

PAPER DETAILS

TITLE: On Cesaro Sums Of Divergent Series

AUTHORS: Ö ÇAKAR

PAGES: 2-12

ORIGINAL PDF URL: <https://dergipark.org.tr/tr/download/article-file/1629486>

COMMUNICATIONS

DE LA FACULTÉ DES SCIENCES
DE L'UNIVERSITÉ D'ANKARA

Série A: Mathématiques, Physique et Astronomie

TOME 23 A

ANNÉE 1974

On Cesaro Sums of Divergent Series

by

Ö. ÇAKAR

2

Faculté des Sciences de l'Université d'Ankara
Ankara, Turquie

Communications de la Faculté des Sciences de l'Université d'Ankara

Comité de Rédaction de la Série A

C. Uluçay, E. Erdik, N. Doğan

Secrétaire de publication

N. Gündüz

La Revue "Communications de la Faculté des Sciences de l'Université d'Ankara" est un organe de publication englobant toutes les disciplines scientifiques représentées à la Faculté: Mathématiques pures et appliquées, Astronomie, Physique et Chimie théorique, expérimentale et technique, Géologie, Botanique et Zoologie.

La Revue, à l'exception des tomes I, II, III, comprend trois séries

Série A : Mathématiques, Physique et Astronomie.

Série B : Chimie.

Série C : Sciences naturelles.

En principe, la Revue est réservée aux mémoires originaux des membres de la Faculté. Elle accepte cependant, dans la mesure de la place disponible, les communications des auteurs étrangers. Les langues allemande, anglaise et française sont admises indifféremment. Les articles devront être accompagnés d'un bref sommaire en langue turque.

Adresse: Fen Fakültesi Tabliğler Dergisi, Fen Fakültesi, Ankara, Turquie.

On Cesaro Sums of Divergent Series

Ö. ÇAKAR

*Dept. of Mathematics, Faculty of Science
(Ankara University, Ankara)*

SUMMARY

Let $\sum_{k=1}^{\infty} a_k$ be an infinite series of real, non-negative numbers and let

$(\varepsilon) = \{\varepsilon_k\}, (k=1,2,\dots, \varepsilon_k = \mp 1)$
be any sequence of signs.

For a given sequence (ε) , we denote the n -th partial sum of the series $\sum \varepsilon_k$ by

$$s_n(\varepsilon) = \sum_{k=1}^n \varepsilon_k a_k$$

and the n -th partial C_1 -sum of the series by

$$\sigma_n(\varepsilon) = \frac{1}{n} \sum_{v=1}^n s_v(\varepsilon).$$

If $\sigma_n(\varepsilon)$ converges then we call

$$\sigma(\varepsilon) = \lim_{n \rightarrow \infty} \sigma_n(\varepsilon)$$

a C_1 -attainable point of $\sum a_k$ and denote the set of all C_1 -attainable points of $\sum a_k$ by $SC(a_k)$.

In this paper we are going to investigate the C_1 -attainable set $SC(a_k)$ of a divergent series $\sum a_k$ and give some theorems on that $SC(a_k) = R$ and $SC(a_k) = \emptyset$, where R is the set of real numbers and $\emptyset$ is the empty set.

1. Introduction

It is known that, if a numerical series is conditionally convergent, then it is possible to sum this series to any value by rearranging its terms, [4], [5].

A similar problem has been investigated for divergent series and some interesting results have been obtained by Bagemihl-Erdős, [3]. Also, Erdős-Hanani got some results for the C_1 -attainable set of a divergent series $\sum a_k$, [1].

In this note we are going to deal with the same type of problems.

2. Notations.

Let $\sum_{k=1}^{\infty} a_k$ be an infinite series of real, non-negative numbers

and let

$$(2.1) \quad (\varepsilon) = \{\varepsilon_k\}, \quad (k=1,2,\dots, \varepsilon_k = \mp 1)$$

be any sequence of signs.

For a given sequence (ε) , we denote the n -th partial sum of the series $\sum \varepsilon_k a_k$ by

$$s_n(\varepsilon) = \sum_{k=1}^n \varepsilon_k a_k$$

and the n -th partial C_1 -sum of the series by

$$\sigma_n(\varepsilon) = \frac{1}{n} \sum_{v=1}^n s_v(\varepsilon).$$

If $\sigma_n(\varepsilon)$ converges then we call

$$\sigma(\varepsilon) = \lim_{n \rightarrow \infty} \sigma_n(\varepsilon)$$

a C_1 -attainable point of $\sum a_k$ and denote the set of all C_1 -attainable points of $\sum a_k$ by $SC(a_k)$.

