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Problems Posed by Prospective Elementary Mathematics Teachers in the Concept of Functions: An Analysis Based on SOLO Taxonomy

İlköğretim Matematik Öğretmeni Adaylarının Fonksiyonlar Konusu İle İlgili Kurdukları Problemler: Solo Taksonomiye Göre Analiz

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Abstract: The aim of this study is to carry out an analysis according to SOLO-taxonomy about problem posing knowledge of prospective elementary mathematics teachers on mathematical functions. The methodology adopted in the current study was a case study. The participants of the study consisted of 67 prospective elementary mathematics teachers. According to the findings of the study, the knowledge of the prospective elementary mathematics teachers on functions in mathematics is open to development through proper teaching methods. Moreover, the majority of their knowledge levels have been grouped as pre-structural level, multi-structural level and relational level while only a few of their knowledge levels were at extended abstract level. Based on the results, it might be beneficiary to train prospective mathematics teachers who can creatively use problem posing activities in their classrooms and who will become a model for their students with their problem-posing performances to overcome their conceptual deficiencies concerned with functions.
Keywords: Mathematical knowledge, problem posing, prospective mathematics teachers, SOLO taxonomy

Öz: Bu çalışmanın amacı ilköğretim matematik öğretmen adaylarının fonksiyonlar kavramı ile ilgili kurdukları problemlerin analizini SOLO taksonomisine göre yapmaktır. Çalışmada kullanılan yöntem durum çalışmasıdır. Çalışmanın katılımcıları 67 ilköğretim matematik öğretmen adayından oluşmaktadır. Araştırmanın bulgularına göre, ilköğretim matematik öğretmenlerinin fonksiyon konusuna ait bilgileri uygun öğretim yöntemlerinin kullanılması ile gelişime açıktır. Ayrıca, öğretmen adaylarının bilgi seviyeleri çoğunlukla tek yönlü, çok yönlü ve ilişkisel yapı seviyelerinde iken soyutlanmış yapı seviyesinde bilgiye sahip olan çok az öğretmen adayı olduğu görülmüştür. Bu çalışmanın sonuçları sınıf için öğretim faaliyetlerinde kurdukları problemlerin öğrenciler için model oluşturacak olan öğretmen adaylarının fonksiyonlarla ilgili kavramsal eksikliklerinin giderilmesi gerekliliğine işaret etmektedir.
Anahtar Kelimeler: Matematiksel bilgi, matematik öğretmeni adayları, problem kurma, SOLO taksonomisi.

Introduction

The concept of function is one of the basic concepts in mathematics learning (Even, 1998; Gagatsis & Shiakalli, 2004). The fact that the concept of function is one of the most important concepts which students come across in secondary and higher education is a widely-recognized conclusion among mathematicians (Eisenberg, 1992; Kalchman & Case, 1998). Sierpinska (1992) asserts that mathematical function is a key component for university educational programs. The concept of function is one of the basic concepts of great importance in mathematics education, which can be applied in almost all the fields of mathematics (Eisenberg, 1991). Despite its great importance in mathematical studies and learning, it is one of the most challenging subjects to the teachers who have difficulty in teaching (Clement, 2001). Moreover,

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several researches carried out with university students have suggested that this is one of the most challenging subjects they have ever had difficulty in learning (Dreyfus & Eisenberg, 1982; Sierpinska, 1992; Tall, 1996).

According to Yamada (2000), the understanding of functions does not appear to be easy because of the diversity of representations associated with this concept and the difficulties presented in the processes of articulating the appropriate systems of representation involved in problem-solving. Therefore, a substantial number of researchers have examined the role of different representations on the understanding and interpretation of functions (Thomas, 2003; Zazkis, Liljedahl & Gadowsky, 2003). Ponte (1990) points out that learning functions requires three forms of representation, which are algorithmic, graphical and conceptual forms, and thus, the difficulty level of this abstract concept increases. The existence of algorithmic, conceptual and graphical questions in examinations has changed classroom instruction by affecting not only students as learners but also teachers as instructors (Baştürk, 2011; Erkan Erkoç, 2011; Kim and Pak, 2002; Maloney, 1994). When it is compared to the questions focusing on the performance on conceptual and algorithmic questions (Coştu, 2007; 2010), it is observed that the studies comparing students' performances on algorithmic, conceptual and graphical questions are less in number. The scores of pre-service teachers on algorithmic, conceptual and graphical questions were compared by Erkan Erkoç (2011) and indicated success on conceptual questions.

