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FINITE ELEMENT MODELLING OF INDUCED POLARIZATION ELECTRIC POTENTIAL FIELD PROPAGATION CAUSED BY ORE BODIES OF ANY GEOMETRICAL SHAPE, IN MOUNTAINOUS RELIEF

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ABSTRACT

In this paper is treated the finite element modelling of anomalous potential electric field generated by induced polarization of polarized bodies, with or without electrical conductivity contrast with surrounding rocks, and with any geometrical shape, in regions with mountainous relief.

INTRODUCTION

Ore bodies, like chalcopyrites, pyrites, chromites with secondary magnetite, pyrite-bearing bauxites, and isolated rock bodies, have any geometrical shape and are mostly situated under accidented relief. Chalcopyrite and pyrite ore bodies of massive texture have a high electrical conductivity as compared with the electrical conductivity of the surrouding rocks, but this contrast of electrical conductivity may not exist, as is the case with disseminated chalcopyrite and pyrite bodies, serpentinites etc.

Thus, the mathematical modelling is realized for these two cases of electrical conductivity contrasts. Different cases of position of source and measurement electrodes are considered, especially when source electrodes A ve B and measurement electrodes M and N are situated on the ground or in the drillholes.

The induced polarization effect Uip is computed with the well known formulae [Bleil D. (1953); Seigel H.O. (1959)]:

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$$Uip = C \int \Delta U \Delta \frac{1}{R} dV \qquad (1)$$

where:

Uip — the potential of induced electric field,

U=Uo+Uip — the potential of resultant electric field,

Uo — the potential of primary electric field,

R — the vector from body point to measurement point,

C — a constant value determined by electric properties of medium.

Under field electrical survey conditions, it is known that $Uip << Uo$, so we have accepted the simplification proposed by Bleil D. (1953) as well as the evaluation made by Komarov V.A. (1972) assuming that:

$$CU = CUo \qquad (2)$$

Following this assumption the computing of Uip is reduced to the calculation of the integral (1) in which the potential Uo of primary electric field has replaced the potential U.

The use made of finite elements while computing this problem was possible because recently in our country powerful programs have been compiled for the application of finite element methods [Frasheri A. (1984, 1987, 1989); Osmani S. (1988); Tole Dh. (1981)].

For the induced polarization problem we have complied some algorithms and seven programs in FORTRAN77 and BASIC.

1. THE PROGRAM "POLARELF" FOR COMPUTING IP EFFECT OVER A POLARIZABLE CONDUCTIVE ORE BODY

Ore body with an electric conductivity different from that of the medium causes anomalies on the primary electric field and the IP electric field. These anomalies affect the distribution of IP electric field to be considered.

We realised the computing of the potential Uo of the primary electric field in a 2D heterogeneous medium by means of finite elements with the program "POLARELF". This program serves to solve the variational problem [Zienkiewicz O.L. R(1977)]:

$$\min\left\{ \frac{1}{2}\int_{s}\left[\gamma x\left(\frac{\partial Uo}{\partial x}\right)^2+\gamma y\left(\frac{\partial Uo}{\partial y}\right)^2\right] ds - \int_{\partial s} Uo\left(\frac{\partial Uo}{\partial n} - a\right) d\partial s \right\} \quad (3)$$

with rectangular finite elements having shape functions:

$$Ni\ (\xi, \eta) = \frac{1}{4}\ (1 + \xi_i\ \xi)\ (1 + \eta_i\ \eta) \quad (4)$$

where:

$i = i,j,k,l$ — indices of the nodes,

(ξ, η) — local coordinates of the basic element $(-1,+1)\times(-1,+1)$,

S — rectangle who serves as a 2D model for the geoelectrical section configuration. The upper edge of the rectangle is curved like the relief of earth surface [Holcombe T.H. (1984)].

The model was constructed for Neuman boundary conditions. Variations on time and space of these conditions were considered in the course of program compiling in compliance with the electrical surveying procedure. To achieve this compatibility the principal algorithm of the program was conceived in the following way (fig. 1):

i) — automatic discretisation of the rectangle S,
— construction of the element matrix,
— construction of the global matrix of the algebraic system (integrated in one single process).

ii) — construction of right hand side of the algebraic system for any case of boundary conditions,
— solving the algebraic system by exploying Gauss method.

iii) — the interpretation of the results.

