PAPER DETAILS

Commun. Fac. Sci. Univ. Ank. Series A1
V. 50 pp. 75-82 (2001)

MULTIDIMENSIONAL INVERSE BOUNDARY PROBLEM FOR THE PARABOLIC TYPE EQUATION IN UNBOUNDED DOMAIN

M.A. GULIYEV

Department of Mathematics, Baku State University, Baku, Azerbaijan.

(Received May 28, 2001; Accepted Dec. 21, 2001)

ABSTRACT

Solvability of the proposed problem is proved by reducing of the multidimensional inverse problem to some problem for the non-linear differential equation.

1. INTRODUCTION

In the paper the problem of solvability of the inverse boundary problems is considered for the parabolic type equation in the unbounded domains (regarding an inverse problem see [4], [5], [6]).

The method is proposed basing on the reducing of the inverse problems to some non-linear infinity system of differential equations. Note that this method allows to prove the existence, stability and uniqueness theorems for the solutions of multidimensional inverse problems on the class of finite smooth functions.

Some results on similar problems for parabolic equations have been obtained in [1], [2], [3].

Consider the problem

$$\frac{\partial u(x,t)}{\partial t} - Au(x,t) = a(x)u(x,t) + h(x)f(x,t), \quad (x,t) \in Q, \tag{1}$$

$$u(x',t) = \varphi(x',t), (x',t) \in \Gamma = (-\infty,\infty) \times S, \tag{2}$$

$$u(x,0) = \psi_1(x), u(x,T) = \psi_2(x), \quad x \in \overline{D}, \tag{3}$$

in the domain $Q = (-\infty,\infty) \times D$, where D-bounded domain in $R^n, S = \partial D \in C^2$, $n \le 3$, $Au = \sum\limits_{i,j=1}^{n} (a_{ij}(x)u_{x_i})_{x_j}, a_{ij}(x) = a_{ij}(x) \in C^4(\overline{D})$, $\sum\limits_{i,j=1}^{n} a_{ij}\xi_i\xi_j \ge \mu_0|\xi|^2$; $\varphi(x,t)$, $f(x,t), \psi_1(x), \psi_2(x)$ are given and $a(x), h(x), u(x,t)$ are seeking functions.

Definition : The triplet of the functions $a(x), h(x), u(x,t)$ we call the solution of problem (1)-(3) if they satisty the following conditions :

1. $a(x), h(x) \in W_2^2(D)$
2. $E(\hat{u}) \equiv \int\limits_{-\infty}^{\infty}\int\limits_{D}(1+|\lambda|)^{2(p-1)}\left[|\hat{u}|^2(1+|\lambda|)^2+|A\hat{u}|^2\right]d\lambda dx<\infty,\ p>\frac{3}{2},$

$$u(x,t)=\int\limits_{-\infty}^{\infty}-\hat{u}(x,\lambda)\exp(-i\lambda t)d\lambda.$$

Conditions (1)-(3) are satisfied in ordinary sense.

Suppose that the functions $\varphi(x,t), f(x,t), \psi_1(x), \psi_2(x)$ satisty the following conditions :

1) $\psi_1(x),\psi_2(x)\in W_2^4(D),\ \psi_1(x)|_s=\varphi(x,t)|_{t=0},\ \psi_2(x)|_s=\varphi(x,t)|_{t=T},$

2) $f(x,t),\ \dfrac{\partial^p f}{\partial t^p}\in L^2(Q),\ \ f(x,0)\in W_2^2(D),$

3) $\varphi(x,t),\ \dfrac{\partial^{p+1}\varphi(x,t)}{\partial t^{p+1}}\in L^2\left(-\infty,\infty;W_2^{7/2}(S)\right),$

4) $|\Delta|=|\psi_1(x)f(x,T)-\psi_2(x)f(x,0)|\ge\delta>0\quad\forall x\in\overline{D}.$

