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Existence and Uniqueness Results for a Computer Virus Spreading Model with Atangana-Baleanu Derivative

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Abstract

A virus is programs that are often hidden inside a file that appears to be harmless and cause unexpected and unwanted events on the computer when the file is run. Viruses, unlike others, are software designed to maliciously damage systems. Viruses have the ability to copy themselves, jump to the files you work with, and delete files, change the file content and make it unavailable. They can do all this without the knowledge of the user and without the need for any command. Therefore, in order to better understanding of computer virus, there are many mathematical models in the literature. In the present study, we consider computer virus spreading model benefiting from Atangana-Baleanu derivative in Caputo sense with nonlocal and non-singular kernels. The solution properties of our fractional model are established benefiting from Arzelo-Ascoli theorem.

Keywords: Arzelo-Ascoli theorem, Atangana-Baleanu derivative, existence and uniqueness results.

1. Introduction

Computer viruses are some computer programs that can damage your computer in different ways when it works. If these programs (or virus codes) are run, they will start damaging your computer according to the way they are programmed. In addition, after viruses are activated in a system, they have the ability to reproduce, spread to other files on your computer, infect other computers over the network and many more. Many mathematical models are being studied to cope with the spread of computer viruses and for better understanding their structure such as SIR model, SEIRS model, SIRS model [1-8].

Due to memory properties of fractional derivatives, fractional operators gain increasing interest from various directions in the modeling of biological process, neural networks, engineering, physics, etc [9-20]. It is clear that Riemann Liouville and Caputo fractional derivatives have singular kernels. To cope with this problem, Atangana and Baleanu have proposed a new fractional derivative named as Atangana-Baleanu (AB) fractional derivative with Mittag-Leffler kernel. There are many extensive treatment and several applications of this recently defined AB derivatives in the literature [21-28].

Taking these motivations and AB derivative into consideration, in this article we examine the existence and uniqueness conditions for the solutions of the below computer virus spreading model presented by [29]:

$$\frac{dS}{dt} = b - \beta SI - \mu S,$$

$$\frac{dE}{dt} = (1 - p)\beta SI - \gamma E + \varepsilon I - \mu E,$$

$$\frac{dI}{dt} = p\beta SI + \gamma E - \varepsilon I - \alpha I - \mu I.$$
(1.1)

We choose this model because it defines a novel virus spreading system considering the possibility of a virus outbreak on a network with restricted antivirus ability. Here, the total number of computers attached the internet are separated into four class: The number of susceptible computers S(t) is the set of external not infected computers which are attached to the network, the number of exposed computers E(t) is the set of all latent computers at which viruses are latent, the number of infected computers I(t) is the set of infected computers I(t)



computers at which viruses are explode. The model parameters are: *b* represents the ratio where external computers are linked with the network, β stands for the ratio where having a link to one latent computer, γ shows the ratio where one latent computer explodes, μ displays the ratio where one computer is out from the network, α displays the recovery ratio of infected computers and connect to the the ability of the anti-virus software. Owing to a possible link with infected computers, susceptible computer is latent with possibility $(1-p)\beta I(t)$, or explodes with possibility $p\beta I(t)$ where p > 0 is a fixed parameter. Since the network's anti-virus ability is restricted, the virus is momentarily suppressed with possibility \mathcal{E} where

In this paper, inspired by the above model, we give properties of the solution of our fractional model with AB derivative by Arzelo-Ascoli theorem.

2. Preliminaries

 $\varepsilon > 0$ is a fixed parameter.

Now, we will give basic definitions related to AB fractional derivative.

Definition 2.1. Let $f \in H^1(a, b)$ be a function, a < band $r \in [0,1]$. The AB derivative in Caputo sense of f is defined as [30]

$${}^{ABC}_{0}D_{r}^{r}f(t) = \frac{K(r)}{1-r}\int_{a}^{t}f'(x)E_{r}\left[-r\frac{(t-x)^{r}}{1-r}\right]dx \quad (2.1)$$

with K(r) is a normalization function with K(0) = K(1) = 1.

