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# New Solutions for the Resonant Nonlinear Schrödinger Equation with Anti-Cubic Nonlinearity

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**Abstract**
In this work, the resonant nonlinear Schrödinger equation (RNLSE) with anti-cubic nonlinearity is considered. The Jacobi elliptic function method (JEFM) has been employed on the RNLSE. The many new forms of dark, dark-bright, singular, combo-singular, bright-singular solitons and periodic solutions for governing model are reached. Furthermore, the graphics of solutions are presented.

## 1. Introduction

The main topic of many scientific studies especially in mathematical physics and engineering is related to the nonlinear equations (NLEs).The NLEs exist in all research fields, such as fluid mechanics, plasma physics, biology, chemistry, and so on. The solitons are obtained by dissolving nonlinear structures. Solitons have been theoretically predicted for more than 50 years. Solitons preserves their shape and speed and continue to maintain these properties after any interaction moment. The optical solitons which are the basis of optical fiber are the most important branches of study in the field of soliton [1]. Optical fibers are commonly used in telecommunication, broadcasting, medical field, defense industry have many commercial and scientific applications. Optical Soliton solutions including bright and dark solitons are a class of exact solutions that have diverse applications in the wide areas of applied sciences from sciences to engineering. The nonlinear Schrödinger equations (NLSE) are one of the basic equations from which optical solitons are derived [2]. Several approaches have been deployed on NLSE to $G'/G$ expansion method [3], Fan sub-equation method [4], generalized projective Riccati equation method [5], the Sub-ODE method [6], the exp-function method [7], the F-expansion method [8], Kudryashov method [9]-[10], modified extended tanh expansion method [11] and so on. One of these analytical techniques, the JEFM with a history of 20 years is an effective method in finding optikal soliton solutions of NLSE [12]-[14].

In this work, we consider the RNLSE having anti-cubic nonlinearity

$$i\Psi_t + \beta\Psi_{xx} + (\delta_1|\Psi|^{-4} + \delta_2|\Psi|^2 + \delta_3|\Psi|^4)\Psi + \sigma\frac{|\Psi|_{xx}}{\Psi}\Psi = 0 \qquad (1)$$

here $\Psi(x,t)$ is a wave profile by complex value. $\sigma$ is the coefficient of resonant term, $\beta$ is the coefficient the group velocity dispersion (GVD). $\delta_1, \delta_2$ and $\delta_3$ are the coefficients of anti-cubic, cubic and quintic terms, respectively [15]-[16].

The aim of the current work is to find some new soliton solutions of the equation (1) by JEFM. In accordance with this purpose, this paper is organized as follows. In section 2, a detailed description of the JEFM is presented. Section 3 is devoted to the applications of the proposed method to the RNLSE with anti-cubic nonlinearity. Section 4 contains discussion and results. Also, Figures of optical

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solitons obtained with technique are shown. Consequently, Section 5 contains Conclusion.

## 2. Material and Method

Let us take up a nonlinear partial differential equation (NPDE) unknown function $u$ and free variable $(x,t)$;

$$N(u,u_x,u_t,u_{xx},...)=0 \quad (2)$$

and by considering the travelling wave transformation

$$u(x,t)=U(\xi),\xi=x-vt \quad (3)$$

[17]. Putting (3) into (2) give in an ODE of the form

$$N(U,U',U'',U''',...)=0. \quad (4)$$

After that, we received general form of (4) as follow

$$U(\xi)=\sum_{i=0}^{n}\alpha_i\phi^i(\xi) \quad (5)$$

where $\alpha_i$ are the constans and $\varphi(\xi)$ satisfies the following auxiliary ODE,

$$\phi'(\xi)=\sqrt{\ell\phi^4(\xi)+\wp\phi^2(\xi)+\varsigma} \quad (6)$$

where $\ell,\wp$ and $\varsigma$ are the real constans. The value of $n$ is found using the balancing principle of homogeneity in (4).

Substitting (5) into (4), we get a polynomial of $\varphi(\xi)$. Equating each coefficient of polynomial to zero. We derived a system of algebraic equations which can be solved by the aid of Mathematica program [18].

It is well-known that (6) has families of Jacobi elliptic functions (JEFs) solutions. In this $sn(\xi,m),ns(\xi,m),sc(\xi,m)$ and so on are the same types of JEFs. In this, $m$ denotes the modulus of JEFs, where $0<m<1$. The JEFs degenerate into hyperbolic function when $m\to 1$ and turn into trigonometric functions $m\to 0$ [19].