Considering the importance of the concept of function, the prospective elementary mathematics teachers are supposed to have highly sound and full knowledge regarding the subject, since they will be solving, even posing problems relative to the functions and the related subjects in the educational classroom activities in the future. Besides, within the professional standards in the education of mathematics, teachers are required to use problem posing and problem-solving skills in their careers (NCTM, 1991). Problem-solving generally involves the ability of a learner to reach a single true answer as well as forming a mathematical structure from a given piece of information, which is totally a matter of comprehension. On the other hand, problem-posing is mainly a process comprising a number of answers, which requires creative thinking (Kojima, Miwa & Matsui, 2009). According to Ticha and Hospesova (2009), problem-posing means producing new problems or redesigning the existing ones. Problem-solving were proven to be effective on critical thinking of the students, dialogue, questioning, participation, investigating the environment in an analytical way, and the student-oriented learning (Kılıç & İncikabı, 2013; Moses, Bjork & Goldenberg, 1990; Nixon-Ponder, 2001). It appears that the researchers focus on solving problems rather than posing them, and the tendency in this direction indicates that there is no trouble with problem-posing. Rather than the levels of the teachers in this subject, the difficulties which the students experience and the recommendations of solutions are emphasized in the conducted studies. In numerous studies, it was pointed out that problem-posing is at the core of mathematical activities as well as being an important component of the mathematical curriculum (Moses, Bjork & Goldenberg, 1990; NCTM, 2000; Silver, 1994).

It is difficult to measure whether or not the students have learned the subject about a topic or a concept. Thus, there is an increasing tendency towards alternative measurement methods in mathematics education (İncikabı & Sancar-Tokmak, 2012). For this purpose, SOLO model can be used as an alternative for measuring and evaluating spatial visualization skills (Baki & Güven, 2007; Dursun, 2010; Göktepe & Özdemir, 2013; Nagy-Kondor, 2014; Özdemir & Yıldız, 2015; Sezen Yüksel & Bülbül, 2014, 2015; Yıldız, Göktepe Körpeoğlu & Körpeoğlu, 2015). SOLO (Structure of the Observed Learning Outcomes) model is a taxonomy used to evaluate students' knowledge (Biggs & Collis, 1991; Pegg & Tall, 2005.) Consisting of five reasoning / thinking stages, SOLO Taxonomy were developed in 1982 by John Biggs and Kevin Collis. These stages correspond to Piaget's Cognitive Development Stages (sensorimotor, pre-operational, concrete operational and formal operational stage and abstract thinking) (Biggs & Collis, 1991; Pegg & Tall, 2005). Each thinking stage in SOLO Model covers the five sub-stages: pre-structural (the lowest level), uni-structural, multi-structural, relational and extended abstract level (the highest level). As the level increases, so do the coherence, associations, and multi-thinking processes (Biggs & Collis, 1991; Chan, Tsui, Chan & Hong, 2002).

SOLO taxonomy is not only used in mathematics but also in other fields to define the understanding of students and their interpretation about the specific concepts (Biggs & Collis, 2014; Lian & Idris, 2006; Money, 2002; Padiotis an& d Mikropoulos, 2010; Pegg & Coady, 1993; Pegg & Davey, 1998; Sheard, Carbone, Lister, Simon, Thompson & Whalley, 2008; Wongyai & Kamol, 2004). It was seen that there are some studies where SOLO taxonomy is used in different subjects within the field of mathematics education. However, as the result of the literature review, the method of evaluation of a problem-posing study according to SOLO taxonomy is considered to be the first. Besides, the types of problems structured in this study were described according to SOLO level. Consequently, it is considered that the insufficiency of prospective teachers in the subject of functions determined through this study will contribute to education a great deal towards developing their problem-posing skills regarding the subject of function.

Based on above literature, the current study aimed to determine the knowledge of the prospective elementary mathematics teachers regarding the concept of function on the basis of the SOLO taxonomy analysis of the problems. Being in line with the stated aim, answers to the following problem sentence have been searched in the study:

- When the problems posed by the prospective elementary mathematics teachers were assessed according to SOLO taxonomy, what are their levels of the knowledge regarding functions?

Method

Being descriptive in nature, the current research utilized case study approach which is one of the qualitative research methods (Meriam, 1988; Stake, 1994). Descriptive research is used in order to determine the behaviours, attitudes and successes of a participant group, and in such studies, the answers to the questions, 'what' and 'how' are searched (McMillan & Schumacher, 2006).

Participants

Participants of the study were chosen according to the convenient sampling method (Patton, 1987). The participants of the study consisted of a total of 67 prospective elementary mathematics teachers attending the 4^{th} grade of the Elementary Mathematics Education Department, in a university located in the North of Turkey in the spring mid-term of 2013-2014 academic year. Since these prospective teachers were in the final grade at university and were about to graduate, they had received all the major field courses. Thus, they were assumed to have had the sufficient knowledge on the subject of functions.

