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On Flying-Handling Qualities of B747-100 Longitudinal Flight based on Gain Scheduling Control

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Abstract: This paper evaluates flying handling qualities (FHQ) for the Boeing 747-100 (B747-100) in longitudinal flight. A genuine control has been realized using the spline gain-scheduling approach and full-state feedback linear quadratic regulator (FSFLQR). Converged steady-state responses have been shown for longitudinal states over Mach number and altitude ranging from 0.2 to 0.9 and sea-level to 12190m respectively. However, the FHQ verifications of such gain-scheduling control design are done using control anticipation parameter (CAP), normalised parameters of pitch rate and Cstar (combination effects of pitch rate, pitch and normal acceleration responses). The controller has been identified at which those conditions to be reformulated so that all the FHQ criteria are being satisfied. The CAP criterion well respects the level I of FHQ boundaries. However, some spectra of normalised pitch rate and Cstar with respect to their steady states slightly infringe the FHQ boundaries. This study has shown such a successful implementation of the gain-scheduling controller in terms of converged normal accelerations as well.

Keywords: Flying handling quality, Tracking longitudinal responses, Gain scheduling, Pitch rate criterion, CAP criterion, Cstar criterion, Normal acceleration

Introduction

As being revealed by Cooper and Harper (Etkins, 1994), ‘‘handling qualities are simply the ease and precision to support an aircraft flight.’’ Adequate handling qualities (HQ) would be required for successful flight performance by considering the numerical pilot rating (1-10) or the Cooper and Harper scale (CHS). For example, considering transport aircraft at low-speed longitudinal controlled flight, pilot rating is specified ten at CHS for deficiency in control performance whereas pilot rating is agreed to be one for adequate performance with tolerable pilot workload and satisfactory control characteristics (Etkins, 1994) and the intermediate ratings are gradually shown how the deficiency being improved. Primarily, the HQ criteria would be evaluated using mathematical models of the aeroplane as well as the pilot interaction control systems. The HQ depends on aircraft dynamics, control system performance, cockpit environment, outside view, and instrument display (Mclean, 1990). The development of HQ criteria has been made by the pilots’ opinions, aircrafts evaluations, in-flight simulation, and ground-based simulators (Jitendra & Jatinder, 2009). Various government agencies are complying with HQ requirements for military aircraft as well as transport aircraft which demand the safe operation rather than the manoeuvrability performance. Modern aircraft development requires full HQ evaluation for different controller modes, loadings, and various operational missions throughout the flight regimes. Flight testing would be time-wasting to cover wide-ranging conditions over the whole flight envelope.

After the 2nd World War with the increase of aircraft flying speed and altitude, the flight envelope has been enlarged a lot and the incidence time lag between the pitch rate response and the normal acceleration response may vary from 0.5s at a high speed and low altitude to 4.0s at low speed and high altitude. Several important

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evaluation criteria of aircraft's longitudinal flying handling qualities are known such as bandwidth criterion, time lag, *CAP* and *Cstar* (*C**) criteria. Bandwidth criterion characterizes the phase roll-off using a phase delay parameter whereas *CAP* clarifies that the initial pitch attitude response of the aircraft ascertains the ultimate response of the flight path (Jitendra, & Jatinder, 2009). Besides, Neal–Smith criterion evaluates the closed-loop performance based on the lead-lag compensation used by the pilot. However, the *Cstar* criterion is widely used by control system designers using a combination of the pitch rate (at low speeds) and the normal acceleration (at high speeds) responses (Jitendra, & Jatinder, 2009).

HQPACK MATLAB package was designed to predict handling qualities and pilot induced oscillation tendencies of aircraft (Shaik, & Chetty, 1998). Also, the HQ of large transport aircraft software (HQLTASW) was evaluated with the NLR's database for Fokker F28/Mk6000 aircraft (Shaik, 2005). Sample software control algorithms were developed to improve the flying qualities of general aviation aircraft and the results also rated the HQs (Rogalski, & Dołega, 2006). Flying qualities and guidance displays were evaluated for an advanced tilt-wing STOL transport aircraft in the final approach and landing (Frost, *et. el.* 2002). The aircraft, control modes, and display combination produced satisfactory flying qualities for all operations excepting that an extremely severe crosswind and wind shear. Effective relationships were found between FQ levels and 52 tests of Mach number, altitude and angle of attack for longitudinal and lateral flight of F/A-18 aircraft (Botez,& Rotaru, 2007).

Many literatures miss out evaluating FHQs of augmented flight control system, particularly for linear quadratic regulator (LQR) and classical control algorithms. For examples: Tosun controlled quadrotor position using the LQR to reach desired attitudes (Tosun, *et. al.*, 2015). LQR control was improved using an integral action for mass-related uncertainties showing the effective stabilisation of the star-shaped Octrotor vehicle (Adîr, & Stoica, 2012). Six degrees of freedom control of a small-scale quadcopter was achieved by an integral LQR for highly tracking and balancing responses (Joukhadar ,*et. el.*, 2015). The linear quadratic Gaussian (LQG) method was used to attenuate the pitching longitudinal noise of cruise aircraft (Shaji, & Aswin, 2015). LQR and LQG controllers showed the accuracy of an attitude microsatellite stabilisation comparing with feedback quaternion and proportional-integral-derivative (PID) designs (Tayebi, *et. el.*, 2017). The LQR method was applied for disturbed longitudinal flight of an unmanned aerial vehicle where the reference speed reached quickly without affecting altitude and pitch angle (Hajiye, *et. el.*, 2015). A good LQR performance was found in real-time pitching stabilisation for reference tracking as high as 55 degrees for a helicopter (Bharathi, & Kumar, 2013). A satisfactory LQR performance was also found for a SUAVE tilt-wing quadrotor during the yaw angles (Oner, *et. el.*, 2009).

