph of Some Graphs

Theorem 1. *For any n, the dominator chromatic number of Line graph of double star graph is,* $\chi_d(L(K_{1,n,n})) = n + 1$.

Proof. Let $K_{1,n.n}$ be a double star graph. The vertex set of $K_{1,n.n}$ is defined as $V(K_{1,n,n}) = \{w\} \cup \{w_i : 1 \le i \le n\} \cup \{u_i : 1 \le i \le n\}$ where w is the root vertex of star graph. Consider $L(K_{1,n,n})$, by the definition of line graph, each edge of $K_{1,n,n}$ taken to be as vertex in $L(K_{1,n,n})$ namely $\{e_i : 1 \le i \le n\} \cup \{s_i : 1 \le i \le n\}$. The vertex set of line graph of double star graph is defined as

$$V(L(K_{1,n,n})) = \{e_i : 1 \le i \le n\} \cup \{s_i : 1 \le i \le n\} .$$

Here the vertices $\{e_1, e_2, \ldots, e_n\}$ induces a clique of order n in $L(K_{1,n,n})$.

Now define a coloring $c : V(L(K_{1,n,n})) \to \{1, 2, 3, \ldots n + 1\}$. The vertex set of $L(K_{1,n,n})$ is colored as follows.

$$c(V(L(K_{1,n,n}))) = \begin{cases} i \text{ for } e_i : 1 \le i \le n \\ n+1 \text{ for } s_i : 1 \le i \le n \end{cases}$$

It is easy to see that above assignment is a dominator coloring with $n + 1$ colors. By the definition of dominator coloring the vertex set $s_i : 1 \le i \le n$ dominates the color class of $e_i : 1 \le i \le n$ and e_i dominates itself. Thus $\chi_d(L(K_{1,n,n})) \le$

$n+1$. Let us assume that $\chi_d(L(K_{1,n,n}))$ is lesser than n+1, i.e., $\chi_d(L(K_{1,n,n}) = n$. Since $e_i : 1 \leq i \leq n$ forms a clique of order n so we have to assign n colors to e_i. To obtain minimum number of coloring, if we assign any preused colors from $e_i : 1 \leq i \leq n$ to the vertices of $s_i : 1 \leq i \leq n$ then it contradicts the definition of dominator coloring. Therefore dominator coloring with n colors is not possible. Hence $\chi_d(L(K_{1,n,n})) = n + 1$. □

Example 2. *The Dominator coloring of $L(K_{1,5,5})$ is shown in the following Figure 1*

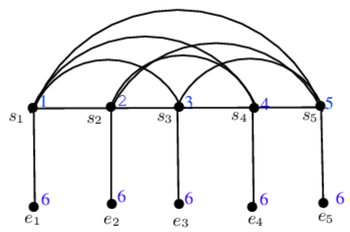

FIGURE 1. $\chi_d(L(K_{1,5,5}))$

Theorem 3. *For any n, the dominator chromatic number of Line graph of Bi-star graph is*

$$\chi_d(L(B_{n,n})) = n + 1.$$

Proof. Consider the Bistar $B_{n,n}$. By definition of Bistar, let $\{u_i : 1 \leq i \leq n\}$ be the n pendant edges attached to the vertex u and $\{v_i : 1 \leq i \leq n\}$ be the another n pendant edges attached to the vertex v. Consider the Line graph of $B_{n,n}$. By the definition of Line graph, the edge set of Bi-star corresponds to the vertex set of $L(B_{n,n})$. The vertex set is defined as

$$V(L(B_{n,n})) = \{w\} \cup \{s_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n\}.$$

In $L(B_{n,n})$ the vertices $s_i : 1 \leq i \leq n$ along with w forms a complete graph of order $n + 1$. Also we see that the vertices $e_i : 1 \leq i \leq n$ together with w forms another complete graph of order $n + 1$.

