ANOTHER WAY TO DETERMINE WEIGHTS OF BALANCED PERFORMANCE EVALUATIONS

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Abstract

In case of multiple inputs and outputs, performance of Decision Making Units (DMU) is defined as the ratio of weighted sum of outputs to weighted sum of inputs. There are two group ways to determine the weights of performance : objective and subjective approaches mainly. In the subjective approaches, weights which will be given to the inputs and outputs are determined based on the opinion of DMUs or experts. In the objective approaches, weights are found via models and calculations which are not based on personal judgments. One of them is the most important and widely used Data Envelopment Analysis (DEA) method. Data Envelopment analysis is a nonparametric and operations research-based technique. DEA, in the performance calculations, assigns weights to multiple inputs and outputs in an objective manner by means of a linear programming model to maximize the performance of each DMU.

There may be two disadvantages for the weights which calculated by this method:

I. To give very small or zero weights to important inputs and outputs.

II. In aggregate evaluation, computed weights generally to be different for each input and output for different decision-makers; in the performance evaluation, importances or weights of the inputs and outputs not to happen same for every DMU.

One way for eliminate the disadvantages mentioned above is to use common weights when calculating the performance of DMUs. Another method is to use the correlation coefficients between inputs and outputs. Mentioned methods in this work will be interpreted by applying to the data of a real-world problem.

Keywords: Data envelopment analysis, common weigths, performans calculations, correlation.

Jell Classification: C02, C44, C61, C67.

DENGELİ PERFORMANS AĞIRLIKLARININ HESAPLANMASINDA DİĞER BİR YOL

Öz

Çok girdi ve çıktı olması durumunda, karar verme birimlerinin (KVB) performans hesabı, ağırlıklı çıktılar toplamı bölü ağırlıklı girdiler toplamı olarak tanımlanır. Performans ağılıklarını belirlemede başlıca iki yol vardır: Sübjektif ve objektif yaklaşımlar. Sübjektif yaklaşımlarda girdi ve çıktılara verilen ağırlıklar KVB'nin ya da uzmanların görüşlerine dayalı belirlenir. Objektif yaklaşımlarda ise ağırlıklar kişisel görüşlere dayanmayarak model ve hesaplamalar yardımıyla tespit edilir. Bunlardan en yaygınca kullanılanı Veri Zarflama Analizi (VZA) yöntemidir. VZA yöntemi parametrik olmayan yöneylem araştırması tabanlı bir tekniktir. VZA performans hesaplamalarında çok girdi ve çok çıktıyı her KVB'nin performansını en büyük yapacak ağırlıkları doğrusal programlamayla objektif biçimde hesaplar.

Bu yöntemle hesaplanan ağırlıklar için iki dezavantaj vardır:

I.Önemli girdi ve çıktılara sıfıra yakın veya sıfır ağırlık vermek.

II. Performans hesaplamalarında her bir girdi ve çıktıya farklı karar vericiler için farklı ağırlıklar ataması

KVB'lerinin performansı hesaplanırken yöntemin yukarıda bahsedilen dezavantajlarını elimine etmenin bir yolu ortak ağırlıklar kullanmaktır. Başka bir yöntem girdilerle çıktılar arasında korelasyonları kullanmaktır.

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Introduction

There are methods for computing performance of decision making units(DMUs). Most popular, most used method is Data Envelopment Analysis (DEA) nowadays. What can be said about the data envelopment analysis. Data Envelopment Analysis (DEA) was developed by Charnes, Cooper, Rhodes (CCR) in 1978 . DEA is a nonparametric method which is used for the evaluation of the efficiency of DMUs. DEA can use multiple inputs and outputs for the evaluation of the efficiency of DMUs. The standard DEA models are run separately for each DMU to calculate the maximum relative efficiency. DEA classify DMUs two classes: efficient or inefficient units. The efficiency score for efficient DMUs are equal to unity or 100. The efficiency score for inefficient DMUs are less than unity or 100. Weight of each input and output for each decision-maker may vary to calculate the maximum relative efficiency. Usually they are different. Generally most of weights of inputs and outputs are zero or very close to zero. Therefore this means that those input(s) and output(s) not taken into account in the calculation of the performance.

