

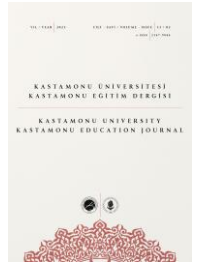
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| Research Article / Araştırma Makalesi |

Investigation of Seventh Grade Students' Written Mathematical Communication Skills

Yedinci Sınıf Öğrencilerinin Yazılı Matematiksel İletişim Becerilerinin İncelenmesi

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Keywords 1. Let's Tell intelligence game (LTIG) 2. Story 3. Written mathematical communication 4. Seventh-grade students	Abstract <i>Purpose:</i> The study aims to examine the written mathematical communication skills of seventh-grade students in the context of angles and lines in the stories they create in the Let's Tell Intelligence Game (LTIG). <i>Design/Methodology/Approach:</i> According to the case study design, this study was conducted with 21 seventh-grade students at a secondary school in the Eastern Black Sea Region. The data of the study were collected by applying LTIG adapted to angles and lines and angles to the students. The stories created by seventh-grade students in the game "Let's Tell a Story" were analyzed concerning their written mathematical communication skills in the context of angles and lines. The stories created in LTIG were analyzed according to the mathematical concepts used, the identification of angles in the visuals, the correct and incorrect use of angles in the visuals, and the formal and informal expression of visual-angle examples, which were determined as written mathematical communication skill criteria. <i>Findings:</i> As a result of the analyses, it was determined that the students had mostly intermediate level written mathematical communication skills, they used a small number of mathematical concepts in their stories and were able to determine only up to 33% of the angles in the visuals. <i>Highlights:</i> It is recommended to use writing practices in mathematics lessons and to conduct them through storytelling.
Anahtar Kelimeler 1. Haydi Anlat zekâ oyunu (HAZO) 2. Hikâye 3. Yazılı matematiksel iletişim becerisi 4. Yedinci sınıf öğrencileri	Öz <i>Çalışmanın amacı:</i> Amaç, yedinci sınıf öğrencilerinin, Haydi Anlat Zekâ oyunu'nda (HAZO) açılar ile doğrular ve açılar konuları bağlamında oluşturdukları hikâyelerdeki yazılı matematiksel iletişim becerilerini incelemektir. <i>Materyal ve Yöntem:</i> Durum çalışmasına göre desenlenen bu araştırma, Doğu Karadeniz Bölgesinde bulunan bir ortaokulda öğrenim gören 21 yedinci sınıf öğrencisiyle gerçekleştirilmiştir. Araştırmanın verileri, açılar ile doğrular ve açılara uyarlanan HAZO'nun öğrencilere uygulanmasıyla toplanmıştır. HAZO'da oluşturulan hikâyeler yazılı matematiksel iletişim beceri kriterleri olarak belirlenmiş olan kullanılan matematik kavramları, görsellerdeki açıların belirlenmesi, görsellerdeki açıların doğru ve hatalı kullanım olma durumları ile görsel-açı örneklerinin formal ve informal ifade olma durumlarına göre analiz edilmiştir. <i>Bulgular:</i> Yapılan analizlerin sonucunda, öğrencilerin çoğunlukla orta düzeyde yazılı matematiksel iletişim becerisine sahip oldukları, oluşturdukları hikâyelerde az sayıda matematik kavramına yer verdikleri ve görsellerdeki açılarının en fazla %33'ünün belirlenebildiği tespit edilmiştir. <i>Önemli Vurgular:</i> Matematik derslerinde yazma uygulamalarının kullanılması ve hikâyeler yoluyla gerçekleştirilmesi önerilmektedir.
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INTRODUCTION

Mathematical communication skill is defined as the ability to express mathematical thoughts verbally or in writing when explaining an algorithm, and to relate everyday language and symbols to mathematical language and symbols (Rajagukguk, 2016). In the same study, it was noted that this skill includes the ability to use different representation forms (concrete models, shapes, graphs, tables, symbols, etc.) when explaining relationships between concepts and mathematical ideas, as well as the awareness that mathematics is a communication tool with its unique symbols and language. For mathematical communication to take place meaningfully, the correct use of mathematical language, the understanding, interpretation, and creation of mathematical discourse (Chapin, O'Connor & Anderson, 2009; Sür & Delice, 2016) and the use of mathematical symbols and models (Sür & Delice, 2016) are largely important. Therefore, it can be said that there is a noticeable and important relationship between mathematical communication and mathematical language. A mathematical language is a communication tool that is formed by combining mathematical words with the language used in real life (Raiker, 2002; cited in Çakmak, 2013). According to Çalikoğlu Bali (2003), mathematical language, unlike other languages, has the ability to express scientific ideas easily and is a collection of applications that contain mathematical concepts, operations, and symbols.

According to the Mathematics Curriculum (for Primary and Secondary Schools, Grades 1-8), published by the Ministry of National Education (MoNE) in 2018, students should be encouraged to engage in individual and interpersonal communication while sharing their methods of constructing mathematical concepts during the teaching process. In addition, in the previous curriculum (MoNE, 2009), it was stated that speaking, writing, and listening activities about mathematics not only improve communication skills but also provide opportunities for students to better understand mathematical concepts. The National Council of Teachers of Mathematics (NCTM) in the United States emphasizes the need for teachers to support students in their learning of mathematical concepts through the languages they bring to school, and to help them develop mathematical terms that facilitate better communication with each other. Therefore, teachers have a special responsibility to help their students learn to speak and write mathematically (Morgan, 2011).

According to Liebars (1997), writing in mathematics provides an opportunity for students to have a clear understanding of the concepts they know and do not know, give feedback, and slow down the thinking process to organize and reflect on their ideas. In addition, writing enables the teacher to "hear" the student and provides the teacher with information about students' schema, misconceptions, and thinking patterns related to concepts (Hoffman & Powell, 1989). These data about students contribute to their higher-order thinking and reasoning skills, provide them with an opportunity to think deeply and make writing an important part of classroom practices (King, Raposo & Gimenez, 2016). Similarly, studies on writing activities in mathematics classes have shown that they contribute to student achievement (Kasa, 2009; Yıldırım, 2016) and increase mathematical communication (Atasoy & Atasoy, 2006), exhibit problem-solving skills, and have a positive impact on student attitudes towards mathematics in an application (Küçük, 2019). Öztürk, Öztürk, and Işık (2016) also stated in their study that teachers believed that writing was a learning process for students and provided learning support. Therefore, it can be seen that the use of writing activities in mathematics classes provides many benefits to students. Some of the writing activities mentioned are expository writing, diary writing, writing a problem situation formed by a scenario, and giving students statements at the end of the lesson and writing their feelings and thoughts against these statements (Atasoy & Atasoy, 2006). Story writing can be considered as one of the writing activities that create an environment where students can apply these writing purposes.

A story is defined as a genre of text that usually describes real or possible events in a short form, typically involving time, place, and characters (Akçay & Akçay, 2017). Mathematical stories, on the other hand, are used as a way to overcome difficulties in understanding and applying mathematics, and to demonstrate the importance of mathematics by forming the basis of mathematical concepts (Franz & Pope, 2005). However, mathematical stories provide students with the opportunity to develop their skills in identifying questions and reasoning (Burton, 1985) and to discover the beauty of mathematics by experiencing new situations (Gadanidis & Hughes, 2011). In studies related to mathematical stories, it has been stated that these stories have a positive impact on learning (Franz & Pope, 2005), can be used to explain difficult mathematical concepts to students (Goral & Gnadinger, 2006), and can be used by students to relate their ideas about mathematics to real-life situations (Healy & Sinclair, 2007). Additionally, the problems created by students using story cards have been examined, and it has been noted that the results are believed to contribute to the teaching of mathematical concepts (Kaya & Ev-Çimen, 2021). The study on the use of story cards brought to mind an intelligent game played with the help of story cubes, and it was thought that students' written communication skills could be determined as well as their ability to write mathematical stories by playing this game.