Mathematical modelling of IP phenomena was based on the finite element presentation of Uo:

$$Uo = N.Uo \quad (5)$$

N — the vector of shape functions,

Uo — the vector of Uo values on the modes of elements.

Through the equation (1), the hypotheses (2) and (3), the potential of electric field of IP was computed according to the equation:

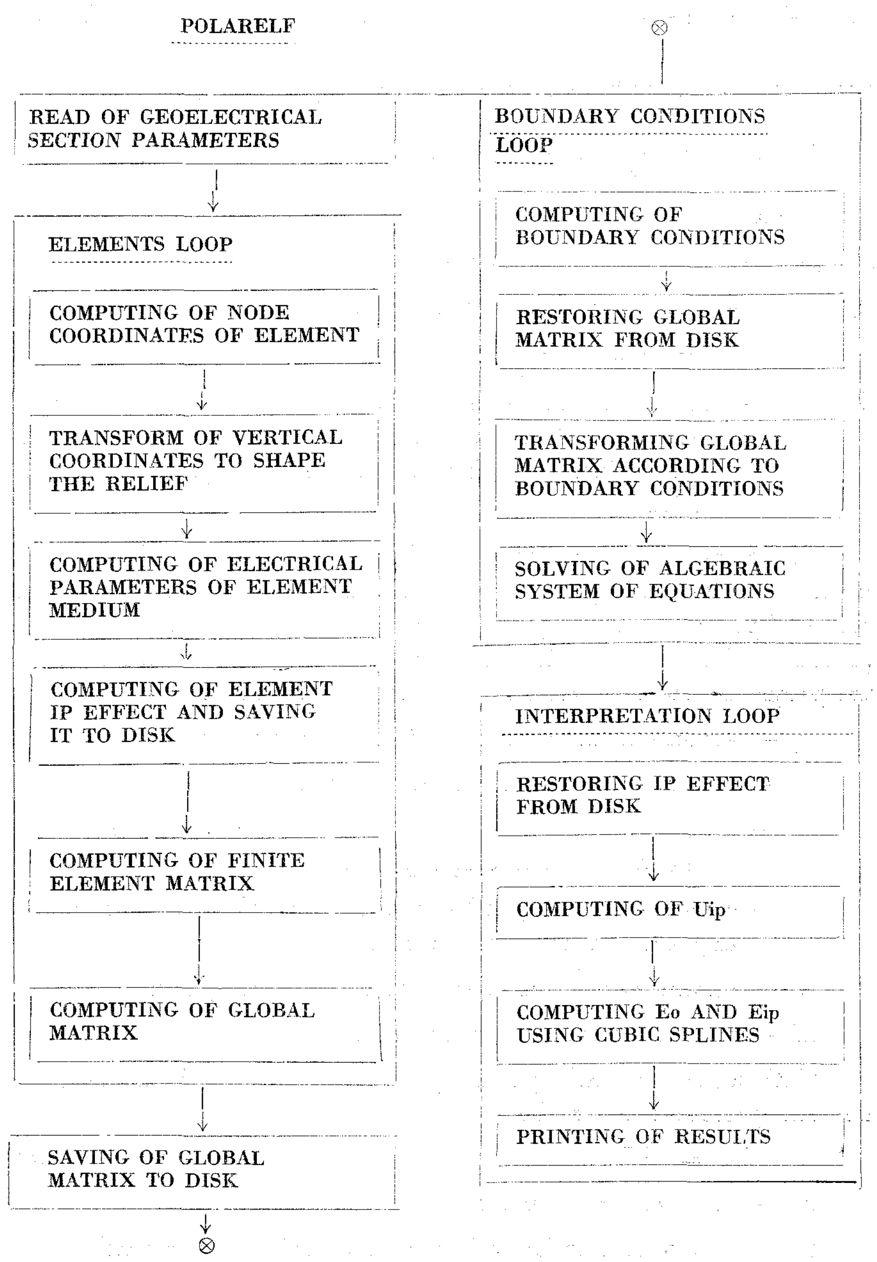


Fig. 1. The principal flow-diagram of POLARELF program.

$$Uip = KC\ Uo \sum_{se} \int_{se} (\Delta N.\vec{R}) \frac{ds}{R^3} \qquad (6)$$

where:

K — the empiric constant of geoelectrical model which expresses the transformation from 3D to 2D model.