### 2.1. Application of Method

We consider the following wave transformation for the conservation of (1) in to the nonlinear ODE,

$$\Psi(x,t)=\hat{g}(\xi)e^{i\varphi},\ \xi=x-vt,\ \varphi=-\kappa x+\omega t+\Theta \quad (7)$$

where $\hat{g}$ is the functional form of the complex wave profile. Substituting (7) into (1), and we get the following forms of the imaginary and real parts as below

$$-v\hat{g}'-2\kappa\beta\hat{g}'=0 \quad (8)$$

and

$$(\beta+\sigma)\hat{g}''+(-\omega-\kappa^2\beta)\hat{g}+\delta_1\hat{g}^{-3}+\delta_2\hat{g}^3+\delta_3\hat{g}^5=0. \quad (9)$$

The imaginary part of (1) yields

$$v=-2\kappa\beta. \quad (10)$$

In real part, by balancing $\hat{g}''$ with $\hat{g}^5$ in (9), we come up $n=1/2$. So

$$\hat{g}(\xi)=\tau(\xi)^{1/2}. \quad (11)$$

By putting (11) into (9) and multiplying by $4\tau^{3/2}$, we obtain the following ODE

$$2(\beta+\sigma)\tau\tau''-(\beta+\sigma)\tau'^2+4(-\omega-\kappa^2\beta)\tau^2+4\delta_1+4\delta_2\tau^3+4\delta_3\tau^4=0. \quad (12)$$

Now balancing $\tau\tau''$ with $\tau'$, we get $n=1$. So from (5)

$$\tau(\xi)=\alpha_0+\alpha_1\phi(\xi) \quad (13)$$

where $\alpha_0$ and $\alpha_1$ are constans and $\alpha_1\neq 0$.

Substituting (13) and its necessary derivatives into (12) and collecting all same powers terms of $\phi(\xi)$. So we acquire the following system

$$4\delta_1-4\omega\alpha_0^2-4\beta\kappa^2\alpha_0^2+4\delta_2\alpha_0^3+4\delta_3\alpha_0^4-\varsigma\beta\alpha_1^2-\varsigma\sigma\alpha_1^2=0$$

$$-8\omega\alpha_0\alpha_1+2\wp\beta\alpha_0\alpha_1-8\beta\kappa^2\alpha_0\alpha_1+2\wp\sigma\alpha_0\alpha_1+12\delta_2\alpha_0^2\alpha_1+16\delta_3\alpha_0^3\alpha_1=0$$

$$-4\omega\alpha_1^2+\wp\beta\alpha_1^2-4\beta\kappa^2\alpha_1^2+\wp\sigma\alpha_1^2+12\delta_2\alpha_0\alpha_1^2+24\delta_3\alpha_0^2\alpha_1^2=0$$

$$4\ell\beta\alpha_0\alpha_1+4\ell\sigma\alpha_0\alpha_1+4\delta_2\alpha_1^3+16\delta_3\alpha_0\alpha_1^3=0$$

$$4\ell\beta\alpha_1^2+3\ell\sigma\alpha_1^2+4\delta_3\alpha_1^4=0.$$

Solving the algebraic equations with the Mathematica, and so results as following

$$\alpha_0=\frac{4\omega-\wp\beta+4\beta\kappa^2-\wp\sigma}{3\delta_2},\ \alpha_1=\frac{\sqrt{-3\ell(\sigma+\beta)}}{2\sqrt{\delta_3}}$$

$$\omega = \frac{1}{32\delta_3}(-9\delta_2^2 + 8\wp\beta\delta_3 - 32\beta\delta_3\kappa^2 + 8\wp\sigma\delta_3)$$

$$\sigma = \frac{1}{96\ell\wp\delta_3^2}(9\wp\delta_2\delta_3 - 96\ell\wp\beta\delta_3^2 + \sqrt{3}\sqrt{27\wp^2\delta_2^4\delta_3^2 - 108\ell\wp\delta_2^4\delta_3^2 - 16384\ell\wp\delta_2\delta_3}).$$

Putting these values in (13), we have solution function of $\tau(\xi)$. Subsequently, considering that (11), exact solution form (1) as follow

$$\Psi(x,t) = \left(\frac{1}{6\delta_2}\left(8\omega - 2\wp\beta + 8\beta\kappa^2 + \frac{3\sqrt{3}\delta_2\sqrt{-\ell(\sigma+\beta)}}{\sqrt{\delta_3}}\right)\phi(\xi)\right)^{1/2} e^{i\varphi}. \tag{14}$$

Considering the JEFs for $\phi(\xi)$, the optical soliton, trigonometric function and singular solutions of (1) are obtained.