R will denote the set of real numbers and $\emptyset$ will denote the empty set.

3. Theorems For $SC(a_k) = R$.

Let us start giving a theorem which is an immediate consequence of Theorem 1 of Erdős - Hanani [1] and Theorem 3 of Yurtsever, [2].

Theorem 3.1. Let $\sum a_k$ be a series of nonnegative terms having a subseries $\sum a_{n_i}$ such that

$$\sum a_{n_i} = \infty, a_{n_i} \rightarrow 0.$$

If (a_k) is monotone and bounded then $SC(a_k) = R$. (2)

Theorem 3.2. Let $\sum a_k = \infty$ be a series of non-negative terms having a subseries $\sum a_{n_i}$ such that

$$\sum a_{n_i} = \infty, a_{n_i} \rightarrow 0.$$

If, for a definite sequence (ε) ,

$$a) \lim_{k \rightarrow \infty} \frac{1}{k+1} \sum_{v=1}^k s_v \varepsilon_{v+1} \text{ exists,}$$

and

(2) During my stay in University of Lancaster in 1969-71, Prof. I. J. Maddox suggested me that Theorem 3.1. can be improved to the following

Theorem 3.1'. Let $\sum a_k$ be a series of non-negative terms having a subseries $\sum a_{n_i}$ such that

$$\sum a_{n_i} = \infty, a_{n_i} \rightarrow 0.$$

If $\sum |\Delta a_k| = \sum |a_k - a_{k+1}| < \infty$, then $SC(a_k) = R$.

Proof. Take $\varepsilon_k = (-1)^k$. Then $\sum \varepsilon_k a_k$ is convergent (and so (C,1) summable), for

$$\sum_{k=0}^n \varepsilon_k a_k = a_n \left(\sum_{k=0}^n \varepsilon_k \right) + \sum_{k=0}^{n-1} \left(\sum_{\mu=0}^k \varepsilon_{\mu} \right) \Delta a_k.$$

Now $\sum |\Delta a_k| < \infty$ implies that a_n tends to a limit, 1 say, as $n \rightarrow \infty$. But $a_{n_i} \rightarrow 0$ implies that $1 = 0$, i.e., $a_n \rightarrow 0$.

Hence

$$\begin{aligned} \sum_{k=0}^n \varepsilon_k a_k &= o(1) \sum_{k=0}^n \varepsilon_k + \sum_{k=0}^{n-1} \left(\sum_{\mu=0}^k \varepsilon_{\mu} \right) \Delta a_k \\ &= o(1) \cdot 0 + \sum_{k=0}^{n-1} 0 \cdot (1) \Delta a_k. \end{aligned}$$

So the result is immediate.

b) the series $\sum s_v \Delta \varepsilon_v$ is C_1 - summable, where

$$s_v = \sum_{\mu=0}^v a_\mu, \text{ and } \Delta \varepsilon_v = \varepsilon_v - \varepsilon_{v+1}, \text{ then } SC(a_k) = R.$$

P r o o f. Take the series $\sum a_k = \infty$ and apply the sequence (ε) . According to the Abel partial summation formula, we have

$$(3.1.) \quad \sum_{k=0}^n \varepsilon_k a_k = \sum_{k=0}^n s_k \Delta \varepsilon_k + s_n \varepsilon_{n+1},$$

$$\text{where } s_n = \sum_{\mu=0}^n a_\mu, s_{-1} = 0 \text{ and } \Delta \varepsilon_k = \varepsilon_k - \varepsilon_{k+1}.$$

If we put

$$S_j = \sum_{k=0}^j \varepsilon_k a_k = \sum_{k=0}^j s_k \Delta \varepsilon_k + s_j \varepsilon_{j+1}, \quad (j=0,1,2,\dots),$$

we easily get

$$(3.2) \quad \lim_{j \rightarrow \infty} \frac{S_0 + S_1 + \dots + S_j}{j+1} \\ = \lim_{j \rightarrow \infty} \frac{1}{j+1} \left(\sum_{k=0}^j s_k \varepsilon_{k+1} + \right. \\ \left. s \Delta \varepsilon_0 + \sum_{k=0}^1 s_k \Delta \varepsilon_k + \dots + \sum_{k=0}^j s_k \Delta \varepsilon_k \right) \\ \lim_{j \rightarrow \infty} \frac{\quad}{j+1}$$

Since the left-hand side of (3.2) is the C_1 - sum of the series $\sum \varepsilon_k a_k$, by Theorem 1 of Erdős-Hanani, [1], the result is straight forward.

