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Proposed Tests for The General Alternative in A Mixed Design Consist of Completely Randomized and Randomized Block Design

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Keywords
Friedman test; Doksum test; Modified Page test; General alternative; Mixed design

Abstract

Two nonparametric tests are proposed in testing for a general alternative under a mixed design consisting of a randomized complete block portion and a completely randomized block portion. In this paper, the proposed tests are a combination of the Doksum test, Modified Page test and Friedman test. A simulation study is conducted to estimate the powers of the tests for 4 and 5 treatment effects under a variety of different location parameters and sample sizes. We compare the performance of the tests in terms of the empirical type I error rates and power of tests. The simulation results show that at least one of the proposed test versions generally has higher power than the other tests. Finally, the usefulness of proposed tests is implemented on a real data set.

Tamamlanmış Blok ve Tamamen Rasgele Tasarımdan Oluşan Bir Karma Tasarımda Genel Alternatif için Önerilen Testler

Anahtar kelimeler
Friedman test; Doksum test; Uyarlanmış Page testi; Genel alternatif; Karma tasarım

Öz

Rasgele tamamlanmış blok ve tamamen rasgele bloktan oluşan bir karma tasarım altında, genel alternatifleri test etmek için iki parametrik olmayan test önerilmiştir. Bu makalede, önerilen testler Doksum testi, uyarlanmış Page testi ve Friedman testlerinin kombinasyonlarıdır. Farklı konum parametreleri ve örnek çapları altında 4 ve 5 işlem etkilerinin testlerinin güçlerini tahmin etmek için bir simülasyon çalışması yapılmıştır. Deneysel I. Tip hata oranı ve testin gücü bakımından testlerin performansları karşılaştırılmıştır. Simülasyon sonuçlarından, önerilen testlerden en az bir tanesinin diğer testlerden daha yüksek güç değerine sahip olduğu görülmüştür. Son olarak, önerilen testlerin kullanışlılığı, gerçek bir veri seti üzerinde uygulanmıştır.

1. Introduction

There are two important branches of hypothesis testing: parametric and nonparametric. Parametric tests require heavy assumptions about the nature of the population from which the data are drawn. If necessary assumptions are not provided, we can use two methods in statistics namely, using data transformation or nonparametric tests. Data transformation is mostly used in hopes of achieving the normality of the data. However, it does not apply in all situations where transformations can correct data to meet assumptions and requirements for a parametric test.

Another way is to use nonparametric tests. The purpose of using nonparametric tests in many experimental studies is to test the effects of treatments by performing hypothesis testing. To do this, researchers need to determine the hypothesis test and the design structure for the test. The null hypothesis is used to test that there are no differences among treatment effects. Though, the alternative hypothesis is at least one treatment is different. The scope of our paper concentrates on general alternative hypotheses. That is,

$H_0: \mu_1 = \mu_2 = \cdots = \mu_t$

$H_1: At\ least\ one\ \mu_i\ is\ different.$

As for the design structure, the researchers have started with one experimental design, they may have to change their designs for a variety of reasons before the experiment is complete. One of these reasons is that a situation may arise when it may not be possible to continue the experiment using a full randomized complete block design (RCBD) for the test. After a while, the researchers may realize that the design is too expensive to continue. So they might need to shift to another design structure.

Another reason is that the researchers wish to conduct their experiment by choosing a RCBD, but issues may arise while collecting data. The cost of obtaining subjects may perhaps be higher than projected. It may be impractical for the researchers to continue using a RCBD. Therefore, the researchers decide to use a completely randomized design (CRD) before the experiment is completed.

At this point, we have a mixed design consists of a RCBD portion and a CRD portion. For example, according to Magel *et al.* (2010), let us suppose there are four graders who grade exams for a biology class and they would like to test and see if there are differences among graders (treatments). The researchers start by having each student exam graded by all four graders. In here, each student paper is a block. So, this part constitutes the RCBD part of the experiment.

After a while, they realized that all of the tests cannot be graded by each of the graders because this method was too expensive to continue. Later, they decide to use CRD in the last part of the experiment instead of RCBD. Each of the remaining papers is graded by only one of four possible graders. In this example, we have a mixed design consisting of RCBD and CRD. The situation in which this mixed design could occur in different combinations that a paired data, two independent sample data, a balanced incomplete block design and a matched pairs design.