**Case 1:** If $\ell = m^2, \wp = -(1+m^2), \varsigma = 1$, then $\phi(\xi) = sn\xi$. JEF solution

$$\hat{g}(\xi) = \left(\frac{4\omega + \beta + m^2\beta + 4\beta\kappa^2 + \sigma + m^2\sigma}{3\delta_2} + \frac{\sqrt{3}\sqrt{-m^2(\beta+\sigma)}}{2\sqrt{\delta_3}} sn(\xi)\right)^{1/2}. \tag{15}$$

We can obtain the dark optical soliton solution $m \to 1$

$$\Psi(x,t) = \left(\frac{4\omega + 2\beta + 4\beta\kappa^2 + 2\sigma}{3\delta_2} + \frac{\sqrt{3(-\beta-\sigma)}}{2\sqrt{\delta_3}}\tanh(x - vt)\right)^{1/2} e^{i\varphi}. \tag{16}$$

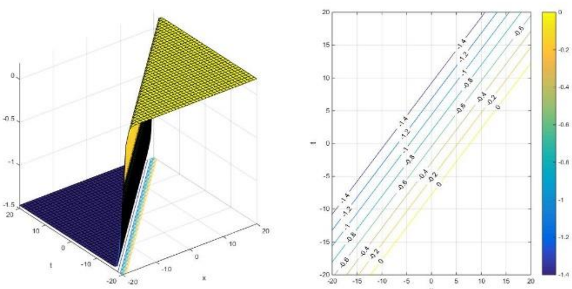

**Figure 1.** The 3D plot and contour plot for solution of Eq. (16). Values chosen are $\delta_1 = -0.1$, $\delta_2 = 2$, $\delta_3 = 1$, $\kappa = 1$ and $\beta = -1$.

**Case 2:** If $\ell = 1, \wp = -(1+m^2), \varsigma = m^2$, then $\phi(\xi) = ns\xi$. So, we get the following Jacobi elliptic function solution

$$\hat{g}(\xi) = \left(\frac{1}{6\delta_2}\left(8\omega + 2\beta + 2m^2\beta + 8\beta\kappa^2 + 2\sigma + 2m^2\sigma + \frac{3\sqrt{3}\delta_2\sqrt{(-\beta-\sigma)}}{\sqrt{\delta_3}} ns(\xi)\right)\right)^{1/2}. \tag{17}$$

We can obtain the singular solution $m \to 1$

$$\Psi(x,t) = \left(\frac{4\omega + 2\beta + 4\beta\kappa^2 + 2\sigma}{3\delta_2} + \frac{\sqrt{3(-\beta-\sigma)}}{2\sqrt{\delta_3}}\coth(x - vt)\right)^{1/2} e^{i\varphi}. \tag{18}$$

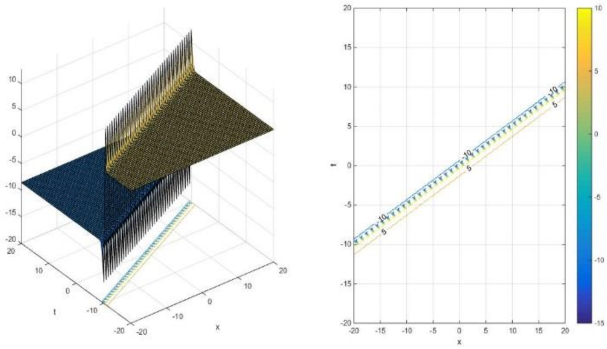

**Figure 2.** The 3D plot and contour plot for solution of Eq. (18). Values chosen are $\delta_1 = -0.1$, $\delta_2 = 5$, $\delta_3 = 1$, $\kappa = 1$ and $\beta = -1$.

**Case 3:** If $\ell = 1 - m^2, \wp = 2 - m^2, \varsigma = 1$, then $\phi(\xi) = sc\xi$. So, we get

$$\hat{g}(\xi) = \left(\frac{1}{6\delta_2}\left((8\omega + 2\beta)(-2 + m^2 + 4\kappa^2) + 2\sigma(-2 + m^2) + \frac{3\sqrt{3}\delta_2\sqrt{(m^2-1)(\beta+\sigma)}}{\sqrt{\delta_3}} sc(\xi)\right)\right)^{1/2}. \tag{19}$$

We obtain the travelling wave solution including of a trigonometric function when $m \to 0$

$$\Psi(x,t) = \left(\frac{1}{6\delta_2}\left(2(4\omega + \beta(-1 + 4\kappa^2) - 2\sigma) + \frac{3\sqrt{3}\delta_2\sqrt{-\beta+\sigma}}{\sqrt{\delta_3}}\tan(x - vt)\right)\right)^{1/2} e^{i\varphi}. \tag{20}$$