Se — the finite element surface inside the ore body.

During the calculation of the finite element matrices we computed the vector:

$$\left(\int (\Delta N.\vec{R}) \frac{ds}{R^3} \right)$$

Which was utilisable while computing the Uip in the results interpretation stage. Here we computed the primary electric field intensity Eo and the intensity Eip of IP field, deriving Uo and Uip with cubic splines. Then we computed the value of overvoltage:

$$\eta_{an} = \frac{Epp}{Eo - Epp}\ 100\ \% \qquad (7)$$

The results of the POLARELF program were tested in the laboratory, in 2D physical modelling, and in field conditions over the known ore bodies. In fig. 2 one example of the comparison of the IP anomally profile over a coppeı sulphide ore body calculated through POLARELF program and surveyed in field conditions is showed. The ore consists of chalcopyrite with electrical ıesistivity of 1 Ohm. m and IP coefficient $\eta = \%\ 60$. It lies between the keratophyres which have a resistivity of 600 Ohm. m and a polarisability of $\eta = \%\ 2.5$. The measurements were caried out under the mountainous relief. A gradient array with AB = 1000 m and MN = 40 m was used.

2. POLARIZ PROGRAM FOR COMPUTING THE IP EFFECT OVER A POLARPIZABLE BODY WITHOUT ELECTRICAL CONDUCIVITY CONTRAST WITH THE MEDIUM.

In all those cases the medium is homogeneous from the poin of view of electrical conductivity, hence to compute the IP potential it is possible to use the well known formulae for the propagation of electric field of point source currents in 3D space.

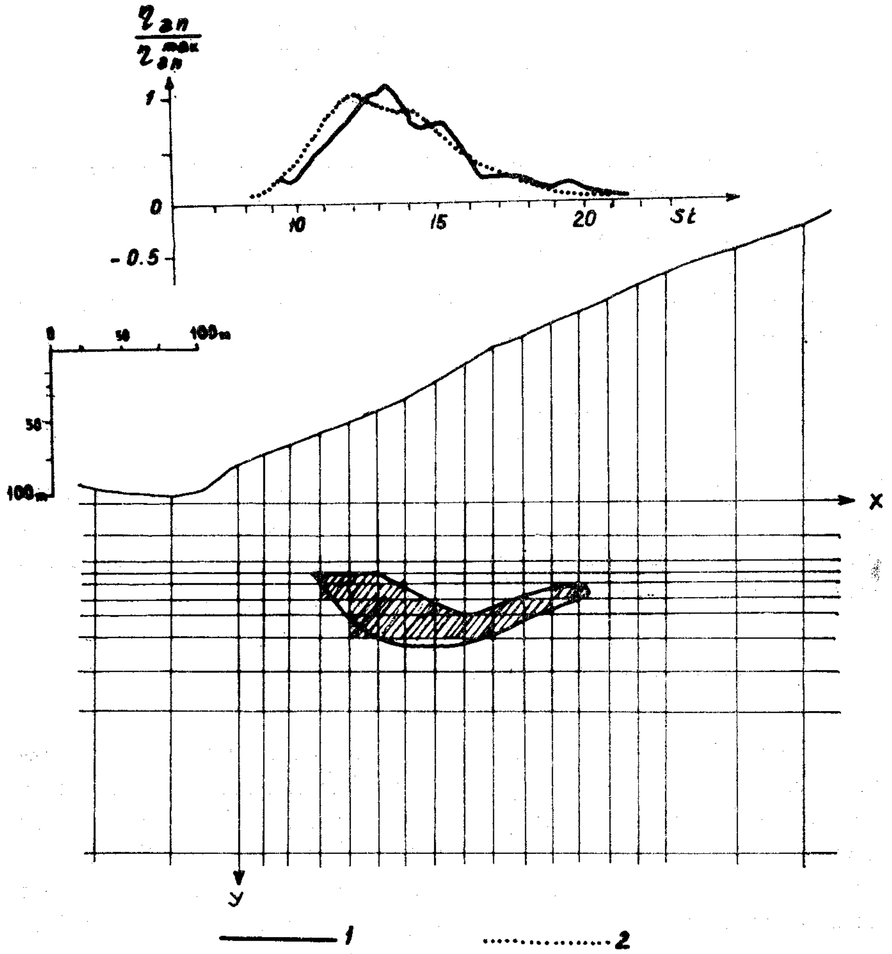
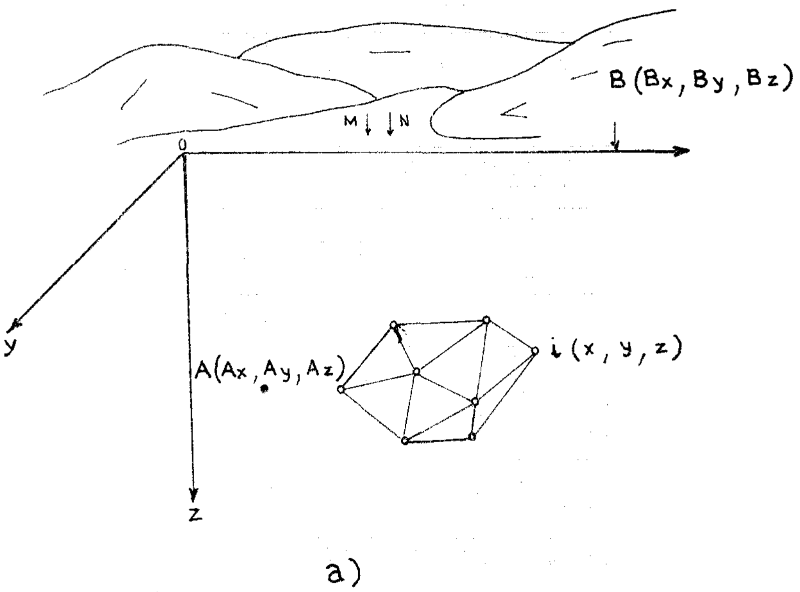


