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Investigation of the Development of the van Hiele Levels of Geometric Thinking in a Computer-Supported Collaborative Learning (CSCL) Environment*

Bilgisayar Destekli İş Birliğiyle Öğrenme Ortamında vanHiele Geometrik Düşünme Seviyelerinin Gelişiminin İncelenmesi

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Abstract: This study investigated the use of a well-designed computer-supported collaborative learning (CSCL) environment, namely Virtual Math Teams (VMT), to develop middle school students' geometric thinking. It also looked into students' VMT discourse to better understand factors leading to higher van Hiele levels of geometric thinking. The participants of the study were selected from middle school students who were at the visual geometric thinking level. For treatment, students were presented with a set of activities on quadrilaterals, which were developed based on van Hiele's phases of learning geometry, within the VMT environment. The data were collected using the van Hiele Geometry Test. The VMT chat logs were qualitatively analyzed using the three-core collaborative problem-solving competencies used in the Programme for International Student Assessment (PISA) 2015. The results showed that the participants significantly developed their van Hiele Geometry Test scores after the intervention. Qualitative results pointed out that collaborative competencies could be essential in developing students' geometric thinking levels within the VMT environment. Considering that in international assessments Turkish students score lower than the international average in geometry and the lowest in the collaborative problem-solving area, it becomes even more important to integrate CSCL environments into Turkish curricula.
Keywords: Computer-supported collaborative learning, virtual math teams, the vanHiele levels of geometric thinking, geometry learning, collaborative competencies, middle school

Öz: Bu çalışmada bilgisayar destekli iş birliğiyle öğrenme aracı olarak geliştirilmiş olanSanal Matematik Takımları (VMT) ortamında ortaokul öğrencilerinin vanHiele geometrik düşünme düzeylerinin gelişimi incelenmiştir. Aynı zamanda, geometrik düşünme düzeylerinin gelişimine yol açan faktörleri daha iyi anlamak için öğrencilerin VMT söylemlerine nitel olarak bakılmıştır. Katılımcılar görsel vanHiele geometrik düşünme düzeyinde olan ortaokul öğrencileri arasından seçilmiştir. Uygulama olarak öğrencilere vanHiele'nin geometriyi öğrenme aşamaları dikkate alınarak dörtgenler konusunda geliştirilen aktiviteler VMT ortamında sunulmuştur. Öğrencilerin geometrik düşünce seviyelerindeki değişiklik vanHiele Geometri Testi kullanılarak değerlendirilmiştir. VMT sohbet kayıtları Uluslararası Öğrenci Değerlendirme Programı (PISA) 2015'te kullanılan üç temel işbirliği yeterliliği göz önüne alınarak nitel olarak analiz edilmiştir. Bulgular uygulama sonunda öğrencilerin vanHiele Geometri Testi skorlarını anlamlı bir şekilde arttırdıklarını göstermiştir. Nitel analiz sonuçları ise işbirliği yeterliliklerinin VMT ortamında öğrencilerin geometrik düşünme seviyelerini geliştirmede önemli bir faktör olabileceğine işaret etmektedir. Türk öğrencilerin uluslararası değerlendirmelerde geometride uluslararası ortalamaların altında ve iş birliğiyle problem çözme alanında sıralamada en sonda yer aldığı göz önüne alındığında, bilgisayar destekli işbirliğiyle öğrenme ortamlarının müfredata entegrasyonu daha da önem kazanmaktadır.
Anahtar Kelimeler: Bilgisayar destekli işbirliğiyle öğrenme, sanal matematik takımları, vanHiele geometrik düşünme seviyesi, geometri öğrenimi, işbirliği yeterlikleri, ortaokul

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Introduction

While geometry is a crucial sub-discipline in the field of mathematics, most students have difficulties with school geometry (Köseleci-Blanchy & Şaşmaz, 2011). One of the explanations for these difficulties with learning geometry is the lack of instruction that is designed based on students'van Hiele levels of geometric thinking, proposed by the two Dutch mathematics educators (Dina van Hiele-Geldof & Pierre van Hiele) in the late 1950s (Usiskin, 1982). The van Hielemodel described five sequential levels of geometric thinking (visual, analysis, informal deduction, deduction, and rigor) that students go through when becoming proficient in geometry (van Hiele, 1999).

Several studies confirmed that the van Hiele levels of geometric thinking scheme were a valid indicator of the achievement in school geometry (e.g., Burger & Shaughnessy, 1986; Senk, 1989; Usiskin, 1982). Not only van did Hieles focus on describing students' cognitive development regarding geometry but also suggested teaching strategies to support this development. Instruction that supports the development of the van Hiele levels of geometric thinking should consist of five learning phases, which are inquiry, direct orientation, explication, free orientation, and integration. Students can pass through one level to the next if instruction based on these phases is provided (Usiskin, 1982).

Researchers have attempted to determine how to increase students' level of geometry understanding (e.g., Abdullah, et al., 2015; Abdullah & Zakaria, 2013; Duatepe-Paksu & Ubuz, 2009; Halat, 2006; Karakuş & Peker, 2015; Kutluca, 2013). They examined the effect of different instructional methods, such as instruction using dynamic geometry software (DGS). In these studies, the instructional method and the effect of social interaction among students became prominent in developing students' understanding of geometry. While the use of DGS and collaboration among students have been highlighted as important factors for learning geometry, to the best knowledge of the authors, there are not any studies that investigated students' van Hiele levels of geometrical thinking in a Computer-Supported Collaborative Learning (CSCL) environment.

CSCL is considered the latest paradigm in the area of educational technology (Koschmann, 1996). Virtual Math Teams (VMT) is a well-known CSCL environment that involves both DGS and collaborative learning as its design elements (Öner, 2016a). VMT provides students a multi-user GeoGebra (an open-source DGS tool) and offers them an interactive learning space where they can work on geometry problems collaboratively. In the current study, along with investigating the role of VMT on improving levels of geometric thinking, we also intended to understand how collaboration among students influenced this development.