Magel *et al.* (2009) proposed nonparametric tests for a mixed design consisting of RCBD and CRD to test an ordered alternative. Their nonparametric test versions are a combination of Page's test (*P*) by Page (1963) and the Jonckheere-Terpstra test (*JT*) by (Terpstra 1952, Jonckheere 1954).

Magel *et al.* (2014) proposed a nonparametric test for a mixed design consisting of a paired sample portion and a two-independent-sample portion to test for differences in treatment effects. The proposed test statistic is the same as the test developed by Dubnicka *et al.* (2002).

Gül and Bayrak (2021) proposed two nonparametric tests in testing for ordered alternatives under a mixed design consisting of CRD and RCBD. Their test statistics are combinations of the Hollander test (*H*) by Hollander (1697) and the *JT* test.

Magel *et al.* (2010) developed a test statistic in mixed design consisting of RCBD and CRD for a general alternative. The proposed test statistic, T_1, is a combination of Friedman's (*F*) test statistic by Friedman (1937) and Kruskal-Wallis (*KW*) test statistic by Kruskal and Wallis (1952). The test statistic, T_1, is

$$T_1 = F + KW$$

The F and KW test statistic are as follows:

$$F = \frac{12}{bk(k+1)} \sum_{j=1}^{k} \left[R_j - \frac{b(k+1)}{2}\right]^2 \qquad (1)$$

and

$$KW = \frac{12}{N(n+1)} \sum_{j=1}^{k} \frac{1}{n_i} \left[R_i - \frac{n_i(N+1)}{2}\right]^2, \qquad (2)$$

respectively. Under the H_0, T_1 has an asymptotic chi-square distribution with $2k-2$ degrees of freedom.

Doksum (D) is a nonparametric test based on Wilcoxon signed ranks for general alternatives in a RCBD Doksum (1967). The D test statistic is

$$D = \sum_{j=1}^{k} \frac{\left[H_{j.} - \{(k-1)/2k\}\right]^2}{(k-1)V_u/2k}, \tag{3}$$

where

$$V_u = \frac{2n-1+(k-2)[7(n-2)+13-6n]}{3kn(n-1)},$$

$$H_{u.} = \sum_{j=1}^{k} \frac{H_{uj}}{k}, \qquad u = 1, \ldots, k,$$

$$H_{uv} = \frac{2(T_{uv} - B_{uv})}{n(n-1)}, \qquad 1 \le u < v \le k,$$

$T_{uv} = \sum_{i=1}^{n} R_{uv}^{i}\, \psi_{uv}^{i}$, $B_{uv} = \sum_{i=1}^{n} \psi_{uv}^{i}$ and

$$\psi_{uv}^{i} = \begin{cases} 1, & if\ X_{iu} < X_{iv} \\ 0, & otherwise \end{cases}.$$

The asymptotic null distribution of the D test statistic is chi-square with *k*-1 degrees of freedom with k being the number of treatments.

Best *et al.* (2009) defined the Modified Page (MP) test for ordered and general alternatives. The MP test statistic is given by

$$MP = \frac{CL^2}{\sum_{i=1}^{t} \lambda_j^2}, \tag{4}$$

where $L = \sum_{j=1}^{t} \lambda_j \bar{R}$ orthogonal trend contrast, λ_j is a linear coefficient, $\bar{R} = {R_j}/{b}$. The *C* and *V* terms are as follows

$$C = \begin{cases} \frac{b(t-1)}{tV}, & ties \\ \frac{12b}{t(t+1)}, & no\ ties \end{cases}$$

$$V = \begin{cases} \left\{\sum_{s=1}^{q} r_s^2 c_s / bt\right\} - (t+1)^2/4, & ties \\ \frac{(t^2-1)}{12}, & no\ ties \end{cases},$$

where r_s and c_s denote the *s*th ranking and its associated count, respectively. The asymptotic null distribution of the *MP* test statistic is chi-square with *k*-1 degrees of freedom.

Our motivation for proposing nonparametric tests is the lack of a general alternative for mixed design in the literature and the applicability of this problem to various fields. In this article, we proposed two nonparametric tests for a general alternative in a mixed design consisting of a combination of a RCBD and then a CRD. We are also assuming that the underlying distributions being sampled from are unknown or that only rank data is available. For this reason, we used nonparametric tests for mixed design.

2. Proposed Tests

In this article, we are proposing two test statistics for a mixed design consisting of RCBD and CRD. Test statistics are considered a general alternative. The test statistics we are proposing is a similar idea to the test developed by Magel *et al.* (2010).