For example, data used in this study is the following:

Table 1: Input and output data of the 17 forest districricts in Taiwan

DMU	budget{I}	Initial STOC{I}	L a - bor{I}	Land{I}	Main Product{O}	Soil conversation{O}	Recretion visit{O}
1	51,62	11,23	49,22	33,52	40,49	14,89	3155,71
2	85,78	123,98	55,13	108,46	43,51	173,93	6,45
3	66,65	104,18	257,09	13,65	139,74	115,96	0,00
4	27,87	107,60	14,00	146,43	25,47	131,79	0,00
5	51,28	117,51	32,07	84,50	46,20	144,99	0,00
6	36,05	193,32	59,52	8,23	46,88	190,77	822,92
7	25,83	105,80	9,51	227,20	19,40	120,09	0,00
8	123,02	82,44	87,35	98,80	43,33	125,84	404,69
9	61,95	99,77	33,00	86,37	45,43	79,60	1252,60
10	80,33	104,65	53,30	79,06	27,28	132,49	42,67
11	250,62	183,49	144,10	59,66	14,09	196,29	16,15
12	82,09	104,94	46,51	127,28	44,87	108,53	0,00
13	202,21	187,74	149,39	93,65	44,97	184,77	0,00
14	67,55	82,83	44,37	60,85	26,04	85,00	23,95
15	72,60	132,73	44,67	173,48	5,55	135,65	24,13
16	84,83	104,28	159,12	171,11	11,53	110,22	49,09
17	71,77	88,16	69,19	123,14	44,83	74,54	6,14

DEA solution of this data is the following:

Table 2: Results Of İnput Oriented Ccr Solutions And Weigths Of İnput And Outputs

DMU	Performance Score	Budget {I}{W}	Initial STOC{I}{W}	Labor {I}{W}	Land {I}{W}	Main Product {O}{W}	Soil Conversation {O}{W}	Recretion visit {O}{W}
1	100,00%	0,000	0,089	0,000	0,000	0,000	0,000	0,000
2	100,00%	0,000	0,007	0,002	0,000	0,000	0,006	0,000
3	100,00%	0,000	0,005	0,000	0,032	0,007	0,000	0,000
4	100,00%	0,004	0,000	0,028	0,003	0,000	0,008	0,000
5	100,00%	0,000	0,004	0,011	0,002	0,011	0,003	0,000
6	100,00%	0,000	0,000	0,008	0,064	0,000	0,005	0,000
7	100,00%	0,000	0,000	0,105	0,000	0,000	0,008	0,000
8	100,00%	0,000	0,012	0,000	0,000	0,000	0,008	0,000
9	100,00%	0,000	0,000	0,028	0,001	0,018	0,000	0,000
10	94,03%	0,000	0,007	0,000	0,004	0,000	0,008	0,000
11	93,46%	0,000	0,005	0,000	0,002	0,000	0,005	0,000
12	82,90%	0,000	0,008	0,004	0,000	0,006	0,007	0,000
13	79,97%	0,000	0,004	0,000	0,002	0,000	0,005	0,000
14	77,33%	0,000	0,008	0,003	0,004	0,006	0,010	0,000
15	76,27%	0,000	0,006	0,003	0,000	0,000	0,007	0,000
16	74,35%	0,001	0,009	0,000	0,000	0,000	0,009	0,000
17	68,73%	0,006	0,002	0,006	0,000	0,022	0,000	0,000

In some DMU's performance calculations some input(s) and output(s) may considered but some DMU's performance calculations not considered. So, in fact, DMUs did not evaluated with the same criterias. Another very important point is this: If multiple DMUs are efficient then DEA gives all of them 1 or 100 same performance score. Therefore, a complete ranking of all DMUs cannot be made. Furthermore, DEA has a rule of thumb for implementation of the technique: the number of decision-making units must be more than 3 times to the number of input plus number of outputs. These are some problems and weak sides of DEA.