Data Collection Tools

Lin (2004) stated that problem-posing studies were useful assessment tools which provide information for teachers about the mathematical learning styles of students. Problem-posing can be used as a significant assessment tool in teaching mathematics (Lin and Leng, 2008). In the study, a ''problem-posing test'' of 3 items was given to participants regarding functions as a data collecting tool which is shown below.

The Problem-Posing Test:

1. Using the sets, A={-2,1,3}, B={0,1,2,3}, write down a problem questioning whether a correlation you will define is a function or not.
2. Pose such a problem sentence that the function $y = 100 - x^2$ can be obtained as the answer.
3. Pose a problem, the answer of which is ''8'', and which asks the value of the reverse of the function to be defined at point 3.

For each item in the test, the candidates were asked to pose a problem which would reflect the proper and creative thinking in line with the secondary education level. The opinions of three math instructors were asked in order to determine whether or not the items in the measurement tool were convenient for the measurement purpose, and by the way, the validity of the questions in terms of language, level, content and scope was ensured. The data of the study were composed of problems posed by 67 prospective elementary mathematics teachers in accordance with the above-mentioned questions. In order to test the reliability of scoring obtained from the problem-posing test, the compatibility among the encoders was taken into account. After the participant answers were encoded independently by three supervisors, the Cohen’s kappa concordance coefficient was calculated for each item. In the concordance /compatibility statistics performed, the general Cohen’s kappa coefficient was found out to be 0.81. It was observed that this result is sufficient regarding the measurement of the consistency of the analysis of the encoders.

Analysis of The Data

In this study, the problems posed by the teacher candidates were analyzed according to SOLO taxonomy in line with the descriptive research method. The problems posed were examined in detail and the knowledge levels of the candidates on the subject of functions were tried to be determined according to SOLO taxonomy. Through SOLO taxonomy, it is possible to determine the knowledge/capability levels of the individuals from their written and / or verbal responses regarding a particular task (Money, 2002; Groth & Bergner, 2006).

The thinking levels of the SOLO taxonomy and how these assessments were done have been explained below (Biggs & Collis 1991):

Pre-structural Level (PSL): This is the lowest level of SOLO Taxonomy on which the student cannot quite understand the question, and the answers s/he gives usually are not related with the desired/expected answers.

Uni-structural Level (USL): In this level, the student has limited understanding with respect to the question. S/He usually focuses on one aspect of the question. Since the focus is only on one aspect, the given answers are, therefore, limited and insufficient.

Multi-Structural Level (MSL): The student, in this level, can use more than one aspect relative to the question but cannot connect these aspects. Thus, the student’s answers are formed of pieces of information unrelated with each other, meaning there is no relational connection between the answers.

Relational-Structural Level (RSL): Here, the student comprehends all the aspects of the question in relation to the answer, their place within the whole and their associations with each other, as the result of which his/her answers show consistency.

Abstracted-Structural Level (ASL): In this level, the student has the ability and skill to think and reason more progressively besides the features of the previous level. S/he can do reasoning beyond the expected task; hence, this level can be considered as a new form of thinking.

The most significant difference between USL and MSL is that the answers of the student (here, the problems) comprise more than one associated data. In MSL, the student can gradually implement the algorithms and follow up the routine processes. Transition from MSL to RSL necessitates not only identifying the information but also the skills to think outside the box with respect to this information. The student should be able to integrate the elements s/he defines within a consistent system. The transition from RSL to ASL is the most desirable but the most challenging phase, during which the student should be able to make inferences beyond the known content by questioning the generalizations s/he formed in the ASL or adding more to them (Pegg & Davey, 1998).

Findings

In this section, the analysis of the problems posed by the prospective teachers for each item in the Problem Posing Test was performed according to SOLO taxonomy. The questions were considered one by one and then assessed; examples were given for some levels of taxonomy for each question. Consequently, there is minimum one sample for each level. The analyses performed according to the problems posed for each of the three items are given together as a summary in Table 1.

Table 1. *SOLO Taxonomy of The Problems Posed By The Prospective Teachers*

SOLO Level	1. Item		2. Item		3. Item		Total	
	(f)	%	(f)	%	(f)	%	(f)	%
Pre-structural Level (PSL)	4	6	8	12	5	8	17	9
Uni-structural Level (USL)	4	6	18	27	19	28	41	20
Multi-structural Level (MSL)	8	12	29	43	33	49	70	35
Relational-Structural Level (RSL)	30	45	8	12	9	13	47	23
Abstracted-Structural Level (ASL)	13	19	4	6	1	2	18	9
Null / Blank	8	12	0	0	0	0	8	4
Total	67	100	67	100	67	100	201	100

Findings on the problems posed for the first item

In this section, the findings of the analysis of the problems posed by the prospective teachers for the first item “Using the sets, A={-2,1,3}, B={0,1,2,3}, write down a problem questioning whether a correlation you will define/identify is a function or not.” were presented.