T h e o r e m 3.3. Let $\sum a_k$ be a series of non-negative terms.

If $\sum a_k = \infty$ and monotonously $a_k \rightarrow 0$, then $SC(a_k) = R$.

P r o o f. Let us write the equality (3.1) in the form of

$$(3.3) \quad \sum_{k=0}^n \varepsilon_k a_k = \sum_{k=0}^n s_k \Delta a_k + s_n a_{n+1},$$

where $s_n = \sum_{\mu=0}^n \varepsilon_\mu$, $s_{-1} = 0$, and $\Delta a_k = a_k - a_{k+1}$.

Put

$$S_j = \sum_{k=0}^j \varepsilon_k a_k.$$

Now, if

$$a') \lim_{j \rightarrow \infty} \frac{1}{j+1} \sum_{k=0}^j s_k a_{k+1} \text{ exists,}$$

and

b') the series $\sum s_k \Delta a_k$ is C_1 -summable,

then

$$(3.4) \lim_{j \rightarrow \infty} \frac{S_0 + S_1 + \dots + S_j}{j+1}$$

exists.

So, we must show, under the given hypothesis, that conditions a') and b') are satisfied.

Choose $\{\varepsilon_k\} = (-1)^k$, ($k = 0, 1, 2, \dots$). Then the partial sums s_k 's are bounded, and since $a_k \rightarrow 0$ monotonously, the series $\sum s_k \Delta a_k$ is convergent. (One can easily see that it is absolutely convergent, in fact.) So, condition b') is satisfied. Namely $SC(a_k) \neq \emptyset$. Condition a') is also satisfied because of the Cauchy's Theorem. The limit exists and equal to zero, (Arithmetic Means), [4], [5].

Therefore, according to Theorem 1 of Erdős-Hanani, [1], $SC(a_k) = R$.

4. A Problem of Erdős - Hanani

In this section, we are going to consider a problem due to Erdős-Hanani, (Problem 1, [1]), and show that the best possible result is $C = 1$.

Theorem 4.1. Let Σa_k be a series of nonnegative terms satisfying $\Sigma a_k = \infty$. If there exists an η_0 with the property that to each η in $0 < \eta \leq \eta_0$ there corresponds an

$$(4.1) \quad n_0 = n_0(\eta)$$

such that for every $n > n_0$,

$$(4.2) \quad \sum_{i=1}^{[\eta n / a_n]} a_{n+i} > a_n + \eta,$$

then $SC(a_k) = R$.

Proof. Let σ be any real number. Then, we are going to construct a sequence (2.1) such that

$$\lim_{n \rightarrow \infty} \sigma_n(\varepsilon) = \sigma.$$

According to (4.1), for every $\eta = 2^{-i}$, ($i = i_0, i_0 + 1, \dots$) there exists a number

$$(4.3) \quad n_i = n_i(2^{-i})$$

such that for every $n > n_i$, (4.2) is satisfied, with $\eta = 2^{-i}$.

Now, choose ε_j arbitrarily for $j = 1, 2, \dots, n_{i_0}-1$. Then, let us put $n_i = j$ and suppose that

$$\sigma_{j-1}(\varepsilon) = \frac{s_1(\varepsilon) + \dots + s_{j-1}(\varepsilon)}{j-1} \leq \sigma.$$

If $s_{j-1}(\varepsilon) \leq \sigma + 2^{-i}$ we take $\varepsilon_j = +1$ to make $\sigma_j(\varepsilon)$ bigger than σ . But, if $s_{j-1}(\varepsilon) > \sigma + 2^{-i}$, then we choose ε_j so as to make $s_j(\varepsilon)$ as small as possible but not less than $\sigma + 2^{-i}$. Continuing this way, suppose that the final partial sum we reached is $s_{k_1}(\varepsilon)$ and let

$$\sigma_{k_1}(\varepsilon) = \frac{s_1(\varepsilon) + s_2(\varepsilon) + \dots + s_{k_1}(\varepsilon)}{k_1}.$$

Now the means must start decreasing and be $\leq \sigma$. Therefore the partial sums must decrease. Then if

$s_{k_1}(\varepsilon) \geq \sigma - 2^{-i}$, we put $\varepsilon_{k_1+1} = -1$;

but, if $s_{k_1}(\varepsilon) < \sigma - 2^{-i}$, we choose ε_{k_1+1} so as to make the left hand side as large as possible but not greater than $\sigma - 2^{-i}$.