This work mainly investigates the FHQ criteria over the B747-100 longitudinal flight envelope. A genuine gain-scheduling FSFLQR controller was already designed over Mach numbers (*M*) and altitudes (*H*) ranging from 0.2 to 0.9 and sea-level to 12190m respectively (Elarbi, *et. el.*, 2019). The gain-scheduling scheme was considered a continuous function of two scheduling variables (*M* and *H*). The gain scheduling methodology is a very effective way to control a nonlinear system since the 1960s (Mclean, 1990). However, gain-scheduling controllers have been widely used in industrial automated machines and aviation since the 1990s (Mclean, 1990). Here, particular attention was paid to longitudinal variables of axial velocity (u), transverse velocity (w), pitch rate (q) and pitch attitude (θ) coupling with elevator (δ_e) and throttle (δ_t). The multivariable dynamics of aircraft control was presented by a linearized state-space (LSS) model. 27 design pairs of *M* and *H* were uniformly sampled using the Latin hypercube approach for an optimised flight landscape (Elarbi, *et. el.*, 2019). A spline gain scheduling interpolation (Chapra, & Canale, 2010) was used to obtain the intermediate responses over the *M-H* ranges meeting velocities and altitudes objectives. Extensive studies related to the FHQs specification will here be undertaken, before being satisfied that the gain scheduling design is acceptable. Normalised pitch rate; *CAP* criteria; and normalised *Cstar* are evaluated over the longitudinal flight envelope. No obvious infringements the level I of FHQ boundaries are seen for the spectra of normalised pitch rate and *Cstar* responses with respect to (wrt) their steady-state values. The *CAP* criteria are well respected the level I of FHQ boundaries. Also, converged normal acceleration responses confirm the successful FSFLQR control based on the gain scheduling design over the B747-100 longitudinal flight envelope.

Analysis Method

Longitudinal Flight Model

The longitudinal flight model cross-coupled with elevator and throttle controls can be given in the LSS form by

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \Delta\dot{\theta} \\ \dot{h} \end{bmatrix} = A(M,H) \begin{bmatrix} u \\ w \\ q \\ \Delta\theta \\ h \end{bmatrix} + B(M,H) \begin{bmatrix} \Delta\delta_e \\ \Delta\delta_t \end{bmatrix} \tag{1}$$

where $A(M,H)$ is A/C dynamics matrix of 5×5 and $B(M,H)$ is a control design matrix of 5×2. These matrices parameters which depend on Mach number and altitude are given below

$$A(M,H) = \begin{bmatrix} X_u & X_w & 0 & -g & 0 \\ Z_u & Z_w & u_0 & 0 & 0 \\ M_{p,u} + M_{p,\dot{w}} Z_u & M_{p,w} + M_{p,\dot{w}} Z_w & M_{p,q} + M_{p,\dot{w}} u_0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & u_0 & 0 \end{bmatrix} \tag{2}$$

$$B(M,H) = \begin{bmatrix} X_{\delta_e} & X_{\delta_t} \\ Z_{\delta_e} & Z_{\delta_t} \\ M_{p,\delta_e} + M_{p,\dot{w}} Z_{\delta_e} & M_{p,\delta_t} + M_{p,\dot{w}} Z_{\delta_t} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{3}$$

where the derivatives of forwarding force (X), transverse force (Z) and pitching moment (M_p) are given wrt axial velocity (u), normal velocity (w), transverse velocity derivative ($\dot{w}$), pitch rate (q), elevator deflection (δ_e) and throttle actuation (δ_t). u_0 is a steady-state velocity and g is a gravity acceleration (9.81m/sec^2). The full-states longitudinal responses can easily be obtained by regarding all the states as system outputs. Thus, the output observation matrix and the state transition matrix were taken unity and nullity matrices respectively.

FSFLQR Algorithm

A full state feedback (FSF) design is obtained by choosing a gain matrix (K (M,H)) which is a linear combination of the longitudinal states. The optimal control law is given as,

$$\begin{bmatrix} \Delta\delta_e \\ \Delta\delta_t \end{bmatrix} = -\mathrm{K}\,(M,H) \begin{bmatrix} u \\ w \\ q \\ \Delta\theta \\ h \end{bmatrix} \tag{4}$$

The LQR control law is typically used to find the optimal control gains for a multivariable large scale system. The controller can be tuned by adjusting the state and control weighting matrices. The cost function ensures that J is non-negative and zero for the optimal tracking system.

$$J = \int_0^{tf} \left[\left([u \quad w \quad q \quad \Delta\theta \quad h] \mathrm{Q}(M,H) \begin{bmatrix} u \\ w \\ q \\ \Delta\theta \\ h \end{bmatrix} \right) + \left([\Delta\delta_e \quad \Delta\delta_t] \mathrm{R}(M,H) \begin{bmatrix} \Delta\delta_e \\ \Delta\delta_t \end{bmatrix} \right) \right] dt \tag{5}$$

Here $\mathrm{Q} = \mathrm{Q}^{\mathrm{T}} \geq 0$ is 5×5 state weighting matrix and $\mathrm{R} = \mathrm{R}^{\mathrm{T}} > 0$ is 2×2 control weighting matrix. t_f is control time. The state feedback gain matrix in Eq. (4) can be obtained from

$$\mathrm{K}(M,H) = \mathrm{R}(M,H)^{-1} \mathrm{B}(M,H)^{T} \mathrm{P} \tag{6}$$

Matrix P is obtained by solving the steady-state algebraic equation below

$$-\mathrm{PA}(M,H) - \mathrm{A}(M,H)^{\mathrm{T}} \mathrm{P} + \mathrm{PB}(M,H) \mathrm{R}(M,H)^{-1} \mathrm{B}(M,H)^{T} \mathrm{P} - \mathrm{Q}(M,H) = 0 \tag{7}$$

Once the optimal LQR gains, K, results in swift longitudinal convergences the autopilot takes place for tracking.