Therefore, the purpose of the current study is to examine the role of working within a CSCL environment (i.e., VMT) on middle school students' van Hiele levels of geometric thinking (both in terms of levels and scores), and to understand how their collaborative competencies influenced their geometry learning while working within the VMT environment.

The van Hiele levels of geometric thinking

According to the van Hieles, there are five levels of understanding in geometry, which are typically numbered from 1 to 5. At Level 1 (visual), shapes are judged according to their appearances rather than their features. For example, students at this level might say that "it is a rectangle because it looks like a box" (van Hiele, 1999, p. 311). At Level 2 (analysis), the properties of a figure become more important than their appearances and children can talk about the mathematical features of geometrical shapes. Burger and Schaughnessy (1986) stated that children at this level could establish the necessary properties of geometrical concepts and judge figures by considering their properties rather than what they look like. However, children cannot logically order the properties of the shapes at this level yet. This is achieved when they are at Level 3 (informal deduction). At the informal deduction level, students can differentiate the necessary and sufficient properties of a concept and logically order them. They can explain why all squares are rectangles by using properties of squares and rectangle. However, this is unlike a formal proof. Although students can make simple deductions, they cannot understand the intrinsic meaning of deduction, such as axioms, postulates, and theorems at this level (van Hiele, 1999). It is at Level 4 (formal deduction) that students can understand the intrinsic meaning of deduction, and roles of axioms, postulates, and proofs in making formal deductions (Usiskin, 1982). Level 5 (rigor) is the highest level of geometric thinking. Students at this level do not need concrete models to study different geometries (Burger & Schaughnessy, 1986). They can go beyond the Euclidean geometry and understand non-Euclidean geometries (Usiskin, 1982).

The van Hiele's phase-based learning

The van Hiele levels of geometric thinking are considered sequential. That is, a student in Level 1 cannot reach Level 3 without passing through Level 2. Students' geometric thinking is not dependent on age or development level. That is, even a university student can be at the visual level (Duatepe-Paksu, 2016). Meanwhile, student's geometry thinking levels can be developed by effective instruction. Usiskin (1982) stated that there are five learning phases that were suggested by the van Hiele model for supporting students to pass from one level to the next. These learning phases are named as inquiry (information), direct orientation, explication, free orientation, and integration. If students are provided with geometry instruction based on these phases, they can move from one level of van Hiele geometric thinking to the next.

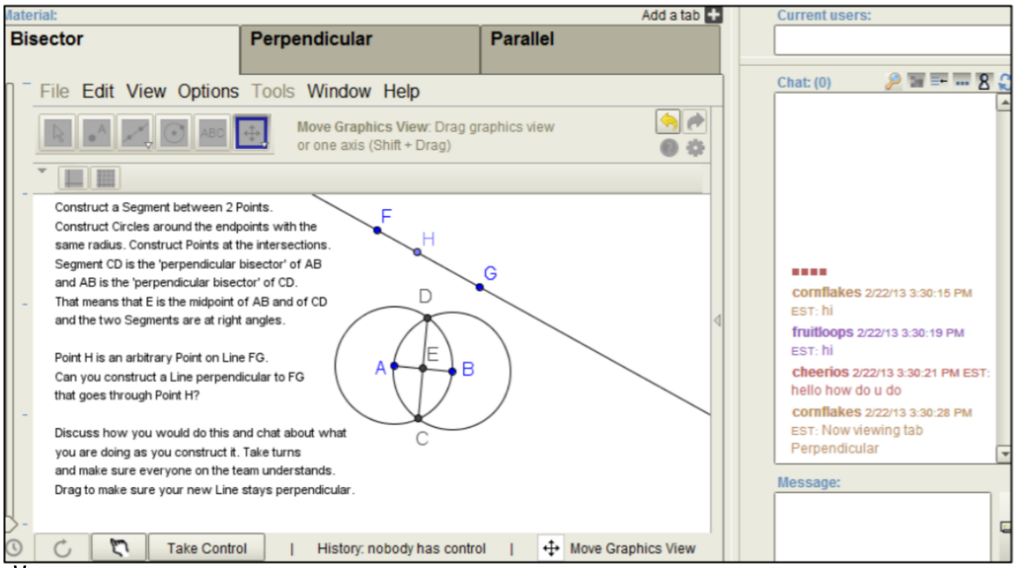
The first of the van Hiele's phases of learning geometry is inquiry (or information). At this phase, the teacher should be in a conversation with students asking questions and encouraging them to make observations about geometrical structures to prepare them for further activities (Crowley, 1987). In the second phase (direct orientation), simple but structured tasks should be presented to students in a way that students can gradually realize the mathematical features of geometrical structures. They should be given the opportunity to change the shapes of given geometric objects in order to explore their features. In the third phase (explication), students are guided to share their opinions about the relationships they have discovered. Teacher introduces the relevant mathematical terminology to aid students' communication of mathematical ideas. In the next phase called free orientation, students are expected to solve more complex tasks with multiple steps. In the last phase, integration, students are led to summarize and relate what they have learned (Crowley, 1987). Teachers should plan the learning tasks carefully, lead students to use the relevant mathematical terminology in their discussions, and encourage them to explain their ideas and problem-solving strategies (van Hiele, 1999).

Use of DGS to improve students' van Hiele levels

Dynamic geometry software (DGS) was defined as the genre of computer software tools that enables students to explore geometric relationships and make conjectures by manipulating geometrical objects on the computer screen (Güven & Kosa, 2008). According to Stahl (2013), the construction of dependencies made clear in dynamic geometry environments. One of the important features of DGS is dragging. In DGS environments, if a figure is constructed properly, the theoretical relationships of the figure remain the same even under dragging (Öner, 2016b). Hence, DGS can support students' exploration of geometrical structures. And by doing that, it could be an important means for the design of activities based on van Hiele's phases of learning geometry.