Lemma 1. Let $\hat{u}(\lambda,x)$ be a solution of the problem

$$-i\lambda\hat{u}(x,\lambda)-A\hat{u}(x,\lambda)=\frac{\hat{u}(x,\lambda)}{\Delta}\left\{f(x,T)\left(-\int\limits_{-\infty}^{\infty}i\lambda\hat{u}(x,\lambda)d\lambda-A\psi_1(x)\right)\right.$$
$$\left.-f(x,0)\left(-\int\limits_{-\infty}^{\infty}i\lambda\hat{u}(x,\lambda)\exp(-i\lambda T)d\lambda-A\psi_2(x)\right)\right\}$$
$$+\frac{\hat{f}(x,\lambda)}{\Delta}\times\left\{\psi_1(x)\left(-\int\limits_{-\infty}^{\infty}i\lambda\hat{u}(x,\lambda)\exp(-i\lambda T)d\lambda-A\psi_2(x)\right)\right.$$
$$\left.-\psi_2(x)\left(-\int\limits_{-\infty}^{\infty}i\lambda\hat{u}(x,\lambda)d\lambda-A\psi_1(x)\right)\right\}\tag{5}$$
$$\hat{u}(x,\lambda)|_s=\hat{\varphi}(x,\lambda)\tag{6}$$

from the class $E(u)=\infty$.

Then the functions

$$u(x,t)=-\int\limits_{-\infty}^{\infty}\hat{u}(x,\lambda)\exp(-i\lambda t)d\lambda,$$

$$a(x)=\frac{1}{\Delta}\left\{f(x,T)\left(-\int_{-\infty}^{\infty} i\lambda\hat{u}(x,\lambda)d\lambda - A\psi_1(x)\right)\right.$$

$$\left.-f(x,0)\left(-\int_{-\infty}^{\infty} i\lambda\tilde{u}(x,\lambda)\exp(-i\lambda T)d\lambda - A\psi_2(x)\right)\right\},$$

$$h(x)=\frac{1}{\Delta}\left\{\psi_1(x)\left(-\int_{-\infty}^{\infty} i\lambda\hat{u}(x,\lambda)\exp(i\lambda T)d\lambda - A\psi_2(x)\right)\right.$$

$$\left.-\psi_2(x)\left(-\int_{-\infty}^{\infty} i\lambda\hat{u}(x,\lambda)d\lambda - A\psi_1(x)\right)\right\}$$

are the solution of inverse problem (1)-(3). The proof of Lemma 1 is obtained from the fact that

$$u(x,0)=\psi_1(x),\quad u(x,T)=\psi_2(x).$$

Now we prove the solvability of problem (5)-(6). Define by (x,λ) the functions

$$A_\psi=0,\ \psi\big|_S=\hat{\varphi},\ \|\psi\|^2_{W_2^4(D)}\le C_1^2\|\hat{\varphi}\|^2_{W_2^{7/2}(D)}.$$

To solve above mentioned problem we use the method of consecutive approximations :

$$-i\lambda v^{(m)}(x,\lambda)-Av^{(m)}(x,\lambda)=\frac{v^{(m-1)}(x,\lambda)+\psi(x,\lambda)}{\Delta}\left[f(x,T).\int_{-\infty}^{\infty} i\lambda v^{(m-1)}(x,\lambda)d\lambda\right.$$

$$-f(x,0)\int_{-\infty}^{\infty} i\lambda v^{(m-1)}(x,\lambda)\exp(-i\lambda T)d\lambda+f(x,T)\int_{-\infty}^{\infty} i\lambda\hat{\psi}(x,\lambda)d\lambda$$

$$\left.-f(x,0)\int_{-\infty}^{\infty} i\lambda\psi(x,\lambda)\exp(-i\lambda T)d\lambda+f(x,T)A\psi_1(x)-f(x,0)A\psi_2(x)\right]$$