Definition 2.3. The fractional integral relevant to AB derivative is given by [30]

$${}^{AB}_{0}I_{r}^{r}f(t) = \frac{1-r}{K(r)}f(t) + \frac{r}{K(r)\Gamma(r)}\int_{r}^{t}f(y)(t-y)^{r-1}dy \qquad (2.2)$$

3. Existence and Uniqueness of the Solutions

Now, we remodel the system (1.1) using AB derivative in Caputo sense.

$${}^{ABC}_{0}D^{r}_{t}S = b - \beta SI - \mu S,$$

$${}^{ABC}_{0}D^{r}_{t}E = (1 - p)\beta SI - \gamma E + \varepsilon I - \mu E,$$

$${}^{ABC}_{0}D^{r}_{t}I = p\beta SI + \gamma E - \varepsilon I - \alpha I - \mu I.$$
(3.1)

In this part, the existence and uniqueness of the solutions for the model (3.1) is considered and proved by using AB derivative. For this aim, we apply AB integral to the system (3.1) and we get

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$$S(t) - S(0) = K \frac{1-r}{K(r)} \{b - \beta SI - \mu S\}$$

$$+ \frac{r}{K(r)\Gamma(r)} \int_{0}^{t} (t - y)^{r-1} \{b - \beta SI - \mu S\} dy,$$

$$E(t) - E(0) = \frac{1-r}{K(r)} \{(1 - p)\beta SI - \gamma E + \varepsilon I - \mu E\}$$

$$+ \frac{r}{K(r)\Gamma(r)} \int_{0}^{t} (t - y)^{r-1} \{(1 - p)\beta SI - \gamma E + \varepsilon I - \mu E\} dy,$$

$$I(t) - I(0) = \frac{1-r}{K(r)} \{p\beta SI + \gamma E - \varepsilon I - \alpha I - \mu I\}$$

$$+ \frac{r}{K(r)\Gamma(r)} \int_{0}^{t} (t - y)^{r-1} \{p\beta SI + \gamma E - \varepsilon I - \alpha I - \mu I\} dy.$$
(3.2)

For the simplification of equations in (3.2), we define our kernels below:

$$s(t, S(t)) = b - \beta SI - \mu S,$$

$$s(t, E(t)) = (1 - p)\beta SI - \gamma E + \varepsilon I - \mu E,$$

$$s(t, I(t)) = p\beta SI + \gamma E - \varepsilon I - \alpha I - \mu I.$$
(3.3)

Now, we will consider the operator $G: H \to H$ defined as

$$GS(t) = \frac{1-r}{K(r)} s(t, S(t)) + \frac{r}{K(r)\Gamma(r)} \int_{0}^{t} (t-y)^{r-1} s(y, S(y)) dy, GE(t) = \frac{1-r}{K(r)} s(t, E(t)) + \frac{r}{K(r)\Gamma(r)} \int_{0}^{t} (t-y)^{r-1} s(y, E(y)) dy, GI(t) = \frac{1-r}{K(r)} s(t, I(t)) + \frac{r}{K(r)\Gamma(r)} \int_{0}^{t} (t-y)^{r-1} s(y, I(y)) dy.$$
(3.4)



Lemma 3.1. Let $M \subset H$ be a bounded set, assume that S,E and I satisfy Lipschitz condition $||S(t_2) - S(t_1)|| \le p ||t_2 - t_1||,$ $||E(t_2) - E(t_1)|| \le r ||t_2 - t_1||,$ $||I(t_2) - I(t_1)|| \le q ||t_2 - t_1||.$

For some positive constants p, q, r. Then $\overline{G(M)}$ is compact.