There are several studies that investigated the effect of instruction with DGS on students' van Hiele levels of geometric thinking. These findings showed that DGS-based instruction supported the development of students' geometric thinking (e.g., Abdullah & Zakaria, 2013; Abdullah et al., 2015; Karakuş & Peker, 2015; Khalil, et al., 2018; Kutluca, 2013). However, it was not only the presence of DGS in the learning environment that made the difference. Some of these studies highlighted the role of the collaborative learning environment in DGS-based instruction as a crucial factor that affected the quality of instruction (e.g., Karakuş & Peker, 2015; Kutluca, 2013).

Computer-supported collaborative learning (CSCL) and virtual math teams (VMT)
CSCL is an area of learning sciences that studies how people learn collaboratively with the help of computers (Stahl, Koschmann & Suthers, 2006). Virtual Math Teams (VMT) is one of the well-designed CSCL environments for learning mathematics that has been around for almost two decades. In the VMT environment, students from all over the world can come together and work on mathematical problems collaboratively. VMT provides a virtual learning environment that affords synchronous text-based chat with an embedded multi-user version of GeoGebra, an open source DGS application (Öner, 2016b). Figure 1 below shows the room interface of VMT.



m interface (Öner, 2016a)

Collaborative competencies
While the notion of collaboration is difficult to operationalize, it is also a very important skill in education. Collaborative problem solving (CPS) has been identified as a crucial and essential skill in the future workforce as the success of the groups depends on collaboration among the group members (Organization for Economic Co-operation and Development [OECD], 2017a). In the Programme for International Student Assessment (PISA) 2015, the three-core collaborative problem-solving competencies were presented as establishing and maintaining shared understanding, taking appropriate action to solve the problem, and establishing and maintaining team organization (OECD, 2017a).

The first collaborative competency is identified as establishing common ground among group members. Students need to build a shared understanding of the task of communicating successfully. Shared understanding is about how students' abilities, knowledge, perspectives interact with those of other members. In order to build and maintain shared understanding among group members, there is a need to create an information flow among group members by communicating the right information at the right time and to attempt to overcome the deficiencies in shared knowledge (OECD, 2017a).

Another indicator of successful collaboration is taking appropriate action to solve the problem or doing the tasks. Students need to try on solving the problem by identifying its sub-tasks and constraints, creating team goals, and taking appropriate communication acts, such as explaining, justifying, negotiating, and debating (OECD, 2017a).

The third collaborative competency is related to team organization. A group cannot be successful without establishing and maintaining group organization. In order to work collaboratively, group organization must be established and maintained. Thus, students must know their role in the group, fulfill the requirements of their role, check whether their teammates performing their roles appropriately, and handle any communication problems. The authority of the group is also important. Group organization may be established by a strong group leader or more democratically based on the type of the problem (OECD, 2017a).

In this study, we used these three-core competencies to understand the role of collaborative competencies of groups as evident in their VMT chat discussions.

Two main research questions guided the current study:

1. Is there a statistically significant difference between the participants' pretest and posttest geometry test scores?
2. How did the collaboration among participants, as defined by the PISA framework, influence their geometry learning while working within the VMT environment?

Method

The present study used a pre-experimental one group pretest-posttest research design aided by qualitative data to address the research questions.

The participants

The participants of the study were selected using a purposive sampling method from two middle schools in Istanbul that were determined based on school administrations and their math teachers' willingness to accommodate the study. They were 24 (13 female) 5^{th} and 7^{th} grade students who were at Level 1 (visual) according to the van Hiele Geometry Test (VHGT) (Usiskin, 1982). Şener-Akbay (2012) has found that 65% of the Turkish 7^{th}-8^{th}grade students were at Level 1. Thus, the participants of this study represent the majority of the middle school students' in Turkey in terms of van Hiele geometric thinking levels. We made sure that each participant had a personal computer and Internet connection at home to participate in the study. Before data collection, IRB approvals and necessary permissions from the participants have been obtained.

For the qualitative analysis, we selected two groups of students using maximal variation sampling (Creswell, 2014) according to their success rates based on the VGHT test. Group 1 was selected as the successful group because two participants out of three in this group have increased their van Hiele level of geometric thinking from Level 1 to Level 2. Group 2 was selected as the unsuccessful group because none of the participants in this group was able to increase their van Hiele level of geometric thinking. The geometric thinking levels of both students stayed the same in this group (Table 1).

Table 1.
The Selected Participants for Qualitative Analysis

Student (pseudonyms)	Group	VGHT pretest	VGHT posttest
Emir	1	Level 1	Level 2
Sude	1	Level 1	Level 2
Lara	1	Level 1	Level 1
Öykü	2	Level 1	Level 1
Naz	2	Level 1	Level 1

Data collection

Before implementing the VMT-based activities, middle school students from the participating schools were given the VHGT test. The study participants (n= 24) were selected from the students who were in Level 1 based on their VHGT pretest scores. As stated before, a student needs to answer at least three questions correctly to reach Level 1 on items 1-5 in the VHGT.

The study participants were informed about the procedure of the study and provided with a Google Drive account and VMT account. Using Google Drive, the participants were able to work on activity worksheets as a group. They were also provided basic instruction on the

VMT environment and Google Sheets. The participants were randomly distributed to the groups of two or three to work on the VMT environment as teams. They completed the VMT activities. After completing VHL-based instruction through VMT, the participants were given the VHGT as a posttest.

The van Hiele levels-based instruction through VMT

Before designing the VMT-based instruction, the MEB middle school mathematics curriculum and its instructional objectives related to the quadrilaterals were examined. Five VMT activities were designed based on the van Hiele's five phases of learning geometry, which included inquiry, direct orientation, explication, free orientation, and integration, to help students who are at Level 1 to reach up to Level 2. The content addressed in these activities involved the properties of five quadrilaterals (trapezoid, parallelogram, rhombus, rectangle, and square) and their relationships with each other. The activities were reviewed by two math education researchers for content and grade level appropriateness. To avoid repetition, these activities will be referred to as "the VHL-based instruction through VMT" (see Table 2).