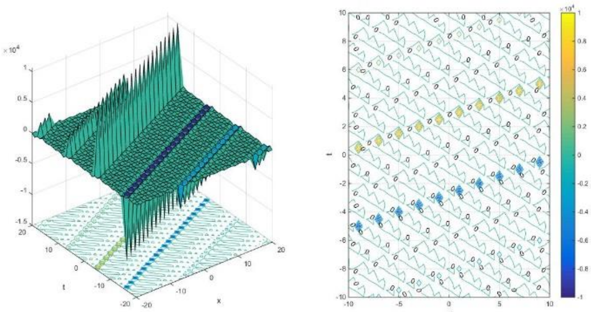

**Figure 3.** The 3D plot and contour plot for solution of Eq. (20). Values chosen are $\delta_1 = 1, \delta_2 = 2$, $\delta_3 = -2, \kappa = 2$ and $\beta = -1$.

**Case 4:** When $\ell = 1, \wp = 2 - m^2, \varsigma = 1 - m^2$, then $\phi(\xi) = cs\xi$. So, from (14)

$$\hat{g}(\xi) = \left( \frac{1}{6\delta_2} \left( 2(4\omega + \beta(-2 + m^2 + 4\kappa^2) + \sigma(-2 + m^2)) + \frac{3\sqrt{3}\delta_2 \sqrt{(-\beta - \sigma)}}{\sqrt{\delta_3}} cs(\xi) \right) \right)^{1/2}. \tag{21}$$

If $m \to 0$, we can obtain the travelling wave solution function

$$\Psi(x,t) = \left( \frac{1}{6\delta_2} \left( 8\omega - 4\beta + 8\beta\kappa^2 - 4\sigma + \frac{3\sqrt{3}\delta_2 \sqrt{-\beta - \sigma}}{\sqrt{\delta_3}} \cot(x - vt) \right) \right)^{1/2} e^{i\varphi}. \tag{22}$$

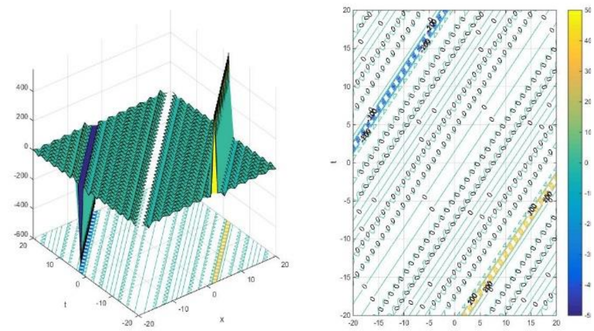

**Figure 4.** The 3D plot and contour plot for solution of Eq. (22). Values chosen are $\delta_1 = 1, \delta_2 = 2$, $\delta_3 = -2, \kappa = 2$ and $\beta = -1$.

**Case 5:** When $\ell = \frac{1 - m^2}{4}, \wp = \frac{1 + m^2}{2}, \varsigma = \frac{1 - m^2}{4}$, then $\phi(\xi) = nc\xi \pm sc\xi$. So, from (14)

$$\hat{g}(\xi) = \left( \frac{1}{6\delta_2} \left( 8\omega - (1 + m^2)\beta + 8\beta\kappa^2 - \sigma(1 + m^2) + \frac{3\sqrt{3}\delta_2 \sqrt{(-1 + m^2)(\beta + \sigma)}}{2\sqrt{\delta_3}} (nc\xi \pm sc\xi) \right) \right)^{1/2}. \tag{23}$$

If $m \to 0$, we can obtain the travelling wave solution function

$$\Psi(x,t) = \left( \frac{1}{6\delta_2} \left( 8\omega - \beta + 8\beta\kappa^2 - \sigma + \frac{3\sqrt{3}\delta_2 \sqrt{-\beta - \sigma}}{2\sqrt{\delta_3}} (\sec(v - vt) \pm \tan(x - vt)) \right) \right)^{1/2} e^{i\varphi}. \tag{24}$$

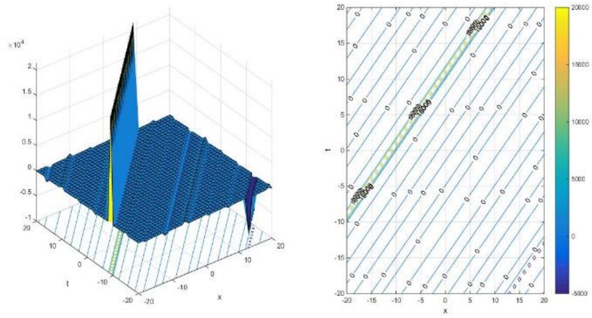

**Figure 5.** The 3D plot and contour plot for solution of Eq. (24). Values chosen are $\delta_1 = 2$, $\delta_2 = 2, \delta_3 = -2, \kappa = 0.1$ and $\beta = -5$.