Proof 3.1 Let
$$P = \max\left\{\frac{1-r}{K(r)} + s(t, S(t))\right\},\$$

 $0 \le S(t) \le K_1$ for some positive constant K_1 . For $S(t) \in M$, then we get

$$\|GS(t)\| = \frac{1-r}{K(r)} \|s(t,S(t))\|$$

+ $\frac{r}{K(r)\Gamma(r)} \int_{0}^{r} (t-y)^{r-1} \|s(y,S(y))\| dy$
$$\leq \frac{1-r}{K(r)} P + \frac{rPt^{r}}{K(r)\Gamma(r+1)}.$$
 (3.5)

Let
$$R = \max\left\{\frac{1-r}{K(r)} + s(t, E(t))\right\}, \quad 0 \le E(t) \le K_2$$

for some positive constant K_2 . For $E(t) \in M$, then we get

$$\|GE(t)\| = \frac{1-r}{K(r)} \|s(t, E(t))\| + \frac{r}{K(r)\Gamma(r)} \int_{0}^{t} (t-y)^{r-1} \|s(y, E(y))\| dy$$

$$\leq \frac{1-r}{K(r)} R + \frac{rRt^{r}}{K(r)\Gamma(r+1)}$$
(3.6)

And similarly, we consider third equation, let

$$Q = \max\left\{\frac{1-r}{K(r)} + s(t, I(t))\right\}, \ 0 \le I(t) \le K_3 \quad \text{for}$$

some positive constant K_3 . For $I(t) \in M$

$$\|GI(t)\| = \frac{1-r}{K(r)} \|s(t, I(t))\|$$

+ $\frac{r}{K(r)\Gamma(r)} \int_{0}^{t} (t-y)^{r-1} \|s(y, I(y))\| dy$
 $\leq \frac{1-r}{K(r)} Q + \frac{rQt^{r}}{K(r)\Gamma(r+1)}.$ (3.7)

Thus from equations (3.5)-(3.7) we can conclude that the operator G is bounded.

Now, we assume $t_1 < t_2$ and $S(t) \in M$. Taking into account that *M* is bounded, for $\mathcal{E} > 0$, if $|t_2 - t_1| < \delta$ we have

$$\begin{split} \|GS(t_{2}) - GS(t_{1})\| &= \left\| \frac{1-r}{K(r)} s(t_{2}, S(t_{2})) + \frac{r}{K(r)} \Gamma(r) \int_{0}^{t_{1}} (t_{2} - y)^{r-1} s(y, S(y)) dy - \frac{1-r}{K(r)} s(t_{1}, S(t_{1})) \\ &- \frac{1-r}{K(r)} s(t_{1}, S(t_{1})) \\ &- \frac{r}{K(r)} \Gamma(r) \int_{0}^{t_{1}} (t_{1} - y)^{r-1} s(y, S(y)) dy \\ &\leq \frac{1-r}{K(r)} \| s(t_{2}, S(t_{2})) - s(t_{1}, S(t_{1})) \| \\ &+ \left\| \frac{r}{K(r)} \Gamma(r) \int_{0}^{t_{2}} (t_{2} - y)^{r-1} s(y, S(y)) dy \\ &- \frac{r}{K(r)} \Gamma(r) \int_{0}^{t_{1}} (t_{1} - y)^{r-1} s(y, S(y)) dy \\ &\leq \frac{1-r}{K(r)} \| s(t_{2}, S(t_{2})) - s(t_{1}, S(t_{1})) \| \\ &\leq \frac{1-r}{K(r)} \| s(t_{2}, S(t_{2})) - s(t_{1}, S(t_{1})) \| \\ &+ \frac{rK_{1}}{K(r)} \left\{ \int_{0}^{t_{1}} (t_{2} - y)^{r-1} dy - \int_{0}^{t_{1}} (t_{1} - y)^{r-1} dy \right\}. \end{split}$$
(3.8)