Fig. 2. Comparison of the IP anomalies calculated throguh POLARELF program (1) and surveyed in the field conditions (2), over a copper sulphide ore body.

The 3D model was constructed for bodies of irregular shape. A 2.1/2D model was constructed for prismatic bodies having an irregular cross section and a horizontal extension (fig.3).

Based on these two models the programs "POLARELF3" and "POLARPRIZ" (fig.4) were compiled in BASIC.

The potential of IP electric field was computed with the equation (1), which after the Green transform and hypothesis (2) was changed to:



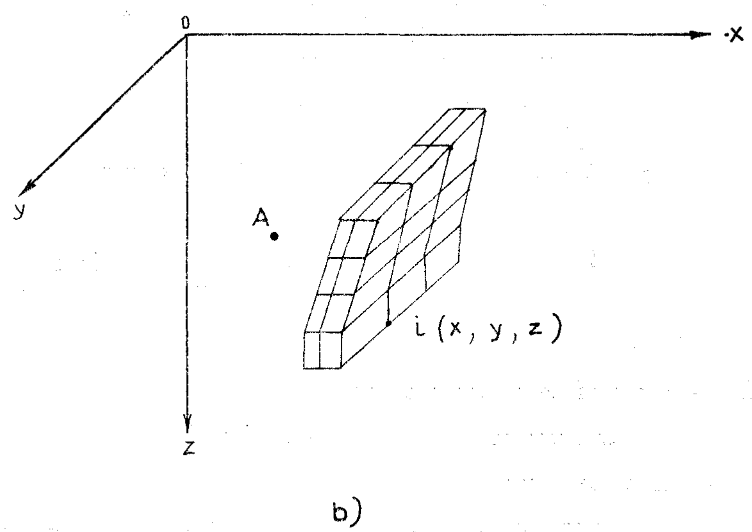


Fig. 3. The 3D and 2.1/2D geoelectrical models for the programs "POLARELF3" and "POLARPRIZ" respectively.