Now consider the color class $C = \{c_1, c_2, c_3, \ldots, c_n\}$. For $1 \leq i \leq n$, assign c_i colors to e_i and s_i. Thus to make the coloring as dominator chromatic one, we should assign only the same set of colours to $e_i : 1 \leq i \leq n$ which we already assigned for $s_i : 1 \leq i \leq n$. Since e_i and s_i forms a complete graph along with w so we have to assign c_{n+1} colors to w. Now all the vertices $s_i, e_i : 1 \leq i \leq n$ and w realizes its own colour, which produces a dominator coloring . Thus by the dominator coloring procedure the above said coloring is dominator chromatic. Hence $\chi_d(L(B_{n,n})) = n + 1$. □

Theorem 4. *Let $m, n \geq 5$. The dominator chromatic number of the Line graph of Bipartite graph is,*

$$\chi_d(L(K_{m,n})) = \begin{cases} m+n-1 \text{ if } m < n \\ 2m \text{ if } m = n \\ m+n-1 \text{ if } m > n. \end{cases}$$

Proof. Let $K_{m,n}$ be the Complete Bi-partite graph with bipartition (X, Y) where $X = \{v_i : 1 \leq i \leq n\}$ and $Y = \{u_i : 1 \leq i \leq n\}$. Consider the Line graph of $K_{m,n}$. Let v_{ij} be the edge between the vertex v_i and u_j for $i = 1, 2, 3, \ldots, m, j = 1, 2, 3, \ldots, n$ i.e. $v_i u_j = v_{ij} : 1 \leq i \leq m, 1 \leq j \leq n$. By the definition of the Line graph, edges in $K_{m,n}$ corresponds to the vertices in $L(K_{m,n})$ i.e.

$$V[L(K_{m,n})] = \{v_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\}.$$

Note that for each i, we say that $< v_{ij} : 1 \leq j \leq n >$ is a complete graph of order n. Also we say for each j, $< v_{ij} : 1 \leq i \leq m >$ forms a complete graph of order m. Clearly the number of cliques in $L(Km, n)$ is $m + n$.

Case 1. when $m < n$

Now consider the color class $C = \{c_1, c_2, c_3, \ldots, c_{m+n}, c_{m+n-1}\}$. Assign the color c_i to v_{i1} for $1 \leq i \leq m$. Next assign color $c_{m+n-j+1}$ to the vertices in the first row of v_{1j}, for $2 \leq j \leq n$ and assign this same colors to each row of v_{ij}, for $2 \leq i \leq m, 2 \leq j \leq n$ in anti clockwise manner.The vertices v_{i1} dominates any one color class of v_{i1} and the remaining vertices $v_{ij} : 2 \leq i \leq m, 1 \leq j \leq n$ dominates the color class $v_{i1} : 1 \leq i \leq m$. By the observation an easy check shows that $\chi_d(B_{m,n}) = m + n - 1$.

Case 2. when $m = n$

Assign the color c_i to $v_{1j}, 1 \leq j \leq n$. Next assign the color c_{2m-j+1} to v_{2j} for $1 \leq j \leq n$ and assign this same colors to each row v_{ij} for $3 \leq i \leq m, 1 \leq j \leq n$ in anti clockwise rotation. By the definition of dominator coloring, here the vertices v_{1j} dominates at least any one of the color class c_i and the remaining vertices $v_{ij} : 2 \leq i \leq m, 1 \leq j \leq n$ dominates the color class of v_{1j} for $2 \leq j \leq n$. Hence an easy observation shows that $\chi_d(B_{m,n}) = 2m$.