We have that

$$\int_{0}^{t_{1}} (t_{2} - y)^{r-1} dy - \int_{0}^{t_{1}} (t_{1} - y)^{r-1} dy$$
$$= \frac{(t_{2} - t_{1})^{r}}{r}.$$
 (3.9)

We study the following

$$\| s(t_2, S(t_2)) - s(t_1, S(t_1)) \|$$

$$\leq (\beta(a+b) + \mu) \| S(t_2) - S(t_1) \|$$

$$\leq J_1 \| t_2 - t_1 \|.$$
 (3.10)

Then, putting Eqs. (3.9)-(3.10) in (3.8), we obtain



$$\|GS(t_{2}) - GS(t_{1})\| \le \frac{1-r}{K(r)}J_{1}\|t_{2} - t_{1}\|$$
$$+ \frac{rK}{K(r)\Gamma(r)}\frac{\|t_{2} - t_{1}\|^{r}}{r}$$

Let
$$\delta_1 = \frac{\varepsilon}{\frac{1-r}{K(r)}J_1 + \frac{rK_1}{K(r)\Gamma(r+1)}}$$
 then we get
 $\|GS(t_2) - GS(t_1)\| \le \varepsilon.$

In the same manner, we can get the following results for the other two functions: If we get $\varepsilon > 0$, we have

$$\begin{split} \delta_2 &= \frac{\varepsilon}{\frac{1-r}{K(r)}J_2 + \frac{rK_2}{K(r)\Gamma(r+1)}},\\ \delta_3 &= \frac{\varepsilon}{\frac{1-r}{K(r)}J_3 + \frac{rK_3}{K(r)\Gamma(r+1)}}, \end{split}$$

then we obtain

$$\left\|GE\left(t_{2}\right)-GE\left(t_{1}\right)\right\| \leq \varepsilon,$$

$$\left\|GI\left(t_{2}\right)-GI\left(t_{1}\right)\right\| \leq \varepsilon.$$

Thus $\overline{G(M)}$ is equicontinuous and so $\overline{G(M)}$ is compact by Arzelo-Ascoli Theorem. \Box

Theorem 3.2. Let $S:[a,b] \times [0,\infty) \rightarrow [0,\infty)$ be increasing for each t in [a,b] and be a continuous function. Suppose that one can find u, v satisfying $M(D)u \leq S(t,u), \qquad M(D)v \geq S(t,v)$ for $0 \leq u(t) \leq v(t)$ and $a \leq t \leq b$. Thus, system (1.1) has a positive solution.

Proof 3.2. Now we handle the fixed point operator *G*. We know that the operator $G: H \to H$ is completely continuous. Let $S_1 \leq S_2, E_1 \leq E_2, I_1 \leq I_2$ then we get

$$GS_{1}(t) \leq \frac{1-r}{K(r)} s(t, S_{1}(t)) + \frac{r}{K(r)\Gamma(r)} \int_{0}^{t} (t-y)^{r-1} \| s(y, S_{1}(y)) \| dy$$
$$\leq GS_{2}(t).$$
(3.11)

Following similar steps, we obtain

$$GE_{1}(t) \leq \frac{1-r}{K(r)} s(t, E_{1}(t)) + \frac{r}{K(r)\Gamma(r)} \int_{0}^{t} (t-y)^{r-1} \| s(y, E_{1}(y)) \| dy$$
$$\leq GE_{2}(t)$$
(3.12)

and

$$GI_{1}(t) \leq \frac{1-r}{K(r)} s(t, I_{1}(t)) + \frac{r}{K(r)} \int_{0}^{t} (t-y)^{r-1} \| s(y, I_{1}(y)) \| dy$$
$$\leq GI_{2}(t)$$
(3.13)

Thus, *G* is increasing operator. From the conjecture, $Gm \ge m$, $Gn \le n$. So $G: \langle m, n \rangle \rightarrow \langle m, n \rangle$ is compact and continuous from Lemma 3.1. Thus, *H* is a normal cone.