In the Secondary School and Imam Hatip Secondary School Intelligence Games Course Curriculum, it is stated that intelligence games are a powerful communication tool and provide the opportunity to communicate with people from all over the world as a universal indicator of knowledge and experience (MoNE, 2013). In this program, intelligence games are defined as a gamified version of all kinds of problems, including real problems, and are discussed in terms of creating different perspectives, developing the habit of focusing on a subject and solution, and developing the skills of reasoning and using logic effectively. The games in the program are handled at three levels: "Beginner Level-D1", "Intermediate Level-D2", and "Advanced Level-D3". In addition, the games in the program are classified into six categories: "Reasoning and Operation Games", "Verbal Games", "Geometric-Mechanical Games", "Memory Games", "Strategy Games", and "Intelligence Questions". Verbal games, one of the categories of intelligence games, are defined as types of games in which players make use of their vocabulary or general culture as well as their

logical inferences, and it is stated that the games can be one-person or team games. In addition, in the aforementioned curriculum, the D1 level of verbal games includes comprehending the basic rules and using words from different fields in games; the D2 level includes deriving appropriate words for the game by using vocabulary and determining basic strategies specific to verbal games; and the D3 level includes reducing the list to be searched by making smart guesses and finding the best solutions in verbal games within the given constraints.

In this context, Let's Tell Intelligence Game (LTIG), which was used in the current study, is in the category of verbal games since it is a game that requires players to create stories by utilizing their existing vocabulary (MoNE, 2013) and is one of the intermediate level intelligence games.

The reasons for preferring this game in the study are that it enables students to create stories during the game process in order to reveal their written mathematical communication skills, it is an easy game, and it is a game that students generally know and like. In studies related to intelligence games, basic geometric concepts and drawings, triangles and quadrilaterals, length and time measurement, data collection and evaluation (Aslan, 2022), patterns and spatial relationships (Esen, 2021), integers, operations with integers, algebraic expressions and area measurement (Demirel, 2015), operations with natural numbers, operations with integers, sets and area measurement (Şanlıdağ, 2020) have been addressed. Therefore, it can be observed that the topics of "Angles" and "Lines and Angles" were not covered in these studies. The adaptation of LTIG in this study focuses on the sub-learning areas of "discovers the properties of adjacent angles, complements, supplements, and vertical angles; solves related problems" and "determines the properties of corresponding, alternate, interior, and exterior angles formed by two parallel lines intersected by a transversal; determines which angles are congruent or supplementary; solves related problems." These learning outcomes were chosen because LTIG includes visuals that are relevant to real-life situations and various angle types. Therefore, the aim is to examine the understanding of the concepts included in these outcomes such as adjacent angles, complementary angles, supplementary angles, vertical angles, corresponding angles, alternate angles, interior angles, and exterior angles.

On the other hand, when the literature is reviewed, no study has been found that examines students' written mathematical communication skills through an intelligence game adapted to mathematics. The current study is important in demonstrating that the verbal intelligence game LTIG can be adapted to mathematics, examining students' written mathematical communication skills through LTIG, and presenting the possibility of using intelligence games with stories in mathematics classes. In this context, the study aims to examine the written mathematical communication skills of seventh-grade students in the stories they create in the context of "Angles" and "Lines and Angles" in LTIG. Therefore, the basic problem of the study can be expressed as follows:

"What are the written mathematical communication skills of seventh-grade students in the stories they create in LTIG?"

METHOD/MATERIALS

The study used a case study design as a qualitative research method. A case study is a research method that focuses on the "how" and "why" questions, and where the researcher has little or no control over the events and the phenomenon is examined in its natural setting (Yıldırım & Şimşek, 2016; Yin, 1984). Additionally, in a case study, a well-defined research topic is described and examined in detail in its real environment (Birinci, Kılıçer, Ünlüer & Kabakçı, 2009). The term "boundary" in this context refers to the ability of the situation to be distinguished from others in terms of time, place, or some physical boundaries (Creswell, 2012). Therefore, the reason why this research is a case study is to provide a rich understanding by examining how seventh grade students' written mathematical communication skills in the stories they created in LTIG, without aiming to generalize.

Sample of the Research

This study consisted of 21 (12 female, 9 male) seventh-grade students who were enrolled in a middle school located in the Eastern Black Sea Region. Criterion sampling, one of the purposive sampling methods, was used to select the participants. The basic understanding of this sampling method is to obtain all cases that meet a predetermined set of criteria. The criteria can be created by researchers or predetermined forms can be used (Teddlie & Yu, 2007; Yıldırım & Şimşek, 2016). In this study, the criteria used to select the participants were being seventh-grade students and having completed classroom activities related to the sub-learning areas of "Angles" and "Lines and Angles". The names of the participating students were kept confidential, and they were coded as "S1, S2, S3, ... S21".



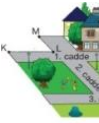












Data Collection Tool
















In this study, the LTIG game material prepared by adapting the concepts of "Angles" and "Lines and Angles" was used as a data collection tool. The original LTIG game and its adaptation to the related concepts are explained below.

Let's Tell Intelligence Game

LTIG is a verbal game played with 12 specially printed wooden cubes. In the game, the cubes are thrown onto a surface randomly. Then, these cubes are arranged in a certain order. The person playing the game tells a story based on the shapes seen on the cube (<https://www.redka.com.tr/redka-haydi-anlat.html>). The LTIG image is given below.

Table 1. Images used in the game and the types of angles to be identified in each image

Images	Types of Angles in the Images	Images	Types of Angles in the Images	Images
I1. Ferris Wheel 	Reverse angle, acute angle, straight angle, right angle.	I2. Scissors 	Inverse angle, acute angle, obtuse angle, complementary angle.	I3. Sketch of a house 
I4. Bicycle 	Acute angle, straight angle, obtuse angle, interior angle, exterior angle, supplementary angle.	I5. Example of a sports branch 	Right angle, acute angle, straight angle.	I6. Paper airplane 
I7. Clock 	Acute angle, straight angle, supplementary angle, vertical angle, obtuse angle.	I8. Window 	Vertical angle, acute angle.	I9. A circus performer 
I10. Desk 	Acute angle, vertical angle, corresponding angle, interior alternate angles, exterior angle, obtuse angle.	I11. Mancala Games 	Obtuse angle, acute angle, vertical angle.	I12. Hairpin 
I13. Glasses 	Obtuse angle, acute angle, vertical angle	I14. Fan 	Straight angle, acute angle, obtuse angle, supplementary angle	I15. Ship 

I16. Chair	Vertical angle, acute angle	I17. Ironing Board	Acute angle, vertical angle, reverse angle, straight angle, obtuse angle	I18. Revolu
				
I19. Car	Acute angle, right angle, obtuse angle, adjacent angles	I20. Hanging monkeys	Acute angle, right angle, Obtuse angle, complementary angle	I21. Spid
				
I22. Goose	Acute angle, Right angle	I23. Kite	Right angle, Acute angle, Obtuse angle, obtuse angle, interior angle.	I24. Play
				
I25. Garden Chair	Acute angle, right angle, obtuse angle, interior angle aç, complementary angles	I26. Football Players	Obtuse angle, Acute angle, right angle	I27. P
				
I28. Runner	Right angle, Acute angle	I29. Ice skate	Acute angle, right angle, obtuse angle	I30. Flan
				

In the second step of the adaptation process of LTIG to mathematics, cubes were made using cardboard in five different colors. The game materials were prepared by pasting the visuals selected from Table 1 on the faces of the cubes and are given in Figure 2.



Figure 2. LTIG material adapted to angles and lines.

The Data Collection and Implementation Process

LTIG was administered to the students in 2 class hours (80 minutes) with the participation of one of the researchers. During this implementation process, the original LTIG was first introduced to the students. Then, the adapted game application was started and the students were asked to take turns on the board and throw the game cubes on an empty table once. When the students threw the game cubes, a 5 cm x 5 cm printout of the image that appeared on the top of the cube was given to the student. In this way, it was ensured that the students did not forget the visuals they obtained while creating the stories. Therefore, each student obtained a total of five images. The visuals used by the students while writing stories in LTIG are given in Table 2.

