PAPER DETAILS

TITLE: N-fuzzy ideals of lattices

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PAGES: 340-348

ORIGINAL PDF URL: https://dergipark.org.tr/tr/download/article-file/774384

Article electronically published on April 15, 2017

Commun.Fac.Sci.Univ.Ank.Series A1 Volume 66, Number 2, Pages 340-348 (2017) DOI: 10.1501/Commua1_0000000824 ISSN 1303-5991 http://communications.science.ankara.edu.tr/index.php?series=A1



N-FUZZY IDEALS OF LATTICES

YILDIRAY ÇELIK

ABSTRACT. In this paper, the new concepts of \mathcal{N} -fuzzy ideals and \mathcal{N} -fuzzy prime ideals of lattices have been introduced. Also, some of theirs basic properties are investigated. Hence, some results about the homomorphic \mathcal{N} -image and pre-image of \mathcal{N} -fuzzy ideals of lattices are established.

1. INTRODUCTION

The concept of fuzzy sets was firstly introduced by Zadeh [16]. Rosenfeld [12] used this concept to formulate the notion of fuzzy groups. Since then many other fuzzy algebraic concepts had been studied by several authors. Yuan and Wu [15] introduced the concepts of fuzzy sublattice and fuzzy ideals of a lattice. Biswas [1] introduced the concept of anti fuzzy subgroups of groups. Shabir and Nawaz [13] introduced the concept of anti fuzzy ideals in semigroups. Khan and Asif [6] characterized different classes of semigroups by the properties of their anti fuzzy ideals. Lekkoksung [10] introduced the concept of an anti fuzzy bi-ideal of ordered Γ -semigroups. Kim and Jun [7] studied the notion of anti fuzzy ideals of a nearring. Datta [2] introduced the concept of anti fuzzy bi-ideals in rings. Anti fuzzy ideals of Γ -rings were studied by Zhou et al. [17]. Srinivas et al. [14] introduced the concept of anti fuzzy ideals of Γ -near-ring. Dheena and Mohanraaj [3] introduced the notion of anti fuzzy right ideal, anti fuzzy right k-ideal and intuitionistic fuzzy right k-ideal in semiring. Hong and Jun [5] introduced the notion of anti fuzzy ideals of BCK algebras. In this paper, we introduce the concepts of \mathcal{N} -fuzzy ideals and \mathcal{N} -fuzzy prime ideals of lattices and investigate some related properties. Also, we give some results about the homomorphic \mathcal{N} -image and pre-image of \mathcal{N} -fuzzy ideals of lattices.

2. Preliminaries

In this section, let X denotes a bounded lattice with the least element 0 and the greatest element 1 unless otherwise specified.

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Received by the editors: May 31, 2016, Accepted: March 02, 2017.

²⁰¹⁰ Mathematics Subject Classification. Primary 06D72; Secondary 20M12.

Key words and phrases. Fuzzy ideal, \mathcal{N} -fuzzy ideal, fuzzy lattice, \mathcal{N} -fuzzy sublattice.

Definition 1. [4] A non-empty subset I of X is called an ideal of X if, for any $a, b \in I$, $a \lor b \in I$, $a \land b \in I$ and, for any $a \in I$ and $x \in X$, $x \land a = x$ implies $x \in I$. A non-empty subset D of X is called a dual ideal of X if, for any $a, b \in D$, $a \land b \in D$ and, for $a \in D$ and $x \in X$, $x \lor a = x$ implies $x \in D$. An ideal P of X is called a proper ideal if $P \neq X$. A proper ideal P of X is called a prime ideal of X if, for any $a, b \in X$, $a \land b \in P$ implies $a \in P$ or $b \in P$.

Definition 2. [11] Let X, Y be two sets and f be a mapping from X to Y. A fuzzy set μ of X (see [16]) is a map from X to [0,1]. If $\mathcal{F}(X)$ is the family of all fuzzy sets of X, then, for all $\mu, \nu \in \mathcal{F}(X), \omega \in \mathcal{F}(Y)$ and $x \in X, y \in Y$, the following operations are defined:

$$\begin{aligned} (\mu \lor \nu)(x) &= \max\{\mu(x), \nu(x)\} \\ (\mu \land \nu)(x) &= \min\{\mu(x), \nu(x)\} \\ (\mu \times \omega)(x, y) &= \min\{\mu(x), \omega(y)\} \\ f(\mu)(y) &= \begin{cases} \bigvee_{f(a)=y} \mu(a) & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise,} \end{cases} \\ f^{-1}(\omega)(x) &= \omega(f(x)) \end{aligned}$$

where $f(\mu)$ and $f^{-1}(\omega)$ are called, respectively, the image of μ under f and the pre-image of ω under f.