$$+\frac{\hat{f}(x,\lambda)}{\Delta}\left[\psi_1(x)\int_{-\infty}^{\infty} i\lambda v^{(m-1)}(x,\lambda)\exp(-i\lambda T)d\lambda-\psi_2(x)\int_{-\infty}^{\infty} i\lambda v^{(m-1)}(x,\lambda)d\lambda\right.$$

$$+\psi_1(x)\int_{-\infty}^{\infty} i\lambda\psi(x,\lambda)\exp(-i\lambda T)d\lambda$$

$$\left.-\psi_2(x)\int_{-\infty}^{\infty} i\lambda\psi(x,\lambda)d\lambda+\psi_1(x)A\psi_2(x)-\psi_2(x)A\psi_1(x)\right]-i\lambda\psi(x,\lambda) \tag{7}$$

$$v_k^{(m)}(x)\big|_S=0,\quad m=0,1,2,.... \tag{8}$$

At first establish the apriory estimations for the solution of problem (7)-(8)

Lemma 2 : Suppose condition (4) is satisfied and

$$K(\mu_0,z_0)\equiv\frac{N_0^2 z_0}{\mu_0}\left[\frac{2C_1^2}{2p-3}\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\|\hat{\varphi}\|^2_{W_2^{7/2}(S)}d\lambda\right.$$

$$\left.+\frac{C_1^2}{2p-3}\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\left\|\hat{f}(x,\lambda)\right\|^2_{C(D)}d\lambda+\|A\psi_1\|^2_{C(D)}+\|A\psi_2\|^2_{C(D)}\right]$$

$$+\frac{2N_0^2}{2p-3}\cdot\frac{z_0^{\frac{6-n}{2}}C_2^4}{\mu_0}\cdot\left\{\frac{z_0}{\mu_0}C_1^2\int_{-\infty}^{\infty}(1+|\lambda|)^{2p+2}\|\hat{\varphi}\|^2_{W_2^{7/2}(S)}d\lambda\right.$$

$$+\frac{N_0^2 z_0 C_1^2}{\mu_0}\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\|\hat{\varphi}\|^2_{W_2^{7/2}(S)}\left(\|A\psi_1\|^2_{C(D)}+\|A\psi_2\|^2_{C(D)}\right)d\lambda$$

$$+\frac{N_0^2}{2p-3}\frac{z_0}{\mu_0}C_1^4\left(\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\|\hat{\varphi}\|^2_{W_2^{7/2}(S)}d\lambda\right)^2$$

$$+\frac{N_0^2C_1^2}{2p-3}\frac{z_0}{\mu_0}\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\|\hat{\varphi}\|^2_{W_2^{7/2}(S)}d\lambda.\int_{-\infty}^{\infty}\int_D(1+|\lambda|)^{2p}\left|\hat{f}(x,\lambda)\right|^2dxd\lambda$$

$$\left.+\frac{N_0^2 z_0}{\mu_0}.\int_{-\infty}^{\infty}\int_D(1+|\lambda|)^{2p}\left|\hat{f}(x,\lambda)\right|^2\left(\|A\psi_1\|^2_{C(D)}+\|A\psi_2\|^2_{C(D)}\right)dxd\lambda\right\}\le\frac{\mu_0}{4}.$$

Then the inequality

$$\int_{-\infty}^{\infty}\int_D(1+|\lambda|)^{2p}\left|\nabla v^{(m)}\right|^2d\lambda dx\le\frac{4}{\mu_0^2}z_0\left\{C_1^2\int_{-\infty}^{\infty}(1+|\lambda|)^{2p+2}\|\hat{\varphi}\|^2_{W_2^{7/2}(S)}d\lambda\right.$$

$$+\frac{N_0^2 z_0 C_1^2}{\mu_0}\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\|\hat{\varphi}\|^2_{W_2^{7/2}(S)}\left(\|A\psi_1\|^2_{C(D)}+\|A\psi_2\|^2_{C(D)}\right)d\lambda$$