Table 2. Visuals used by students in LTIG

Participants	Used Visuals
S ₁	Clock (G7), Car (G19), Kite (G23), Runner (G28), Ice Skating (G29)
S ₂	Glasses (G13), Ironing Board (G17), Car (G19), Spider Web (G21), Runner (G28)
S ₃	Carousel (G1), Window (G8), Origami Ship (G15), Chair (G16), Kite (G23)
S ₄	Sketch Example (G3), Bicycle (G4), Clock (G7), Garden Chair (G25), Flamingo (G30)
S ₅	Circus Show Example (G9), Hanger (G12), Glasses (G13), Children's Playground (G24), Football Players (G26)
S ₆	Scissors (G2), Bicycle (G4), Children's Playground (G24), Garden Chair (G25), Runner (G28)
S ₇	Sketch Example (G3), Row (G10), Mancala Game (G11), Goose (G22), Football Players (G26)
S ₈	Glasses (G13), Origami Ship (G15), Ironing Board (G17), Car (G19), Spider Web (G21)
S ₉	Book Pages (G6), Glasses (G13), Ironing Board (G17), Spider Web (G21), Flamingo (G30)
S ₁₀	Sports Branch Example (G5), Fan (G14), Ironing Board (G17), Revolving Door (G18), Children's Playground (G24)
S ₁₁	Chair (G16), Ironing Board (G17), Revolving Door (G18), Car (G19), Kite (G23)
S ₁₂	Mancala Game (G11), Glasses (G13), Car (G19), Monkey Bars (G20), Goose (G22)
S ₁₃	Sketch Example (G3), Sports Branch Example (G5), Circus Show Example (G9), Football Players (G26), Flamingo (G30)
S ₁₄	Sketch Example (G3), Desk (G10), Mancala Game (G11), Fan (G14), Monkey Bars (G20)

S15	Sketch Example (G3), Sports Branch Example (G5), Circus Show Example (G9), Hanger (G12), Garden Chair (G25)
S16	Glasses (G13), Fan (G14), Origami Ship (G15), Ironing Board (G17), Ice Skating (G29)
S17	Bicycle (G4), Clock (G7), Children's Playground (G24), Football Players (G26), Runner (G28)
S18	Scissors (G2), Bicycle (G4), Goose (G22), Garden Chair (G25), Flamingo (G30)
S19	Sports Branch Example (G5), Chair (G16), Monkey Bars (G20), Football Players (G26), Flamingo (G30)
S20	Bicycle (G4), Origami Ship (G15), Car (G19), Kite (G23), Children's Playground (G24)
S21	Book Pages (G6), Circus Show Example (G9), Revolving Door (G18), Children's Playground (G24), Garden Chair (G25)

Data Analysis

LTIG, which was prepared to collect the data of the study, was applied to 21 students and each of them was asked to write a story. It was decided to evaluate the stories written by the students according to the following four criteria in the context of expert opinion:

The stories written by the students were evaluated according to the following four criteria:

1. Mathematics concepts used in the story
2. Identification of angles in the visuals used in the story
3. Whether the angle examples used in the story are correct or incorrect
4. Whether the visual-angle examples used in the story are formal or informal expressions

The stories obtained in the study were analyzed independently by the researchers according to the relevant criteria. In the analyses, the researchers analyzed the participants' stories in the context of the criteria given above. In the second step, the researchers came together and compared their analyses and decided on the final version of the analysis after eliminating the differences between the analyses.

Mathematics concepts used in the story:

With this criterion, it was attempted to determine how many different concepts the students used when explaining an event in their story. The different concepts used while reading the story (acute angle, right angle, obtuse angle, straight angle, reflex angle, complete angle, supplementary angle, complementary angle, adjacent angle) were underlined, and the concept count was determined by counting them. However, when determining the concept counts, only the concepts used for angles and lines were taken into account, and concepts used for all topics of mathematics were considered. In addition, if the same concept was used repeatedly, it was not taken into account.

Determining angles in the used visuals in the story:

The second evaluation criterion is the determination of angles in the visuals used by the students in each story. Firstly, the total number of angle types in the visuals used by the students while writing their stories was calculated using Table 1 and presented in Table 2. Then, the angles used in the stories were determined, and their ratios were examined based on the total number of angle types, and the percentage values for angle determination were calculated (Table 5).

The correct and incorrect use of angle examples in the story:

With this criterion, sentences in which students described angles in any visual and expressed them in relation to the relevant parts of the visuals were evaluated as correct or incorrect. Sentences in which angle examples related to visuals were identified in the story were underlined and evaluated separately. For example, sentences such as "My game on a wide-angle barbecue" and "We saw flamingos with right angles on their legs" were coded as correct visual-angle examples. However, students who mentioned angle examples that were not present in the visuals were evaluated as incorrect visual-angle examples, such as "The stem of the glasses is an example of the u-rule or complementary angle." In this context, the analyses were classified according to the mathematical communication skills assessment rubric developed by Wahyuningrum and Suryadi (2014), which was modified by the researchers. In the original rubric, student performances were divided into five and scored as "0, 1, 2, 3, 4." In this study, four of these performances were used, category names were assigned to the performances, and

the relevant rubric is presented in Table 3. When determining the levels based on the number of correct and incorrect visual-angle examples used in the students' stories, the equation "the width of the series is divided by the desired number of groups (Tekin, 2007)" was taken into account.

Table 3. Rubric for assessing mathematical written communication skills in the context of the "visual-angle" example

Category	Written Mathematical Communication Skill
Incorrect or insufficient narrative (0)	All of the "visual-angle" examples provided by the student are incorrect or no concept is included.
Low-level communication (1)	The use of mathematical language (terms, symbols, signs or representations) is partially correct.
Medium-level communication (2)	Use of mathematical language (terms, symbols, signs, or representations) is correct and detailed.
High-level communication (3)	Use of mathematical language (terms, symbols, signs, or representations) is correct and highly detailed.

Formal and informal expressions of the visual-angle examples used in the narrative:

With this criterion, it was examined whether the descriptions of angles in the sentences explaining the angle examples in the visuals were made by paying attention to the basic properties of these concepts and whether the expressions were formal or informal. Therefore, the student's knowledge of the basic properties of the angle examples used in the narrative was analyzed. For example, the statement "the fan made a right angle of 180° ." was considered as a formal expression since the basic features of the concept were taken into account in the visual-angle example, such as the right angle being 180° . On the other hand, the statement "one day the monkeys were sitting on vines with counterfactual angles." was considered as an informal expression of the visual-angle examples since the concept of counterfactual angle does not make sense in the context of the Angles topic.

Validity and Reliability of the Study

In order to ensure the quality of a qualitative study, it should possess credibility, transferability, dependability, and confirmability (Lincoln & Guba, 1985). In this context, credibility and consistency in the study were ensured by analyzing the data separately by the researchers and analyzing them four times in total, with one week between the analyses. In this context, the researchers worked on examining the stories written by the students according to the criteria specified in the analysis. As mentioned, during these studies, which were carried out at different times with a one-week interval, the classifications made for the students' stories were compared with each other, the similarities and differences between these classifications were examined and revised again in the context of expert opinions. In this way, the quality of the current study was attempted to be improved. Confirmability of the study was attempted to be ensured by including as many stories as possible written by the 7th-grade students at LTIG during the presentation of the research. Transferability was attempted to be achieved by describing and interpreting the study in a clear and detailed manner. Within this scope, the mathematical concepts used in the stories by the students, angle examples in the visuals used in the stories, the percentages of the determination of these examples, and whether they were in formal or informal expressions were classified, organized, interpreted, and presented with example stories in the findings section. Additionally, the sampling method used to select the students included in the study was explained in detail to ensure the transferability of the study.

FINDINGS

This section provides examples of mathematical concepts and numbers, percentages for determining angles in visuals, correct and incorrect use of angle examples in visuals, as well as formal and informal expressions of the numbers and visual-angle examples used in the stories created by participants in the LTIG.

The findings related to mathematical concepts used in the story

The concepts were identified by examining the expressions used by seventh-grade students while creating their stories in LTIG and presented in Table 4.

Table 4. Mathematical concepts and numbers used in the story

Participants	Mathematics Concepts in the Story Created in LTIG	Number of Mathematics Concepts in the Story Created in LTIG
S ₁	Right angle, obtuse angle, 360 degrees.	3
S ₂	Diameter, vertex, area, triangle, trapezoid, square, isosceles triangle, interior opposite angle, right angle, 120 degrees	10
S ₃	Circle, hexagon, rectangle, vertex, equilateral triangle, right triangle, straight angle, interior angle, supplementary angle, corresponding angle.	10
S ₄	Interior opposite angle, supplementary angle, exterior opposite angle, co-interior angle, obtuse angle, right angle, corresponding angle.	7
S ₅	Acute angle, right angle, obtuse angle.	3
S ₆	Right angle, complementary angle, co-interior angle, supplementary angle, obtuse angle.	5
S ₇	Obtuse angle, acute angle, right angle.	3
S ₈	Interior opposite angle, corresponding angle.	2
S ₉	Right angle, adjacent supplementary angle, straight angle, obtuse angle, acute angle, circle, corresponding angle, 5 minutes.	8
S ₁₀	Acute angle, obtuse angle, 150 degrees.	3
S ₁₁	Supplementary angle, interior opposite angle, acute angle, corresponding angle.	4
S ₁₂	Obtuse angle, acute angle, right angle, corresponding angle.	4
S ₁₃	Supplementary angle, obtuse angle, co-interior angle, interior opposite angle, exterior opposite angle, right angle, straight angle, 20 seconds.	8
S ₁₄	Right angle, straight angle, obtuse angle, acute angle.	4
S ₁₅	Co-interior angle, supplementary angle, acute angle, obtuse angle, interior opposite angle, exterior opposite angle, 2 days, 180 degrees.	8
S ₁₆	Acute angle, obtuse angle, supplementary angle, opposite angle, corresponding angle.	5
S ₁₇	Acute angle, complementary angle, right angle, 2 months, 30 seconds, 2 meters.	6
S ₁₈	Acute angle, right angle, obtuse angle, interior opposite angle, supplementary angle, 2 days.	6
S ₁₉	Acute angle, right angle, complementary angle, obtuse angle, supplementary angle, parallel, Z rule (interior opposite angle rule).	7
S ₂₀	Right angle, equilateral triangle, square, trapezoid, circle, hexagon, 360 degrees.	7
S ₂₁	Acute angle, right angle, obtuse angle.	3