Gain Scheduling Design

The interpolations of the longitudinal flight variables will be applied over the discretized flight envelope. Physical design plans are chosen as $M \in [0.2, 0.9]$ and $H \in [0, 12190]$

$$M^{(i)}, H^{(j)} = \frac{(M^{(i)}, H^{(j)}) - (M_l, H_l)}{(M_u, H_u) - (M_l, H_l)} \quad (8)$$

where M_u and M_l are the upper and lower bounds of M respectively whereas H_u and H_l are the upper and lower bounds of H respectively. The cubic splines will be used to determining intermediate responses for a group of systematic data points which can be defined as (Chapra, & Canale, 2010),

$$f(M) = f(M_i) + \frac{f(M_{i+1}) - f(M_i)}{M_{i+1} - M_i}(M - M_i) \qquad M_1 \leq M \leq M_n \quad (9)$$

$$f(H) = f(H_i) + \frac{f(H_{i+1}) - f(H_i)}{H_{i+1} - H_i}(H - H_i) \qquad H_1 \leq H \leq H_n \quad (10)$$

where data points $i = 1, 2, 3 \ldots, n$ and n is number of intervals. These equations are used to predict the LQR gains and state-space models at the selected intervals. The n^{th} equations can then be employed to compute values within each interval. The scheme was implemented using built-in MATLAB functions which result in a more memory-efficient implementation than a lookup table.

FHQ Criteria

Flying handling qualities describe the easy, precise and rapid level when pilot controlling an aircraft to conduct flying tasks, such as flight refuel, landing and rolling. The MIL-F-8785 standard 'Military Specification, Flying Qualities of Piloted Airplane' based on enormous flying experience data and flight simulator tests were created by the military aeronautic organization, and the last version was released in 1980 in which aircraft is described in a linear mathematical model. Of the numerical requirements is system parameters based on aircraft's mathematical model, for example, natural frequency and damping ratio. Three important FHQ criteria used in this work are represented next

Pitch Rate Criteria

Pitch rate FHQ criterion is evaluated using the pitch rate history in the normalized form concerning the steady-state pitch rate which should be sited within specified boundaries. The steady-state pitch rate can be obtained using the final value theory as below

$$\frac{q_{ss}}{\Delta\delta} = lim_{s\to 0}\left(\frac{q(s)}{\Delta\delta}\right) \quad (11)$$

The normalized pitch rate concerning steady-state pitch rate is given by

$$\frac{q}{q_{ss}} = \frac{q}{\Delta\delta} \cdot \frac{\Delta\delta}{q_{ss}} \quad (12)$$

CAP Criteria

Control anticipation parameter is the ratio of the initial pitch acceleration to the steady-state normal acceleration. The CAP criteria measures how the trimmed flying condition coincides with what the pilot expected. The maximum CAP boundaries indicate the manoeuvrability constraints in terms of short-term natural frequency. Large CAP value specifies sensitive and abrupt aircraft response and small CAP means sluggish and overshooting aircraft (Mclean, 1990). CAP can be expressed by the formula below (Mclean, 1990),

$$CAP = \frac{sq(\infty)}{az(0)} = \frac{{\omega_{sp}}^2}{az_{\propto}} \tag{13}$$

From the flying geometric model and ignoring the effect of gravity, normal acceleration at the aircraft centre of gravity for perturbed motion is defined as

$$\frac{az(s)}{\Delta\delta} = \frac{sw(s)}{\Delta\delta} - u_0 \frac{q(s)}{\Delta\delta} \tag{14}$$

where ω_{sp} is short period frequency. Normal load factor wrt angle of attack can be expressed as,

$$az_{\propto} = \frac{az(s)}{\Delta\delta} . \frac{\Delta\delta}{\Delta\propto(s)} = \frac{1}{g} . \frac{az}{\Delta\propto} \tag{15}$$

Cstar Criteria

The *Cstar* criteria assess the dynamic response of the aircraft longitudinal motion wrt the normal acceleration and pitch acceleration. The pitch rate of aircraft should be laid between specific *Cstar* criteria limits (Mclean, 1990). The *Cstar* criteria may be arranged so;

$$\frac{Cstar(s)}{\Delta\delta} = \frac{1}{g}\left(\frac{az(s)}{\Delta\delta} + V_c \frac{q(s)}{\Delta\delta}\right) \tag{16}$$

where V_c is a crossover velocity. The steady-state *Cstar* can be found by

$$\frac{Cstar_{ss}}{\Delta\delta} = lim_{s\to 0}\left(\frac{Cstar}{\Delta\delta}\right) \tag{17}$$

The normalized *Cstar* criteria may now be rearranged so

$$\frac{Cstar}{Cstar_{ss}} = \frac{Cstar}{\Delta\delta} . \left(\frac{Cstar_{ss}}{\Delta\delta}\right)^{-1} \tag{18}$$

Discussions of Results

Flight Envelope Discretization

A symmetric longitudinal flight manoeuvre was considered under small perturbations with almost comparable stall speed and cruise speed. The B747-100 flight envelope for the M range from 0.2 to 0.9 and the H range from sea-level to 12190m is shown in Fig. 1.