$$+\frac{N_0^2}{2p-3}C_1^4\left(\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\|\hat{\varphi}\|^2_{W_2^{7/2}(S)}d\lambda\right)^2+\frac{N_0^2C_1^2}{2p-3}\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\|\hat{\varphi}\|^2_{W_2^{7/2}(S)}d\lambda$$

$$\times\int_{-\infty}^{\infty}\int_D(1+|\lambda|)^{2p}\left|\hat{f}(x,\lambda)\right|^2dxd\lambda+N_0^2\left(\|A\psi_1\|^2_{C(D)}+\|A\psi_2\|^2_{C(D)}\right)$$

$$\left.\times\int_{-\infty}^{\infty}\int_D(1+|\lambda|)^{2p}\left|\hat{f}(x,\lambda)\right|^2dxd\lambda\right\}\equiv\frac{4}{\mu_0}\gamma_0,$$

where

$$C_1:\|y\|^2_{C(D)}\le C_1^2\|y\|^2_{W_2^2(D)};\qquad C_2:\|y\|_{L^4}\le C_2\|y\|^2_{W_2^1(D)}\cdot\|y\|_{L_2(D)},$$

is true for the consecutive approximations given by (7), (8), where $0\ne\frac{1}{z_0}$ – the minimal eigenvalue of the problem

$$\Delta u = -\frac{1}{z^2}u; \; u\big|_S = 0.$$

Proof. One may obtain from (7), (8)

$$\mu_0 \int_{-\infty}^{\infty}\int_D (1+|\lambda|)^{2p} \left|\nabla v^{(m)}(x,\lambda)\right|^2 dx d\lambda \le \frac{9\varepsilon}{2} \int_{-\infty}^{\infty}\int_D (1+|\lambda|)^{2p} \left|v^{(m)}(x,\lambda)\right|^2 dx d\lambda$$

$$+ \frac{9}{\varepsilon} \int_{-\infty}^{\infty}\int_D (1+|\lambda|)^{2p} (1+|\lambda|)^2 |\psi(x,\lambda)|^2 dx d\lambda$$

$$+ \frac{9}{2\varepsilon} \frac{N_0^2 C_2^4}{2p-3} \left(\int_{-\infty}^{\infty}\int_D (1+|\lambda|)^{2p} \left|\nabla v^{(m-1)}(x,\lambda)\right|^2 dx d\lambda \right)^{n/2}$$

$$\times \left(\int_{-\infty}^{\infty}\int_D (1+|\lambda|)^{2p} |v(x,\lambda)|^2 dx d\lambda \right)^{\frac{4-n}{2}} + \frac{9N_0^2 C_1^2}{2\varepsilon(2p-3)} \int_{-\infty}^{\infty} (1+|\lambda|)^{2p} \|\hat{\varphi}\|^2_{W_2^{7/2}(S)} d\lambda$$

$$\times \int_{-\infty}^{\infty}\int_D (1+|\lambda|)^{2p} \left|v^{(m-1)}(x,\lambda)\right|^2 dx d\lambda + \frac{9N_0^2}{\varepsilon} \int_{-\infty}^{\infty}\int_D (1+|\lambda|)^{2p} |\psi(x,\lambda)|^2$$

$$\times \left[\|A\psi_1\|^2_{C(D)} + \|A\psi_2\|^2_{C(D)} + \frac{2C_1^2}{2p-3} \int_{-\infty}^{\infty} (1+|\lambda|)^{2p} \|\hat{\varphi}\|^2_{W_2^{7/2}(S)} d\lambda \right] dx d\lambda$$

$$+ \frac{9N_0^2}{\varepsilon} \frac{C_1^2}{2p-3} \int_{-\infty}^{\infty} (1+|\lambda|)^{2p} \left|\hat{f}(x,\lambda)\right|^2 d\lambda \int_{-\infty}^{\infty}\int_D (1+|\lambda|)^{2p} \left|v^{(m-1)}(x,\lambda)\right|^2 dx d\lambda$$