**Case 6:** If $\ell = \frac{1}{4}, \wp = \frac{m^2 - 2}{2}, \varsigma = \frac{m^2}{4}$, then $\phi(\xi) = ns\xi \pm ds\xi$. So, from (14)

$$\hat{g}(\xi) = \left( \frac{1}{6\delta_2} \left( 8\omega + (2 - m^2 + 8\kappa^2)\beta - \sigma(-2 + m^2) + \frac{3\sqrt{3}\delta_2 \sqrt{(-\beta - \sigma)}}{2\sqrt{\delta_3}} (ns\xi \pm ds\xi) \right) \right)^{1/2}. \tag{25}$$

So, while $m \to 1$, we can obtain the combo singular soliton solution

$$\Psi(x,t) = \left( \frac{1}{6\delta_2} \left( 8\omega + \beta(1 + 8\kappa^2) + \sigma + \frac{3\sqrt{3}\delta_2 \sqrt{-\beta - \sigma}}{2\sqrt{\delta_3}} (\coth(v - vt) \pm \operatorname{csc}h(x - vt)) \right) \right)^{1/2} e^{i\varphi}. \tag{26}$$

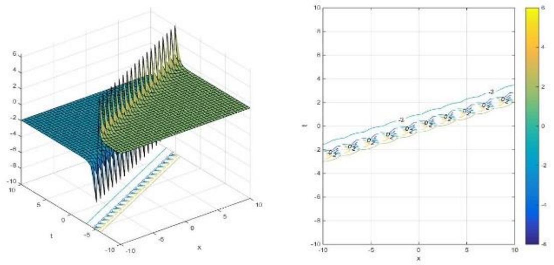

**Figure 6.** The 3D plot and contour plot for solution of Eq. (26). Values chosen are $\delta_1 = 2$, $\delta_2 = 2$, $\delta_3 = -2$, $\kappa = 0.1$ and $\beta = -5$.

**Case 7:** If $\ell > 0, \wp < 0, \varsigma = \dfrac{m^2 \wp^2}{(1+m^2)^2 \ell}$, and this conditions, $\phi(\xi) = \sqrt{-\dfrac{m^2 \wp}{(1+m^2)^2 \ell}}\, sn\left(\sqrt{\dfrac{-\wp}{1+m^2}}\,\xi\right)$. So, from (14)

$$\hat{g}(\xi) = \left(\left(\frac{8\omega + \beta + 4\beta\kappa^2\sigma}{3\delta_2} + \frac{\sqrt{3}\sqrt{(-\beta-\sigma)}}{2\sqrt{\delta_3}}\right)\left(\sqrt{-\frac{m^2\wp}{(1+m^2)^2\ell}}\, sn\left(\sqrt{\frac{-\wp}{1+m^2}}\,\xi\right)\right)\right)^{1/2}. \tag{27}$$

Specially, if $\ell > 0, \wp < 0$ and $m \to 1$, we can obtain the dark optical soliton solution as follow

$$\Psi(x,t) = \left(\frac{4\omega + \beta + 4\beta\kappa^2 + \sigma}{3\delta_2} + \frac{\sqrt{3}\sqrt{(-\beta-\sigma)}}{2\sqrt{2\delta_3}} \tanh\left(\frac{x-vt}{\sqrt{2}}\right)\right)^{1/2} e^{i\varphi}. \tag{28}$$

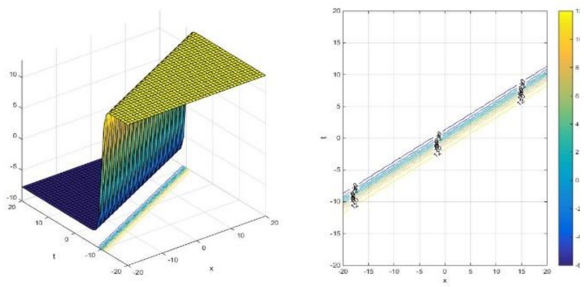

**Figure 7.** The 3D plot and contour plot for solution of Eq. (28). Values chosen are $\delta_1 = 2$, $\delta_2 = 2$, $\delta_3 = -1$, $\kappa = 0.5$ and $\beta = -2$.