We denote $\mu \leq \nu$ if and only if $\mu(x) \leq \nu(x)$ for every $x \in X$. For $T \subseteq X$, $\chi_T \in \mathcal{F}(X)$ is called characteristic function of T, and defined by $\chi_T(x) = 1$ if $x \in T$ and $\chi_T(x) = 0$ otherwise for all $x \in X$.

Definition 3. [8] Let μ be a fuzzy set of X. Then the complement of μ , denoted by μ^c , is the fuzzy set of X given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$. For $t \in [0, 1]$, the set $\mu_t^- = \{x \in X | \mu(x) \le t\}$ is called a lower t-level cut of μ and $\mu_t^+ = \{x \in X | \mu(x) \ge t\}$ is called an upper t-level cut of μ .

It is clearly seen that $\mu_t^- = (\mu^c)_{1-t}^+$ for all $t \in [0,1]$.

Definition 4. [8] Let $\{\mu_i | i \in \Lambda\}$ be a family of fuzzy sets in X, then the union $(\forall \mu_i)_{i \in \Lambda}$ is defined by $(\forall \mu_i)_{i \in \Lambda}(x) = \sup\{\mu_i(x) | i \in \Lambda\}$ for each $x \in X$.

Definition 5. [4] A fuzzy set μ of X is proper if it is a non constant function.

Definition 6. [9] A fuzzy set μ of X is called a fuzzy sublattice of X if $\mu(x \wedge y) \ge \min\{\mu(x), \mu(y)\}$ and $\mu(x \vee y) \ge \min\{\mu(x), \mu(y)\}$ for all $x, y \in X$.

Definition 7. [9] A fuzzy sublattice μ of X is called a fuzzy ideal of X if $\mu(x \lor y) = \min\{\mu(x), \mu(y)\}$ for all $x, y \in X$.

Definition 8. [9] A proper fuzzy ideal μ of X is called fuzzy prime ideal of X if $\mu(x \lor y) \le \max\{\mu(x), \mu(y)\}$ for all $x, y \in X$.



FIGURE 1. .

Theorem 1. [9] A nonempty subset P of X is a prime ideal of X if and only if χ_p is a fuzzy prime ideal of X.

3. \mathcal{N} -Fuzzy Ideals of Lattices

Definition 9. A fuzzy set μ of X is called an \mathcal{N} -fuzzy sublattice of X if $\mu(x \wedge y) \leq \max\{\mu(x), \mu(y)\}$ and $\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$ for all $x, y \in X$.

Example 1. Consider the lattice $X = \{0, \alpha, \beta, 1\}$ given by as follows Let define a fuzzy set μ of X by $\mu(0) = 0.1$, $\mu(\alpha) = 0.2$, $\mu(\beta) = 0.3$ and $\mu(1) = 0.3$. Then μ is an N-fuzzy sublattice of X.

Theorem 2. A fuzzy set μ of X is an \mathcal{N} -fuzzy sublattice of X if and only if μ^c is a fuzzy sublattice of X.

Proof. Let μ be an \mathcal{N} -fuzzy sublattice of X. Then for $x, y \in X$,

$$\mu^{c}(x \wedge y) = 1 - \mu(x \wedge y)$$

$$\geq 1 - max\{\mu(x), \mu(y)\}$$

$$= min\{1 - \mu(x), 1 - \mu(y)\}$$

$$= min\{\mu^{c}(x), \mu^{c}(y)\}$$

$$\mu^{c}(x \vee y) = 1 - \mu(x \vee y)$$

$$\geq 1 - max\{\mu(x), \mu(y)\}$$

$$= min\{1 - \mu(x), 1 - \mu(y)\}$$

$$= min\{\mu^{c}(x), \mu^{c}(y)\}$$

Hence μ^c is a fuzzy sublattice of X. Similarly, the converse can be proved.