$$+ \frac{9N_0^2}{\varepsilon} \frac{C_1^2}{2p-3} \int_{-\infty}^{\infty} (1+|\lambda|)^{2p} \|\hat{\varphi}\|^2_{W_2^{7/2}(S)} d\lambda \int_{-\infty}^{\infty}\int_D (1+|\lambda|)^{2p} \left|\hat{f}(x,\lambda)\right|^2 dx d\lambda$$

$$+ \frac{9N_0^2}{\varepsilon} \int_{-\infty}^{\infty}\int_D (1+|\lambda|)^{2p} \left|\hat{f}(x,\lambda)\right|^2 \left(\|A\psi_1\|^2_{C(D)} + \|A\psi_2\|^2_{C(D)} \right) dx d\lambda$$

$$+ \frac{N_0^2 z_0}{\mu_0} \int_{-\infty}^{\infty}\int_D (1+|\lambda|)^{2p} \left|v^{(m-1)}(x,\lambda)\right|^2 \left(\|A\psi_1\|^2_{C(D)} + \|A\psi_2\|^2_{C(D)} \right) dx d\lambda$$

$$+ \frac{N_0^2 z_0}{\mu_0} \frac{C_1^2}{2p-3} \int_{-\infty}^{\infty} (1+|\lambda|)^{2p} \|\hat{\varphi}\|^2_{W_2^{7/2}(S)} d\lambda \int_{-\infty}^{\infty}\int_D (1+|\lambda|)^{2p} \left|v^{(m-1)}(x,\lambda)\right|^2 dx d\lambda,$$

where

$$N_0 = \max\left\{ \left\| \frac{f(x,T)}{\Delta} \right\|_{C(D)}, \left\| \frac{f(x,0)}{\Delta} \right\|_{C(D)}, \left\| \frac{\psi_1(x)}{\Delta} \right\|_{C(D)}, \left\| \frac{\psi_2(x)}{\Delta} \right\|_{C(D)} \right\}.$$

Taking $\varepsilon = \dfrac{\mu_0}{9z_0}$ in (8) we get

$$\frac{\mu_0}{2}\int_{-\infty}^{\infty}\int_D (1+|\lambda|)^{2p}\left|\nabla v^{(m)}(x,\lambda)\right|^2 dx d\lambda \le \frac{z_0}{\mu_0}C_1^2\int_{-\infty}^{\infty}(1+|\lambda|)^{2p+2}\|\hat\varphi\|^2_{W_2^{7/2}(S)}d\lambda$$

$$+\frac{N_0^2 z_0 C_1^2}{\mu_0}\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\|\hat\varphi\|^2_{W_2^{7/2}(S)}\left(\|A\psi_1\|^2_{C(D)}+\|A\psi_2\|^2_{C(D)}\right)d\lambda$$

$$+\frac{N_0^2}{2p-3}\frac{z_0}{\mu_0}C_1^4\left(\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\|\hat\varphi\|^2_{W_2^{7/2}(S)}d\lambda\right)^2$$

$$+\frac{N_0^2C_1^2}{2p-3}\frac{z_0}{\mu_0}\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\|\hat\varphi\|^2_{W_2^{7/2}(S)}d\lambda.\int_{-\infty}^{\infty}\int_D(1+|\lambda|)^{2p}\left|\hat f(x,\lambda)\right|^2dxd\lambda$$

$$+\frac{N_0^2 z_0}{\mu_0}\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\left|\hat f(x,\lambda)\right|^2\left(\|A\psi_1\|^2_{C(D)}+\|A\psi_2\|^2_{C(D)}\right)dxd\lambda$$

$$+\left[\frac{N_0^2C_1^2}{2p-3}\frac{2z_0}{\mu_0}\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\|\hat\varphi\|^2_{W_2^{7/2}(S)}d\lambda+\frac{N_0^2z_0}{2p-3}\frac{C_1^2}{\mu_0}\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\left|\hat f(x,\lambda)\right|^2_{C(D)}d\lambda\right.$$