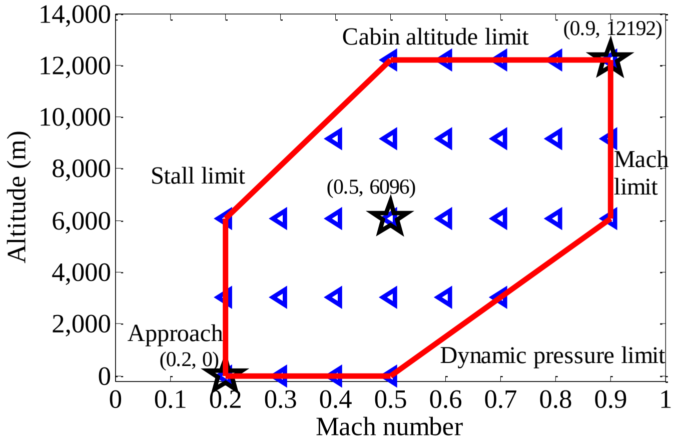


Figure 1. The B747-100 flight envelope

The Latin hypercube design (Forrester., *et. al.*, 2008) was used to select operating points for Mach number of 0.1 and altitude of 3048m. The black stars signify equilibrium points: (0.2, 0), (0.5, 6096m) and (0.9, 12190m). 27 points over the flight envelope were chosen to avoid overlapping gain scheduling, the regions of the dynamic pressure and stall limits. The spline approach was applied in connecting lower-order polynomials of subsets. Then the gain scheduling approach was used to interpolate based on the equilibrium points' simulations. Therefore, all the flight and stability derivatives were obtained to fulfil the control law of the most flight scenarios.

Pitch Rate Criteria Responses

The FHQ pitch rate time-history criterion evaluations over the M-H flight envelope are shown in Fig. 2. The pitch rate transfer function and the steady-state pitch rate were found over the flight envelope ($H_i = 0 - 12190$m and $M_i = 0.2 - 0.9$) below.

$$\frac{q}{\Delta\delta} = \frac{\begin{array}{c}(12.8H + 2\times10^5M + 6.39HM - 3.1\times10^5)s^4 + \\ (49.37H + 8.75\times10^5M + 25.43HM - 9.94\times10^5)s^3 + \\ (56.88H + 8.38\times10^5M + 28.03HM - 1.46\times10^6)s^2 + \\ (91.09H + 0.24\times10^5M + 1.36HM + 8.72\times10^6)s \\ (35.38H + 4.56\times10^5M + 16.96HM - 1.03\times10^6)\end{array}}{\begin{array}{c}(-1.04\times10^{-5}H + 0.219M - 2.37\times10^{-6}HM + 0.956)s^5 \\ +(1.55H + 7.2\times10^4M + 1.13HM + 5.14\times10^4)s^4 + \\ (182.53H + 5.35\times10^6M + 109.67HM + 2.41\times10^5)s^3 + \\ (-50.64H + 9.73\times10^5M - 12.26HM + 4.48\times10^6)s^2 + \\ (-90.43H - 1.16\times10^6M - 43.34HM + 2.63\times10^6)s + \\ (-231.43H - 2.89\times10^6M - 110.22HM + 6.91\times10^6)\end{array}}$$

$$\left.\frac{q_{ss}}{\Delta\delta}\right|_{\forall(M,H)} = \frac{(35.38H + 4.56\times10^5M + 16.96HM - 1.03\times10^6)}{(-231.43H - 2.89\times10^6M - 110.22HM + 6.91\times10^6)}$$

Table 1 valuates the steady-state pitch rate wrt the combined control inputs over the flight envelope. The "*s*" variable terms in numerator and denominator were eliminated by applying the final value theory on the transfer function terms. The steady-state pitch rate wrt one degree of control input over the flight envelope varies from -80.7×10^{-3} to -148×10^{-3} sec^{-1}. Although small pitch rates were found authorizing the steady-state longitudinal flight the negative signs indicated violent nose-down tendencies due to the reverse pitch damping.