Definition 10. Let μ be an \mathcal{N} -fuzzy sublattice of X. Then μ is an \mathcal{N} -fuzzy ideal of X if $\mu(x \lor y) = max\{\mu(x), \mu(y)\}$ for all $x, y \in X$.

Example 2. Let consider the lattice $X = \{0, \alpha, \beta, 1\}$ given by in the Figure 1. Let define a fuzzy set μ of X by $\mu(0) = 0.1$, $\mu(\alpha) = 0.2$, $\mu(\beta) = 0.3$ and $\mu(1) = 0.3$. Then μ is an \mathcal{N} -fuzzy ideal of X.

Every \mathcal{N} -fuzzy sublattice of X need not be \mathcal{N} -fuzzy ideal of X.

Example 3. Let consider the lattice $X = \{0, \alpha, \beta, 1\}$ given by in the Figure 1. Let define a fuzzy set μ of X by $\mu(0) = 0$, $\mu(\alpha) = 0.2$, $\mu(\beta) = 0.3$ and $\mu(1) = 0.2$. Then μ is an \mathcal{N} -fuzzy sublattice of X, but μ is not an \mathcal{N} -fuzzy ideal of X as $\mu(\alpha \lor \beta) = \mu(1) \neq \max\{\mu(\alpha), \mu(\beta)\}$.

Remark 1. Let μ be an \mathcal{N} -fuzzy ideal of X. As $\mu(x) = \mu(x \vee 0) = max\{\mu(x), \mu(0)\}$, then we get $\mu(0) \leq \mu(x)$ for any $x \in X$.

Theorem 3. A fuzzy set μ of X is an \mathcal{N} -fuzzy ideal of X if and only if μ^c is a fuzzy ideal of X.

Proof. Let μ be an \mathcal{N} -fuzzy ideal of X. By Theorem 2, μ^c is a fuzzy sublattice of X. For $x, y \in X$,

$$\mu^{c}(x \lor y) = 1 - \mu(x \lor y)$$

= $1 - max\{\mu(x), \mu(y)\}$
= $min\{1 - \mu(x), 1 - \mu(y)\}$
= $min\{\mu^{c}(x), \mu^{c}(y)\}$

Thus, μ^c is a fuzzy ideal of X. Similarly, the converse can be proved.

Theorem 4. A fuzzy set μ of X is an N-fuzzy ideal of X if and only if μ_t^- is an ideal of X for each $t \in [\mu(0), 1]$.

Proof. Let μ be an \mathcal{N} -fuzzy ideal of X and $t \in [\mu(0), 1]$. Then by Theorem 3, μ^c is a fuzzy ideal of X. Hence $\mu_t^- = (\mu^c)_{1-t}^+$ is an ideal of X.

Conversely, μ , μ_t^- is an ideal of X for each $t \in [\mu(0), 1]$ and $s \in [0, 1 - \mu(0)] = [0, \mu^c(0)]$. Then $1 - s \in [\mu(0), 1]$ and $(\mu^c)_s^+ = \mu_{1-s}^-$ is an ideal of X. Hence $(\mu^c)_s^+$ is an ideal of X for all $s \in [0, \mu^c(0)]$, and μ^c is a fuzzy ideal of X. This shows that μ is an \mathcal{N} -fuzzy ideal of X.

Theorem 5. If I is an ideal of X, then for each $t \in [0, 1]$, there exists an \mathcal{N} -fuzzy ideal μ of X such that $\mu_t^- = I$.

Proof. Let I be an ideal of X and $t \in [0, 1]$. Let define a fuzzy set μ of X by

$$\mu(x) = \begin{cases} t, & \text{if } x \in I \\ 1, & \text{if } x \notin I \end{cases}$$

for each $x \in X$. Then $\mu_s^- = I$ for any $s \in [t, 1) = [\mu(0), 1)$, and $\mu_1^- = X$. Thus μ_s^- is an ideal of X for all $s \in [\mu(0), 1]$. Hence, from Theorem 4, μ is \mathcal{N} -fuzzy ideal of X and $\mu_t^- = I$.

Let μ be a fuzzy set of X and let define $X_{\mu} = \{x \in X | \mu(x) = \mu(0)\}$. We then get the following theorem.

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Theorem 6. If μ is an \mathcal{N} -fuzzy ideal of X, then X_{μ} is an ideal of X.