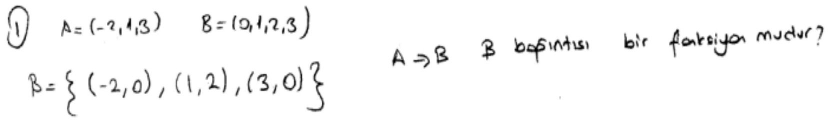
According to the data on Table 1 for the first item, it is obvious that the thinking levels of teacher candidates regarding the first item are high. According to the table, there are 30 candidates in the relational-structural level (RSL), while there are 13 prospective teachers in the abstracted structural level, which makes up 43 prospective teachers constituting 64% of the participants. However, a total of 16 prospective teachers together with other 8 prospective teachers, whose thinking levels are pre-structural (n=4) and uni-structural (n=4) and who also left this item blank, constitute 24% of the participants. It may be suggested that the group does not fully comprehend the knowledge “whether the relation defined between the given two sets is a function or not”, which is handled within the first item of the problem-posing test, and therefore, it can be said that they cannot pose the problems that were expected from them.

Here, it was observed that the candidates (n=4), concerning the first item, whose thinking levels were on the Pre-structural Level (PSL) did not comprehend the situation in the problems they posed, and therefore, they could not pose the problems expected from them. It was observed that the candidates (n=4) from the Uni-structural Level (USL) group had limited comprehension on the requirements about the first item in the problem posing test and they focused on only one aspect regarding the problems they posed.

In the problems posed by the candidates (n=8) whose thinking levels were in the Multi-Structural Level (MSL), the different aspects of the subject were detected; however, they did not constitute a meaningful outcome.

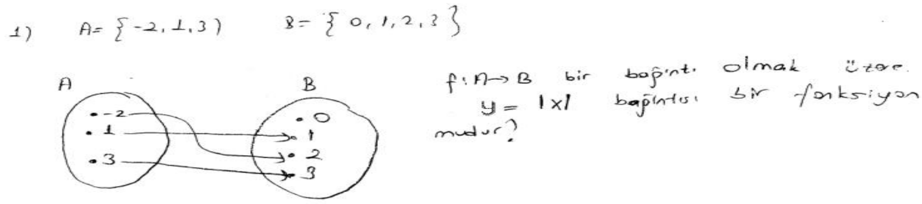
It was observed that the problems posed by the prospective teachers (n=30) in the Relational-Structural Level (RSL) are consistent and coherent with the given data. In Figure 1-a, there is a problem sample of a candidate from the RSL group. Here, the candidate defined a correlation with the given two sets and asked whether this correlation was a function or not. However, s/he did not fully explain what the expression “A→B” in the question sentence meant which shows that s/he failed to abstract the situation through the way s/he used the notations. The candidate is questioning the conceptual knowledge in his/her problem.

a)



[Does β relation define a function?]

b)



[Given that $f: A \rightarrow B$ is a relation, does $y = |x|$ relation define a function?]

Figure 1. *Students' Sample Answers*

Consequently, it was observed that the prospective teachers (n=13) in the Abstracted-Structural Level (ASL) have more advanced thinking skills and are able to pose different problems with their inferences beyond the demanded levels. In Figure 1-b, there is an example problem of a candidate in this group. As seen in the example, the candidate defines a correlation between the two given sets; and here, there is a situation that has never been seen in the problems of the candidates from the other thinking level groups. The relation defined by the candidate consists of a rule. The candidate is questioning the conceptual knowledge in his/her problem. It is observed that the candidates in the Abstracted-Structural Level (ASL) group also posed similar problems.

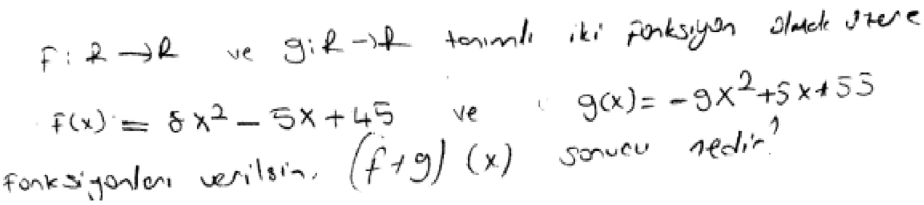
Findings on the problems posed for the second item

The analysis findings of the problems posed by the prospective teachers for the second item “Pose such a problem sentence that the function $y = 100 - x^2$ can be obtained as the answer.” have been presented here.