When Table 4 is examined, it is seen that in the stories written by the students in LTIG, S8 included at least 2 concepts, namely, internal inverse angle and counterfactual angle; S2 included at most 10 concepts, namely, diameter, corner, area, triangle, trapezoid, trapezoid, square, isosceles triangle, internal inverse angle, right angle, 120 degrees; and S3 included at most 10 concepts, namely, circle, hexagon, rectangle, corner, equilateral triangle, right triangle, acute angle, interior angle, integral angle, counterfactual angle. However, it is seen that the concepts used by the students are not limited to the topics of angles and lines and angles, and that there are also concepts related to the topics of Circle and Circle, Triangles and Quadrilaterals and Time Measurement in the stories.

Findings Related to Determining the Angles in Visuals Used in the Story

Table 5 provides the percentage values for determining the angles in the visuals used by the students while writing stories.

Table 5. Percentage values of students for determining angles in visuals

Participants	Percentage Values	Participants	Percentage Values
S ₁	%11	S ₁₂	%24
S ₂	%11	S ₁₃	%33
S ₃	%25	S ₁₄	%18
S ₄	%25	S ₁₅	%27
S ₅	%14	S ₁₆	%28
S ₆	%21	S ₁₇	%13
S ₇	%17	S ₁₈	%20
S ₈	%10	S ₁₉	%30
S ₉	%29	S ₂₀	%4
S ₁₀	%9	S ₂₁	%15
S ₁₁	%20		

When examining Table 5, it can be seen that S13 identified the highest number of angle types by determining 33% of the angles used in the visuals they used in their story, while S20 discovered the least number of angle types by identifying only 4% of the angles in the visuals.

The Findings on Examples of Correct and Incorrect Use of Angles in Visuals Used in the Story

The 21 stories written by the students in LTIG were analyzed with the help of the evaluation rubric given in Table 3. The written communication levels of the statements specified for each criterion are given in Table 6.

Table 6. The number of correct and incorrect use of angle examples in the visuals used in the story and the level of written communication

Participants	Correct Usage Examples of Angle Illustrations Used in the Story	Incorrect Usage Examples of Angle Illustrations Used in the Story	Written Communication Level
S ₁	Runner (right angle), ice skating (obtuse angle)	-	Intermediate level communication
S ₂	Spider web (alternate interior angle), runner (right angle), car (right angle)	-	Intermediate level communication
S ₃	Ship origami (right angle), window (right angle)	Ferris wheel (full angle), chair (opposite angles), kite (complementary angles)	Low-level communication

S ₄	Garden chair (obtuse angle), clock (obtuse angle/straight angle/right angle), bicycle (exterior angle), sketch (corresponding angle), flamingo (obtuse angle/vertical angle)	Flamingo (opposite angles)	High-level communication
S ₅	Glasses (acute angle), circus performance (right angle), soccer players (wide angle)	-	Intermediate level communication
S ₆	Runner (right angle), children's playground (corresponding angle), garden chair (obtuse angle), scissors (complementary angle), bicycle (vertical angle)	-	High-level communication
S ₇	Mancala (acute angle/wide angle), goose (acute angle), sketch (right angle/acute angle), soccer players (wide angle)	-	High-level communication
S ₈	Spider web (interior angle)	Glasses (corresponding angles), car (corresponding angles), origami ship (corresponding angles), ironing board (corresponding angles)	Low-level communication
S ₉	Flamingo (right angle), spider web (acute angle), ironing board (right angle/wide angle/acute angle)	Glasses (U rule), book pages (adjacent supplementary angles), ironing board (Z rule)	Intermediate level communication
S ₁₀	Fan (acute angle), children's playground (wide angle)	-	Intermediate level communication
S ₁₁	Revolving door (obtuse angle), kite (obtuse angle), chair (acute angle)	Car (opposite angles), ironing board (opposite angles)	Intermediate level communication
S ₁₂	Mancala (wide angle), goose (acute angle), glasses (wide angle), car (right angle)	Monkey bars (corresponding angles),	Intermediate level communication
S ₁₃	Flamingo (obtuse angle), circus performance (wide angle), soccer players (right angle), sketch (corresponding angles)	Sketch (external alternate angles)	Intermediate level communication
S ₁₄	Desk (right angle), fan (acute angle), sketch (obtuse angle), monkey bars (right angle/acute angle/obtuse angle), mancala (obtuse angle)	Sketching (right angle/wide angle)	High-level communication
S ₁₅	Sketch (adjacent angles), garden chair (acute angle), hanger (acute angle), circus show (acute angle/obtuse angle)	Examples of a sports branch (reflex angle), hanger (obtuse angle), circus show (interior alternate angle/exterior alternate angle/corresponding angle)	Low-level communication
S ₁₆	Glasses (acute angle), ice skating (acute angle), ship origami (obtuse angle), fan (complementary angle), ironing board (obtuse angle)	Glasses (vertical angles)	Intermediate level communication
S ₁₇	Children's playground (acute angle), bicycle (acute angle), clock (multiple	-	High-level communication

	angles), soccer players (right angle), runner (acute angle)		
S ₁₈	Goose (acute/right angle), flamingo (right angle), garden chair (obtuse angle), bicycle (acute angle), scissors (straight angle)	Scissors (alternate interior angle)	High-level communication
S ₁₉	Monkey bars (acute/obtuse/straight angle), flamingo (right angle), soccer players (right/obtuse angle)	Monkey bars (parallel angles), chair (Z rule/wide angle), an example of sports branch (supplementary angles/wide angle)	Low-level communication
S ₂₀	-	-	Wrong or insufficient story
S ₂₁	Book pages (acute angle), circus performance (acute angle), garden chair (right angle), revolving door (obtuse angle)	-	Intermediate level communication

When Table 6 is examined, when the stories written by the students in LTIG are classified according to the number of correct and incorrect angle examples in the visuals, it is seen that they mostly exhibited a moderate level of mathematical communication ($n=10$), and only one student's written communication level was insufficient because he did not include any visual-angle examples in his story. However, 6 students had high level written communication skills and 4 students had low level written communication skills.

The stories in LTIG that contain examples of visual angles are provided as high-level communication (Figure 3), moderate-level communication (Figure 4), low-level communication (Figure 5), and incorrect or inadequate responses (Figure 6) examples below.

MAKASIN İNTİKAMI

Yolda yürürken ünlü kadın kosucu, bacağımda dik açı olan Abdulpakize'yle tanıştım. Onu karizmamla tavladım. O da bana Starbucks'tan kahve ısmarladı. Kahveciden ayırdık ve 15 bin TL'lik Cool, tümler açılı bisikletime eve bıraktım. Ondan sonra 15 bin TL'lik Cool, tümler açılı bisikletime yoldaş açılı parka gittim. Altın kaydıraktan kaydım. Parkta bütünler açılı makas görüp kimseye sormadan hırsız edasıyla makası çarice alırken, güzel, damarlı, ellerini makas kesti. Ağlayarak iPhone 18 pro max 130 binlik telefonumu babamı aradım ve beni almasını söyledim. 3 trilyonluk evime vardığımda ters açılı, Altından 1 milyonluk Salıncakımda sallanırken uyuya kalmışım. Rüyanda bütünler açılı makasın 90° açılıp, benden intikam almak için beni kesmeye çalıştığını gördüm ve korkarak uyanırdığımda Makas yandı...