Proof. Let μ be an \mathcal{N} -fuzzy ideal of X and $x, y \in X_{\mu}$. Then $\mu(x) = \mu(0)$ and $\mu(y) = \mu(0)$. So $\mu(x \lor y) = max\{\mu(x), \mu(y)\} = \mu(0)$. Hence $x \lor y \in X_{\mu}$. Now let $x \le a, x \in X$ and $a \in X_{\mu}$. Then $x \lor a = a$ and $\mu(a) = \mu(0)$. As μ is an \mathcal{N} -fuzzy ideal of X, $\mu(x \lor a) = max\{\mu(x)\mu(a)\}$. Thus $\mu(a) = max\{\mu(x), \mu(a)\}$. Therefore $\mu(x) \le \mu(a) = \mu(0)$. Also by Remark 1, $\mu(0) \le \mu(x)$. So we get $\mu(x) = \mu(0)$. Hence $x \in X_{\mu}$. This shows that X_{μ} is an ideal of X.

Theorem 7. If $\{\mu_i | i \in \Lambda\}$ a family of \mathcal{N} -fuzzy ideals of X, then so is $(\forall \mu_i)_{i \in \Lambda}$.

Proof. Let $\{\mu_i, | i \in \Lambda\}$ be a family of \mathcal{N} -fuzzy ideals of X. Let $x, y \in X$.

$$\begin{aligned} (\vee\mu_i)_{i\in\Lambda}(x\wedge y) &= \sup\{\mu_i(x\wedge y)|i\in\Lambda\}\\ &\leq \sup\{\max\{\mu_i(x),\mu_i(y)\}\}\\ &= \max\{\sup\{\mu_i(x)\},\sup\{\mu_i(y)\}\}\\ &= \max\{\{\psi_i)_{i\in\Lambda}(x),\forall\mu_i\}_{i\in\Lambda}(y)\}\\ (\vee\mu_i)_{i\in\Lambda}(x\vee y) &= \sup\{\mu_i(x\vee y)|i\in\Lambda\}\\ &\leq \sup\{\max\{\mu_i(x),\mu_i(y)\}\}\\ &= \max\{\sup\{\mu_i(x)\},\sup\{\mu_i(y)\}\}\\ &= \max\{\forall\mu_i)_{i\in\Lambda}(x),\forall\mu_i\}_{i\in\Lambda}(y)\}\end{aligned}$$

Hence $(\forall \mu_i)_{i \in \Lambda}$ is an \mathcal{N} -fuzzy sublattice of X. Also,

$$\begin{aligned} (\lor \mu_i)_{i \in \Lambda}(x \lor y) &= \sup\{\mu_i(x \lor y) | i \in \Lambda\} \\ &= \sup\{\max\{\max\{\mu_i(x), \mu_i(y)\}\} \\ &= \max\{\sup\{\mu_i(x)\}, \sup\{\mu_i(y)\}\} \\ &= \max\{(\lor \mu_i)_{i \in \Lambda}(x), (\lor \mu_i)_{i \in \Lambda}(y)\} \end{aligned}$$

Hence $(\lor \mu_i)_{i \in \Lambda}$ is an \mathcal{N} -fuzzy ideal of X.

Theorem 8. Let $f : X \to Y$ be a lattice homomorphism where Y is a bounded lattice. Let μ be an \mathcal{N} -fuzzy ideal of Y. Then $f^{-1}(\mu)$ is an \mathcal{N} -fuzzy ideal of X.

Proof. Let $x, y \in X$. Then

$$\begin{aligned} f^{-1}(\mu)(x \wedge y) &= & \mu(f(x \wedge y)) \\ &= & \mu(f(x) \wedge f(y)) \\ &\leq & max\{\mu(f(x)), \mu(f(y))\} \\ &= & max\{f^{-1}(\mu)(x), f^{-1}(\mu)(y)\} \end{aligned}$$

Thus $f^{-1}(\mu)(x \wedge y) \le \max\{f^{-1}(\mu)(x), f^{-1}(\mu)(y)\}.$

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Similarly we can prove $f^{-1}(\mu)(x \vee y) \leq \max\{f^{-1}(\mu)(x), f^{-1}(\mu)(y)\}$. Hence $f^{-1}(\mu)$ is an \mathcal{N} -fuzzy sublattice of X. Also for $x, y \in X$

$$\begin{aligned} f^{-1}(\mu)(x \lor y) &= & \mu(f(x \lor y)) \\ &= & \mu(f(x) \lor f(y)) \\ &= & max\{\mu(f(x)), \mu(f(y))\} \\ &= & max\{f^{-1}(\mu)(x), f^{-1}(\mu)(y)\} \end{aligned}$$

Thus $f^{-1}(\mu)(x \vee y) = max\{f^{-1}(\mu)(x), f^{-1}(\mu)(y)\}$. This shows that $f^{-1}(\mu)$ is an \mathcal{N} -fuzzy ideal of X.