Accordingly, we get

$$\sigma_{j_2}(\varepsilon) = \frac{s_1(\varepsilon) + \dots + s_{k_1}(\varepsilon) + \dots + s_{j_2}(\varepsilon)}{j_2} \leq \sigma.$$

Then, it follows that the sequence $(\sigma_v(\varepsilon))$ attains alternately minimas $\sigma_{j_h}(\varepsilon)$, $(h = 1, 2, \dots)$ and maximas $\sigma_{k_h}(\varepsilon)$, $(h = 1, 2, \dots)$, with $j_1 < k_1 < j_2 < k_2 < \dots$ such that

$$\sigma_{j_h}(\varepsilon) \leq \sigma \text{ and } \sigma_{k_h}(\varepsilon) > \sigma, (h = 1, 2, \dots).$$

Therefore the sequence $(\sigma_v(\varepsilon))$ is monotonically increasing for $j_h \leq v \leq k_h$ and monotonically decreasing for $k_h \leq v \leq j_{h+1}$.

To prove the theorem, it is enough to show that the difference between σ and maxima $\sigma_{k_h}(\varepsilon)$ (or, σ and minima $\sigma_{j_h}(\varepsilon)$) tends to zero as $n \rightarrow \infty$. So we must show the existence of a number j_0 such that for every $k_h > j_0$

$$(4.4) \quad 0 < \sigma_{k_h}(\varepsilon) - \sigma < \eta$$

holds.

Let i be an integer such that

$$(4.5) \quad 2^{-i} < \eta/6$$

and let n_i be the corresponding index fixed by (4.1) :

$$n_i = n_i(2^{-i}).$$

Further, let h be an integer such that $k_{h-1} > n_i$ and m the greatest index providing $j_h < m \leq k_h$ such that $\varepsilon_m = 1$.

According to our construction, we write

$$(4.6) \quad \sigma_{m-1}(\varepsilon) \leq \sigma.$$

And if

$$(4.7) \quad s_{m-1}(\varepsilon) \leq \sigma + 2^{-i}$$

then, for $m \leq j \leq k_h$, we get

$$(4.8) \quad \sigma < s_j(\varepsilon) < \sigma + 2^{-i} + 2 a_m.$$

Also, in the case

$$(4.9) \quad s_{m-1}(\varepsilon) > \sigma + 2^{-i}$$

the relation (4.8) is still valid.

Now, if $s_{m-1}(\varepsilon) > \sigma + 2^{-i}$, then we are going to suppose that

$$(4.10) \quad s_{m-1}(\varepsilon) - (\sigma + 2^{-i}) < 2^{-i}.$$

So, under this assumption, we can put the following

L e m m a .

$$(4.11) \quad \sum_{j=m+1}^{k_h} a_j < 2^{-i} + a_m.$$

P r o o f .

1°) Let $s_{m-1}(\varepsilon) \leq \sigma + 2^{-i}$. Since

$$\sigma < s_m(\varepsilon) \leq \sigma + 2^{-i} + a_m,$$

by (4.8), we can write

$$\sigma < s_m(\varepsilon) - \sum_{j=m+1}^{k_h} a_j \leq \sigma + 2^{-i} + a_m.$$

Therefore, we get

$$\sigma + \sum_{j=m+1}^{k_h} a_j < s_m(\varepsilon) \leq \sigma + 2^{-i} + a_m$$

$$\text{and} \quad \sum_{j=m+1}^{k_h} a_j < 2^{-i} + a_m.$$

2°) Let $s_{m-1}(\varepsilon) > \sigma + 2^{-i}$. Then

$$s_{m-1}(\varepsilon) = \sigma + 2^{-i} + \alpha,$$

where $0 < \alpha < 2^{-i}$. Therefore, we get

$$s_{m-1}(\varepsilon) + a_m > \sigma + 2^{-i}$$

$$s_{m-1}(\varepsilon) + a_m - \sum_{j=m+1}^{k_h} a_j > \sigma + 2^{-i}$$

$$\sigma + 2^{-i} + \alpha + a_m - \sum_{j=m+1}^{k_h} a_j > \sigma + 2^{-i}$$

$$\sum_{j=m+1}^{k_h} a_j < \alpha + a_m$$

or

$$\sum_{j=m+1}^{k_h} a_j < 2^{-i} + a_m .$$

This completes the proof of the Lemma.