Overcome this problem, authors was doing some studies and these attempts even are continuing up to these days. No doubt that these attempts will continue in the future. Some of these are the followings:

- Anderson–Peterson (1993) proposed a method based on super efficiency scores. The super efficiency score is more than unity for the extremely efficient DMUs and is equal to unity for the non-extremely efficient DMUs. It is less than unity for the inefficient DMUs.
- Thompson et al.(1990) defined the restriction of the input weights independent of the output weights

- Mecit and Alp (2013) offered a new AR Aproach (the ARIII (COR)) which is restricted by correlation coefficients, is developed. The purpose of the proposed approach is to assign weights according to the correlations between the input and output variables in the weight restriction.

- Authours of some other studies:

Bal and Örkçü (2011), Cook et al.(2009), Torgersen et al.(1996), Mehrabian et al.(1999), Roll et al. (1991) , Li et al. (2008), Podinovski(2007), Sinuany-Stern et al. (1994) , Wong and Beasley(1990), Cooper and Tone (1997), Kao and Hung (2005), Adler et al (2002)., Angulo-Meza and Estellita Lins (2002), Doyle and Green (1994), Ganley and Cubbin (1992), Troutt (1995)…

1. A Close Look To Performance Calculation

Evaluations of performance and it is especially concerned with evaluating the activities of organizations such as business firms, government agencies, hospitals, educational institutions, etc. Such evaluations take a variety of forms in customary analyses.

Examples include

- cost per unit,
- profit per unit,
- satisfaction per unit, and so on,

which are measures stated in the form of a ratio like the following,

$$\frac{\textbf{OUTPUT}}{\textbf{INPUT}}$$

This is a commonly used measure of efficiency. The usual measure of "productivity" also assumes a ratio form when used to evaluate worker or employee performance.

- "Output per worker hour" or
- "output per worker employed"

are examples with sales, profit or other measures of output appearing in the numerator. Such measures are sometimes referred to as "partial productivity measures.

This terminology is intended to distinguish them from "total factor productivity measures," because the latter attempt to obtain an output-to input ratio value which takes account of all outputs and all inputs.

$$\text{Virtual input} = v_1 x_{10} + v_2 x_{20} + \ldots + v_m x_{m0}$$
$$\text{Virtual output} = u_1 y_{10} + u_2 y_{20} + \ldots + u_s y_{s0}$$

Then efficiency is equal to:

$$\frac{\text{Virtual Output}}{\text{Virtual input}} = \frac{u_1 y_{10} + u_2 y_{20} + \ldots + u_s y_{s0}}{v_1 x_{10} + v_2 x_{20} + \ldots + v_m x_{m0}}$$

Where ui: weight of ith output,

vi: weight of ith input,

s: number of output,

m: number of input,

No doubt that units of inputs and outputs are very different from each others.

In this equation weights of inputs and outputs can be determined with two ways: Subjective or Objective manner.

Subjective approach: If weights of inputs and outputs determined based on preference of DMU or opinion of an expert then this way is a subjective approach. Objective approach: If weights of inputs and outputs determined by a model or true scientific method then this way is an objective approach.

The computation of weights with this two ways can be named as direct calculation of performance

So also there are indirect ways: In these approaches, inputs and outputs used for performance calculation are considered as variables or independent variables. Obtained results can be use as performans score of DMUs after applaying related statistical technic. And this scores can be use in ranking of DMUs. Some of these technics are the followings:

a. Regression line/ surface: Regression line or surface fits a single function to the data collected on the basis of average behavior of this pre-specified functional form. A rank number can be given to each DMU according to the distance between DMU and regression line or surface. With this way, evaluation has been based on the average DMU, not the best.