Definition 11. Let $f : X \to Y$ be a mapping where Y is a non-empty set. Let μ be a fuzzy set of X. Then \mathcal{N} -image of μ under f is a fuzzy set $f(\mu)$ of Y defined by $f(\mu)(y) = \inf\{\mu(x)|x \in X \text{ and } f(x) = y\}$ for all $y \in Y$.

Theorem 9. Let $f : X \to Y$ be an onto lattice homomorphism where Y is a bounded lattice. Let μ be \mathcal{N} -fuzzy ideal of X. Then $f(\mu)$ is an \mathcal{N} -fuzzy ideal of Y.

Proof. Let μ be an \mathcal{N} -fuzzy ideal of X and $a, b \in Y$. As f is onto, there exist $p, q \in X$ such that f(p) = a and f(q) = b. Also $a \wedge b = f(p) \wedge f(q) = f(p \wedge q)$ and

$$\begin{array}{lll} f(\mu)(a \wedge b) &=& \inf\{\mu(z)|z \in X \ and \ f(z) = a \wedge b\} \\ &\leq& \inf\{\mu(p \wedge q)|f(p) = a \ and \ f(q) = b\} \\ &\leq& \inf\{\max\{\mu(p), \mu(q)\}|f(p) = a \ and \ f(q) = b\} \\ &=& \max\{\inf\{\mu(p)/f(p) = a\}, \inf\{\mu(q)|f(q) = b\}\} \\ &=& \max\{f(\mu)(a), f(\mu)(b)\} \end{array}$$

Thus $f(\mu)(a \wedge b) \leq max\{f(\mu)(a), f(\mu)(b)\}$. Similarly we can prove that $f(\mu)(a \vee b) \leq max\{f(\mu)(a), f(\mu)(b)\}$. Hence $f(\mu)$ is an \mathcal{N} -fuzzy sublattice of Y.

Again let $x, y \in Y$. As f is onto, there exist $r, s \in X$ such that f(r) = x and f(s) = y. Also $x \lor y = f(r) \lor f(s) = f(r \lor s)$ and

$$\begin{split} f(\mu)(x \lor y) &= \inf\{\mu(z)|z \in X \text{ and } f(z) = x \lor y\} \\ &= \inf\{\mu(r \lor s)|f(r) = x \text{ and } f(s) = y\} \\ &= \inf\{\max\{\mu(r), \mu(s)\}|f(r) = x \text{ and } f(y) = s\} \\ &= \max\{\inf\{\mu(r)|f(r) = x\}, \inf\{\mu(s)|f(s) = y\}\} \\ &= \max\{f(\mu)(x), f(\mu)(y)\} \end{split}$$

Thus $f(\mu)(x \vee y) = max\{f(\mu)(x), f(\mu)(y)\}$. This shows that $f(\mu)$ is an \mathcal{N} -fuzzy ideal of Y.

Theorem 10. Every \mathcal{N} -fuzzy ideal of X is order preserving.

Proof. Let μ be an \mathcal{N} -fuzzy ideal of X. Let $x, y \in X$ such that $x \leq y$. Then $\mu(y) = \mu(x \lor y) = max\{\mu(x), \mu(y)\}$. Thus $\mu(x) \leq \mu(y)$. \Box

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Definition 12. Let λ and μ be fuzzy sets of X. The \mathcal{N} Cartesian product $\lambda \times \mu$: $X \times X \rightarrow [0,1]$ is defined by $\lambda \times \mu(x,y) = max\{\lambda(x),\mu(y)\}$ for all $x, y \in X$.

Theorem 11. If λ and μ are N-fuzzy ideals of X, then $\lambda \times \mu$ is N-fuzzy ideal of $X \times X$.