In Table 1 for second item, the minority of the prospective teachers (n=6) whose levels are at the Abstracted-Structural Level (ASL) draws the attention. Here, it was observed that a total of 47 prospective teachers from the Multi-Structural Level (MSL) and Uni-structural Level (USL) form a percent of 70%. Therefore, it can be said that most of them could not succeed in progressing towards advanced thinking/reasoning levels in the second item. It was also seen that while the candidates were posing a problem statement/sentence giving the function $\mathrm{y = 100 - x^2}$ as an answer, they usually could not define the image sets and the domains accurately in their functions.

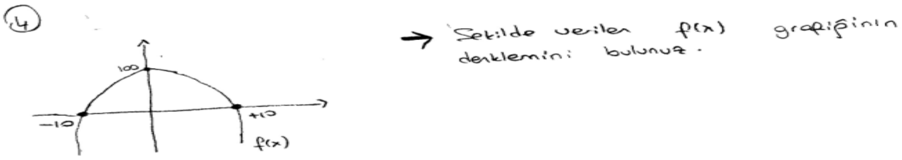
A sample problem of a candidate from the Multi-Structural Level (MSL) group (n=29) is seen in Figure 2-a. In the problem, two functions were defined and their summation was asked. All components were used properly in the question, and the answer yields the desired information. However, the problem cannot be assessed as a product of advanced thinking, since it is sufficient to sum up two functions for the answer of the problem and it requires a completely operational skill.

a)



[Let $f(x) = 8x^2 - 5x + 45$ and $g(x) = -9x^2 + 5x + 55$ be two functions defined in $f: R \rightarrow$R and $g: R \rightarrow$R. What is the value of $(\mathrm{f+g})(x)$?]

b)



[Please provide the equation of the graph given in the figure]
Figure 2. *Students' Sample Answers*

In Figure 2-b, on the other hand, there is the problem sample of the prospective teachers (n=4) from the group of the Abstracted-Structural Level (ASL). The candidate provided a graphic in the problem and asked for the function defined by the graphic. Here, the candidate expects to be able to read the graphic from the problem solver and then write a function based on that graphic. For this reason, the candidate was observed to specifically define the points at which the curve in the graphic intersects the axes. For the second item, there are only 4 candidates who can pose problems with similar properties. In order to solve the problems in this group; operational skills, conceptual and graphical knowledge are required, thus, it can be stated that the thinking/reasoning levels of these 4 candidates are of abstract structure (ASL).

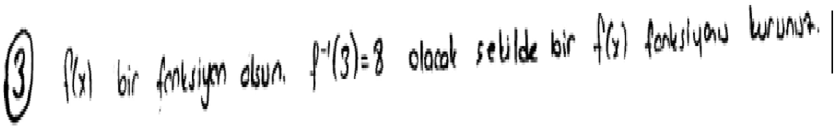
Findings on the problems posed for the third item

The analysis findings of the problems posed by the prospective teachers for the third item "Pose a problem, the answer of which is ''8'', and which asks the value of the reverse of the function to be defined by you at point 3." have been presented here.

According to Table 1 for third item, there are 15 teacher candidates in total, 9 candidates in the Relational-Structural Level (RSL) group and 1 candidate in Abstracted-Structural Level (ASL) group. This situation indicates that the percentage of the problems posed for this item which requires multi-level thinking is 15%. As in the second item, it was also observed in here that in the Multi-Structural Level (MSL) and Uni-structural Level (USL) groups, there are 52 teacher candidates forming a percent of 77%. Therefore, it can be concluded that most of the teacher candidates failed to perform the transition to the advanced reasoning level in the third item, as well. The candidates usually focused only on the fact that the result should be 8 as was demanded, without paying particular attention to the fact that the function they defined in general has to be one-to-one and onto function while posing a problem asking for the value of the reverse of the function at the point 3.

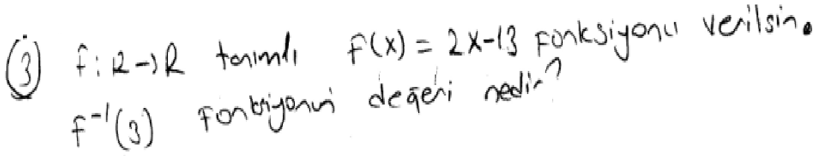
A sample of the problems posed by the prospective teachers, whose related thinking levels regarding the third item were in the Pre-Structural Level (PSL) group (n=5), is given in Figure 3-a. Here, it was observed that this candidate posed a problem sentence by writing down what was asked in the third item in his/her own way. On the other hand, one sample of the problems posed by the prospective teachers (n=19) whose involved thinking levels regarding the third item were in the Uni-structural Level (USL) is given in Figure 3-b. In this problem, the candidate applies the reverse of a linear function. Hence, the solution will require a simple operation.

a)



[Let f(x) be a function. Please provide a function of f(x) such that $\boldsymbol{f^{-1}(3) = 8}$]

b)



[Let f(x) = 2x-13 be a function defined in f:$R \rightarrow$R. What is the value of $f^{-1}(3)$?]