Different time schedules were created for each group according to the students' available time. Each group participated in the five VMT sessions at different times by using their own personal computers either at home or at school. Each session lasted about one hour.The first author was available in each online session to guide students, facilitate group work, and solve any technical problems students might have.

Table 2.
The VHL-Based Instruction Through VMT

Activity #	Phase	Content of activity
1	inquiry	Explore a pre-made GeoGebra sketch and the types of quadrilaterals (rectangle, square, parallelogram, rhombus, and trapezoid); discuss their properties using present geometric vocabulary; fill out a worksheet as a group
2	direct orientation	Explore pre-made GeoGebra sketches (see Figure 2 as an example) regarding the properties of types of quadrilaterals in terms of side lengths, diagonal lengths, angle measures; share observations and opinions with teammates; fill out a worksheet as a group
3	explication	Students are provided with relevant geometrical terminology with definitions; expected to use the new terminology (e.g. right angle, opposite sides) expressing the properties of each type of quadrilaterals with new terminology
4	free orientation	Explore the pre-made GeoGebra sketch and generate quadrilaterals (a square, a rectangle, a parallelogram, a rhombus, and a trapezoid) changing measures of side lengths and angles by using the slider tool provided on the VMT screen; each member is expected to construct each quadrilateral by considering their theoretical properties; fill out a worksheet as a group
5	integration	Review and summarize the properties of types of quadrilaterals; fill out a worksheet as a group

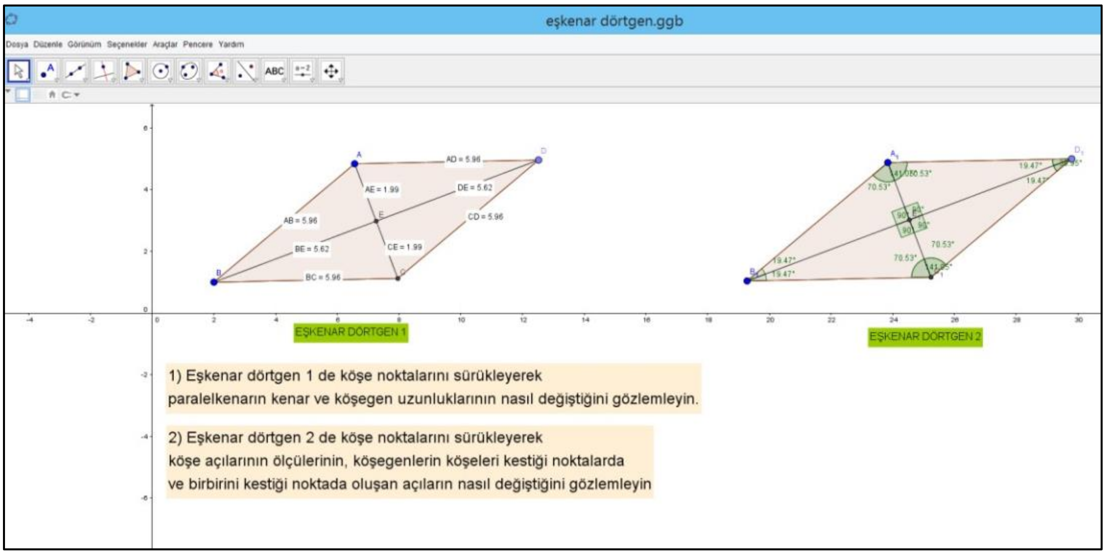


Figure 2. Pre-made GeoGebra Sketch Rhombus Screen (in Activity 2)

Data collection instrument: The van Hiele Geometry Test

In order to evaluate students' geometric thinking levels, we used the VHGT that was translated into Turkish by Duatepe (2000). The 25-item test was originally developed by Usiskin (1982). The first five questions address Level 1, the second five questions address Level 2, and so on for levels 3, 4, and 5. In this study, the first 15 items of the VHGT were used since middle school students can only reach up to Level 3 (van Hiele, 1986). In addition, Şener-Akbay (2012) found that in her sample (434 middle school students in Turkey) none of the students achieved Level 4. The Cronbach alpha reliability measures of the VHGT Turkish adoption was found to be as 82, .51, .70, .72, and .59 for each level of the test respectively (Duatepe, 2000).

Data analysis

In order to investigate the change in van Hiele levels of geometric thinking and VHGT scores, students' VHGT pretest and posttest scores were calculated. Using the grading system suggested by Usiskin (1982), students received 1 point for each correct answer and 0 points for each incorrect answer. Since only 15 questions were considered in the study, the VHGT scores ranged between 0 and 15. Usiskin (1982) stated that a student needed to give at least three correct answers to be successful at a certain level. For example, if a student gets at least three correct answers on items 1-5 (Level 1) but doesn't get at least three answers on items 6-10 (Level 2), the van Hiele level of geometric thinking of the student is determined as Level 1. After checking the parametric test assumptions, the paired sample t-test was used to compare the students' geometric thinking scores before and after the intervention.

In order to examine the role of collaborative competencies on students' van Hiele level of geometric thinking, the VMT chat logs of two groups of students were qualitatively analyzed. As explained in more detail in the 'participants' section, these two groups were selected based on the maximal variation sampling method.

The chat logs of two groups of students were qualitatively analyzed using *directed content analysis* approach (Hsieh & Shannon, 2005). In directed content analysis, predetermined codes which are derived from either theories or relevant research findings guide the analysis. In this analysis, the codes came from the PISA 2015 collaborative problem-solving framework. As stated before,in this framework, three-core collaborative competencies (CCC) were presented as: (1) establishing and maintaining shared understanding, (2) taking appropriate action to solve the problem, (3) establishing and maintaining group organization. Since the activities that were used in the present study were not typical problem-solving activities, we changed "taking appropriate action to solve the problem" to "taking appropriate action to complete the tasks."