THE REVENGE OF THE SCISSORS

While walking on the street, I met a famous female runner named Abdulpakize, who had a right angle on her leg. I charmed her with my charisma and she bought me coffee from Starbucks. We left the coffee shop and I went to the corresponding angle park on my expensive bike with complementary angles, worth 15,000 TL. I slid down the golden slide. When I saw supplementary angle scissors in the park, I acted like a thief and tried to take them without asking anyone. However, the scissors cut my beautiful, veiny hands. I cried and called my father with my iPhone 18 Pro Max, which costs 130,000 lira, asking him to pick me up. When we arrived at our house, which costs 3 trillion lira, I fell asleep while swinging on my 1 million lira golden swing with reverse angles. In my dream, I saw the complementary angle scissors trying to cut honey at a 70-degree angle to take revenge on me, and when I woke up in fear, the scissors were next to me...

Bir pazar sabahı kahvaltıda gidiyorduk. Gittiğimiz bahçesinde gagası der ağılı ördekler vardı ve sürü gibi bacakları dik açıya sahip flamingolar vardı. Kahvaltıda hazır olana kadar direkleri ters ağılı salıncakda sallandım. Kahvaltıda yedikten sonra kordonla demiri der ağılı bisikletlere bindik. Oradaki insanlar da ters ağılı makasla ördeğin tüylerini kestiler. Gagaları dik açıyla olan ördeklerde direkleri ters ağılı salıncakta sallandılar. İlk gün sonra babaannemize gittik. Bir de ne göreyim ahırda bacakları dik ağılı flamingolardan brodada var. Amcalar köye gelirken gagası der ağılı ördeklerden almışlar. Demiri der ağılı bisikletle evin bahçesine gittik. Bahçedeki çiçeklerin kurumuş yapraklarını bütünler ağılı makasla kestik.

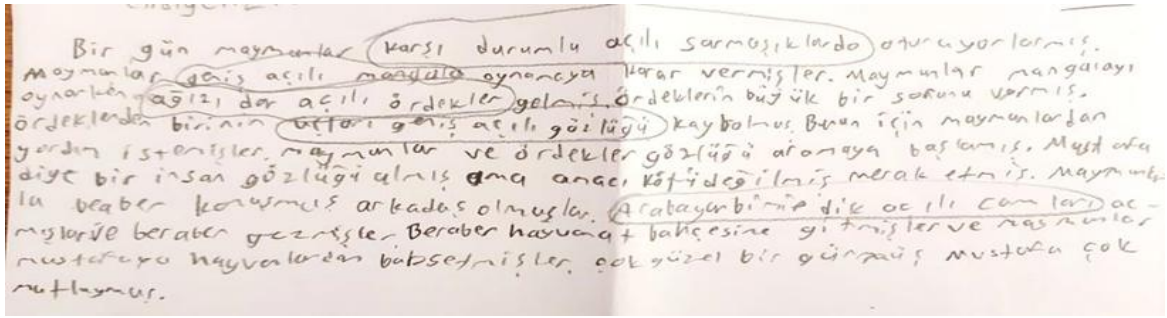
One Sunday morning, we set out for breakfast. There were ducks with acute-angled beaks and flamingos with legs at right angles in the tea garden where we went. While waiting for breakfast to be ready, I swung on a swing with reverse-angle hanging ropes. After breakfast, my brother and I rode on our iron bicycles with acute angles. The people there cut the feathers of the duck with an interior angle. The ducks with right-angled beaks climbed onto the reversed-angled swing. Two days later, we went to my grandmother's house and I saw that there were also flamingos with right-angled legs in the barn. My uncle had bought the ducks with acute-angled beaks when they came to the village. We went to the house on an acute-angled iron bicycle. We used supplement-angled scissors to cut the dried leaves of the flowers in the garden.

Figure 3. Examples of high-level communication (S6 and S18)

The student coded S18, whose written explanation is given in Figure 3, correctly identified 6 out of 7 visual-angle examples in the visuals he used while writing the story, and the student coded S6 correctly identified all 5 visual-angle examples in the visuals he used while writing the story. From this point of view, it was seen that the terms used in the stories about the use of mathematical language were correct and detailed by explaining them with angle examples. Therefore, it is seen that T6 and T18 exhibited a high level of mathematical communication by including correct and highly detailed expressions in the use of mathematical language.

günlerden Çarşamba'dı okula girmek için Sabah erkenden kalbim gıcık kıyafetlerimi giyip aksamı kontrol ettikten sonra evden çıkardım ve arabaya binerim. Don okula arabamın aynalarının karşı durumlu açısı olduğunu öğrenmişim. Bugün sınavım vardı ve çok heyecanlıydım, ben konuştuktan okula varmışık bile. Okul çıkışı: babam gelip beni alır babama neride gittiğimizi sorardı. Oda Annem'e gideceğimizi söyledi ve annem'e varmışık bile. Annem'in kapısı dikliğini aynasına ve babama kapıda ters açı olduğunu söyledim. Babamda: Peki Sırdaki kocamın aynasında hangi açı var diye sordu. Ben de: İki ters açı olduğunu söyledim. Annem'e girdiğim de iki direkte mağazalar karşıladı. Orda bir mağaza dikliğini aynasına mağazanın içinde aynası vardı. Babama buda karşı durumlu açısı olduğunu söyledim. Hem anneler gence garklaşıyordu annem aynası aynamı diye sordu ve babamda: Olur dedi: babam Sırdaki köşkte hangi açı olduğunu bilersen orda alican dedi: bade dar açı olduğunu söyledim ama malesef heyecandan onu bilemedim.

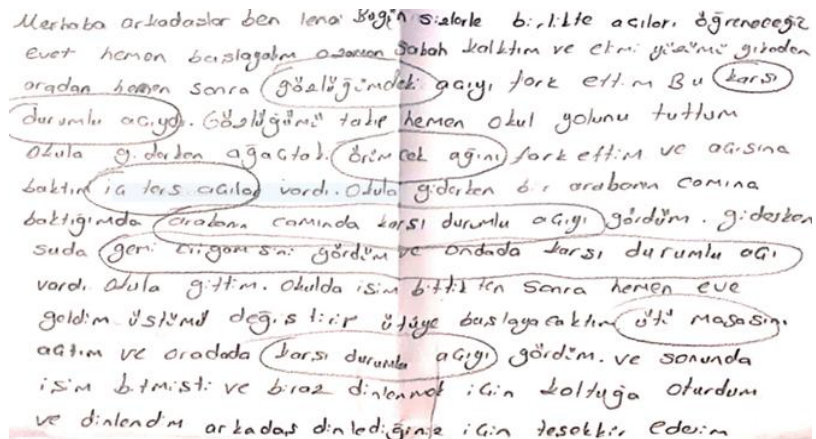
It was Wednesday and I woke up early in the morning to go to school. After putting on my daily clothes and checking my bag, I left the house and got into the car. Yesterday, I learned that the windows in the car were in vertical angles while I was at school. Today, I was very excited because I had an exam. We arrived at school while I was talking. After school, my dad came to pick me up and I asked him where we were going. He said we were going to the shopping mall and we had already arrived, I noticed that the door of the mall was in reverse position. I told my dad that the door was in the reverse position. Then he asked me, "What about the angle of the kite that boy is flying over there?" I said it was an alternate interior angle. When we entered the shopping mall, we saw the stores. One store caught my attention, it had an ironing board inside, so I told my dad that the ironing board had vertical angles of the legs. Since Mother's Day was approaching, I asked my dad if we should buy an ironing board for my mom. He agreed and said, "If you can tell me the angle of that chair over there, I will buy it too." I said it was an acute angle, but unfortunately, due to excitement, I couldn't remember it accurately.



One day, the monkeys were sitting on vines that had a vertical angles. The monkeys decided to play a wide-angled game of Mangala. While the monkeys were playing Mangala, ducks with acute-angled beaks came along. The ducks had a big problem; one of them had lost their obtuse-angled glasses. So, they asked the monkeys for help. The monkeys and ducks started to search for the glasses. A human named Mustafa got glasses, but his intentions were not bad, he was just curious. Mustafa became friends with the monkeys. They got into the car and opened the right angled windows, and drove around together. They went to the zoo and the monkeys told Mustafa about the animals. It was a beautiful day, and Mustafa was very happy.

Figure 4. Examples of intermediate-level communication (S11 and S12)

S11 coded student, explained in Figure 4, correctly identified 3 out of 5 visual-angle examples used in their story, while S12 coded student correctly identified 4 out of 5 visual-angle examples used in their story. Therefore, it is seen that they exhibited a moderate level of mathematical communication by including mostly correct and detailed statements in the use of mathematical language.