Figure 3. Students' Sample Answers

Considering Table 1, it is seen that the teacher candidates whose thinking levels are in the Multi-structural level (MSL) predominate by 35%. Moreover, if the "Null/ Blank" group is eliminated, it is observed that the number of people in other groups indicate a distribution close to the normal one. This situation is also given in Figure 4.

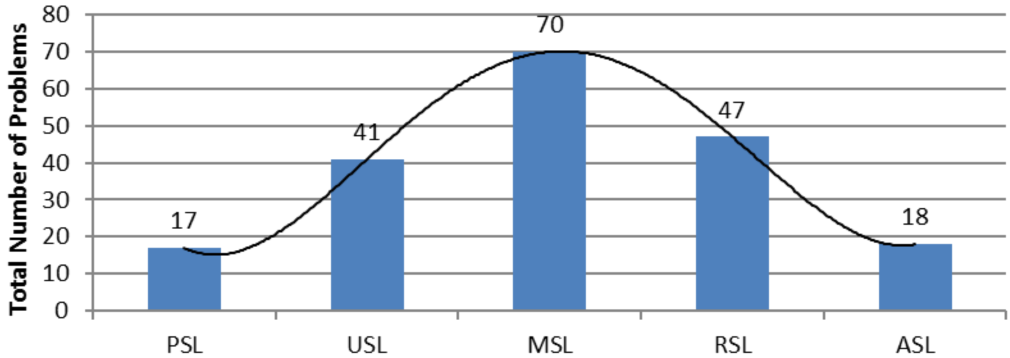


Figure 4. *Distribution curve*

Results and Discussions

In this study, the problems on functions posed by prospective elementary mathematics teachers were analyzed with the SOLO taxonomy, and their knowledge levels regarding the concept of function were also analyzed.

The analysis of the problems by means of this taxonomy gives insight not only regarding the incompetency of the candidates in conceptual understanding but also their knowledge levels on functions. This situation indicates that SOLO taxonomy can be used as a tool to measure the knowledge levels necessary to pose mathematical problems. Using SOLO taxonomy for concept understanding provides a powerful tool for assessing the problem solving processes of the students and their understanding the concepts in this matter (Lian & Idris, 2006; Pegg & Tall, 2005). According to the findings of the research, the majority of the prospective teachers preferred to pose problems which require only operational competency. For the problems posed in this way, the thinking levels of the prospective teachers were grouped as Pre-structural Level (PSL), Uni-structural Level (USL) and Multi-Structural Level (MSL). In such problems, it has been observed that the prospective teachers try to avoid the situations questioning the conceptual knowledge. This outcome supports the conclusions that the problems posed by the prospective teachers are mostly predictable, simple, not well-structured, and insolvable, as acquired from other similar studies (İncikabı, Biber, Takıcak & Bayam, 2015; İncikabı, Tuna & Biber, 2012; Nicol, 1999; Stein, Smith, Henningsen & Silver, 2000).

Minority of the students have posed problems asking the conceptual information. Such kinds of problems have been posed by the prospective teachers mostly related with Multi-Structural Level (MSL) and Relational-Structural Level (RSL). Those prospective teachers have written problems which require both conceptual and operational knowledge. Conceptual and operational information are two dependent components which complete each other. Both operational and conceptual information are very essential in order to be successful in mathematics (Hiebert & Carpenter, 1992).

Furthermore, it was also observed in this research that the graphs used by minority of prospective teachers and few of them are used properly. This situation coincides with the result of the research of Elia and Spyrou (2006) “More success is detected in indicating the function algebraic rather than indicating it graphically.” Dreyfus and Eisenberg (1991) state that the interpretation of visual representations among others requires advanced cognitive performances. This point of view can be the reason why the success of the prospective teachers in posing problems for visual representations is lower.

Moreover, it was understood that prospective teachers were not sure about the solidity of their knowledge on functions in general. Few prospective teachers, on the other hand, posed problems questioning conceptual knowledge. Such problems were mostly posed by candidates whose thinking levels were at Multi-Structural (MSL) and Abstracted-Structural Levels (ASL). These prospective teachers wrote out problems requiring both conceptual and operational knowledge. Moreover, few of prospective teachers in this group however, used graphics in their problem-posing. It was concluded that the candidates posing questions that checked the conceptual, operational and graphical knowledge in problems were at the Abstracted-Structural Level (ASL) of thinking.