Case 3. when $m > n$

Assign the color c_i to v_{1j} for $1 \leq j \leq n$. Next assign color $c_{m+n-i+1}$ to v_{i1} for $2 \leq i \leq m$ and assign this same colors to each column in anti clockwise manner v_{ij} for $3 \leq i \leq m, 1 \leq j \leq n$ in anti clockwise rotation. By the definition of dominator coloring, the vertices $v_{1j} : 1 \leq j \leq n$ dominate any one color class of v_{1j} and the remaining vertices $v_{ij} : 2 \leq 1 \leq m, 1 \leq j \leq n$ dominate the color class $vi1 : 1 \leq i \leq m$. By the observation an easy check shows that $\chi_d(B_{m,n}) = m+n-1$. This completes the proof of the theorem. □

Theorem 5. *For the Line graph of $K_{1,n}, \chi_d(C(L(K_{1,n}))) = n + 1$.*

Proof. Consider the star graph of $K_{1,n}$. Let $V(K_{1,n}) = \{v\} \cup \{v_i : 1 \leq i \leq n\}$. By the definition of the Line graph, edge set in $K_{1,n}$ corresponds to the vertex set of $K_{1,n}$ i.e. $V(L(K_{1,n})) = \{e_i : 1 \leq i \leq n\}$. Here in $L(K_{1,n})$, the vertices $\{e_i : 1 \leq i \leq n\}$ forms a clique of order n.

Next consider the Central graph of Line graph of the star graph . By the definition of central graph, $C(L(K_{1,n}))$ is obtained by subdividing each edge e_i exactly only once and joining all the non-adjacent vertices of $L(K_{1,n})$ in $C(L(K_{1,n}))$. The vertex set is defined as,

$$V(C(L(K_{1,n})) = \{e_i : 1 \leq i \leq n\} \cup \left\{e_i^{'} : 1 \leq i \leq n(n-1)/2\right\}$$

Now consider the color class $C = \{c_1, c_2, c_3, \ldots, c_{n+1}\}$. For $1 \leq i \leq n$, assign the color c_i to e_i and assign the color c_{n+1} to $e_i^{'} : 1 \leq i \leq n(n-1)/2$. Here every vertex in $e_i^{'} : 1 \leq i \leq n(n-1)/2$ dominates atleast any one of the color class of $e_i : 1 \leq i \leq n$ and all the vertices of e_i dominates itself. Hence an easy check shows that $\chi_d(C(L(K_{1,n}))) = n + 1$. □

Theorem 6. *Let $n \geq 2$. The dominator chromatic number of Central graph of Line graph of $K_{1,n,n}$ is ,*

$$\chi_d(C(L(K_{1,n,n}))) = \begin{cases} 3\lfloor n/2 \rfloor + 1 & \text{if } n \text{ is even} \\ 3\lfloor n/2 \rfloor + 2 & \text{if } n \text{ is odd} \end{cases}$$

Proof. Consider the star graph of $K_{1,n,n}$. Let

$$V(K_{1,n,n}) = \{w\} \cup \{w_i : 1 \leq i \leq n\} \cup \left\{w_i^{'} : 1 \leq i \leq n\right\}.$$

By the definition of the Line graph, edge set in $K_{1,n,n}$ corresponds to the vertex set of $K_{1,n,n}$ i.e. $V(L(K_{1,n,n})) = \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\}$. Now consider the Central graph of Line graph of the star graph . By the definition of central graph $C(L(K_{1,n,n}))$ is obtained by subdividing each edge $v_i, u_i : 1 \leq i \leq n$ exactly only once and joining all the non-adjacent vertices of $L(K_{1,n,n})$ in $C(L(K_{1,n,n}))$. The vertex set is defined as

$$\begin{aligned} V(C(L(K_{1,n,n}))) &= \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n(n-1)/2\} \\ &\cup \left\{e_i^{'} : 1 \leq i \leq n\right\}. \end{aligned}$$

Case 1. when n is even.
Now we define coloring $c : V[C(L(K_{1,n,n}))] \to \{1, 2, 3, \ldots, 3\lfloor n/2 \rfloor + 1\}$. The vertex set of $C(L(K_{1,n,n}))$ is colored as follows.