Proof. Let (x_1, y_1) and $(x_2, y_2) \in X \times X$. Then

$$\begin{split} \lambda \times \mu((x_1, y_1) \wedge (x_2, y_2)) &= \lambda \times \mu(x_1 \wedge x_2, y_1 \wedge y_2) \} \\ &= \max\{\lambda(x_1 \wedge x_2), \mu(y_1, y_2)\} \\ &\leq \max\{\max\{\lambda(x_1), \lambda(x_2)\}, \{\max\{\mu(y_1), \mu(y_2)\}\} \} \\ &= \max\{\max\{\lambda(x_1), \mu(y_1)\}, \{\max\{\lambda(x_2), \mu(y_2)\} \} \\ &= \max\{\lambda \times \mu(x_1, y_1), \lambda \times \mu(x_2, y_2)\} \\ &\lambda \times \mu((x_1, y_1) \vee (x_2, y_2)) &= \lambda \times \mu(x_1 \vee x_2, y_1 \vee y_2) \} \\ &= \max\{\lambda(x_1 \vee x_2), \mu(y_1 \vee y_2) \} \\ &= \max\{\lambda(x_1), \lambda(x_2)\}, \max\{\mu(y_1), \mu(y_2)\} \} \\ &= \max\{\max\{\lambda(x_1), \lambda(x_2)\}, \max\{\lambda(x_2), \mu(y_2)\} \} \\ &= \max\{\lambda \times \mu(x_1, y_1), \lambda \times \mu(x_2, y_2)\} \end{split}$$

Thus $\lambda \times \mu$ is an \mathcal{N} -fuzzy sublattice of $X \times X$. Also,

$$\begin{split} \lambda \times \mu((x_1, y_1) \lor (x_2, y_2)) &= \lambda \times \mu(x_1 \lor x_2, y_1 \lor y_2) \\ &= \max\{\lambda(x_1 \lor x_2), \, \mu(y_1 \lor y_2) \\ &= \max\{\max\{\lambda(x_1), \lambda(x_2)\}, \max\{\mu(y_1), \mu(y_2)\}\} \\ &= \max\{\max\{\lambda(x_1), \mu(y_1)\}, \max\{\lambda(x_2), \mu(y_2)\}\} \\ &= \max\{\lambda \times \mu(x_1, y_1), \, \lambda \times \mu(x_2, y_2)\} \end{split}$$

Hence $\lambda \times \mu$ is an \mathcal{N} -fuzzy ideal of $X \times X$.

Definition 13. Let μ be an \mathcal{N} -fuzzy ideal of X. Then μ is called an \mathcal{N} -fuzzy prime ideal of X if $\mu(x \wedge y) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in X$.

Example 4. Consider the lattice $X = \{0, \alpha, \beta, 1\}$ given by in the Figure 1. Let define a fuzzy set μ of X by $\mu(0) = 0.2$, $\mu(\alpha) = 0.2$, $\mu(\beta) = 0.3$ and $\mu(1) = 0.3$. Then μ is an N-fuzzy prime ideal of X.

Every \mathcal{N} -fuzzy ideal of X need not be \mathcal{N} -fuzzy prime ideal of X.

Example 5. Consider the lattice $X = \{0, \alpha, \beta, 1\}$ given by in the Figure 1. Let define a fuzzy set μ of X by $\mu(0) = 0.1$, $\mu(\alpha) = 0.2$, $\mu(\beta) = 0.3$ and $\mu(1) = 0.3$. Then μ is an N-fuzzy ideal of X, but μ is not an N-fuzzy prime ideal of X as $\mu(\alpha \land \beta) = \mu(0) \not\geq \min\{\mu(\alpha), \mu(\beta)\}.$

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Theorem 12. Let P be a non-empty subset of X and $r, t \in [0, 1]$ such that r < t. Let μ_p be a fuzzy subset of X such that

$$\mu_p(x) = \begin{cases} r, & \text{if } x \in P \\ t, & \text{if } x \notin P \end{cases}$$

for all $x \in X$. Then P is a prime ideal of X if and only if μ_p is an \mathcal{N} -fuzzy prime ideal of X.