b. Canonical correlation analysis: Canonical correlation approach (CCA) is an enlargement of the regression analysis. Regression line or surface explains one output/dependent variable with one or multiple inputs/independent variables but canonical correlation analyzes multiple inputs and multiple outputs. CCA searches for a single vector weight for the inputs and outputs, common to all the units.

c. Factor analysis: Obviously it is expect that there exist a relationship between input(s) and output(s). Factor analysis (FA) is a statistical method used to describe variability among observed, correlated variables (inputs and outputs) in terms of a potentially lower number of unobserved variables called factors. Rank numbers are obtained by multiplying solution of factor analysis with the standardized data of each DMU. FA also searches for a single vector weight for the inputs and outputs, common to all the units.

d. Discriminant analysis: Discriminant function analysis is used to determine which variables discriminate between two or more naturally occurring groups. DMUs are divided into two groups as efficient and inefficient sets with lowest error. Using traditional discriminant analysis of efficient and inefficient sets an alternative one-dimensional linear function is constructed that name of this new function is Lineer discriminant function.

2. A New Algorithm For Direct Calculations

At this point I am offering a new algorithm for finding a Common Set of Weights (CSW). This new algorithms is based on correlation of inputs and outputs. Obviously, a correlation between inputs and outputs is expected. It should be expected too. Steps of this new algorithm is the following:

Step 1. Data normalization. In this paper, it is supposed that there is no outlier data in inputs and outputs. For convenience of comparison, the inputs xio (i=1,2,…,m) and output Yro(r=1,2,…,s) of DMUo (j=1,2,.o.,n) are normalized as follows:

$$X_{io}=x_{io}/max(x_{ij}) \qquad j\{1,2,...,n\}$$

$$y_{ir}=x_{iro/}max(y_{rj}) \qquad j\{1,2,...,n\}$$

In order to compare different inputs and outputs, the data should be normalized first. And by the normalization of inputs and outputs, the following input and output data and correlatiıns would all become dimensionless and comparable.

Step 2. Compute correlations between inputs and output and their totals. Get absolute value of correlation matrix:

$$r=abs(r)$$

Table 3: Correlation of inputs and outputs

Input(s)\Output(s)	Y_1	Y_2	…	Y_s	Total
X_1	r_{11}	r_{12}	…	r_{1s}	V_1
X_2	r_{21}	r_{22}	…	r_{2s}	V_2
.	.	.	…	.	.
.	.	.	…	.	.
.	.	.	…	.	.
X_m	r_{m1}	r_{m2}	…	r_{ms}	V_m
Total	U_1	U_2	…	U_s	

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Step 3. Compute every weight vector of input and output. Weight vectors of inputs and outputs can be calculate with two ways:

1-Arithmetical average of correlations

2 Median values of correlations.

If there exist so small or so big values than median values more balanced than arithmetical average.

For weights are computed via arithmetical average as follows:

$$U_r/m \qquad r=1,2,\ldots,s,$$

$$v_i=V_i/s \qquad i=1,2,\ldots,m.$$

Step 4. Compute every efficiency scores of DMUs. The efficiency measure for DMU o is computed as follows:

$$E_0=\sum_{r=1}^{s} u_r\, y_{ro} \,/\, \sum_{i=1}^{m} v_i\, x_{io} \qquad j=1,\ldots,n$$

where the weights, and, are non-negative values. It is customary to use 0 to 1.0 intervals for easy evaluation in the performance calculation for efficiency scores when applied to all DMUs. If it exists one or more values greater than 1.0 then divide all values to maximum of this set.

$$E_o= E_o/\max(E_j) \qquad j\in\{1,2,\ldots,n\}.$$

3. Illustrative Example

In this section, a numerical example is provided as the illustration and examination of proposed methodology. Kao and Hung illustrated their compromising solution approach for CSWs problem by expressing an example that is derived from Kao and Hung (2005) and later Makui et al.(2008) also examined this example by their goal programming method for finding CSWs and compared their results with Kao and Hung (2005). Here, this example is analyzed with proposed method and its results are compared with Kao and Hung(2005), Makui et al.(2008) and Razavi et all (2014) results. Thus,the study is related to evaluation of 17 forest districts. Four inputs: budget (in US dollars), initial stocking (in cubic meters), labor (in number of employees) and land (in hectares), and three outputs: main product (in cubic meters), soil conservation (in cubic meters), and recreation (in number of visits) are considered to measure the efficiency. Table 1 contains the original data.