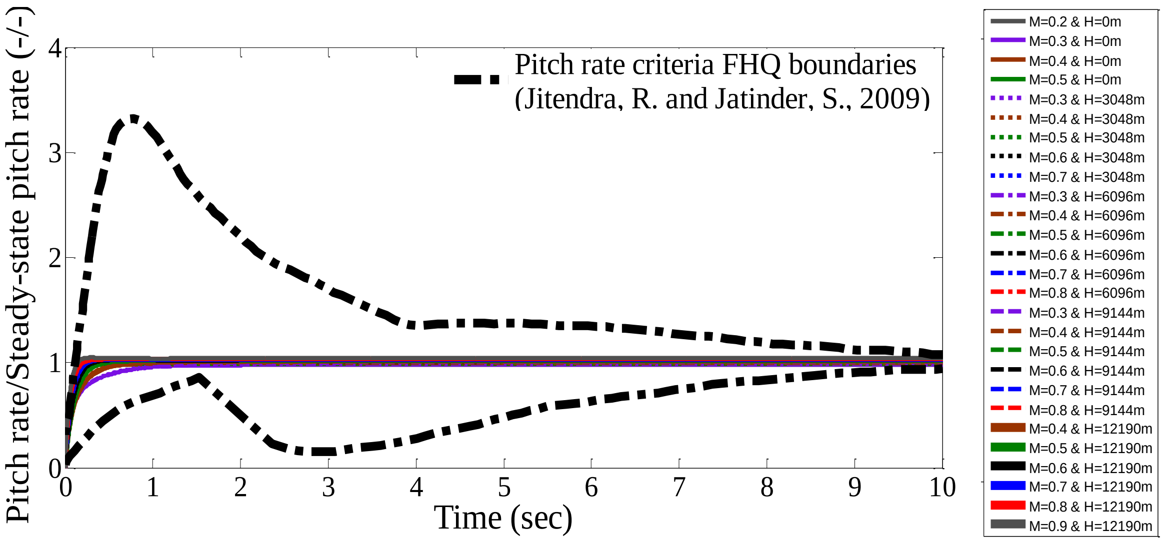


Figure 2. FHQ pitch rate criteria over the B747-100 longitudinal flight envelope

Table 1. The steady-state pitch rate over the flight envelope

M	0.2	0.3	0.4	0.5	0.3	0.4	0.5	0.6	0.7	0.3	0.4	0.5	0.6	0.7
H (m)	0	0	0	0	3048	3048	3048	3048	3048	6096	6096	6096	6096	6096
$\frac{q_{ss}}{\Delta\delta}$ (10^{-3}sec^{-1})	-148	-147	-147	-146	-147	-146	-145	-144	-143	-145	-144	-143	-142	-140

M	0.8	0.3	0.4	0.5	0.6	0.7	0.8	0.4	0.5	0.6	0.7	0.8	0.9
H (m)	6096	9144	9144	9144	9144	9144	9144	12190	12190	12190	12190	12190	12190
$\frac{q_{ss}}{\Delta\delta}$ (10^{-3}sec^{-1})	-138	-144	-142	-140	-138	-134	-129	-139	-135	-129	-119	-97.6	-80.7

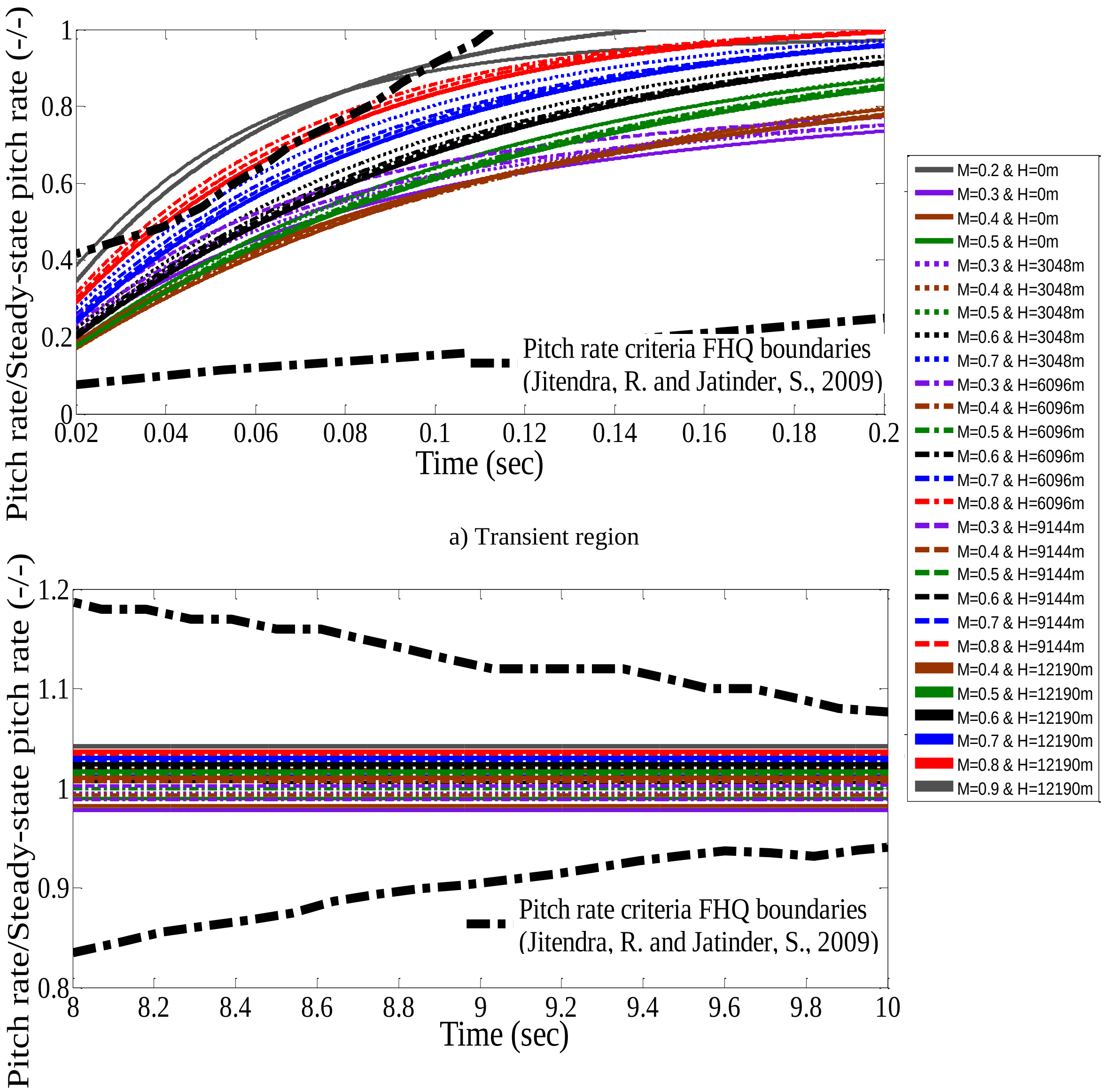
The normalized pitch rate wrt steady-state pitch rate over the flight envelope $(q/q_{ss})|_{\forall (M,H)}$ is discussed next. The normalized parameter should lay within boundaries specified in Fig. 3. Figure 3 a) shows the close-up transient region of FHQ pitch rate criteria. All the normalized rates' settled to almost levelled responses in the range of 0.002 to -0.004 which satisfies the longitudinal trimmed merits of straight levelled flight. The responses of Mach number higher than 0.8 and altitude higher than 6096m infringed slightly the upper boundaries of the transient region. However, all the responses placed inside the lower boundaries. Figure 3 b) shows the close-up steady-state region of FHQ pitch rate criteria. All the response spectra well placed inside the boundaries.