The three CCC used in the analysis, their definitions, and corresponding proficient behaviors were given in Table 3.

The unit of analysis was identified as sections in VMT chat logs in which participants talked about a single issue. Each of these sections were coded in terms of the components of the collaborative problem-solving framework. The frequency tables of the codes were created and the collaborative competencies of the groups were compared. To establish the reliability of the coding, another researcher, who has been trained in the coding scheme, independently coded 25 % of the whole data. The agreement between the two coders is found to be 90%.

Table 3.
The Three Core Collaborative Competencies, Their Definitions and Proficient Behaviors in PISA 2015 (OECD, 2017a)

Collaborative competency	A brief explanation of the collaborative competency	Proficient behavior
A. Establishing and maintaining a shared understanding	Creating an information flow among themselves by communicating the right information at the right time, and to attempt to overcome the deficiencies in shared knowledge.	A1. Discovers others' abilities and shares information about own ability A2. Discusses the tasks - asks questions, responds to others' questions A3. Communicates during monitoring and resolution of group work
B. Taking appropriate action to complete the tasks	Making an effort on completing the task by understanding the task assignments properly.	B1. Understands the type of interaction needed, makes sure to know who does what B2. Describes and discusses tasks and task assignment B3. Enacts plans together with others and performs the actions of the assigned role B4. Monitors and evaluates others' work
C. Establishing and maintaining team organization	Being aware of their role in the group, fulfill the requirements of their role, check whether their teammates performing their roles appropriately, and handle with the communication problems	C1. Acknowledges and enquires about roles C2. Follows rules of engagement - complies with a plan, ensures others comply with the plan C3. Monitors team organization - notices issues, suggests ways to fix them

Findings

The development of the van Hiele levels of geometric thinking and VHGT scores

After the intervention, of the 24 participants who started at Level 1, 11 of them increased their level to Level 2, and one participant to Level 3 (Table 4).

Table 4.
Participants' van Hiele Levels of Geometric Thinking Before and After The Treatment

	Before the treatment	After the treatment
Level 0	0	1
Level 1	24	11
Level 2	0	11
Level 3	0	1

The descriptive statistics related to the VHGT pretest and posttest scores showed that the participants performed better after the intervention (M = 7.83, SD = 1.34) as opposed to before the intervention (M = 6.67, SD = 1.31). The paired-samples t-test (Table 5) was used to examine whether the difference between the means was statistically significant (the Shapiro-Wilk's test showed that the data were normally distributed). The t-test analysis showed that the improvement in VHGT scores was statistically significant, ($t_{(23)} = 3.83$, $p < .01$, $d = .78$). The effect size ($d = .78$) indicated a large effect (Cohen, 1988).

Table 5.
The Paired-Samples T-Test Statistics

	t	df	Sig. (2-tailed)	Cohen's d
PostVHGT- PreVHGT	3.826	23	.001	.78

The role of collaborative competencies

Although there was a statistically significant increase in terms of the VHGT scores, not all students were able to improve their geometric thinking levels. Before the intervention, all students were at the visual level (Level 1). After they completed the VHL-based instruction through VMT, half of the students improved their van Hiele level of geometric thinking. However, the other half could not do so. In order to understand the reasons lying behind the difference, we performed an in-depth qualitative analysis of the VMT chat logs of two groups of students that represented successful and unsuccessful groups regarding the development of their van Hiele levels of geometric thinking.

Table 6.
Frequencies of The Three CCC in Two Groups' VMT Chat Logs

CCC (codes)	Sub-codes (proficient CCC behaviors)	Group 1 (successful)	Group 2 (unsuccessful)
A. Shared understanding	A1	0	0
	A2	12	5
	A3	6	3
	Total	18	8
B. Taking appropriate action	B1	0	0
	B2	1	2
	B3	5	6
	B4	4	4
	Total	10	12
C. Group organization	C1	6	4
	C2	5	3
	C3	4	1
	Total	15	8

Table 6 shows the frequencies of the three CCC evident in the two groups' VMT chat logs. According to these, there was more evidence of shared understanding and group organization in VMT chat logs of Group 1 (successful group) compared to Group 2 (unsuccessful group). On the other hand, there was not much difference between the two groups in terms of taking appropriate action.

The role of collaborative competencies

Establishing and maintaining a shared understanding

Group 1 (the successful group) showed the proficient behaviors A2 and A3 mostly. That is, the group members discussed their opinions about the properties of quadrilaterals, asking questions and responding to others' questions (A2), and kept the communication going to maintain group work (A3). However, we did not find any evidence showing that they discovered others' abilities and informed others about their own ability (A1). Here is an example of how the students in Group 1 discussed the tasks - asked questions, responded to others' questions (see Table7). Here they were talking about properties about rectangles (Activity 3). Sude claimed that the lengths of all sides of the rectangle are not equal (see line 252). On the other hand, Emir claimed that the lengths of all sides of a rectangle are equal (see line 253). First, Lara agreed with Emir (see line 256). However, after Sude contested that, Lara changed her mind and accepted that the lengths of opposite sides of the rectangle are equal (see lines 258, and 259).

Table 7.

Line	Participant	Chat Posting
252	Sude	The lengths of all sides of the rectangle are not equal.
253	Emir	The lengths of all sides of the rectangle are equal.
254	Sude	How can they be equal?
255	Sude	They cannot.
256	Lara	They can be. Why not?
257	Sude	One of the side lengths must be smaller and the other one must be larger.
258	Lara	It is sensible.
259	Lara	It is so sensible.
260	Emir	The lengths of all sides of a rectangle are equal.
261	Sude	The lengths of opposite sides are equal.
262	Sude	No.
263	Lara	Think again.
264	Sude	The lengths of all sides are not equal.
266	Lara	Ok. The lengths of opposite sides are equal.