In addition, it was understood that the candidates are not sure about the strength of their knowledge on functions as a whole. When posed problems are taken into consideration, most of the candidates consider the functions as a correlation performing one-to-one correspondence between the elements of two sets (Dubinsky & Harel, 1992), that is to say, they misperceive the correspondence of each element in the domain with only one single element in the codomain (range). For this reason, the candidates do not accept the correlations corresponding more than one element in the domain with the same element in the codomain as functions, or they can match one element in the codomain with more than one element in the codomain in the way that there will be no elements left alone.

It can be inferred from the results of this study that there is a need for the prospective mathematics teachers who will pose problems in the prospective educational classroom activities, and these prospective teachers will become a model for their students with their problem-posing performances to overcome their conceptual deficiencies concerned with functions. Qualitative researches may be conducted in order to put forward the reasons for the conceptual deficiencies of teacher candidates regarding functions. In addition, the skill of problem-posing provides students with teaching mathematical reasoning, discovering mathematical models and being able to express the mathematical modes properly, either in verbal/oral or written form. It is required that teachers having the basic knowledge and skills, with the awareness of the importance of problem-posing approach should be trained in this matter. It is considered that instead of the habitual educational model, providing opportunities for students to pose their own problems by apprehending the complementary significance of problem-posing in their mathematical curriculums will make great contribution to education.

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Uzun Öz

Giriş

Öğrencilerin matematik dersine ilişkin bir konuyu ya da kavramı öğrenip öğrenmediklerini ölçmek oldukça zor bir iştir. Bu yüzden son yıllarda eğitimde dolayısıyla matematik eğitiminde alternatif ölçme değerlendirme tekniklerine bir yöneliş vardır. Bu amaçla öğretmen adaylarının herhangi bir konuda bilgi seviyelerinin ölçülmesinde ve değerlendirilmesinde SOLO taksonomi alternatif olarak kullanılabilir. SOLO (Structure of the Observed Learning Outcomes) farklı konu alanı ve seviyelerde öğrencilerin becerileri ve bilişsel bilgilerini değerlendirmek amacıyla kullanılan bir taksonomidir (Biggs ve Collis, 1991; Pegg ve Tall, 2004). Bu araştırmada da öğretmen adaylarının fonksiyonlar konusundaki bilgileri SOLO taksonomi ile değerlendirilmiştir. SOLO Taksonomisi, John Biggs ve Kevin Collis tarafından 1982 yılında geliştirilmiş olup beş düşünme evresinden oluşmaktadır. Bu evreler, Piaget'nin bilişsel gelişim evrelerine (duyusal-motor, işlem öncesi, somut işlemler, soyut işlemler) karşılık gelmektedir (Biggs ve Collis, 1991; Pegg ve Tall, 2005). SOLO taksonomisinde her düşünme evresi, belirli bir soruya öğrencilerin verdikleri cevapları, yapısal karmaşıklığına göre sınıflandıran beş alt evreyi içerir; yapı öncesi (en düşük seviye), tek yönlü yapı, çok yönlü yapı, ilişkilendirilmiş yapı ve soyutlanmış yapı (en yüksek seviye). Seviye arttıkça tutarlılık, ilişkilendirmeler ve çok yönlü düşünme de artmaktadır (Biggs ve Collis, 1991; Chan Tsui,Chan ve Hong, 2002).

Araştırmanın Amacı

Bu çalışmanın amacı; kurdukları problemlerin SOLO taksonomisine göre ilköğretim matematik öğretmen adaylarının fonksiyon kavramına ilişkin bilgi seviyelerini betimlemektir.

Araştırmanın Problemi

Araştırmada aşağıda verilen probleme cevap aranmıştır,

- Kurdukları problemler SOLO taksonomisine göre değerlendirildiğinde, ilköğretim matematik öğretmeni adaylarının fonksiyonlar konusundaki bilgi seviyeleri nasıldır?

Yöntem

Nitel bir çalışma olan bu araştırmada durum çalışması yaklaşımı kullanılmıştır (Meriam, 1988; Stake, 1994). Çalışmanın verileri betimsel analiz yöntemi temel alınarak incelenmiştir. Buna göre araştırmanın katılımcıları, 2012-2013 öğretim yılı bahar yarıyılında Türkiye'nin kuzeyinde bir üniversitenin Eğitim Fakültesi İlköğretim Matematik Öğretmenliği 4. sınıfında öğrenim gören toplam 67 öğretmen adayından oluşmaktadır. Çalışmada katılımcılara fonksiyonlar konusu ile ilgili aşağıda verilen 3 maddelik bir "problem kurma testi" veri toplama aracı olarak kullanılmıştır.

Problem Kurma Testi:

1- $A = \{-2,1,3\}$, $B = \{0,1,2,3\}$ kümelerini kullanarak tanımlayacağınız bir bağıntının fonksiyon olup olmadığını yoklayan bir problem yazınız.

2- Öyle bir problem cümlesi kurunuz ki, cevap olarak $y = 100 - x^2$ fonksiyonu elde edilsin.