Hello friends, I'm Lena. Today we are going to learn about angles together. Yes, let's start right away then! I woke up in the morning and washed my face, and while putting on my glasses, I noticed the angle in my glasses. It was a vertical angle. I put on my glasses and set off to school immediately. On the way to school, I noticed a spider web on a tree and looked at its angles. There were interior angles. When I looked at a car window while going to school, I saw the vertical angle in the car window. While on the way, I saw a paper boat in the water, and it also had a vertical angle. I arrived at school. After finishing my work at school, I quickly came home. I changed my clothes and was about to start ironing. I noticed the vertical angle on the ironing board. Finally, my work was done, and I sat down on the couch to rest. Thank you for listening.

Figure 5. An example of low-level communication (S8)

The student with the code S8, described in Figure 5, incorrectly identified all 5 visual-angle examples in the images used in their story. Therefore, in the story written by S8, it is seen that the terms, symbols, signs or representations related to the use of mathematical language are incorrect and the student exhibits a low level of written mathematical communication.

Bir gün arabayla köye gittik. Ve derede kağıtlan gemi yaptım. Eve geri geldiğimizde bisiklete binip pedal 360° derece dandüverek parka gittim. Ve eve gelirken 90° geldiğim düz geldim. Köyde gemi yaparken geminin yelkenin teki eşkenar üçgen di teki ise kare idi yamuk almıştı ama his batmadan gitmişti. Arabanın tekerleği çember gibiydi yusyuvarlak. Uçurtma ise b gendi parkta ise bir çocuk salın cakta 360° dönüyodu ve bugünüm Matematik üzerineydi. Güzel geçmişti ve eve geldim televizyonu açtığımda arka Sokaklar 1208. bölümü iyi bir bölümü çok iyi sahneler ve efektler vardı biraz telefona bakıp yatmaya gittim.

One day, we went to the village by car. I made a paper boat in the creek. When we got back home, I rode my bike and went to the park by pedaling 360 degrees. And when I came back home, I came straight as a right angle. While making the boat in the village, one sail was an equilateral triangle and the other was square but then it was a trapezium, yet it managed to sail without sinking. The wheel of the car was like a circle. The kite was hexagonal and there was a child swinging 360 degrees in the park, and today everything was about math. It was a nice day and when I turned on the TV, it was the 1208th episode of "Arka Sokaklar". It was a good episode with great action scenes and effects. After checking my phone for a while, I went to bed.

Figure 6. An example of a wrong or insufficient answer (S20)

As explained in Figure 6, the student with the code S20 was unable to correctly identify any of the angle examples used in their story. Therefore, since all of S20's angle examples are incorrect and the concepts related to angles are not used correctly, this story is considered incorrect or insufficient in terms of mathematical communication and is not included at any level of mathematical communication.

The Findings on the Formal and Informal Expressions of the Visual-Angle Examples Used in the Story

It has been observed that there are correct and incorrect uses in identifying angle examples in the visuals of the stories written by the students in LTIG, and therefore, formal and informal expressions related to angle concepts are included. In the formal definition, the properties of the terms are taken into consideration in the expressions related to angles, and they are used correctly. In the informal definition, it is seen that some of the properties of this subject are used, leading to incomplete or incorrect expressions. Therefore, in this section, the formal and informal states of the expressions in the visual-angle examples are discussed. Below are some examples of formal expressions found in the stories.

vacı ordeklerin üstüne binerek dereden geçtik. Annaneme
 vardığımda Annanemin evinin yanındaki Sahada Annanem
 futbol oynuyordu. Bende Annanemin evine gidip dar acılı
 mangala oynadım. Kuşda vacı. Dik acılı sıralar bana eslik
 etti. Ordekler geldi. dar acılı ordeklerde bana eslik etti annanem
 eve geldi ve beni dik acılı ordek görmek için annanemle
 sudan çıktı. ve dar acılı ordekler gelik. geniş acılı sıralarda
 yerimize geçtik annanem beni bırakır bırakmaz. geniş
 acılı sahaya geri gitti ve vacı ordeklerde geldi.
 ve hoca sınıfa girdi. hoca sınıfa girdiği an geniş acılı
 mangala hocanın kafasına dokundu. hoca çok sinirlendi.
 ve annanem birden sınıfa girdi beni alıp çıktı. ve bulcadı.

We crossed the stream by riding on the flying ducks. When I arrived at my grandmother's house, my grandparents were playing soccer in the field next to my grandmother's house. I played a game of acute-angled backgammon at my grandmother's house. The right-angled desks in the sky accompanied me. Ducks came, and the acute-angled ducks accompanied me. My grandmother came home and we left the house to take me to the right-angled school. We arrived at the acute-angled school and sat at the wide-angle desks. As soon as my grandmother left me, she went back to the wide-angle field and the flying ducks also came. Then the teacher entered the class. As soon as the teacher entered the class, the wide-angle backgammon fell on the teacher's head. The teacher got very angry. Suddenly, my grandmother entered the classroom, took me, and left.

Ahmed Akkaleti kınıfa giden bir çocuktu.
 Ahmed okulları sonra parka gitmeye çok sevdi.
 Yöni öyle bir günde annesi ve karısıyla parka
 gittimlerde parka dar acılı yerlerden geçenler
 gittimlerde düştü ve elleri paralandı. Ve o gün yaklaştı
 2ay boyunca geçmedi. 2ay sonra bir futbol maçı
 amacıyla bisiklete bindi ve futbolu gitti
 bisikleti orda dar acılı gitti. Bu da onun eski
 bisikletini anımsatmıştı. Futbol sahasına geline
 maçı başlamasına yaklaştı. 30 saniye kalmıştı saat.
 12.30 da Ahmed matematik dersinde bu konuyu
 görmüşü saat 13.00'de aya banyoya. Hoca Lendile
 ve son saatlarında Ahmed okuma bir ruhsatı aldı
 ve gitti. eve geldiğinde maçı tekrarıma katıldı
 ve ayağının parmağı dik dar acılı olduğunu gördü. Ve yarınki
 kızı görmek için erkenden yattı. Sadece kollarında
 saatini ~~görmüşü~~ görünce saatın geç olduğunu gördü
 dü ve kızı yarınki kızı kızı gitti. ayağına katıldı.
 en andaki insan ayağı çok hızlı koşmak için
 ayakkabı, neydi? 2m saat, ve bisikletinde ko-
 şmaları arasında kolları acı. Dar acılı. Ve anladık
 kızı... her yerde vardı. Bisiklet, saat katıldı.

Ahmed was a fourth-grade student who had a great fondness for going to the park after school. One day, he went to the park with his parents and while sliding through some acute places, he accidentally fell and injured his knee. Despite receiving treatment for two months, the wound did not heal. Two months later, he rode his bike to attend a football match and noticed that the bike had an acute angle. Upon arrival at the football field, he realized that the match was starting in 30 seconds at 12:30 pm. During his math class, Ahmed noticed that the clock resembled a supplementary angle. The match began and a goal was scored. Upon returning home, he reviewed the match and noticed that he had a right angle on his foot. He went to bed early for a running competition the next day. When he woke up and checked the time, he realized he was late, but still managed to rush to the race. Starting off in the lead, The angle he was stuck in between the bushes to speed up was an acute angle. And he realized that running was everywhere...

Figure 7. Examples of formal expressions (S7 and S17)

When examining Figure 7, it is seen that student S7 correctly identified the relevant angles in the images by considering the basic characteristics of acute, obtuse, and right angles. Similarly, it is observed that student S17 correctly identified the relevant angles in the images by taking into account the information related to acute, complementary, and right angles. Therefore, it can be stated that students with codes S7 and S17 used formal expressions in their stories.

The informal expressions and explanations used by students in their stories are given in Table 7.