CAP Criteria Responses

The steady state velocity was assumed equivalent to the crossover velocity of 150m/sec. The normal acceleration transfer function for ($H_i = 0 - 12190$m and $M_i = 0.2 - 0.9$) was found by Eq. (14),

$$\frac{az}{\Delta\delta} = \frac{\begin{array}{c}(67.01H + 1.024\times10^{6}M + 33.3HM - 1.65\times10^{6})s^5 + \\ (-0.31\times10^{3}H + 0.04\times10^{7}M - 1.12\times10^{2}HM + 1.77\times10^{7})s^4 + \\ (-2.76\times10^{3}H - 3.89\times10^{7}M - 1.35\times10^{3}HM + 7.39\times10^{7})s^3 + \\ (-5.49\times10^{3}H - 8.96\times10^{7}M - 2.76\times10^{3}HM + 12.4\times10^{7})s^2 + \\ (-2.94\times10^{3}H - 3.78\times10^{7}M - 1.4\times10^{3}HM + 8.5\times10^{7})s\end{array}}{\begin{array}{c}(-1.04\times10^{-5}H + 0.219M - 2.37\times10^{-6}HM + 0.956)s^5 + \\ (1.55H + 7.2\times10^{4}M + 1.13HM + 5.14\times10^{4})s^4 + \\ (182.53H + 5.35\times10^{6}M + 109.67HM + 2.41\times10^{5})s^3 + \\ (-50.64H + 9.73\times10^{5}M - 12.26HM + 4.48\times10^{6})s^2 + \\ (-90.43H - 1.16\times10^{6}M - 43.34HM + 2.63\times10^{6})s + \\ (-231.43H - 2.89\times10^{6}M - 110.22HM + 6.91\times10^{6})\end{array}}$$

Perturbed normal acceleration at the A/C centre of gravity over *M-H* flight envelope is shown in Fig. 4. Converged responses took longer to almost settle the normal acceleration in the range of 0.034 to -0.0069m/sec^2 which well agreed with the main merits of longitudinal trimmed straight levelled flight. However, these responses were tipped at 50sec to 100sec during transient regions. However, the angle of attack transfer function for ($H_i = 0 - 12190$m and $M_i = 0.2 - 0.9$) was already obtained (Elarbi, *et. el.*, 2019),



a) Transient region

b) Steady-state region

Figure 3. Close-up FHQ pitch rate criteria

$$\frac{\Delta\propto}{\Delta\delta}=\frac{\begin{array}{c}(67.01H+1.024\times10^{6}M+33.3HM-\ 1.65\times10^{6})s^{4}+\\(1.61\times10^{3}H+3.04\times10^{7}M+8.41\times10^{2}HM-\ 2.88\times10^{7})s^{3}+\\(4.65\times10^{3}H+9.23\times10^{7}M+2.47\times10^{3}HM-7.52\times10^{7})s^{2}+\\(3.04\times10^{3}H+3.61\times10^{7}M+1.44\times10^{3}HM-9.47\times10^{7})s+\\(2.37\times10^{3}H+3.06\times10^{7}M+1.14\times10^{3}HM-6.93\times10^{7})\end{array}}{\begin{array}{c}(-8.41H-1.15\times10^{6}M-\ 4.1HM+2.32\times10^{5})s^{4}+\\(4.18\times10^{3}H+\ 1.21\times10^{7}M+250.89HM+3.96\times10^{5})s^{3}+\\(2.85\times10^{4}H+7.58\times10^{8}M+\ 1.65\times10^{4}HM-\ 1.06\times10^{8})s^{2}+\\(-3.49\times10^{3}H-2.02\times10^{7}M-1.49\times10^{3}HM+1.47\times10^{8})s+\\(-5.57\times10^{3}H-7.11\times10^{7}M-\ 2.66\times10^{3}HM+1.63\times10^{8})\end{array}}$$

The FHQ *CAP* criterion was then easily obtained using Eq. (13) and is shown over the flight envelope in Fig. 5. The upper and lower limits are straight lines, each with a slope of 0.5 on the log-log plot. The plot is defined by short-period frequency versus normal load factor per unit angle of attack. *CAP* parameter slightly passes close to the upper limit which indicates excellent dynamic response characteristics in executing flight tasks. A zooming-in view of *CAP* parameter is also shown on the upper left corner.