$$\left.+\frac{N_0^2z_0}{\mu_0}\left(\|A\psi_1\|^2_{C(D)}+\|A\psi_2\|^2_{C(D)}\right)\right]\cdot\int_{-\infty}^{\infty}\int_D(1+|\lambda|)^{2p}\left|v^{(m-1)}(x,\lambda)\right|^2dxd\lambda$$

$$+\frac{N_0^2}{2p-3}\cdot\frac{z_0}{\mu_0}.z_2^{\frac{4-n}{2}}.C_2^4\left(\int_{-\infty}^{\infty}\int_D(1+|\lambda|)^{2p}\left|\nabla v^{(m-1)}(x,\lambda)\right|^2dxd\lambda\right).$$

From this follows the statement of Lemma 2

Lemma 3. Let the condition of Lemma 1 be satisfied. Then the following estimations are true for the consecutive approximations given by (7), (8)

$$\int_{-\infty}^{\infty}\int_D(1+|\lambda|)^{2p-2}\left|Av^{(m-1)}(x,\lambda)\right|^2dxd\lambda\le C_3(\gamma_0)$$

$$\int_{-\infty}^{\infty}\int_D(1+|\lambda|)^{2p-2}\left[(1+|\lambda|)^{2p}.\left|v^{(m)}(x,\lambda)-v^{(m-1)}(x,\lambda)\right|^2\right.$$

$$\left.+\left|A\left(v^{(m)}(x,\lambda)-v^{(m-1)}(x,\lambda)\right)\right|^2\right]dxd\lambda\le const\ q^m,\ q<1.$$

Proof. From (7), (8) and Lemma 2 we get

$$\int_{-\infty}^{\infty}\int_D(1+|\lambda|)^{2p}\left|Av^{(m)}(x,\lambda)\right|^2d\lambda dx\le\frac{9N_0^2z_0^{\frac{4-n}{2}}C_2^4}{2p-3}.\frac{16\gamma_0^2}{\mu_0^2}$$

$$+\frac{9N_0^2C_1^2}{2p-3}\frac{4\gamma_0}{\mu_0}\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\|\hat\varphi\|^2_{W_2^{7/2}(S)}d\lambda$$

$$+9N_0^2\left(\|A\psi_1\|^2_{C(D)}+\|A\psi_2\|^2_{C(D)}\right)\frac{4\gamma_0}{\mu_0}+\frac{36\gamma_0}{\mu_0}$$

$$+\frac{9N_0^2C_1^2}{2p-3}\frac{4\gamma_0}{\mu_0}\cdot\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\left\|\hat{f}(x,\lambda)\right\|_{C(D)}^2 d\lambda+\frac{9N_0^2C_1^2}{2p-3}\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\|\hat{\varphi}\|_{W_2^{7/2}(S)}^2 d\lambda$$

$$\times\int_{-\infty}^{\infty}\int_D(1+|\lambda|)^{2p-2}\left|\hat{f}(x,\lambda)\right|^2 dxd\lambda+9N_0^2\left(\|A\psi_1\|_{C(D)}^2+\|A\psi_2\|_{C(D)}^2\right)$$

$$\times\int_{-\infty}^{\infty}\int_D(1+|\lambda|)^{2p-2}\left|\hat{f}(x,\lambda)\right|^2 dxd\lambda+9C_1^2\int_{-\infty}^{\infty}(1+|\lambda|)^{2p}\|\hat{\varphi}\|_{W_2^{7/2}(S)}^2 d\lambda\equiv C_3(\gamma_0).$$

Similarly for the substraction $v_k^{(m)}-v_k^{(m-1)}$ from (7), (8), considering the estimation from Lemma 2 we get

$$\int_{-\infty}^{\infty}\int_D(1+|\lambda|)^{2p-2}\left[(1+|\lambda|)^2\cdot\left|v_k^{(m)}(x,\lambda)-v_k^{(m-1)}(x,\lambda)\right|^2\right.$$

$$\left.+\left|A\left(v^{(m)}(x,\lambda)-v^{(m-1)}(x,\lambda)\right)\right|^2\right]dxd\lambda\le const\ q^m,\ q<1.$$

which proves Lemma 3.