POLARPRIZ

READING OF GEOELECTRICAL SECTION PARAMETERS

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ELEMENTS LOOP

COMPUTING OF LOCAL GEOMETRIC PARAMETERS OF ELEMENTS

↓

TRANSFORM OF GEOMETRIC PARAMETERS FROM LOCAL TO GLOBAL COORDINATES

↓

PROFILE LOOP

COMPUTING IP ELEMENT EFFECT TO PROFILE

↓

INTERPRETATION LOOP

COMPUTING OF Eo AND Eip USING CUBIC SPLINES

↓

PRINTING OF RESULTS

Fig. 4. The principal flow-diagram of POLARPRIZ program.

$$Uip = C\left[\int_s \frac{1}{R} \cdot \frac{\partial U}{\partial n} \cdot dS - \int_V \frac{1}{R} \cdot \Delta Uo.dV\right] \quad (8)$$

where:

S — the surface of the ore body V,

N — the unit vector, normal to the surfase S.

Δ — Laplace operator.

The potential Uo of primary electric field is determined by Laplace equation so the second integral in (8) is zero and the formula is simplified:

$$Uip = C \int_s \frac{1}{R} \cdot \frac{\partial Uo}{\partial n} \cdot dS \tag{9}$$

To compute the integral (9) the idea of finite elements was used. We divided the body surface on triangular elements for the body with irregular shape, and on rectangular elements for the prismatic body. Using these elements the formula (9) was changed to:

$$Uip = C \sum_e \int_{se} \frac{1}{R} \cdot \frac{\partial Uo}{\partial n} \cdot dS \tag{10}$$

where: Se — the surface of element 'e'.

The integration on the element was realized using the Gauss method with three and four points of integration depending on the element shape.

In the formula (10) we replaced:

$$\frac{\partial Uo}{\partial n} = \Delta Uo.\vec{n} = Co \left(\frac{\vec{R}_A}{R^3{}_A} - \frac{\vec{R}_B}{R^3{}_B} \right) \cdot \vec{n} \tag{11}$$

where:

$Co = \delta I/2.\pi$

δ — resistivity of the medium,

I — intensity of current floxw in the earth,

Ra,Rb — the vectors fiom current electrodes A, B to the integration point of Gauss method.

At last we expressed the IP potential Uip in the form:

$$Uip = CCo \sum_e \sum_i \frac{1}{R_i} \cdot \vec{n} \left(\frac{\vec{R}_{Ai}}{R^3{}_{Ai}} - \frac{\vec{R}_{Bi}}{R^3{}_{Bi}} \right) \cdot Wi.Se \tag{12}$$

where:

i — the number of integration point in the element e,

Wi — the weight of integration for the point i,

Se — the surface of the element e.

We computed also the differences of the potential of primary electric field in the same points. Through these values we computed the intensity Eo of primary electric field and Eip of the IP field, using cubic splines. With the formula (7) we computed the value of overvoltage.

The POLARPRIZ and POLARELF3 programs were tested comparing their results with ones obtained by physical 2D modelling and with so called geometrical factor F according to the equations presented by KOMAROV (1972).

In fig. 5 is presented one example of the IP anomaly profiles calculated with POLARPRIZ program for a 2.1/2D geoelectrical model of a copper ore deposit, a chalcopyrite body with resistivity 500 Ohm. m and polarizability $\eta = \%\ 50$ in a diabase medium with resistivity 500 Ohm. m and polarizability $\eta = \%1$, in the cases when measurements are carried out in mountainous (1) and flatten (2) relief. In fig. 6 the IP contour map for the same model as in fig. 5, but in a flatten relief is given.

In table I the values calculated with the integral (10) using POLARPRIZ program and the values of factor F, for the anomaly of a vertical prism without resistivity contrast with surrounding homogeneous medium are given.

The dimensions of the prism for this example were $b = c = 0.5a$, the depth of its upper part was $H = b$ and the remnant polarizability was $\eta = \%20$. In fig.7 are shown the IP anomalies calculated according POLARPRIZ program, using factor F and surveyed in 2D physical modelling.