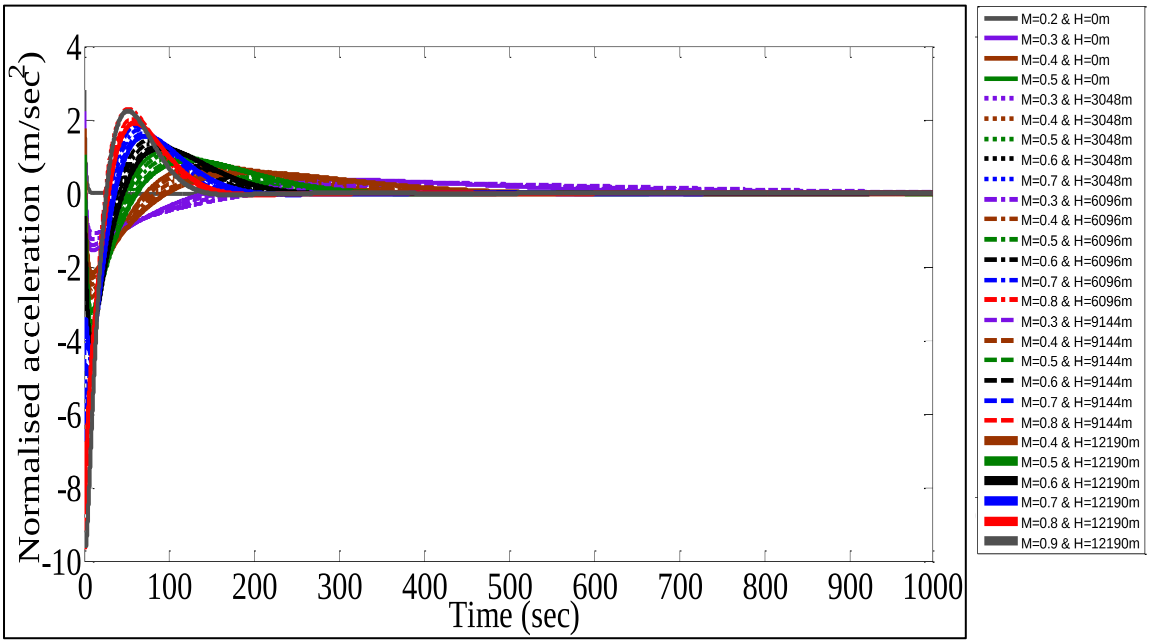


Figure 4. B747-100 normal accelerations over the longitudinal flight envelope

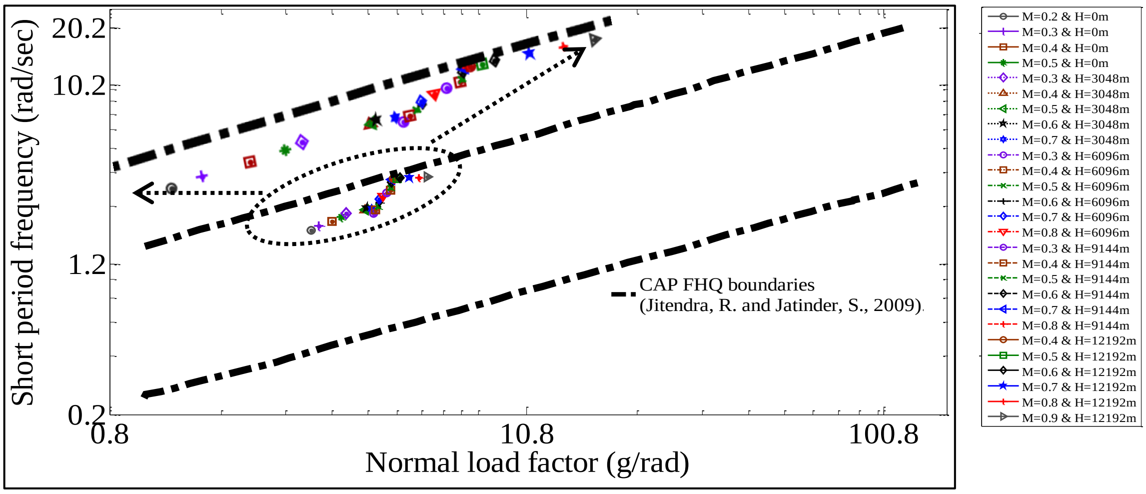


Figure 5. FHQ *CAP* criteria over the B747-100 longitudinal flight envelope

Cstar Criteria Responses

The *Cstar* criterion was obtained by Eq. (16) in terms of transfer function wrt control inputs over the flight envelope ($H_i = 0 - 12190$m and $M_i = 0.2 - 0.9$) using the assumption earlier made, i.e, $V_c = 150$m/sec.

$$\frac{Cstar(s)}{\Delta\delta} = \frac{\begin{gathered}(67.01H + 1.024\times10^6 M + 33.3HM - 1.65\times10^6)s^5 + \\ (-0.12\times10^3 H + 0.35\times10^7 M - 0.14\times10^2 HM + 1.29\times10^7)s^4 + \\ (-2.01\times10^3 H - 2.55\times10^7 M - 0.96\times10^3 HM + 5.87\times10^7)s^3 + \\ (-4.62\times10^3 H - 7.68\times10^7 M - 2.33\times10^3 HM + 10.17\times10^7)s^2 + \\ (-2.4\times10^3 H - 3.1\times10^7 M - 1.4\times10^3 HM + 5.9\times10^7)s + \\ 0.00467\end{gathered}}{\begin{gathered}(-1.04\times10^{-5} H + 0.219M - 2.37\times10^{-6} HM + 0.956)s^5 + \\ (1.55H + 7.2\times10^4 M + 1.13HM + 5.14\times10^4)s^4 + \\ (182.53H + 5.35\times10^6 M + 109.67HM + 2.41\times10^5)s^3 + \\ (-50.64H + 9.73\times10^5 M - 12.26HM + 4.48\times10^6)s^2 + \\ (-90.43H - 1.16\times10^6 M - 43.34HM + 2.63\times10^6)s + \\ (-231.43H - 2.89\times10^6 M - 110.22HM + 6.91\times10^6)\end{gathered}}$$