Table 7. Informal expressions in Visual-Angle examples

Participants	Informal Expressions in Visual-Angle Examples	Explanations
S ₃	1."Since the Ferris wheel is circular, there is a full angle at the center."	When examining Figure 1, it can be seen that S3 provides an e concept related to angles and therefore lacks a concept explan be due to the presence of a straight angle in the relevant visual angle".
	2."A kite shaped like a hexagon has 6 equilateral triangles inside due to the sticks, and the sum of the interior angles of the triangles becomes a straight angle."	In the case of the kite (I23), S3 states that the sum of the interior angle. It is thought that the student knows that the sum of the i S3 states that the sum of the measures of two angles that measu the concept of full angle. Therefore, it has been determined tha of full angle.
	3."Since the legs of the chair are in the middle, angles that correspond to each other are formed between the arms of the chair and the ground."	Finally, S3 mentions an example of "corresponding angle" which therefore lacks a concept explained in the chair visual (I16).
S ₄	1."He noticed that there are alternate angles in the legs of flamingos."	According to the analysis, it is observed that Ö4 stated the exist flamingo visual (I30). The reason for this may be the presence of
	1."Then I noticed the angle on my glasses. It was a corresponding angle."	
S ₈	2."On my way to school, when I looked at a car window, I saw a corresponding angle on the window."	It can be observed that S8 has identified a type of angle called story. However, it has been revealed that the student provided can be found in many visuals (I13, I15, I17, I19). Therefore, it can learning on the topic of angles, despite the presence of many an to "opposite angles" instead.
	3."While I was walking, I saw a paper boat origami on the water, and it had corresponding angles."	
	4."I opened the ironing board and saw corresponding angles there too."	
S ₉	1."The eyeglasses temple is an example of the U-rule, which refers to opposite congruent angles."	S9 stated that the glasses image (I13) demonstrates the "U-rule reason for the student's thinking may be the presence of corres
	2."The pages of the book are an example of adjacent complementary angle"	S9 indicated that the image of book pages (I6) contains "adjacen the student determined this type of angle without consideri neighboring and adjacent to whole angles are called neighbo
	3."The legs of ironing boards are an example of the Z-rule."	

Participants	Informal Expressions in Visual-Angle Examples	Explanations
		conceptual deficiencies regarding neighboring wholes angles, as neighboring wholes angles.
		S9 stated that the image of the ironing board (I17) demonstrated this type of angle as the "Z-rule" because it is as revealed that S9 selected a visual-angle example without considering
S ₁₁	<p>1. "Yesterday at school, I learned that the windows of the car have opposite congruent angles."</p> <p>2. "There was an ironing board in the store, and I told my dad that there are opposite congruent angles here."</p>	<p>It appears that S11 identified a type of angle as "opposite congruent angles". It was observed that the student gave examples of this angle type. It appears that the student determined the angles as "opposite congruent angles" concepts related to angles, despite the presence of many angle</p>
S ₁₂	<p>1. "One day, monkeys were sitting on vines with opposite congruent angles."</p>	<p>S12 identified the angle type of "opposite congruent angles" in "opposite congruent angles". This may be due to the presence of angles facing the same direction. It appears that the student named these angles as "opposite congruent angles" explanations.</p>
S ₁₃	<p>1. "I mistakenly marked the sketch, and then I saw that there were alternate exterior angles in it."</p>	<p>Based on the given information, it seems that S13 identified one type of angle. Alternate exterior angles are angles formed when a transversal intersects two parallel lines. However, there is no external information. It appears that S13 selected a visual-angle example without considering</p>
S ₁₄	<p>1. "While I was making a fan for myself, we drove through a street in a car, first passing through a wide-angle 3rd Avenue, and then through a right-angle 2nd Avenue."</p>	<p>S14 identified wide angle and right angle as examples for the angles. However, wide angles and right angles are not present in this example. The reason for the student's misconceptions about wide angles and right angles.</p>
S ₁₅	<p>1. "She saw an athlete on TV extending their leg 180 degrees, which reminded her of the straight angle."</p> <p>2. "There were a wide variety of angles on the hangers."</p>	<p>It can be observed that S15 provided a wrong example of angle in "180 degrees". An athlete's leg forming a 180° angle creates a complementary angle. It appears that the student had a conceptual misunderstanding by stating that this angle was a complementary angle. It appears that the student had a conceptual misunderstanding by stating that this angle was a complementary angle. It appears that the student had a conceptual misunderstanding by stating that this angle was a complementary angle.</p>

Participants	Informal Expressions in Visual-Angle Examples	Explanations
	<i>3."When they arrived home, they saw people performing acrobatics in front of their doors, including alternate interior, alternate exterior, and corresponding angles."</i>	<p>S15 also made an incorrect statement about the existence of m lack of understanding of this concept.</p> <p>Furthermore, S15 indicated that there were interior alternate, e performance example visual (I9). However, as there were no su the student misidentified the types of angles.</p>
S ₁₆	<i>1."Glasses- Opposite congruent angles."</i>	<p>S16 has indicated the presence of "corresponding angles" in the thinking may be the presence of angles that are opposite to each</p>
S ₁₈	<i>1"The people there cut the feathers of the duck with scissors that have an interior angle."</i>	<p>It is observed that S18 indicated the presence of an interior angle that an interior angle is an angle between parallel lines that lies it can be stated that there is no interior angle in the scissors visu in the scissors visual, was found to have a wrong or incomplete u</p>
S ₁₉	<i>1.In the first picture, there were monkeys and we found parallel angles related to them.</i> <i>2.In the third paper, there was a stool and I said there was a Z angle, while my sibling said there was a wide angle.</i> <i>3.In the last paper, there was a male gymnast and my sibling found complementary and obtuse angles in it.</i>	<p>In relation to the visual representation of monkey clusters (I20), to as a "parallel angle." However, it can be observed that S19 concept of parallelism, which pertains to the relative positions o evident that S19 has conceptual deficiencies regarding the relat</p> <p>Regarding the chair visual (I16), S19 has indicated the presence is presumed that S19 associates this angle type with the concep obtuse angle in the given visual, it becomes apparent that S19 ha to the accurate concept of interior angles and obtuse angles.</p> <p>In the sports example visual (I5), Ö19 has expressed the existen believed that S19 holds an erroneous belief that the athlete's leg Moreover, the absence of an actual obtuse angle in the visual in concepts.</p>

DISCUSSION, CONCLUSION AND RECOMMENDATIONS

In this study, the written mathematical communication skills of seventh-grade students in their stories created in LTIG were investigated. In this context, LTIG was adapted to the topics of angles and lines and applied to the students.

The present study investigates the written mathematical communication skills of seventh grade students in their stories created in LTIG. In this context, LTIG was adapted to the topics of angles, lines, and angles and applied to the students. It was found that students included at least two and up to 10 concepts in their stories written in LTIG, and these concepts were not limited to the topics of angles and lines but also related to other topics in mathematics. However, even though there were many concepts that could be included in the visuals used by students in LTIG, being limited to a maximum of 10 concepts indicates that there were not enough concepts used in the stories. Similarly, Dur (2010) stated that students used a small number of mathematical relationships and concepts when expressing their ideas mathematically in writing. Therefore, based on the findings of the present study, it is consistent with the results of previous relevant research that students are able to use only a small number of mathematical concepts when expressing their thoughts in writing in LTIG. Writing activities have been stated to allow students to recall their experiences related to mathematics, reflect on these experiences and establish a relationship between the desired problem situation and experiences (Powell, 1997), provide feedback by slowing down the thinking process and organizing ideas (Liebars, 1997), and thus contribute to the development of higher-order cognitive skills (Smoak, 2017). The study by Öztürk, and Işık (2016) revealed that teachers believed writing practices could support learning. Therefore, it can be said that incorporating writing activities such as story writing into the teaching process can facilitate learning by contributing to students' cognitive and metacognitive skills. In line with the recommendation of using event-based applications such as stories, fairy tales, and riddles in math classes (Hacısalihoğlu Karadeniz, 2018), it is suggested that writing practices related to the concepts taught in math classes should be used in the form of stories, acrostic poems, short stories, riddles, etc. In this way, it is believed that students' thinking processes about the newly learned concept can be developed and meaningful and permanent learning can be achieved rather than memorizing a concept as a set of rules.

According to the study, when the percentages of students' ability to determine angles in the visuals used in LTIG were examined, it was revealed that only up to 33% of the angles in the visuals could be identified. Therefore, it can be said that students were also unsuccessful in determining angles in visuals selected from real-life situations, and they could not establish a relationship between the knowledge they acquired about angles and their ability to determine angles in visuals. It is believed that the reason for this is that students cannot relate their angle concepts to everyday life situations. Kabaal and Ata-Baran (2016) stated that teachers' perceptions regarding the use of mathematical language in different disciplines and in real-life situations play a significant role in the development of their mathematical communication skills. Thus, it can be argued that teachers' failure to relate mathematics to real-life situations has a negative impact on students' ability to establish connections between concepts and their applications. Therefore, teachers should focus on conceptual understanding by emphasizing the relationship between the concept of angles and their real-life applications in learning environments (Yavuz-Mumcu, 2018). Based on this, it is recommended to conduct studies to explore angle examples in everyday life and determine the measurements and types of angles in objects selected from real-life situations in teaching the concept of angles.