Now, by the definition of $\sigma_{k_h}(\varepsilon)$, we have

$$\sigma_{k_h}(\varepsilon) = \frac{1}{k_h} \left[(m-1) \sigma_{m-1} + \sum_{j=m}^{k_h} s_j(\varepsilon) \right]$$

and by (4.6) and (4.8)

$$(4.12) \quad \sigma_k(\varepsilon) < \sigma + \frac{1}{k_h} (2^{-i} + 2 a_m) (k_h - m + 1) .$$

If $a_m \leq 2^{-i}$, then we easily get

$$\sigma_{k_h}(\varepsilon) - \sigma < \eta/2 .$$

If $a_m > 2^{-i}$, then obviously $m > n_i$. So. (4.1) and (4.11) give

$$k_h - m < 2^{-i} \cdot \frac{m}{a_m}$$

and, we also have

$$1 < 2^{-i} \cdot \frac{m}{a_m} .$$

Therefore, from (4.12), we get

$$\sigma_{k_h}(\varepsilon) - \sigma < \frac{1}{k_h} (2^{-i} + 2a_m) (k_h - m + 1)$$

$$\sigma_{k_h}(\varepsilon) - \sigma < \frac{1}{k_h} \cdot 3 a_m \cdot 2 \cdot 2^{-i} \cdot \frac{m}{a_m}$$

which implies, by (4.5), that

$$\sigma_{k_h}(\varepsilon) - \sigma < \eta .$$

In a similar way, we can show that the difference between σ and minima $\sigma_{j_h}(\varepsilon)$ tends to zero as $h \rightarrow \infty$.

5. A Theorem For $SC(a_k) = \emptyset$

In this chapter we are going to prove a theorem which gives a sufficient condition for $SC(a_k) = \emptyset$. This theorem will be based on Cauchy's general convergence principle. It is known that, if the sequence

$$\sigma_n(\varepsilon) = \frac{s_1(\varepsilon) + s_2(\varepsilon) + \dots + s_n(\varepsilon)}{n}$$

where $s_n(\varepsilon) = \sum_{v=1}^n \varepsilon_v a_v$, is divergent, then the series can

not be C_1 — summable. So, what we need is having that, for at least one $k \geq 1$ and for each n

$$| \sigma_{n+k}(\varepsilon) - \sigma_n(\varepsilon) | > \eta$$

where $\eta > 0$.

Take $1 \leq k \leq n$, and write

$$\begin{aligned} | \sigma_{n+k}(\varepsilon) - \sigma_n(\varepsilon) | &= \\ &= \frac{n[s_{n+1}(\varepsilon) + \dots + s_{n+k}(\varepsilon)] - k[s_1(\varepsilon) + \dots + s_n(\varepsilon)]}{n(n+k)} \end{aligned}$$