The first proposed test statistic, denoted by T_{first}, is given in (5):

$$T_{first} = D + KW$$

$$T_{first} = \sum_{j=1}^{k} \frac{\left[H_{j.} - \left\{\frac{(k-1)}{2k}\right\}\right]^2}{\frac{(k-1)V_u}{2k}} + \frac{12}{N(n+1)} \sum_{j=1}^{k} \frac{1}{n_i}\left[R_i - \frac{n_i(N+1)}{2}\right]^2. \tag{5}$$

The test statistic, T_{first}, is the sum of the *D* test statistic and the *KW* test statistic. Under H_0, T_{first} has an asymptotic chi-square distribution with 2*k*-2 degrees of freedom.

The second proposed test statistic, denoted by T_{second}, is given in (6):

$$T_{second} = MP + KW$$

$$T_{second} = \frac{CL^2}{\sum_{i=1}^{t} \lambda_j^2} + \frac{12}{N(n+1)} \sum_{j=1}^{k} \frac{1}{n_i}\left[R_i - \frac{n_i(N+1)}{2}\right]^2. \tag{6}$$

The test statistic, T_{second}, is the sum of the *MP* test statistic and the *KW* test statistic. Under H_0, T_{second} has an asymptotic chi-square distribution with 2*k*-2 degrees of freedom.

3. Simulation Study

A simulation study was designed to estimate and compare the powers of the proposed tests and the powers of the test, T_1, constructed by Magel *et al.* (2010). The underlying population distribution is considered as standard normal. The number of treatments was taken 4 and 5. A simulation of size

5000 was implemented for each combination of the different equal and unequal sample sizes, and locations parameters arrangements. We also considered 24, 30 and 48 blocks for the randomized complete block portion and 6, 10, 12, 15 and 24 for the completely randomized portion. Programs for simulation are coded using MATLAB (R2018b).

Table 1. Estimated powers for T_{first}, T_{second}, T_1, *F*, *D*, and *MP* tests for k=4; normal populations with variance=1; sample sizes RCBD portion 24 blocks and CRD portion 6 blocks.

Location parameters				*F*	*D*	*MP*	T_1	T_{first}	T_{second}
0.00	0.00	0.00	0.00	0.0520	0.0558	0.0544	0.0424	0.0534	0.0426
0.00	0.25	0.50	0.75	0.4626	0.6042	0.6590	0.4478	0.5816	0.5350
0.00	0.00	0.50	0.50	0.3680	0.4966	0.4748	0.3510	0.4780	0.3632
0.00	0.25	0.25	0.50	0.2078	0.2842	0.3034	0.1754	0.2514	0.2038
0.00	0.20	0.30	0.90	0.6302	0.7604	0.7454	0.6270	0.7590	0.6574
0.00	0.10	0.70	0.90	0.7714	0.8864	0.8734	0.7762	0.8848	0.8238
0.00	0.70	0.80	0.90	0.6810	0.8116	0.7574	0.6836	0.8126	0.6672
0.00	0.10	0.50	0.80	0.5966	0.7252	0.7536	0.5862	0.7310	0.6570
0.00	0.50	0.60	0.90	0.6056	0.7452	0.7490	0.5862	0.7238	0.6404
0.00	0.30	0.60	0.90	0.6552	0.7878	0.8156	0.6410	0.7750	0.7224
0.00	0.90	0.90	0.90	0.7716	0.8896	0.7092	0.7762	0.8904	0.6638

Table 2. Estimated powers for T_{first}, T_{second}, T_1, *F*, *D*, and *MP* tests for k=4; normal populations with variance=1; sample sizes RCBD portion 24 blocks and CRD portion 12 blocks.

Location parameters				*F*	*D*	*MP*	T_1	T_{first}	T_{second}
0.00	0.00	0.00	0.00	0.0504	0.0594	0.0500	0.0422	0.0474	0.0468
0.00	0.25	0.50	0.75	0.4812	0.6104	0.6624	0.5612	0.6762	0.6524
0.00	0.00	0.50	0.50	0.3972	0.5262	0.4772	0.4696	0.5784	0.4836
0.00	0.25	0.25	0.50	0.2094	0.2840	0.3074	0.2340	0.3078	0.2664
0.00	0.20	0.30	0.90	0.6514	0.7818	0.7578	0.7628	0.8510	0.7932
0.00	0.10	0.70	0.90	0.7656	0.8852	0.8728	0.8724	0.9420	0.9066
0.00	0.70	0.80	0.90	0.6890	0.8216	0.7486	0.8080	0.8904	0.8080
0.00	0.10	0.50	0.80	0.5824	0.7278	0.7514	0.7126	0.8108	0.7710
0.00	0.50	0.60	0.90	0.6040	0.7506	0.7510	0.7246	0.8280	0.7788
0.00	0.30	0.60	0.90	0.6426	0.7850	0.8090	0.7642	0.8546	0.8290
0.00	0.90	0.90	0.90	0.7818	0.8974	0.7102	0.8856	0.9494	0.8210