The last one represents a thin flatten horizontaly electrolitic layer with dimensions 1500 mm $\times$ 100 mm, 3–5 mm thick and consists of diluted pure CuSO4 electrolyte.

As a model of the prism serves a chalcopyrite prism 5 mm thick. A gradient array with AB = 1000 mm, MN = 100 mm and a 1 mm survey step was used.

A little discrepancy between physical and mathematical modelling of this example, as regards the width of the anomaly, is subject of the different model dimensions used (2D and 3D).

As a conclusion, we may say that the use of finite elements to compute IP effect over a polarizable body with any geometrical shape, with or without conductivity contrast with the medium, situated on regions with any kind of relief, yields good results. It is possible to use 2D models as well, using an empiric constant to express the differences with 3D model. The use of finite element idea to integrate on irregulaı space surfaces offers the posibility of compiling simple and powerful programs to compute IP effect for homogeneous geoelectrical models.

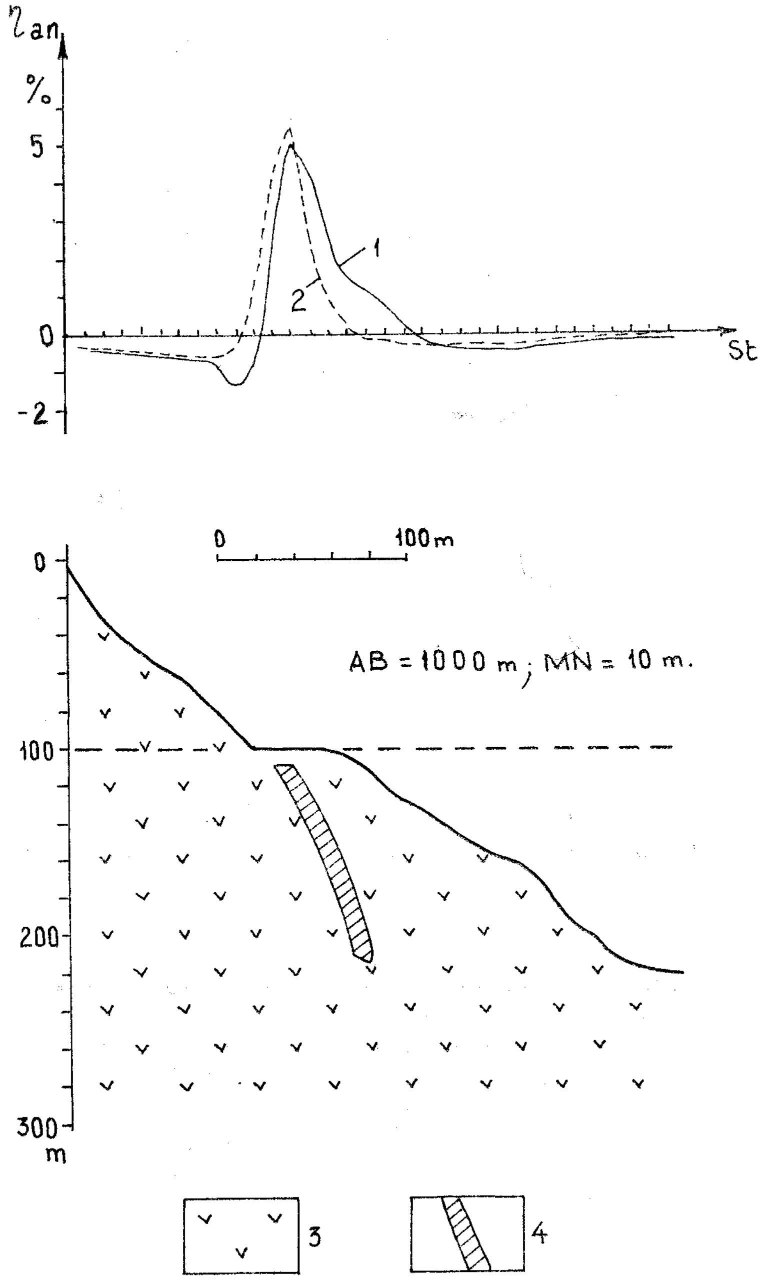

Fig. 5. 2.1/2D geoelectrical model of a chalcopyrite ore body and the IP anomalies calculated with POLARPRIZ program in the case of a mountainous relief (1) and in the flatten relief (2).