**Case 8:** If $\ell = 1, \wp = m^2 + 2, \varsigma = 1 - 2m^2 - 4m^4$, and this conditions, $\phi(\xi) = \dfrac{dn\xi\, cn\xi}{sn\xi}$. So, we get the following function

$$\hat{g}(\xi) = \left(\frac{1}{6\delta_2}\left(8\omega - (2+2m^2)\beta + 8\beta\kappa^2 - 2\sigma(2+m^2) + \frac{3\sqrt{3}\delta_2\sqrt{(-\beta-\sigma)}}{\sqrt{\delta_3}}\left(\frac{dn\xi\, cn\xi}{sn\xi}\right)\right)\right)^{1/2}. \tag{29}$$

In this, if $m \to 1$, we can obtain the bright-singular optical soliton solution as follow

$$\Psi(x,t) = \left(\frac{1}{6\delta_2}\left(8\omega - 6\beta + 8\beta\kappa^2 - 6\sigma + \frac{3\sqrt{3}\delta_2\sqrt{(-\beta-\sigma)}}{\sqrt{\delta_3}}\left(\operatorname{sech}(x-vt)\operatorname{csch}(x-vt)\right)\right)\right)^{1/2} e^{i\varphi}. \tag{30}$$

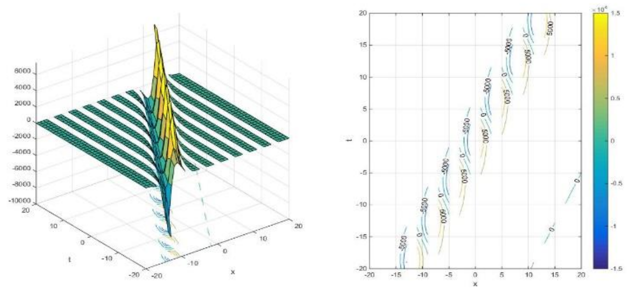

**Figure 8.** The 3D plot and contour plot for solution of Eq. (30). Values chosen are $\delta_1 = 2$, $\delta_2 = 3$, $\delta_3 = -2$, $\kappa = 0.2$ and $\beta = -2$.

**Case 9:** When $\ell = \dfrac{1}{4}, \wp = \dfrac{1-2m^2}{2}, \varsigma = \dfrac{1}{4}$, and $\phi(\xi) = \dfrac{sn\xi}{1 \pm cn\xi}$. So, we get the following function

$$\hat{g}(\xi) = \left(\left(\frac{8\omega + \beta(-1+2m^2+8\kappa^2) + (2m^2-1)\sigma}{6\delta_2} + \frac{\sqrt{3}\sqrt{(-\beta-\sigma)}}{4\sqrt{\delta_3}}\right)\left(\frac{sn\xi}{1 \pm cn\xi}\right)\right)^{1/2}. \tag{31}$$

If $m \to 1$, we can obtain the dark-bright optical soliton solution as follow

$$\Psi(x,t) = \left(\frac{1}{6\delta_2}\left(8\omega + \beta + 8\beta\kappa^2 + \sigma + \frac{3\sqrt{3}\delta_2\sqrt{(-\beta-\sigma)}}{2\sqrt{\delta_3}}\left(\frac{\tanh(x-vt)}{1 \pm \operatorname{sech}(x-vt)}\right)\right)\right)^{1/2} e^{i\varphi}. \tag{32}$$

Also here, while $m \to 0$, we get travelling wave solution with the including of trigonometric function

$$\Psi(x,t) = \left( -\frac{1}{6\delta_2} \left( -8\omega + \beta - 8\beta\kappa^2 + \sigma - \frac{3\sqrt{3}\delta_2\sqrt{(-\beta-\sigma)}}{2\sqrt{\delta_3}} \tan\left( \frac{x-vt}{2} \right) \right) \right)^{1/2} e^{i\varphi}. \tag{33}$$

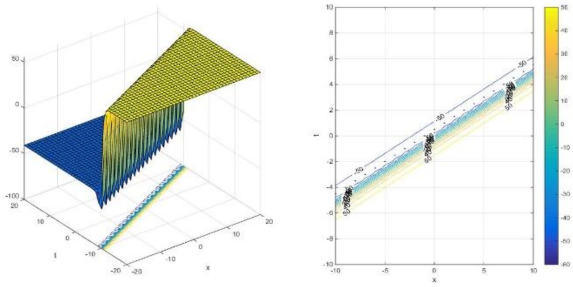


**Figure 9.** The 3D plot and contour plot for solution of Eq. (33). Values chosen are $\delta_1 = 2$, $\delta_2 = 1$, $\delta_3 = -2$, $\kappa = 1$ and $\beta = -1$.