Table 3. Estimated powers for T_{first}, T_{second}, T_1, *F*, *D*, and *MP* tests for k=4; normal populations with variance=1; sample sizes RCBD portion 48 blocks and CRD portion 12 blocks.

Location parameters				*F*	*D*	*MP*	T_1	T_{first}	T_{second}
0.00	0.00	0.00	0.00	0.0526	0.0554	0.0482	0.0456	0.0572	0.0424
0.00	0.25	0.50	0.75	0.8136	0.9164	0.9186	0.8272	0.9272	0.8842
0.00	0.00	0.50	0.50	0.6962	0.8378	0.7534	0.7146	0.8380	0.6998
0.00	0.25	0.25	0.50	0.3950	0.5376	0.5152	0.3876	0.5128	0.4156
0.00	0.20	0.30	0.90	0.9298	0.9816	0.9568	0.9466	0.9844	0.9480

0.00	0.10	0.70	0.90	0.9818	0.9978	0.9934	0.9890	0.9986	0.9912
0.00	0.70	0.80	0.90	0.9582	0.9906	0.9558	0.9682	0.9928	0.9528
0.00	0.10	0.50	0.80	0.9132	0.9746	0.9678	0.9304	0.9788	0.9494
0.00	0.50	0.60	0.90	0.9126	0.9738	0.9552	0.9298	0.9792	0.9434
0.00	0.30	0.60	0.90	0.9394	0.9836	0.9830	0.9488	0.9846	0.9706
0.00	0.90	0.90	0.90	0.9852	0.9980	0.9414	0.9880	0.9988	0.9548

Table 4. Estimated powers for T_{first}, T_{second}, T_1, *F*, *D*, and *MP* tests for k=4; normal populations with variance=1; sample sizes RCBD portion 48 blocks and CRD portion 24 blocks.

Location parameters				*F*	*D*	*MP*	T_1	T_{first}	T_{second}
0.00	0.00	0.00	0.00	0.0544	0.0550	0.0508	0.0484	0.0592	0.0452
0.00	0.25	0.50	0.75	0.8150	0.9156	0.9192	0.9134	0.9606	0.9466
0.00	0.00	0.50	0.50	0.7108	0.8458	0.7678	0.8306	0.9116	0.8324
0.00	0.25	0.25	0.50	0.3916	0.5260	0.5194	0.4722	0.5774	0.5226
0.00	0.20	0.30	0.90	0.9312	0.9796	0.9596	0.9828	0.9950	0.9858
0.00	0.10	0.70	0.90	0.9772	0.9960	0.9912	0.9974	0.9998	0.9982
0.00	0.70	0.80	0.90	0.9550	0.9892	0.9594	0.9902	0.9988	0.9884
0.00	0.10	0.50	0.80	0.9104	0.9662	0.9584	0.9702	0.9888	0.9796
0.00	0.50	0.60	0.90	0.9198	0.9756	0.9644	0.9768	0.9926	0.9852
0.00	0.30	0.60	0.90	0.9364	0.9816	0.9816	0.9864	0.9962	0.9942
0.00	0.90	0.90	0.90	0.9842	0.9986	0.9392	0.9974	0.9999	0.9882

Table 5. Estimated powers for T_{first}, T_{second}, T_1, *F*, *D*, and *MP* tests for k=4; normal populations with variance=1; sample sizes RCBD portion 30 blocks and CRD portion 10 blocks.