On the other hand, Group 2 had some problems on establishing a shared understanding of the tasks. Although they tried to discuss the properties of the quadrilaterals at the beginning of the study, they could not communicate about the same task. For example, in Activity 3, both students tried to discuss what they explored about the properties of quadrilaterals (see Table 8, lines 96, 100 and 104). Furthermore, they asked questions to each other (see line 100, 104 and 105). However, they did not focus on the same task. Hence, they did not understand what the other was talking about (107, and 113). This situation prevented them to establish a shared understanding.

In the rest of the session, the discussion between Group 2 members continued. However, there is not any important evidence in their chat logs that shows the discussion on the properties of given quadrilaterals. Thus, they did not find an opportunity to check with each other whether their ideas about the properties were correct or not. They mostly focused on completing the tasks individually changing the position of the given quadrilaterals and writing missing values about quadrilaterals individually (side lengths, angles, etc.) on the worksheet.

Table 8.

Line	Participant	Chat Posting
96	Öykü	The lengths of opposite sides are equal and it is a trapezoid.
97	Öykü	Naz, do you see?
98	Naz	Öykü, is this you?
99	Öykü	Yes.
100	Naz	What do you say? I said that the first question is “not equal” What do you think?
101	Öykü	Naz?
102	Öykü	We dragged the corner points.
103	Öykü	Did you realize?
104	Öykü	The lengths of opposite sides are equal.
105	Naz	What do you think about the answer?
106	Naz	The answer?
107	Öykü	Why do you say “they are not equal”?
108	Öykü	The lengths of opposite sides are equal
109	Naz	For the first question.
110	Naz	What about you?
111	Naz	What?
112	Öykü	Do you realize what I did, Naz?
113	Naz	What are you talking about?

Taking appropriate action to complete the tasks

Regarding the second CCC, taking appropriate action to complete the tasks, members of Group 1 monitored each other’s work, asked for explanations if necessary, made necessary plans to complete the tasks as a group, and displayed an effort to do their own part. For example, in Table 9, Emir observed that Lara was doing her part very slowly (line 84). Lara had to apologize for explaining why that was the case (problem with her computer). From this example, we can understand that Emir was monitoring Lara’s work. Eventually, Lara said she was taking the control again to complete her part in the task.

Table 9.

Line	Participant	Chat Posting
84	Emir	Lara is so slow.
85	Lara	Emir, I have a problem with my computer.
86	Lara	I am sorry.
87	Lara	Really…
88	Lara	Sude, did you complete?
89	Lara	I am taking the control.

When the chat logs of Group 2 were qualitatively analyzed based on taking appropriate action to complete the tasks, there is evidence that shows that the members of Group 2 explained the tasks to each other (B2), they made the necessary plan to complete the tasks as a group, displayed effort to do their own part (B3); evaluated the others’ work and warned each other if necessary (B4). For example, in Table10, we can see how the members planned to do the task assignments in Activity 4 (free orientation) and fulfilled their responsibility in the group work. Öykü and Naz enacted the plan together to complete the task assignments (see lines 279, 282, and 284) and informed each other that they followed the plan (see lines 280, 283, and 284).

Table 10.

Line	Participant	Chat Posting
279	Öykü	Let's construct a square right now.
280	Naz	Ok, it is your turn.
281	Öykü	I have constructed a rectangle.
282	Naz	Ok. Start! That is your turn.
283	Öykü	Ok.
284	Öykü	I am writing on the table right now.
285	Naz	Ok.

In summary, both groups seemed to enact their plans together and fulfilled their own responsibilities. There is evidence that students in both groups monitored and evaluated each other's work in their respective groups. One can conclude that there were not any noteworthy differences between the two groups regarding the aspect of taking appropriate action to complete the tasks.

Establishing and maintaining a group organization

In regarding to the group organization collaborative competency, evidence shows that the members of Group 1 tried to fulfill their responsibilities about their role in the group work (C1), engaged in the group work, stuck the group plan, ensured that others followed the plan (C2), kept eye on group organization, and proposed a way to fix any problem in group organization (C3).

For example, we can see from Table11 how team organization in Group 1 was established before starting Activity 2 (free orientation). The teams were required to explore the characteristic features of the quadrilaterals by dragging the corner points of the quadrilaterals given in a pre-made GeoGebra sketch and change their shapes in three different positions. Here, they needed to have a plan to complete the activity successfully. In this task, Sude determined in which order the activity would be done (see lines 45, 46, and 47). She also determined how much time each member could have the control and complete the task (see line 48). The other students agreed with Sude (see lines 49, and 51) and accepted their roles in the group work establishing group organization.

Table 11.

Line	Participant	Chat Posting
45	Sude	Emir is starting first.
46	Sude	Then, me.
47	Sude	Then, Lara.
48	Sude	Two minutes for each of us…
49	Lara	I agree.
51	Emir	I am the first.

Regarding group organization, Group 2 members had some problems in establishing and maintaining the group organization. For instance, while Öykü was working on Activity 1 (see Table12, line 76), Naz was dealing with the Activity 2 (see line 74), focusing on different tasks at the same time. Even later, after some time has passed, they were not able to solve the problem about the group organization.

Table 12.

Line	Participant	Chat Posting
68	Öykü	I think the first one is parallelogram.
69	Naz	But it was asked that "are they equal?"
70	Öykü	I think the lengths of the opposite sides are equal.

71	Naz	Öykü, are you in the parallelogram part?
72	Öykü	I think the lengths of all sides are equal for Quadrilateral C and D.
73	Naz	Is it "Yes"?
74	Naz	We are not there, we are in trapezoid part.
75	Öykü	The lengths of opposite sides are equal for Quadrilateral A.
76	Öykü	We did not complete the Activity 1.