The normalized $Cstar$ of FHQ criteria is shown in Fig. 6. Slightly infringements the upper boundaries at trainset region are obtained at low Mach numbers and altitudes. However, the steady-state region showed passable convergences in which the evaluations of normalized $Cstar$ wrt steady-state $Cstar$ pass well between the upper and lower boundaries. The steady-state $Cstar$ and the normalized $Cstar$ were obtained using Eqs. (17) and (18) respectively as below,

$$\frac{Cstar_{ss}}{\Delta\delta} = \frac{0.00467}{-231.43H - 2.89\times10^{6}M - 110.22HM + 6.91\times10^{6}}$$

$$\frac{Cstar}{Cstar_{ss}} = (-0.05H - 618.84M - 0.024HM + 1479.66) \times 10^{6}. \frac{\begin{gathered}(67.01H + 1.024\times10^{6}M + 33.3HM - 1.65\times10^{6})s^{5} + \\ (-0.12\times10^{3}H + 0.35\times10^{7}M - 0.14\times10^{2}HM + 1.29\times10^{7})s^{4} + \\ (-2.01\times10^{3}H - 2.55\times10^{7}M - 0.96\times10^{3}HM + 5.87\times10^{7})s^{3} + \\ (-4.62\times10^{3}H - 7.68\times10^{7}M - 2.33\times10^{3}HM + 10.17\times10^{7})s^{2} + \\ (-2.4\times10^{3}H - 3.1\times10^{7}M - 1.4\times10^{3}HM + 5.9\times10^{7})s + \\ 0.00467\end{gathered}}{\begin{gathered}(-1.04\times10^{-5}H + 0.219M - 2.37\times10^{-6}HM + 0.956)s^{5} + \\ (1.55H + 7.2\times10^{4}M + 1.13HM + 5.14\times10^{4})s^{4} + \\ (182.53H + 5.35\times10^{6}M + 109.67HM + 2.41\times10^{5})s^{3} + \\ (-50.64H + 9.73\times10^{5}M - 12.26HM + 4.48\times10^{6})s^{2} + \\ (-90.43H - 1.16\times10^{6}M - 43.34HM + 2.63\times10^{6})s + \\ (-231.43H - 2.89\times10^{6}M - 110.22HM + 6.91\times10^{6})\end{gathered}}$$

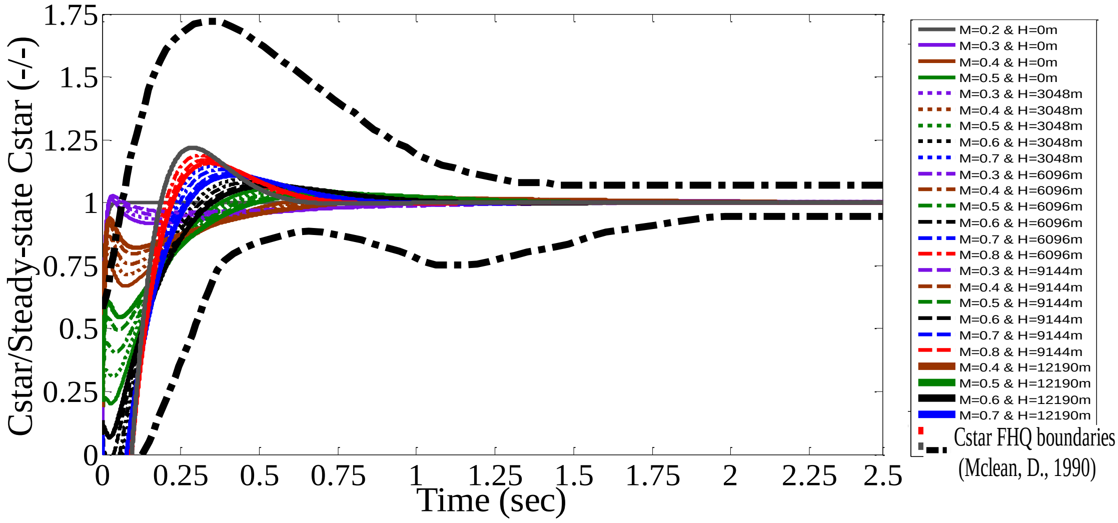


Figure 6. Normalized $Cstar$ FHQ criteria over the B747-100 longitudinal flight envelope

Conclusions

Only the flying handling characteristics of the B747-100 longitudinal flight control have been taken into account through the design cycle of in-flight stressing control law authentications. The gain scheduling design has been arranged to assure acceptable flying qualities criteria within the operational envelope for a safely manoeuvrable flight from one steady-state condition to another. Local FSFLQR controllers have been scheduled at the combinations of Mach numbers and altitudes. Feasibly global control based on the combined elevator and throttle has been obtained governing the whole longitudinal flight envelope. Time responses of velocity, pitch rate, pitch attitude and altitude healthily match the performance specifications of negligible steady-state errors and swift responses of small overshoots and fast transitions. Realistic satisfactions of the FHQs' requirements are achieved based on normalized pitch rate criteria, CAP criteria and normalized $Cstar$ criteria. No obvious infringements the FHQ limits are seen from large-scale assessments conducted within the flight envelope. In addition to the quality of gain scheduling approach being confirmed in producing a uniquely stabilizing control law, it is also shown the success in controlling the normal acceleration which is not primarily state variable.

Finally, the implementation of FSF normal acceleration control would be useful in case of altering the sensed normal accelerations of less than 1g and then to correct the aircraft nose-down attitude.

Recommendations

Further FHQ criteria should be evaluated to validate the performance of the FSFLQR gain scheduling control design. Of those criteria could be the pitch attitude bandwidth and flight-path bandwidth in the frequency domain, and Gibson's dropback criterion in the time domain.

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