Location parameters				*F*	*D*	*MP*	T_1	T_{first}	T_{second}
0.00	0.00	0.00	0.00	0.0504	0.0586	0.0556	0.0436	0.0562	0.0464
0.00	0.25	0.50	0.75	0.5900	0.7292	0.7662	0.6314	0.7550	0.7046
0.00	0.00	0.50	0.50	0.4714	0.6230	0.5756	0.5010	0.6320	0.5074
0.00	0.25	0.25	0.50	0.2536	0.3548	0.3854	0.2568	0.3430	0.2842
0.00	0.20	0.30	0.90	0.7440	0.8730	0.8442	0.7890	0.8918	0.8146
0.00	0.10	0.70	0.90	0.8688	0.9508	0.9418	0.9174	0.9692	0.9368
0.00	0.70	0.80	0.90	0.7984	0.9136	0.8460	0.8414	0.9298	0.8292
0.00	0.10	0.50	0.80	0.7186	0.8482	0.8536	0.7606	0.8692	0.8158
0.00	0.50	0.60	0.90	0.7092	0.8482	0.8406	0.7564	0.8688	0.8080
0.00	0.30	0.60	0.90	0.7506	0.8748	0.8970	0.7984	0.8972	0.8612
0.00	0.90	0.90	0.90	0.8694	0.9516	0.8054	0.9178	0.9706	0.8340

Table 6. Estimated powers for T_{first}, T_{second}, T_1, *F*, *D*, and *MP* tests for k=4; normal populations with variance=1; sample sizes RCBD portion 30 blocks and CRD portion 15 blocks.

Location parameters				*F*	*D*	*MP*	T_1	T_{first}	T_{second}
0.00	0.00	0.00	0.00	0.0464	0.0584	0.0538	0.0470	0.0548	0.0476
0.00	0.25	0.50	0.75	0.5706	0.7162	0.7628	0.6928	0.7996	0.7724
0.00	0.00	0.50	0.50	0.4852	0.6318	0.5754	0.5832	0.6916	0.5870
0.00	0.25	0.25	0.50	0.2598	0.3582	0.3746	0.2916	0.3824	0.3300

0.00	0.20	0.30	0.90	0.7468	0.8744	0.8478	0.8574	0.9246	0.8732
0.00	0.10	0.70	0.90	0.8666	0.9506	0.9390	0.9528	0.9806	0.9642
0.00	0.70	0.80	0.90	0.7994	0.9044	0.8390	0.9006	0.9500	0.8918
0.00	0.10	0.50	0.80	0.7084	0.8350	0.8404	0.8226	0.8990	0.8630
0.00	0.50	0.60	0.90	0.7130	0.8558	0.8464	0.8364	0.9138	0.8732
0.00	0.30	0.60	0.90	0.7444	0.8630	0.8848	0.8636	0.9294	0.9148
0.00	0.90	0.90	0.90	0.8732	0.9512	0.8156	0.9550	0.9820	0.9124

Table 7. Estimated powers for T_{first}, T_{second}, T_1, *F*, *D*, and *MP* tests for k=5; normal populations with variance=1; sample sizes RCBD portion 24 blocks and CRD portion 6 blocks.

Location parameters					*F*	*D*	*MP*	T_1	T_{first}	T_{second}
0.00	0.00	0.00	0.00	0.00	0.0530	0.0580	0.0484	0.0456	0.0528	0.0454
0.00	0.25	0.50	0.75	0.90	0.6872	0.8002	0.8754	0.6844	0.7964	0.7800
0.00	0.00	0.50	0.50	0.50	0.4148	0.5304	0.5218	0.3972	0.5018	0.4016
0.00	0.25	0.25	0.50	0.50	0.2488	0.3216	0.3984	0.2238	0.2922	0.2552
0.00	0.20	0.30	0.40	0.90	0.6226	0.7336	0.7794	0.6048	0.7116	0.6558
0.00	0.10	0.50	0.70	0.90	0.7492	0.8484	0.9082	0.7368	0.8390	0.8246
0.00	0.60	0.70	0.80	0.90	0.6586	0.7708	0.7744	0.6470	0.7572	0.6598
0.00	0.10	0.30	0.60	0.80	0.6172	0.7252	0.8130	0.6140	0.7178	0.7046
0.00	0.50	0.60	0.75	0.90	0.6336	0.7420	0.7948	0.6202	0.7248	0.6798
0.00	0.20	0.40	0.60	0.80	0.5548	0.6640	0.7766	0.5366	0.6424	0.6376
0.00	0.90	0.90	0.90	0.90	0.7824	0.8644	0.6500	0.7844	0.8690	0.5954

Table 8. Estimated powers for T_{first}, T_{second}, T_1, *F*, *D*, and *MP* tests for k=5; normal populations with variance=1; sample sizes RCBD portion 24 blocks and CRD portion 12 blocks.