Using

$$s_n(\varepsilon) = \sum_{v=1}^n \varepsilon_v a_v$$

we get

$$\begin{aligned} | \sigma_{n+k}(\varepsilon) - \sigma_n(\varepsilon) | &= \\ &= \left| \frac{k}{n(n+k)} \sum_{v=2}^{n+1} (v-1) \varepsilon_v a_v + \frac{1}{n+k} \sum_{v=1}^{k-1} (k-v) \varepsilon_{n+1+v} a_{n+1+v} \right| \\ &= \left| \frac{k}{n(n+k)} \sum_{v=0}^{n+1} (n-v) \varepsilon_{n+1-v} a_{n+1-v} \right| \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{n+k} \sum_{v=1}^{k-1} (k-v) \varepsilon_{n+1+v} a_{n+1+v} \quad | \\
& = \frac{k}{n+k} \left[\varepsilon_{n+1} a_{n+1} + \sum_{v=1}^{n-1} \frac{n-v}{n} \varepsilon_{n+1-v} a_{n+1-v} \right. \\
& \quad \left. + \sum_{v=1}^{k-1} \frac{k-v}{k} \varepsilon_{n+1+v} a_{n+1+v} \right] \\
& = \frac{k}{n+k} \left[\varepsilon_{n+1} a_{n+1} + \sum_{v=1}^{k-1} \frac{n-v}{n} \varepsilon_{n+1-v} a_{n+1-v} \right. \\
& \quad + \sum_{v=k}^{n-1} \frac{n-v}{n} \varepsilon_{n+1-v} a_{n+1-v} \\
& \quad \left. + \sum_{v=k}^{k-1} \frac{k-v}{k} \varepsilon_{n+1+v} a_{n+1+v} \right] \\
& \geq \frac{k}{n+k} \left[a_{n+1} - \sum_{v=1}^{k-1} \frac{n-v}{n} a_{n+1-v} - \sum_{v=k}^{n-1} \frac{n-v}{n} a_{n+1-v} \right. \\
& \quad \left. - \sum_{v=1}^{k-1} \frac{k-v}{k} a_{n+1+v} \right] \\
& \geq \frac{k}{n+k} \left[a_{n+1} - \sum_{|v|=1}^{k-1} a_{n+1+v} - \sum_{v=k}^{n-1} a_{n+1-v} \right] \\
& > \eta .
\end{aligned}$$

So, we can express the following

Theorem 5.1. Let $\sum a_k$ be a series of nonnegative terms. If there exists a number k ($1 \leq k \leq n$) and an $\eta > 0$ such that for every n satisfying

$$a_{n+1} - \sum_{|v|=1}^{k-1} a_{n+1+v} > (1 + \binom{n}{k}) \eta + \sum_{v=k}^{n-1} a_{n+1-v}$$

REFERENCES

- [1] Erdős, P. - Hanani, H., On C_1 - summability of series, Michigan Math. J., 9, (1963), 1-14,
 [2] Yurtsever, B., Über die C-Summierbarkeit der unendlichen Reihen, Communication, V, (1953), 1-11,
 [3] Bagemihl, F. - Erdős, P., Rearrangements of C_1 -summable series, Acta Math., 92, (1954), 35-53,
 [4] Knopp, K., Theory and Applications of Infinite Series, Blackie & Son Limited, 1964,
 [5] Hardy, G. H., Divergent Series, Oxford, 1949.

ÖZET

$\sum_{k=1}^{\infty} a_k$ reel ve negatif olmayan terimli bir nümerik sonsuz seri ve

$$(\varepsilon) = \{\varepsilon_k\}, (k = 1, 2, \dots, \{\varepsilon_k\} = \pm 1)$$

herhangi bir işaret dizisi olsun.

Verilen bir (ε) dizisi için $\sum \varepsilon_k a_k$ serisinin n 'inci kısmi toplamını

$$s_n(\varepsilon) = \sum_{k=1}^n \varepsilon_k a_k$$

ve n 'inci kısmi C_1 -toplamını

$$\sigma_n(\varepsilon) = \frac{1}{n} \sum_{v=1}^n s_v(\varepsilon)$$

ile belirttik ve $\sigma_n(\varepsilon)$ 'un yakınsak olması halinde

$$\sigma(\varepsilon) = \lim_{n \rightarrow \infty} \sigma_n(\varepsilon)$$

'a $\sum a_k$ serisinin bir C_1 -erişilir noktası adını verdik. $\sum a_k$ nın bütün C_1 -erişilebilir noktalar cümlesini $SC(a_k)$ ile gösterdik.

Bu araştırmamızda ise ıraksak bir $\sum a_k$ serisinin bütün C_1 -erişilir noktaları cümlesi olan $SC(a_k)$ cümlesini ele alıp $SC(a_k) = R$ ve $SC(a_k) = \emptyset$ olması hakkında bazı teoremler verdik, burada R reel sayılar cümlesini ve $\emptyset$ ise boş cümleyi ifade etmektedir.

Prix de l'abonnement annuel

Turquie : 15 TL; Étranger: 30 TL.

Prix de ce numéro : 5 TL (pour la vente en Turquie).

Prière de s'adresser pour l'abonnement à : Fen Fakültesi

Dekanlığı Ankara, Turquie.