In summary, the result of the qualitative analysis showed that the members of Group 1 displayed more collaborative competency behaviors compared to the members of Group 2. The main differences between the two groups were most notably identified in terms of the two core collaborative competencies: “shared understanding” and “group organization.” More specifically, Group 1 (successful group) engaged in group work, followed the plan, checked each other’s work, and solved the problems about group organization when there was a problem. Furthermore, the members discussed their opinions based on their explorations from the activities and tried to maintain a shared understanding. On the other hand, the members of Group 2 (unsuccessful group) were not able to successfully deal with the problems in group organization, mostly preferred to divide the tasks between them, and completed the tasks individually without sharing their ideas and discussing the task. These aspects of their collaboration might have prevented them to learn from each other and improve their van Hielelevels of geometric thinking.

Discussion and Conclusions

This study investigated the role of working within a CSCL environment (i.e., VMT) on middle school students’ van Hiele levels of geometric thinking (both in terms of levels and scores), and looked into students’ collaborative competencies while working within the VMT environment.

In the previous literature, there were a number of studies that found DGS environments to be effective in developing students’ geometric thinking (Abdullah et al., 2015; Abdullah & Zakaria, 2013; Karakuş & Peker, 2015; Kutluca, 2013). The results of the current study corroborated these findings showing that students’ geometric thinking levels could be developed in a CSCL environment where a multi-user DGS was embedded, even when the treatment is not long.

Researchers observed that freely sharing and discussing ideas in a learning environment affected students’ learning positively (e.g., Karakuş & Peker, 2015; Kutluca, 2013). Essentially, they implied the importance of collaboration among students. However, there were not any studies that examined students’ geometric thinking in a well-designed CSCL environment. In this respect, the results of the present study expanded the previous literature by finding that van Hiele levels of geometric thinking can also be developed in a CSCL setting, where supporting collaboration among students has been the guiding design element of the learning environment.

The results of the current study also supported the previous literature about the effectiveness of van Hiele phased-based instruction (e.g., Siew, Chong & Abdullah, 2013). In the present study, the learning tasks were designed for the VMT environment by considering van Hiele’s phases of learning geometry. We found that half of the students improved their van Hiele levels of geometric thinking, and students’ geometric thinking scores significantly increased.

One important factor that affected students’ geometric thinking development in the VMT environment could be students’ collaborative competencies, which are considered essential for success in modern societies (Cukurova, et al., 2018). We investigated the VMT chat logs of two groups (Group1 and Group 2) who were both at Level 1 initially but showed different improvements in terms of their van Hiele levels of geometric thinking after the intervention. The result of the qualitative analysis showed that members of Group 1 (the successful group in terms of developing van Hiele levels of geometric thinking) displayed more collaborative competencies compared to Group 2 members (the unsuccessful group).

More specifically, the members of Group 1 expressed and discussed their ideas about the characteristic features of quadrilaterals and met on common ground. Also, they planned how to complete the activities and executed the plan during the team work. They followed each other's work and fixed the problems about group organization. On the other hand, Group 2 members did not share much information about what they explored. They could not deal with the deficiencies in the shared understanding. Hence, they were not able to realize each other's misunderstandings and help each other. Furthermore, they were not able to fix the problem about group organization. The problems about group organization might have affected their learning negatively.

Based on the qualitative analysis, we speculated that effective collaboration among students could be an important factor that supported their geometric thinking development. While the design of VMT supported collaborative work, this feature does not make students automatically more collaborative. Students still need to develop and use collaborative competencies to take advantage of such learning environments for learning school subjects. Teachers who want to integrate CSCL environments into their classes will still need to teach and model the indicators of collaborative work.

These findings have several implications for the design of geometry learning environments. Firstly, mathematics teachers can consider incorporating both the DGS-based activities on quadrilaterals and CSCL environments, such as VMT, in geometry instruction. The present study provides ready-to-use DGS-based VMT activities on quadrilaterals based on van Hiele phase-based instruction for the middle school students.

International assessment studies, such as PISA and TIMMS, have shown that the geometry level of students in Turkey has been below the international average. According to the TIMMS data in 2011, for example, the level of students at the 4th grade in Turkey was below the international average and this difference was found to be statistically significant (Oral & McGivney, 2013). The geometry level of students at the 8th grade in Turkey was also below the international average based on the TIMSS data in 2011. Oral and McGivney (2013) claimed that these results pointed to important problems in the quality of geometry education in Turkey. It is argued that the geometry curricula of elementary and middle schools in Turkey misguided students by leading them to memorize definitions and properties of geometric shapes (Olkun, Sinoplu & Deryakulu, 2009). Students are not expected to make reasoning about geometrical shapes and their features. Olkun et al. (2009) further stated that teachers in Turkey lacked technology-based learning materials, such as DGS-based instruction, and knowledge about how to use those materials. Thus, teachers can integrate the DGS-based activities designed in this study into their classes to teach geometry more effectively.

In PISA 2015, in which the first time collaborative problem-solving has been assessed, Turkish students scored the lowest among the 35 OECD countries (OECD, 2017b). While collaboration is viewed as an essential 21st century skill, it is not explicitly taught in schools, and Turkish curricula are not an exception. The qualitative results of the current study suggested that students who collaborated more effectively were more likely to improve their geometric thinking. Hence, teachers and curriculum developers should focus on developing students' collaborative competencies using CSCL environments so that Turkish students would develop more competitiveskills for the future workforce. The results of this study show that such efforts would result in not only developing collaborative competencies but also students' geometric thinking.

The present study had some limitations. In the current study, the groups were not formed by considering students' collaborative competencies and a true experimental design was not used. Therefore, we cannot talk about the real "effect" of collaborative competencies on the development of geometric thinking skills.In addition, the intervention was designed for only the middle school students who were at the visual level (Level 1). Future studies can focus on forming groups based ontheir collaborative competencies and designing instruction for students at other geometric thinking levels and grades.