Location parameters					*F*	*D*	*MP*	T_1	T_{first}	T_{second}
0.00	0.00	0.00	0.00	0.00	0.0466	0.0490	0.0448	0.0476	0.0508	0.0470
0.00	0.25	0.50	0.75	0.90	0.6928	0.8046	0.8752	0.8064	0.8754	0.8760
0.00	0.00	0.50	0.50	0.50	0.4354	0.5454	0.5496	0.5090	0.5948	0.5250
0.00	0.25	0.25	0.50	0.50	0.2524	0.3298	0.3872	0.2854	0.3456	0.3354
0.00	0.20	0.30	0.40	0.90	0.6170	0.7276	0.7702	0.7252	0.8014	0.7728
0.00	0.10	0.50	0.70	0.90	0.7570	0.8516	0.9020	0.8488	0.9146	0.9094
0.00	0.60	0.70	0.80	0.90	0.6652	0.7708	0.7714	0.7772	0.8448	0.7956
0.00	0.10	0.30	0.60	0.80	0.5990	0.7236	0.8054	0.7096	0.8084	0.7912
0.00	0.50	0.60	0.75	0.90	0.6252	0.7408	0.7900	0.7316	0.8156	0.7874
0.00	0.20	0.40	0.60	0.80	0.5568	0.6760	0.7840	0.6610	0.7532	0.7570
0.00	0.90	0.90	0.90	0.90	0.7864	0.8732	0.6632	0.8834	0.9314	0.7692

Table 9. Estimated powers for T_{first}, T_{second}, T_1, *F*, *D*, and *MP* tests for k=5; normal populations with variance=1; sample sizes RCBD portion 48 blocks and CRD portion 12 blocks.

Location parameters					*F*	*D*	*MP*	T_1	T_{first}	T_{second}
0.00	0.00	0.00	0.00	0.00	0.0464	0.0592	0.0502	0.0438	0.0506	0.0456
0.00	0.25	0.50	0.75	0.90	0.9570	0.9916	0.9940	0.9706	0.9926	0.9866
0.00	0.00	0.50	0.50	0.50	0.7634	0.8646	0.8252	0.7644	0.8660	0.7410

0.00	0.25	0.25	0.50	0.50	0.4964	**0.6216**	0.6924	0.4846	0.6004	0.5430
0.00	0.20	0.30	0.40	0.90	0.9238	0.9742	0.9750	0.9324	0.9750	0.9492
0.00	0.10	0.50	0.70	0.90	0.9760	0.9958	0.9956	0.9858	0.9968	0.9930
0.00	0.60	0.70	0.80	0.90	0.9426	0.9804	0.9720	0.9576	0.9880	0.9550
0.00	0.10	0.30	0.60	0.80	0.9206	0.9728	0.9806	0.9356	0.9766	0.9618
0.00	0.50	0.60	0.75	0.90	0.9288	0.9714	0.9738	0.9426	0.9758	0.9528
0.00	0.20	0.40	0.60	0.80	0.8918	0.9524	0.9758	0.9006	0.9596	0.9480
0.00	0.90	0.90	0.90	0.90	0.9846	0.9976	0.9286	0.9930	0.9980	0.9290

Table 10. Estimated powers for T_{first}, T_{second}, T_1, *F*, *D*, and *MP* tests for k=5; normal populations with variance=1; sample sizes RCBD portion 48 blocks and CRD portion 24 blocks.

Location parameters					*F*	*D*	*MP*	T_1	T_{first}	T_{second}
0.00	0.00	0.00	0.00	0.00	0.0474	0.0552	0.0520	0.0472	0.0524	0.0488
0.00	0.25	0.50	0.75	0.90	0.9632	0.9884	0.9920	0.9912	0.9964	0.9962
0.00	0.00	0.50	0.50	0.50	0.7668	0.8656	0.8308	0.8744	0.9264	0.8642
0.00	0.25	0.25	0.50	0.50	0.4952	0.6156	0.6810	0.5928	0.6898	0.6564
0.00	0.20	0.30	0.40	0.90	0.9262	0.9748	0.9754	0.9820	0.9954	0.9870
0.00	0.10	0.50	0.70	0.90	0.9792	0.9950	0.9970	0.9972	0.9992	0.9994
0.00	0.60	0.70	0.80	0.90	0.9444	0.9838	0.9726	0.9856	0.9958	0.9892
0.00	0.10	0.30	0.60	0.80	0.9190	0.9688	0.9818	0.9738	0.9880	0.9864
0.00	0.50	0.60	0.75	0.90	0.9280	0.9756	0.9774	0.9778	0.9932	0.9884
0.00	0.20	0.40	0.60	0.80	0.8854	0.9498	0.9788	0.9590	0.9834	0.9804
0.00	0.90	0.90	0.90	0.90	0.9848	0.9978	0.9330	0.9980	0.9998	0.9876