$$c[V(C(L(K_{1,n,n})))] = \begin{cases} i \text{ for } u_i : 1 \leq i \leq n \\ n + i/2 \text{ for } v_i \text{ where } i = 2, 4, 6, \ldots, n \\ 2 \text{ for } e_i : 1 \leq i \leq n(n-1)/2, i \neq 2, \\ 3 \text{ for } e_2 \\ 2 \text{ for } e_i^{'} : 1 \leq i \leq n \\ 3\lfloor n/2 \rfloor + 1 \text{ for } v_i \text{ where } i = 1, 3, 5, \ldots, n-1 \end{cases}$$

Thus the above coloring assignment is a dominator coloring with $3\lfloor n/2 \rfloor + 1$ colors. By the definition of dominator coloring, every vertex in $u_i : 1 \leq i \leq n$ dominates atleast any one of the color class of $u_i : 1 \leq i \leq n$ and all the vertices of v_i, e_i and $e_i^{'}$ are dominated by the atleast anyone of the color class of $u_i : 1 \leq i \leq n$. or $e_i : 1 \leq i \leq n(n-1)/2$. Hence an easy check shows that $\chi_d(C(L(K_{1,n,n}))) = 3\lfloor n/2 \rfloor + 1$.

Case 2. when n is odd
Now we define coloring

$$c : V[C(L(K_{1,n,n}))] \to \{1, 2, 3, \ldots, 3\lfloor n/2 \rfloor + 2\}.$$

The vertex set of $C(L(K_{1,n,n}))$ is colored as follows.

$$c[V(C(L(K_{1,n,n})))] = \begin{cases} i \text{ for } u_i : 1 \leq i \leq n \\ n + i/2 \text{ for } v_i \text{ where } i = 2, 4, 6, \ldots, n-1 \\ 2 \text{ for } e_i : 1 \leq i \leq n(n-1)/2, i \neq 2, \\ 3 \text{ for } e_2 \\ 2 \text{ for } e_i^{'} : 1 \leq i \leq n \\ 3\lfloor n/2 \rfloor + 2 \text{ for } v_i \text{ where } i = 1, 3, 5, \ldots, n \end{cases}$$

It is easy to see that above coloring assignment is a dominator coloring with $3\lfloor n/2 \rfloor + 2$ colors. By the definition of dominator coloring here every vertex in $u_i : 1 \leq i \leq n$ dominates the any one color class of $u_i : 1 \leq i \leq n$ and all the vertices of v_i, e_i and $e_i^{'}$ dominates atleast any one the color class of u_i or $v_i : 1 \leq i \leq n(n-1)/2$. Hence an easy observation shows that $\chi_d(C(L(K_{1,n,n}))) = 3\lfloor n/2 \rfloor + 2$. □

3. Dominator Chromatic Number of Jump Graph of Some Graphs

Theorem 7. *Let $n \geq 4$, the dominator chromatic number of Jump graph of Sunlet graph is, $\chi_d(J(S_n)) = n$.*

Proof. Let us define the vertex set V of S_n as $V(S_n) = \{v_i : \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\}$ where v_i are the vertices of cycles taken in cyclic order and u_i are the pendent vertices. By the definition of jump graph, the vertex set are defined as $V(J(S_n)) =$

$\{e_i : 1 \leq i \leq n\} \cup \left\{e_i^{'} : 1 \leq i \leq n\right\}$. Now we define coloring

$$c : V[J(S_n)] \to \{1, 2, 3, \dots, n.\}.$$

The vertex set of $J(S_n)$ is colored as follows.

$$c[V(J(S_n))] = \begin{cases} i \text{ for } e_i^{'} : 1 \leq i \leq n \\ i+1 \text{ for } e_i : 1 \leq i \leq n-1 \\ 1 \text{ for } e_n. \end{cases}$$

It is not hard to see that above assignment is dominator coloring with n colors. Here every vertex in $e_i, e_i^{'}$ dominates atleast any of the color class of $e_i : 1 \leq i \leq n$ and $e_i^{'} : 1 \leq i \leq n$. On the other hand we can not assign $n-1$ colors to $e_i^{'}$ because $e_i^{'} : 1 \leq i \leq n$ forms a clique of order n. Hence $\chi_d(J(S_n)) = n$. □