**Case 10**: When

$\ell = \dfrac{2 - m^2 - 2\sqrt{1-m^2}}{4}$, $\wp = \dfrac{m^2}{2} - 1 - 3\sqrt{1-m^2}$,

$\varsigma = \dfrac{2 - m^2 - 2\sqrt{1-m^2}}{4}$, and

$\phi(\xi) = \dfrac{m^2 sn\xi cn\xi}{sn^2\xi(1+\sqrt{1-m^2}dn\xi - 1 - \sqrt{1-m^2})}$. So, we get the following function

$$\hat{g}(\xi) = \left( \frac{1}{12\delta_2} \left( 16\omega + 2\beta(2 - m^2 + 6\sqrt{1-m^2} + 8\kappa^2) + (2m^2 - 1)\sigma + \frac{\sqrt{3}\sqrt{(-\beta-\sigma)}}{4\sqrt{\delta_3}} \right) \left( \frac{sn\xi}{1 \pm cn\xi} \right) \right)^{1/2}. \tag{34}$$

If $m \to 1$, we can obtain the dark-bright optical soliton solution as follow

$$\Psi(x,t) = \left( \frac{1}{6\delta_2} \left( 8\omega + \beta + 8\beta\kappa^2 + \sigma + \frac{3\sqrt{3}\delta_2\sqrt{(-\beta-\sigma)}}{\sqrt{\delta_3}} \left( \frac{\tanh(x-vt)}{1 \pm \mathrm{sech}(x-vt)} \right) \right) \right)^{1/2} e^{i\varphi}. \tag{35}$$

Also here, while $m \to 0$, we get travelling wave solution with the including of trigonometric function

$$\Psi(x,t) = \left( -\frac{1}{6\delta_2} \left( -8\omega + \beta - 8\beta\kappa^2 + \sigma - \frac{3\sqrt{3}\delta_2\sqrt{(-\beta-\sigma)}}{\sqrt{\delta_3}} \tan\left( \frac{x-vt}{2} \right) \right) \right)^{1/2} e^{i\varphi}. \tag{36}$$

## 3. Results and Discussion

This section contain the graphical representation of some new exact traveling wave solutions of the equation (1). The software Mathematica is used to describe the behavior of wave solutions. These solutions include dark solitons, singular soliton, trigonometric function solutions, combo singular soliton, bright-singular soliton and dark-bright optical solitons. The 3D and contour plots for different wave solutions of (1) are demonstrated in Figs. 1-9. $|\Psi(x,t)|^2$ received while drawing figures. Also, $\beta\kappa < 0$ and $\delta_1\delta_3 < 0$ are necessary conditions for all waves to occur. Under these basic conditions, figures are drawn by appropriate selection of the values of arbitrary parameters.

In Fig. 1, (16) shows dark optical soliton. Fig. 2 represents the graph of solution given in (18), which is singular soliton. In Figs. 3,4 and 5, the graphs for (20), (22) and (24) illustrating trigonometric function solutions are shown. Fig.6 represents the graph of solution given in (26), which is a combo singular soliton solution. In Figs. 7 and 8, the graphs for (28) and (30) illustrating dark and bright- singular soliton solutions respectively are shown. Similarly in Fig. 9, the graph for (32) is presented dark-bright optical soliton solutions.

## 4. Conclusion and Suggestions

In this work, we investigate the optical soliton and wave solutions of the RNLSE with anti-cubic nonlinearity throughthe use of the JEFM. The approach is very powerful scheme that first transforms the NLSE to an ODE through a complex wave transformation. So the coefficients of equal power and compared in the obtained ODE's. Finally, the obtained algebraic equation system is solved in Mathematica. The 3D and contour plots of dark solitons, singular soliton solutions, periodic solutions, combo singular solutions, bright-singular soliton and dark-bright soliton solutions are also provided along with suitable choice of values of arbitrary parameters. As a result of the calculations, it has been seen that $\beta\kappa < 0$ and $\delta_1\delta_3 < 0$ are necessary conditions for the formation of soliton and periodic waves. The results presented in this research are novel and can be a valuable addition in the literature.

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**Conflict of Interest Statement**

There is no conflict of interest between the authors.

**Statement of Research and Publication Ethics**

The study is complied with research and publication ethics

**References**

[1] Y. S. Kivshar and G. P. Agrawal, *Optical solitons: From fibers to photonic crystals*. Elsevier Science & Technology, 2003

[2] A. Biswas and S. Konar, *Introduction to non-Kerr law optical solitons*. London, England: CRC Press, 2020.