Table 11. Estimated powers for T_{first}, T_{second}, T_1, *F*, *D*, and *MP* tests for k=5; normal populations with variance=1; sample sizes RCBD portion 30 blocks and CRD portion 10 blocks.

Location parameters					*F*	*D*	*MP*	T_1	T_{first}	T_{second}
0.00	0.00	0.00	0.00	0.00	0.0482	0.0478	0.0490	0.0482	0.0498	0.0446
0.00	0.25	0.50	0.75	0.90	0.8112	0.9022	0.9186	0.8514	0.9432	0.9126
0.00	0.00	0.50	0.50	0.50	0.5292	0.6494	0.6316	0.5552	0.6634	0.5446
0.00	0.25	0.25	0.50	0.50	0.3242	0.4104	0.4938	0.3316	0.4078	0.3866
0.00	0.20	0.30	0.40	0.90	0.7294	0.8406	0.8554	0.7706	0.8662	0.8004
0.00	0.10	0.50	0.70	0.90	0.8504	0.9284	0.9476	0.8926	0.9562	0.9352
0.00	0.60	0.70	0.80	0.90	0.7706	0.8680	0.8626	0.8128	0.8870	0.8164
0.00	0.10	0.30	0.60	0.80	0.7322	0.8276	0.9008	0.7746	0.8600	0.8448
0.00	0.50	0.60	0.75	0.90	0.7464	0.8504	0.8704	0.7832	0.8818	0.8270
0.00	0.20	0.40	0.60	0.80	0.6714	0.7902	0.8642	0.7178	0.8138	0.8098
0.00	0.90	0.90	0.90	0.90	0.8778	0.9420	0.7614	0.9154	0.9576	0.7844

Table 12. Estimated powers for T_{first}, T_{second}, T_1, *F*, *D*, and *MP* tests for k=5; normal populations with variance=1; sample sizes RCBD portion 30 blocks and CRD portion 15 blocks.

Location parameters					*F*	*D*	*MP*	T_1	T_{first}	T_{second}
0.00	0.00	0.00	0.00	0.00	0.0476	0.0556	0.0508	0.0470	0.0494	0.0464
0.00	0.25	0.50	0.75	0.90	0.8126	0.8974	0.9458	0.9060	0.9486	0.9478

0.00	0.00	0.50	0.50	0.50	0.5226	0.6410	0.6306	0.6248	0.7138	0.6322
0.00	0.25	0.25	0.50	0.50	0.3178	0.4046	0.4920	0.3614	0.4390	0.4210
0.00	0.20	0.30	0.40	0.90	0.7250	0.8252	0.8570	0.8410	0.8956	0.8718
0.00	0.10	0.50	0.70	0.90	0.8520	0.9264	0.9598	0.9366	0.9692	0.9646
0.00	0.60	0.70	0.80	0.90	0.7748	0.8726	0.8688	0.8792	0.9322	0.8938
0.00	0.10	0.30	0.60	0.80	0.7222	0.8338	0.8946	0.8322	0.8982	0.8918
0.00	0.50	0.60	0.75	0.90	0.7494	0.8498	0.8836	0.8678	0.9170	0.8980
0.00	0.20	0.40	0.60	0.80	0.6702	0.7846	0.8708	0.7792	0.8542	0.8576
0.00	0.90	0.90	0.90	0.90	0.8764	0.9450	0.7604	0.9496	0.9770	0.8718

The empirical type I error rates were within acceptable values ranging between 0.0424 and 0.0594 for all number of treatments and sample size combinations.

When the sample size for the CRD portion was 1/2 and 1/3 that of the RCBD portion, T_{first} is superior to the other tests. When the sample size for the CRD portion was 1/4 that of the RCBD portion, in Tables 3 and 9, T_{first} has higher power than the other tests. In this scenario, *MP*, *D and* T_{first} have higher power than the other tests, especially in Tables 1 and 7. When the sample size increases, it is seen that the power of all tests is getting higher.