Now, we will investigate the uniqueness of solutions. To manage this, we study the followings:

$$\|GS_{1}(t) - GS_{2}(t)\| \leq \frac{1-r}{K(r)} \|s(t, S_{1}(t)) - s(t, S_{2}(t))\|$$

+ $\frac{r}{K(r)\Gamma(r)} \int_{0}^{t} (t-y)^{r-1} \|s(y, S_{1}(y)) - s(y, S_{2}(y))\| dy$
 $\leq \frac{1-r}{K(r)} F_{1} \|S_{1}(t) - S_{2}(t)\|$
+ $\frac{r}{K(r)\Gamma(r)} F_{1} \int_{0}^{t} (t-y)^{r-1} \|S_{1}(y) - S_{2}(y)\| dy$
which gives

$$\|GS_{1}(t) - GS_{2}(t)\| \leq \left\{\frac{1-r}{K(r)}F_{1} + \frac{rF_{1}b'}{K(r)\Gamma(r+1)}\right\} \times \|S_{1}(t) - S_{2}(t)\|$$
(3.14)

By a similar method, we obtain

$$\|GE_{1}(t) - GE_{2}(t)\| \leq \left\{ \frac{1-r}{K(r)}F_{2} + \frac{rF_{2}b'}{K(r)\Gamma(r+1)} \right\} \times \|E_{1}(t) - E_{2}(t)\|.$$
(3.15)

$$\|GI_{1}(t) - GI_{2}(t)\| \leq \left\{\frac{1-r}{K(r)}F_{3} + \frac{rF_{3}b^{r}}{K(r)\Gamma(r+1)}\right\} \times \|I_{1}(t) - I_{2}(t)\|.$$
(3.16)



Therefore, if the following conditions satisfy

$$\frac{1-r}{K(r)}F_{1} + \frac{rF_{1}b'}{K(r)\Gamma(r+1)} < 1,$$

$$\frac{1-r}{K(r)}F_{2} + \frac{rF_{2}b'}{K(r)\Gamma(r+1)} < 1,$$

$$\frac{1-r}{K(r)}F_{3} + \frac{rF_{3}b'}{K(r)\Gamma(r+1)} < 1,$$

the mapping G is a contraction, so it has a fixed point by Banach fixed-point theorem. Thus the new model has a unique positive solution.

4. Numerical Results

Here, in order to observe how fractional order *r* affects behavior of the fractional model (1.1), some numerical simulations of this model are depict benefiting from the numerical method presented by Toufik and Atangana in the paper [31]. We select the parameters b=5, $\beta = 0.007$, $\mu = 0.6$, $\gamma = 0.25$, $\varepsilon = 0.3$, p = 0.7 with the initial conditions S(0)=3, E(0)=1, I(0)=1 as given in [29]. From Figure 1, we see the behavior of susceptible, exposed and infected computers. From Figure 2, it is visible that as *r* goes up, the number of susceptible computers S(t) increases while the number of exposed computers E(t) and infected computers I(t) decreases.



Figure 1. Numerical simulations for the model (1.1) at r=0.95 and r=0.6, respectively.



Figure 2. The behavior of the fractional computer virus spreading model components for distinct values *r*.

5. Conclusion

In this study, Atangana–Beleanu derivative with Mittag-Leffler kernel has been applied in reformulating the computer virus spreading model presented by Xu and Ren [29]. Then, Arzelo-Ascoli theorem is used to prove the existence and uniqueness properties of the considered model. Benefiting from Toufik-Atangana method [31], some numerical simulations are depicted with several values r and briefly interpreted. We expect that the present study will be more helpful to construct computer virus spreading model with fractional derivatives.

Author's Contributions

Sümeyra Uçar drafted the manuscript, compiled information from the literature, and designed the figures and tables.

Ethics

There are no ethical issues after the publication of this manuscript.

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