[3] E. Ulutas, "Travelling wave and optical soliton solutions of the Wick-type stochastic NLSE with conformable derivatives," *Chaos Solitons Fractals*, vol. 148, no. 111052, p. 111052, 2021.

[4] N. Cheemaa and M. Younis, "New and more general traveling wave solutions for nonlinear Schrödinger equation," *Waves Random Complex Media*, vol. 26, no. 1, pp. 30–41, 2016.

[5] A. M. Shahoot, K. A. E. Alurrfi, I. M. Hassan, and A. M. Almsri, "Solitons and other exact solutions for two nonlinear PDEs in mathematical physics using the generalized projective Riccati equations method," *Adv. Math. Phys.*, vol. 2018, pp. 1–11, 2018.

[6] M. Mirzazadeh, R. T. Alqahtani, and A. Biswas, "Optical soliton perturbation with quadratic-cubic nonlinearity by Riccati-Bernoulli sub-ODE method and Kudryashov's scheme," *Optik (Stuttg.)*, vol. 145, pp. 74–78, 2017.

[7] K. Ayub, M. Y. Khan, and Q. Mahmood-Ul-Hassan, "Solitary and periodic wave solutions of Calogero–Bogoyavlenskii–Schiff equation via exp-function methods," *Comput. Math. Appl.*, vol. 74, no. 12, pp. 3231–3241, 2017.

[8] W. B. Rabie and H. M. Ahmed, "Cubic-quartic solitons perturbation with couplers in optical metamaterials having triple-power law nonlinearity using extended F-expansion method," *Optik (Stuttg.)*, vol. 262, no. 169255, p. 169255, 2022.

[9] N. A. Kudryashov, "Method for finding highly dispersive optical solitons of nonlinear differential equations," *Optik (Stuttg.)*, vol. 206, no. 163550, p. 163550, 2020.

[10] N. A. Kudryashov, "Highly dispersive solitary wave solutions of perturbed nonlinear Schrödinger equations," *Appl. Math. Comput.*, vol. 371, no. 124972, p. 124972, 2020.

[11] A. Zafar, M. Raheel, and A. Bekir, "Exploring the dark and singular soliton solutions of Biswas–Arshed model with full nonlinear form," *Optik (Stuttg.)*, vol. 204, no. 164133, p. 164133, 2020.

[12] N. Z. Petrović and M. Bohra, "General Jacobi elliptic function expansion method applied to the generalized (3 + 1)-dimensional nonlinear Schrödinger equation," *Opt. Quantum Electron.*, vol. 48, no. 4, 2016.

[13] T. A. Khalil, N. Badra, H. M. Ahmed, and W. B. Rabie, "Bright solitons for twin-core couplers and multiple-core couplers having polynomial law of nonlinearity using Jacobi elliptic function expansion method," *Alex. Eng. J.*, vol. 61, no. 12, pp. 11925–11934, 2022.

[14] A. Biswas, A. Sonmezoglu, M. Ekici, A. S. Alshomrani, and M. R. Belic, "Highly dispersive singular optical solitons with Kerr law nonlinearity by Jacobi's elliptic ds function expansion," *Optik (Stuttg.)*, vol. 192, no. 162954, p. 162954, 2019.

[15] A. U. Awan, H. U. Rehman, M. Tahir, and M. Ramzan, "Optical soliton solutions for resonant Schrödinger equation with anti-cubic nonlinearity," *Optik (Stuttg.)*, vol. 227, no. 165496, p. 165496, 2021.

[16] K. S. Nisar, K. K. Ali, Mustafa Inc, M. S. Mehanna, H. Rezazadeh, and L. Akinyemi, "New solutions for the generalized resonant nonlinear Schrödinger equation," *Results Phys.*, vol. 33, no. 105153, p. 105153, 2022.

[17] S. Tarla, K. K. Ali, R. Yilmazer, and M. S. Osman, "New optical solitons based on the perturbed Chen-Lee-Liu model through Jacobi elliptic function method," *Opt. Quantum Electron.*, vol. 54, no. 2, 2022.

[18] L. Gürgöze, "Exact Solutions With Jacobi Elliptic Function Method of Some Nonlineer Equations," Firat University, 2022.

[19] E. M. E. Zayed, R. M. A. Shohib, A. Biswas, Y. Yıldırım, F. Mallawi, and M. R. Belic, "Chirped and chirp-free solitons in optical fiber Bragg gratings with dispersive reflectivity having parabolic law nonlinearity by Jacobi's elliptic function," *Results Phys.*, vol. 15, no. 102784, p. 102784, 2019.