In the study, expressions identifying visual-angle examples from students' created stories were evaluated according to the "evaluation rubric for written communication skills in the context of visual-angle examples." As for students' written mathematical communication skills, it is observed that 48% of them have a moderate level of mathematical language proficiency, 28% have a high level, 19% have a low level, and 5% have insufficient use of mathematical language. Thus, it can be said that the majority of students have a moderate level of mathematical communication skills. Similarly, it has been stated that the level of understanding and using mathematical language for exponentials among eighth-grade students is at a moderate level (Güzel & Yılmaz, 2020). Furthermore, Yılmaz and Türkmen (2019) reported that seventh-grade students' use of mathematical language regarding lines and angles is generally at a moderate level. These studies support the results of the present study. However, Yılmaz and Türkmen (2019) stated that the highest level of students' mathematical language use was only 11.1%, which differs from the result of the present study.

In the study, it was found that 19% of the students used mathematical language at a low level and 5% had inadequate use of mathematical language due to the absence of visual-angle examples in the visuals they used in their stories. Similarly, it has been stated that the mathematical language skills of seventh-grade students in the learning area of "Algebra" need to be developed (Akarsu-Yakar & Yılmaz, 2017; Yalvaç, 2019), there are deficiencies in the mathematical language used by students regarding quadrilaterals (Kula Yeşil, 2015), the use of mathematical language by students is limited and not clear (Açıl & Zeybek, 2017), and students have insufficient ability to use mathematical language in story writing (Dur, 2010). In addition, it has been stated that eighth-grade students have deficiencies in their use of mathematical language in expressions involving square roots (Yılmaz & Güzel, 2020), and students have difficulty using appropriate mathematical language when expressing their mathematical thoughts (Yüzerler, 2013). Therefore, this result of the study is supported by previous research. Yıldız (2016) stated that every expression used by the teacher in class has an important role in the process of creating schemas in students' minds regarding the concepts they have learned. In the same study, it was noted that teachers' failure to pay attention to the language they use during the teaching process could lead to the incorrect or inadequate formation of mathematical language in students. Additionally, the low level of proficiency in using mathematical language by students may be related to the lack of an environment that promotes the

use of this language in the teaching and learning process, as well as teachers' focus on operational skills rather than ensuring the proper use and implementation of mathematical language (Güldal, 2022). During classroom activities, allowing students to freely express their ideas and verbally and/or in writing explain their problem-solving methods with reasons could advance their mathematical communication skills (Aksoy, 2021). Furthermore, it is suggested that the use of certain words, word patterns, and corresponding explanations by the teacher can help students become familiar with mathematical language, develop their expression styles, and effectively communicate math concepts using language (Jamison, 2000).

In the study, it is stated that S9 in I6 "adjacent angles", S13 in I3 "exterior angles", S14 in I3 "obtuse angles and right angles", S15 in I12 "obtuse angles", and in I9 "alternate interior angles, alternate exterior angles, and corresponding angles", S18 in I2 "interior angles" and in S19 in I30 "vertical angles", and in I16 and I15 "obtuse angles" are mentioned. However, these angles are not found in the relevant visuals. Therefore, it can be said that these students have made incomplete or incorrect learning regarding these angles and cannot determine the angles in the visuals. In addition, Ünal (2013) has also shown that students have difficulty identifying exterior angles. Ünal (2013) states that students confuse obtuse angle with reflex angle and often identify adjacent angles as exterior angles when determining them on parallel lines. Yılmaz (2011) found that 45% of seventh grade students could not identify alternate angles, 28.3% could not identify interior angles, 43.3% could not identify exterior angles, and 60% could not identify corresponding angles. In a study by Özbellek (2003), it was determined that sixth and seventh grade students had difficulty in demonstrating the concepts of obtuse angle, interior angle, exterior angle, and alternate angle on a diagram. Therefore, these results are consistent with the findings of this study.

In the current study, it was found that students used both formal and informal language when identifying angles in the images used in LTIG. In the formal definitions, the terms were correctly used by considering their properties, while in the informal definitions, some terms were used incompletely or incorrectly by considering their properties. In this context, it was revealed that the students used informal language such as "perigon", "opposite angle", "Z Rule", "corresponding angle", "U Rule" and "parallel angle" that do not belong to any of the concepts related to angles in their stories. It has been determined that the concepts in question are not included in various mathematics textbooks approved by the Board of Education or in the current curriculum (MoNE, 2018). It is thought that the Z Rule among these concepts is related to "interior alternate angles" and that the parallel angle is related to "three lines on the same plane being parallel to each other." In a study where students were asked to explain the concept of angles, it was stated that although students' knowledge of the "angle" concept was insufficient, teachers talked about angles without explaining what they were (Dickson, Brown, Gibson, 1993; cited in Yeşildere, 2007). This situation emphasizes the need for teachers to be careful in their use of mathematical language during the concept teaching process.

The study includes statements such as "The sum of the interior angles of a triangle is a whole angle" for S3 and "I saw an athlete on TV with his leg at 180 degrees, which reminded me of the concept of a whole angle" for S15. However, it is seen that S19 stated that the sport with a 180° angle is an example of a whole angle, even though it is not a whole angle. Based on this, it has been revealed that these students have incomplete, weak, and incorrect learning about the concept of "whole angle" without considering the explanation that "an angle consisting of two angles whose measures add up to 180 degrees is called a whole angle". Similarly, Kula Yeşil (2015) stated that students have difficulty expressing the concept of "whole angle" mathematically, and Yılmaz (2011) stated that students mostly cannot determine whole angles. In addition, it has been found that there are concept misconceptions related to the "Algebra" learning area in the study that focuses on the mathematical language usage of seventh-grade students (Akarsu, 2013). Yalvaç (2019), in his study examining the mathematical language usage skills of eighth-grade students, revealed that students have concept misconceptions in addition to operations with algebraic expressions. The study on the effects of concept cartoons applications on fifth-grade students' use of mathematical language (Aygün, 2018) reported that students had a misconception about the representation of dots by using lowercase letters. In their study examining students' level of mathematical language use and their opinions about the language related to square root expressions, Yılmaz and Güzel (2020) found that students had a misconception about square root expressions. Therefore, the related studies on students' mathematical language skills support the detection of misconceptions in the present study.

In conclusion, the study found that seventh grade students included only a small number of mathematical concepts, up to 10, in their stories written in LTIG, and were able to determine only 33% of the angles in the visuals provided. Despite the limited use of mathematical concepts and the low success rate in identifying angles, the study revealed that students had a moderate level of proficiency in written mathematical communication skills. This is due to the fact that most of the expressions used by the students in describing the visual angle examples were correct. Previous literature has shown that mathematical stories have many benefits for students (Franz & Pope, 2005; Goral & Gnadinger, 2006; Healy & Sinclair, 2007). In light of the data obtained from this study, it is emphasized that writing applications used in learning environments can be used in various forms in math lessons. Especially, it is recommended to write stories related to the taught concept. Thus, the accuracy of the information acquired by learners can be checked, and incomplete or incorrect learning and conceptual misunderstandings can be detected. In addition, it has been shown that the incorrect use of mathematical language by mathematics teacher candidates has a negative impact on students' ability to use mathematical language correctly (Kula Ünver & Bukova Güzel, 2019). Moreover, it has been stated that these language deficiencies are one of the main reasons why students find it difficult to understand mathematical concepts (Moore, 1994). In this context, it is considered important for teacher candidates to use correct mathematical language and to have a high level of mathematical communication skills. Therefore, it is recommended that teacher education programs provide information

to candidates on the advantages/disadvantages of correct/incorrect usage of mathematical language and raise awareness on the importance of this issue.

In future studies, one of the limitations of the current study can be addressed by including additional geometry concepts such as polygons, right triangles, acute triangles, obtuse triangles, isosceles triangles, equilateral triangles, scalene triangles, parallelograms, quadrilaterals, trapezoids, and diagonals in the LTIG process. In this adaptation, students' ability to identify both angle types and related concepts in real-life visuals can be assessed through written communication skills or mathematical language contexts. As a final remark, it is hoped that this study will contribute to future studies in the context of mathematical written communication, mathematical language and educational games (traditional children's games, intelligence games) adapted to mathematics.

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Statements of publication ethics

I/We hereby declare that the study has not unethical issues and that research and publication ethics have been observed carefully.

Researchers' contribution rate

The study was conducted and reported with equal collaboration of the researchers.

Ethics Committee Approval Information

The research proposal titled "Analyzing the Written Mathematical Communication Skills in the Stories Created by Seventh Grade Students in the Let's Tell Intelligence Game" was discussed by the Giresun University Ethics Committee Commission at the meeting dated 09.05.2023 and numbered 05/04 and was unanimously approved. The approval code for the study is E-50288587-050.01.04-154547.

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