4. Illustrative Example

In this section, we present a real data set to show the usefulness of proposed test statistics, T_{first} and T_{second}, for a mixed design consisting of RCBD and CRD. The data set represents four graders grading exams of 35 students taken from Magel et al. (2010). The data are given in Figure 1.

	Grader			
Student	*1*	*2*	*3*	*4*
1	80	83	74	85
2	62	67	60	64
3	73	80	69	82
4	85	90	75	88
5	50	57	48	60
6	79	84	71	87
7	90	95	86	97
8	88	93	84	92
9	83	90	80	91
10	79	78	81	84
11	66	67	60	69
12	74	78	70	79
13	82	79	85	78
14	91	94	89	95
15	83	80	81	84
16	66	.	.	.
17	75	.	.	.
18	88	.	.	.
19	77	.	.	.
20	92	.	.	.
21	.	70	.	.
22	.	77	.	.
23	.	90	.	.
24	.	76	.	.
25	.	95	.	.
26	.	.	65	.
27	.	.	86	.
28	.	.	72	.
29	.	.	89	.
30	.	.	74	.
31	.	.	.	80
32	.	.	.	90
33	.	.	.	70
34	.	.	.	94
35	.	.	.	78

Figure 1. Four graders grading exams of 35 students.

All four graders are graded for 15 students (randomized block portion). Figure 2 gives the ranks for randomized block portion.

	Grader			
Student	*1*	*2*	*3*	*4*
1	2	3	1	4
2	2	4	1	3
3	2	3	1	4
4	2	4	1	3
5	2	3	1	4
6	2	3	1	4
7	2	3	1	4
8	2	4	1	3
9	2	3	1	4
10	2	1	3	4
11	2	3	1	4

12	2	3	1	4
13	3	2	4	1
14	2	3	1	4
15	3	1	2	4

Figure 2. Ranks of the completely randomized block design.

If these four graders continue to be applied to all students, it will be a waste of time and money. Therefore, the remaining twenty exams to each of the four graders are assigned randomly (completely randomized portion). Figure 3 gives the ranks for completely randomized portion.

	Grader			
Student	*1*	*2*	*3*	*4*
16	2	.	.	.
17	7	.	.	.
18	14	.	.	.
19	9.5	.	.	.
20	18	.	.	.
21	.	3.5	.	.
22	.	9.5	.	.
23	.	16.5	.	.
24	.	8	.	.
25	.	20	.	.
26	.	.	1	.
27	.	.	13	.
28	.	.	5	.
29	.	.	15	.
30	.	.	6	.
31	.	.	.	12
32	.	.	.	16.5
33	.	.	.	3.5
34	.	.	.	19
35	.	.	.	11

Figure 3. Ranks of the completely randomized design.

T_{first} test statistic is computed by using the following formula:

$$D = \sum_{j=1}^{k} \frac{\left[H_{j.} - \left\{\frac{(k-1)}{2k}\right\}\right]^2}{\frac{(k-1)V_u}{2k}} = 2.29,$$

$$KW = \frac{12}{N(n+1)} \sum_{j=1}^{k} \frac{1}{n_i} \left[R_i - \frac{n_i(N+1)}{2}\right]^2 = 1.57,$$

$$T_{first} = D + KW = 2.28 + 1.57 = 3.85.$$

T_{second} test statistic is computed by using the following formula:

$$MP = \frac{CL^2}{\sum_{i=1}^{t} \lambda_j^2} = 3.96$$

$$T_{second} = MP + KW = 3.96 + 1.57 = 5.53.$$

5. Conclusion

Two nonparametric test statistics were introduced for the mixed design consisting of a CRD portion and a RCBD portion. We compared them with the *F*, *D*, *MP* and T_1 in terms of type I error rate and power of the tests. A significant level of 0.05 was considered for all the proposed tests based on the asymptotic standard normal distribution of the test statistics under the null hypothesis. A variety of location parameters configurations were considered for the power of tests.

The empirical type I error rates of all tests are close to the nominal level. When more randomized complete blocks are present in the mixed design, the results show that T_{first} is significantly more powerful than the other tests. When more completely randomized blocks are used in the mixed design, T_{first}, *MP* and *D* have a greater power than the other tests. As in the case, the differences between the power of the tests are not pronounced. So, T_{first} is preferable when more randomized complete blocks are used.

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