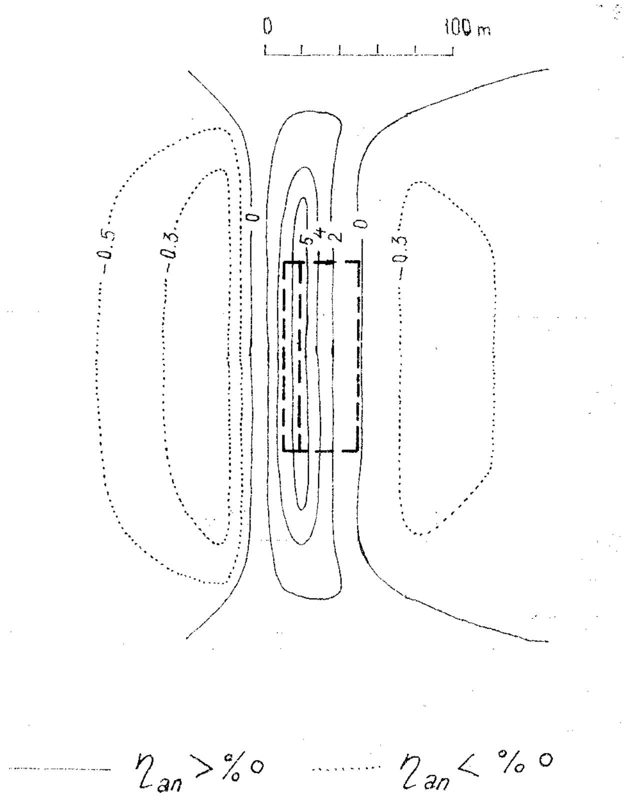

Fig. 6. The IP anomaly contour map over the geoelectrical model as in fig. 5

Table I. The values calculated using POLARPRIZ program and the values of the factor F for the anomaly of a vertical prism without resistivity contrast with surrounding homogeneous medium.

STATION	CALCULATED POLARPRIZ	VALUES OF FACTORF
0	0.3560	0.3590
1	0.1296	0.1232
2	-0.0996	-0.1012
3	-0.0784	-0.9761
4	-0.0431	-0.0435
5	-0.2490	-0.2355
6	-0.0126	-0.0158

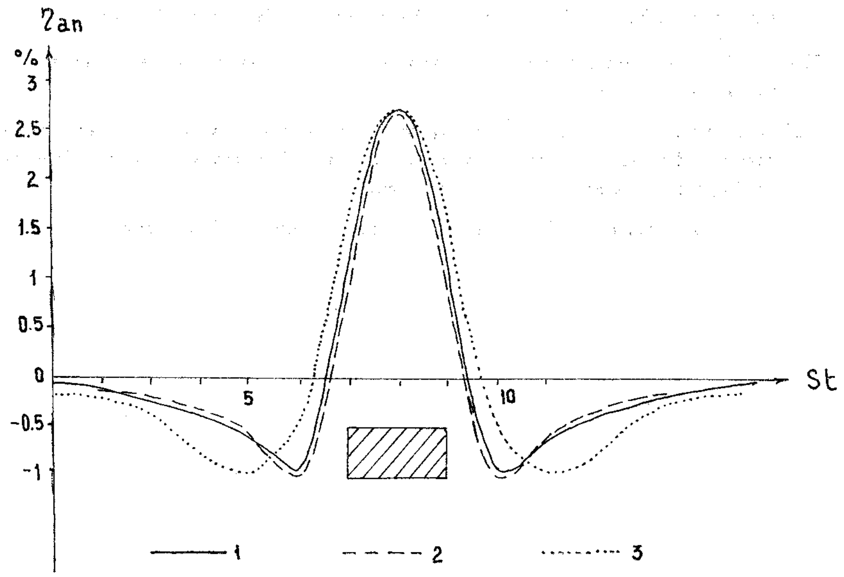

Fig. 7. Comparison of IP anomalies calculated with the POLARPZIR program (1), with the factor F (2) and surveyed in a 2D physical modelling (3).

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