Theorem 8. *Let $n \geq 3$, the dominator chromatic number of Jump graph of Triple star graph is, $\chi_d(J(K_{1,n,n,n})) = n+1$.*

Proof. Let us define the vertex set V of $K_{1,n,n,n}$ as

$$\begin{aligned} V(K_{1,n,n,n}) &= \{w\} \cup \{w_i : 1 \leq i \leq n\} \cup \left\{w_i^{'} : 1 \leq i \leq n\right\} \\ &\cup \left\{w_i^{''} :: 1 \leq i \leq n\right\}. \end{aligned}$$

By the definition of jump graph, the vertex set of $J(K_{1,n,n,n})$ is defined as

$$V[J(K_{1,n,n,n})] = \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n\}.$$

Now we define coloring $c : V(J(K_{1,n,n,n})) \to \{1, 2, 3, \dots, n, n+1\}$.

The vertex set of $V(J(K_{1,n,n,n}))$ is colored as follows.

$$c[V(J(K_{1,n,n,n})] = \begin{cases} i \text{ for } e_i, u_i : 1 \leq i \leq n \\ n+1 \text{ for } v_i : 1 \leq i \leq n \end{cases}$$

The above assignment gives lead to be a dominator coloring with $n+1$ colors. By the definition of dominator coloring , every vertex in $v_i : 1 \leq i \leq n$ dominates atleast any one of the color classes of $e_i, u_i : 1 \leq i \leq n$. On the other hand, if we assign n colors to u_i, v_i, e_i then the sub graph of atleast any two vertices receives the same colors. Therefore an easy check shows that dominator coloring with n color is not possible. Hence $\chi_d(J(K_{1,n,n,n})) = n+1$. □

Theorem 9. *Let $n \geq 3$, the dominator chromatic number of Jump graph of cycle graph is*

$$\chi_d(J(C_n)) = \lceil n/2 \rceil.$$

Proof. Let $V(C_n) = \{v_i : 1 \leq i \leq n\}$. By the definition, of jump graph the vertex set is defined as $V(J((C_n)) = \{u_i : 1 \leq i \leq n\}$ Now we define coloring $c : V[J(C_n)] \to \{1, 2, 3, \ldots, \lceil n/2 \rceil\}$. The vertex set of $V(J(C_n))$ is colored as follows.

$$c[V(J(C_n))] = \begin{cases} \lceil i/2 \rceil \text{ for } u_i \text{ when } i = 1, 3, 5, \ldots, 2k-1, 1 \leq k \leq \lceil n/2 \rceil \\ i/2 \text{ for } u_i \text{ when } i = 2, 4, 6, \ldots, 2k, 1 \leq k \leq \lceil n/2 \rceil \end{cases}$$

Thus the above coloring assignment leads to be a dominator coloring with n colors. By the definition of dominator coloring every vertex in $u_i : 1 \leq i \leq n$ dominates at least any one color class of u_i Thus $\chi_d(J(C_n)) \leq \lceil n/2 \rceil$.

To prove $\chi_d(J(C_n)) \geq \lceil n/2 \rceil$. Let us assume that $\chi_d(J(C_n))$ is lesser than $\lceil n/2 \rceil$ If we assign less than$\lceil n/2 \rceil$ colors to u_i then a subgraph of atleast any two of the vertices receives same color and it contradicts the definition of dominator coloring . Hence an easy observation shows that dominator coloring with less than $\lceil n/2 \rceil$ color is not possible. Hence $\chi_d(J(C_n)) = \lceil n/2 \rceil$. □

Conclusion 10. *In this paper, we obtained the dominator chromatic number for the Central graph of Line graph of Star graph and Central graph of Line graph of Double Star graph, also the dominator chromatic number for Jump graph of some graphs such as* $J(S_n), J(C_n)$ *and* $J(K_{1,n,n,n})$ *Graphs are obtained. This paper can further be extended by identifying graph families of graphs for which these chromatic numbers are equal to other kinds of chromatic numbers.*

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