Applying first, second, third steps of proposed algorithm it is obtained that CSW of inputs and outputs as follows:

U={0.268, .758, .326, .411}

V={.528, .471,.322 }.

Solution of given data were obtained with applying fourth step of proposed algorithm and shown in the last column of Table 4. Table 4 shows the results of solving this district problem. Second column of Table 2 shows the results obtained from original CCR model that is run for data. Based on CCR model results, there are 9 efficient DMUs among which the model cannot discriminate, and all efficient DMUs are categorized as efficient.

Table 4: Results of analyzing data with different CSWs methods

DMU	CCR	MAD	MSE	MAX	Makui	Razavietall	Proposed Algorithm
1	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
2	1,0000	1,0000	1,0000	1,0000	1,0000	0,3739	,0297
3	1,0000	1,0000	0,9989	0,7231	1,0000	0,7200	,0344
4	1,0000	1,0000	0,9927	0,8987	1,0000	0,5211	,0245
5	1,0000	0,9747	0,9866	1,0000	1,0000	0,5072	,0312
6	1,0000	0,8534	0,9123	0,8692	0,9654	1,0000	,1059
7	1,0000	0,9244	0,8849	0,7432	0,8743	0,4028	,0182
8	1,0000	0,8954	0,8707	0,8939	0,8469	0,2707	,0645
9	1,0000	0,6619	0,6690	0,7230	0,6783	0,5202	,0168
10	0,9403	0,8721	0,8768	0,8761	0,8779	0,3059	,0300
11	0,9346	0,6398	0,6518	0,6577	0,6526	0,1377	,0189
12	0,8290	0,7456	0,7282	0,7594	0,7175	0,2829	,0221
13	0,7997	0,6229	0,6260	0,6453	0,6227	0,1888	,0195
14	0,7733	0,7140	0,7142	0,7406	0,7126	0,2631	,0255
15	0,7627	0,7245	0,7210	0,6410	0,7215	0,2364	,0181
16	0,7435	0,6996	0,6811	0,4665	0,6696	0,1965	,0164
17	0,6873	0,6310	0,6068	0,5908	0,0593	0,2589	,0190

The solutions of the common weight algorithms presented in the literature as an alternative to the CCR solution are shown in Table 4, 3-7th Column. And the solution according to the proposed algorithm in this study is in the last column of Table 4. The Spearman rank correlation test of the solutions was performed and is shown in Table 5.

Table 5: Pairwise Spearman rank correlation between different CSWs models and CCR

Proposed Model	CCR	MAD	MSE	MAX	Makui	Razavi etal.
Algorithm 1	0,549	0,620	0,760	0,740	,643	0,590

Other algorithms are the CCR-based algorithms but I proposed a statistical-correlation based algorithm. For this reason, high Spearman correlation coefficients between CCR solutions and other algorithms should already be expected. In fact, according to logic of solution, Spearman rank correlation coefficient between the given algorithm and the CCR should be small.

4. Conclusion And Discussion

As it was said at the beginning of this study, in DEA calculations, weight of each input and output for each decision-maker may vary to compute the maximum relative efficiency. Usually they are different. Generally most of weights of inputs and outputs are zero or very close to zero.Therefore this means that those input(s) and output(s) not taken into account in the calculation of the performance. All inputs and outputs evaluated by the newly proposed algorithm are assigned the same weights for each decision maker. And since the weights will usually be greater than zero, all inputs and outputs will be included in the performance account.

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