Lemma 4. Let the conditions of Lemma be satisfied. Then for each fixed m the only solution of problem (7), (8) exists from the class

$$\int_{-\infty}^{\infty}\int_D(1+|\lambda|)^{2p-2}\left[(1+|\lambda|)^2\cdot\left|v^{(m)}(x,\lambda)\right|^2+(1+|\lambda|)^2\left|\nabla v^{(m)}(x,\lambda)\right|^2+\left|Av^{(m-1)}(x,\lambda)\right|^2\right]dxd\lambda<\infty.$$

Proof. The proof of this lemma is carried out by the Galyorkin's method.

Theorem : Let conditions 1)-4) be satisfied and

$$K(\mu_0,z_0)\le\frac{\mu_0}{4},$$

$$\mu_0-z_0\left[6N_0^2\left(\|A\psi_1(x)\|_{C(D)}^2+\|A\psi_2(x)\|_{C(D)}^2\right)+\frac{12C_1^2N_0^2}{2p-1}C_3(\gamma_0)\right.$$

$$\left.+\frac{12C_1^2N_0^2}{2p-1}\int_{-\infty}^{\infty}(1+|\lambda|)^{2p+2}\|\hat{\varphi}\|_{W_2^{7/2}(S)}^2 d\lambda\right]>0.$$

Then the only solution exists of inverse problem (1)-(3), where $C_3(\gamma_0)$ is defined in Lemma 3, z_0,C_1,C_2,N_0 in Lemma 2.

Proof. The solvability of problem (5), (6) follows from Lemma 2 and Lemma 3. It needs to prove

$$u|_{t=0}=\psi_1(x),\quad u(x,t)|_{t=T}=\psi_2(x).$$

Suppose $u|_{t=0}=\widetilde{\psi}_1(x)$ and consider the function $\widetilde{\psi}_1(x)-\psi_1(x)$.

$$A(\widetilde{\psi}_1(x)-\psi_1(x))=b(x)(\widetilde{\psi}_1(x)-\psi_1(x)),\quad (\widetilde{\psi}_1(x)-\psi_1(x))|_S=0;$$

$$|b(x)|\le 6N_0^2\left(\|A\psi_1(x)\|_{C(D)}^2-\|A\psi_2(x)\|_{C(D)}^2\right)+\frac{12C_1^2N_0^2}{2p-1}C_3(\gamma_0)$$

$$+\frac{12C_1^2N_0^2}{2p-1}\int_{-\infty}^{\infty}(1+|\lambda|)^{2p-2}\|\hat{\varphi}\|^2_{W_2^{7/2}(S)}d\lambda$$

From these relations and conditions of the theorem follows $(\widetilde{\psi}_1(x)-\psi_1(x)=0$.
Similarly one may prove $u(x,T)=\psi_2(x)$.

REFERENCES

[1] Bubnov, B.A., References on solvability of multidimensional inverse problems for the parabolic equations, Preprint 714, Novosibirsk 1990 (in Russian).

[2] Guliyev, M.A., The multidimensional boundary inverse problem for the parabolic equation in the bounded domain, Vestnik Bakinscogo Universitata No.1, 1997, S. 106-113 (in Russian).

[3] Iskenderov, A.D., Multidimensional inverse problems for linear and non-linear parabolic equations, Dokl. AN. SSSR, 1975, 225, No 5 (in Russian).

[4] Lavrentev, M.M., Romanov, V.G. and Shishatskii, S.P., III-posed problems of mathematical physics and analysis, AMS, Providence, RI, 1986.

[5] Namazov, G.K., Obratnie zadachi teori uravneniy matematicheskoy fiziki, Baku, 1984 (in Russian).

[6] Romanov, V.G., Inverse problems of mathematical physics, VNU, Utrecht, the Netherlands, 1987.