3- Tanımlayacağınız bir fonksiyonun tersinin 3 noktasındaki değerini soran ve cevabı "8" olan bir problem kurunuz.

Bulgular

Bu bölümde Problem Kurma Testinde yer alan her bir madde için öğretmen adaylarının kurmuş olduğu problemlerin SOLO taksonomisine göre analizleri yapılmıştır. Sorular sıra ile ele alınarak değerlendirilmiştir, her soruda SOLO taksonomideki seviyeleri temsil eden örnekler verilmiştir. Buna göre her seviye için en az bir örnek bulunmaktadır.

Tablo 1. *Öğretmen Adaylarının Kurdukları Problemlerin SOLO Taksonomiye Göre Dağılımı*

Düşünme seviyesi	1. Madde		2. Madde		3. Madde		Toplam	
	(f)	%	(f)	%	(f)	%	(f)	%
Yapı Öncesi (YÖ)	4	6	8	12	5	8	17	9
Tek Yönlü Yapı (TYY)	4	6	18	27	19	28	41	20
Çok Yönlü Yapı (ÇYY)	8	12	29	43	33	49	70	35
İlişkilendirilmiş Yapı (İY)	30	45	8	12	9	13	47	23
Soyutlanmış Yapı (SY)	13	19	4	6	1	2	18	9
Boş	8	12	0	0	0	0	8	4
Toplam	67	100	67	100	67	100	201	100

Kurulan problemler birlikte analiz edildiğinde Tablo 1'de "Toplam" sütununa göre düşünme seviyesi çok yönlü yapıda olan öğretmen adaylarının % 35 ile çoğunlukta olduğu görülmektedir.

Tartışma ve Sonuç

Bu çalışmada ilköğretim matematik öğretmen adaylarının fonksiyonlar konusunda kurdukları problemler SOLO taksonomisi yöntemiyle analiz edilerek, adayların fonksiyon kavramına ilişkin bilgi seviyeleri betimlenmiştir. Araştırmadan elde edilen bulgulara göre öğretmen adaylarının fonksiyon konusundaki bilgi seviyeleri normal dağılıma yakın bir dağılım göstermektedir. Bu durum uygun öğretim yöntemleri ile adayların fonksiyon konusu ile ilgili bilgi seviyelerinin gelişime açık olduğu şeklinde yorumlanabilir.

Problemlerin bu taksonomi ile analizi, adayların fonksiyon konusuyla ilgili bilgi düzeylerini belirlemekten öte, kavramsal anlamaya yönelik eksiklikleri hakkında bilgi vermektedir. Bu durum SOLO taksonominin matematiksel problem kurmada gerekli olan bilgi seviyelerini ölçmek için bir araç olarak kullanılabileceğini göstermektedir. Ayrıca Pegg ve Tall (2005) ve Lian ve Idris (2006) bu taksonominin kavramlarla ilgili olarak öğrencilerin anlama ve problem çözmelerini değerlendirmek için güçlü bir araç sunduğunu belirtmiştir. Araştırmaya göre adayların çoğunun fonksiyonlarla ilgili bilgi seviyelerinin çok yönlü yapıda olduğu görülmüştür. Bu durum benzer çalışmalardan (Groth ve Berner, 2006; Lian ve Idris, 2006) elde edilen bulgularla paralellik arz etmektir.

Araştırma bulgularına göre adayların büyük bir kısmı sadece işlemsel beceri gerektiren kolay problemler kurmayı tercih etmişlerdir. Bu şekilde kurulan problemlerde adaylar ağırlıklı olarak Tek Yönlü (TY), Çok Yönlü (ÇY) ve İlişkilendirilmiş (İY) olarak gruplandırılmıştır. Ayrıca bu tip problemlerde adayların kavramsal bilgileri sorgulayan durumlardan kaçınmaya çalıştıkları görülmüştür. Bu sonuç benzer çalışmalarda ortaya çıkan (Crespo, 2003; Nicol, 1999; Stein, Smith, Henningsen ve Silver, 2000;) öğretmen adaylarının ürettikleri problemlerin çoğunlukla tahmin edilebilir, basit, iyi yapılandırılmamış ve çözülemez şeklinde olduğu bulgusunu desteklemektedir.

Bu çalışmanın sonuçları sınıf içi öğretim faaliyetlerinde problem kuracak ve kurdukları problemlerin öğrenciler için model oluşturacak olan öğretmen adaylarının fonksiyonlarla ilgili kavramsal eksikliklerinin giderilmesi gerekliliğine işaret etmektedir. Ayrıca öğretmen adaylarının fonksiyonlarla ilgili kavramsal eksikliklerinin sebeplerini ortaya koyabilmek için nitel araştırmalar da yapılabilir.