Proof. Let P be a prime ideal of X and $x, y \in X$. If $x \wedge y \in P$, then $\mu_p = (x \wedge y) = r \leq \max\{\mu_p(x), \mu_p(y)\}$. If $x \wedge y \notin P$, then $x \notin P$ and $y \notin P$. Then $\mu_p(x \wedge y) = t$, $\mu_p(x) = t$ and $\mu_p(y) = t$. Hence $\mu_p(x \wedge y) \leq \max\{\mu_p(x), \mu_p(y)\}$. Therefore we have $\mu_p(x \wedge y) \leq \max\{\mu_p(x), \mu_p(y)\}$. Similarly we can prove that $\mu_p(x \vee y) \leq \max\{\mu_p(x), \mu_p(y)\}$. This shows that μ_p is an \mathcal{N} -fuzzy sublattice of X.

Now let $x, y \in X$. If $x \vee y \in P$, then $x \in P$ and $y \in P$. Therefore $\mu_p(x \wedge y) = r$, $\mu_p(x) = r$ and $\mu_p(y) = r$. Hence $\mu_p(x \vee y) = max\{\mu_p(x), \mu_p(y)\}$. If $x \vee y \notin P$, then $x \notin P$ or $y \notin P$. Therefore $\mu_p(x \wedge y) = t$, $\mu_p(x) = t$ or $\mu_p(y) = t$. Hence $\mu_p(x \vee y) = max\{\mu_p(x), \mu_p(y)\}$. This shows that μ_p is an \mathcal{N} -fuzzy ideal of X.

Again let $x, y \in X$. If $x \wedge y \in P$, then $x \in P$ or $y \in P$ (since P is a prime ideal of X). Therefore $\mu_p(x \wedge y) = r$, $\mu_p(x) = r$ or $\mu_p(y) = r$. Hence $\mu_p(x \vee y) \ge \min\{\mu_p(x), \mu_p(y)\}$. If $x \wedge y \notin P$, then $x \notin P$ and $y \notin P$ (since P is an ideal of X). Therefore $\mu_p(x \wedge y) = t$, $\mu_p(x) = t$ and $\mu_p(y) = t$. Hence $\mu_p(x \wedge y) \ge \min\{\mu_p(x), \mu_p(y)\}$. This shows that μ_p is \mathcal{N} -fuzzy prime ideal of X.

Conversely, let μ_p be \mathcal{N} -fuzzy prime ideal of X and $x, y \in P$. As μ_p is \mathcal{N} -fuzzy sublattice of X, $\mu_p(x \wedge y) \leq max\{\mu_p(x), \mu_p(y)\} = r$. Hence $x \wedge y \in P$. Similarly we can prove that $x \vee y \in P$. This shows that P is a sublattice of X. Now let $a \in P, x \in X$ such that $x \wedge a = x$. Thus $x \vee a = (x \wedge a) \vee a = a$. Therefore $r = \mu_p(a) = \mu_p(x \vee a) = max\{\mu_p(x), \mu_p(a)\}$. Hence $\mu_p(x) = r$ and so $x \in P$. This shows that P is an ideal of X.

Again let $x \wedge y \in P$. Then $\mu_p(x \wedge y) = r$. As μ_p is \mathcal{N} -fuzzy prime ideal of $X, \ \mu_p(x \wedge y) \geq \min\{\mu_p(x), \mu_p(y)\}$. Therefore $\mu_p(x) = r$ or $\mu_p(y) = r$. Hence $x \in P$ or $y \in P$. This shows that P is a prime ideal of X. \Box

4. Conclusion

In this paper we defined the notions of \mathcal{N} -fuzzy sublattice, \mathcal{N} -fuzzy ideal and \mathcal{N} -fuzzy prime ideal of a bounded lattice. We showed that the complement of \mathcal{N} -fuzzy sublattice of a bounded lattice is a fuzzy sublattice. We also showed that the union a family of \mathcal{N} -fuzzy ideals of a bounded lattice is also \mathcal{N} -fuzzy ideal of a bounded lattice. We stated how the homomorphic \mathcal{N} -images and inverse images of \mathcal{N} -fuzzy ideals of a bounded lattice become \mathcal{N} -fuzzy ideal of a bounded lattice. We also investigated how the \mathcal{N} -Cartesian product of \mathcal{N} -fuzzy ideals of a bounded lattice becomes \mathcal{N} -fuzzy ideal of a bounded lattice. Our future work will focus on studying the intuitionistic \mathcal{N} -fuzzy ideals of a bounded lattice.

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