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Uzun Öz

Giriş

Geometri matematikte önemli bir alt disiplin olmasına rağmen pek çok öğrencinin güçlük çektiği bir alandır (Köseleci-Blanchy ve Şaşmaz, 2011). Geometri öğrenimi ile ilgili yaşanan zorlukların bir sebebi öğrencilere sunulan öğretimin vanHiele geometrik düşünce seviyelerine uygun hazırlanmayışıdır. Geometrik düşünme düzeyleri Hollandalı iki matematik eğitimcisi (Dina vanHiele-Geldof ve Pierre vanHiele) tarafından önerilmiş ve öğrencilerin geometri öğrenimlerinde neden zorluk çektiklerine açıklama getiren bir modeldir. Buna göre öğrenciler geometri öğrenirken bir dizi ardışık düzeyden (görsel, betimsel, basit çıkarım, çıkarım ve sistematik düşünme) geçerek ilerlemektedir (vanHiele, 1999). Bir öğrencinin herhangi bir geometrik düşünme düzeyinde değerlendirilebilmesi için ondan önce gelen tüm düzeylerden geçmiş olması gerekir.

Araştırma sonuçları vanHiele geometrik düşünme düzeylerinin geometride başarıyı açıklayan geçerli bir gösterge olduğunu doğrulamaktadır (örn. Burger ve Shaughnessy, 1986; Senk, 1989; Usiskin, 1982). Van Hiele modeline göre geometri düzeyinde ilerleme yaşa bağlı değildir. Öğrencilerin bir düzeyden diğerine ilerleyebilmesi belli adımları takip eden bir geometri öğretimi ile mümkündür (Usiskin, 1982). vanHiele modeline dayalı öğretim süreci şu öğrenme aşamalarını içermelidir: araştırma (görüşme), doğrudan yöneltme, netleştirme (açıklama), serbest çalışma, bütünleme.

Öğrencilerin geometrik anlama seviyelerini etkileyen faktörleri açıklamaya yönelik pek çok araştırma bulunmaktadır (Abdullah ve diğerleri, 2015; Abdullah ve Zakaria, 2013; Duatepe-Paksu ve Ubuz, 2009; Halat, 2006; Karakuş ve Peker, 2015; Khalil ve diğerleri, 2018; Kutluca, 2013). Bu araştırmalarda çoğunlukla dinamik geometri yazılımlarının (Dynamic Geometry Software [DGS]) kullanıldığı öğretimin etkili olduğu gösterilmekle beraber, öğrenciler arasındaki sosyal etkileşim ve iş birliğinin de geometri öğreniminde önemli olduğu vurgulanmıştır. Buna rağmen alan yazında bu iki bileşene sahip olan bilgisayar destekli iş birliğiyle öğrenme (Computer-Supported Collaborative Learning [CSCL]) ortamlarının öğrencilerin geometri düşünme düzeylerinin geliştirilmesindeki rolüne dair bir çalışma bulunmamaktadır.

Sanal Matematik Takımları (Virtual Math Teams [VMT]), İnternet üzerinden ücretsiz olarak erişilebilen ve iş birliği içinde öğrenmeyi desteklemek amacıyla, tasarım temelli bir araştırma projesi sonucunda geliştirilmiş olan bir CSCL öğrenme ortamıdır (Öner, 2016a). Sohbet ara yüzüne ek olarak bir DGS programını (GeoGebra) ortak kullanmayı mümkün kılan ilk platformdur. İlk olarak 2015 yılında iş birliğiyle problem çözme alanının OECD'nin Uluslararası Öğrenci Değerlendirme Programı (PISA) testinde yer almış olması, Sanal Matematik Takımlarının Türkiye'de tanıtılması ve yaygınlaşması için teşvik edici bir unsur olarak görülmektedir (Öner, 2016a).

Bu çalışmada bilgisayar destekli iş birliğiyle öğrenme aracı olarak,tasarım temelli bir araştırmanın sonucunda geliştirilmiş Sanal Matematik Takımları (VMT) ortamında ortaokul öğrencilerinin vanHiele geometrik düşünme düzeylerinin gelişimi incelenmiştir. Aynı zamanda, geometrik düşünme düzeylerinin gelişimine yol açan faktörleri daha iyi anlamak için öğrencilerin VMT söylemlerinin nitel analizi yapılmıştır.

Yöntem

Bu çalışma öntest son test deneysel öncesi araştırma desenine göre nitel veri analizi ile de desteklenerek tasarlanmıştır.Katılımcılar amaçlı örnekleme yöntemiyle belirlenen vanHiele geometrik düşünme düzeyi görsel seviyede olan, iki farklı okuldan seçilen (13'ü kız) 24 ortaokul öğrencisidir. Öğrenciler ders programlarına uygun olarak 2 veya 3 kişilik takımlara ayrılmışlardır. Uygulama olarak takımlara vanHiele modeline dayalı öğretim süreçlerini (araştırma, doğrudan yöneltme, netleştirme, serbest çalışma, bütünleme) gözeterek dörtgenler konusunda geliştirilen beş aktivite VMT ortamında sunulmuştur. Tüm takımlar ayrı zamanlarda (okulda veya evde) ve birinci yazarın da moderatör olarak bulunduğu seanslarda VMT üzerinde buluşarak tüm beş aktiviteyi tamamlamıştır. Her bir seans yaklaşık 1 saat sürmüştür. Uygulama sonunda öğrencilerin geometrik düşünce seviyelerindeki değişiklik Duatepe (2000) tarafından Türkçeye çevrilen vanHiele Geometri Testi (Usiskin, 1982) kullanılarak değerlendirilmiştir. Ayrıca, maksimum çeşitlilik örnekleme yöntemine göre belirlenen iki takımın VMT sohbet kayıtları, Ekonomik Kalkınma ve İşbirliği Örgütü (OrganisationforEconomicCo-operationand Development [OECD]) tarafından yürütülen Uluslararası Öğrenci Değerlendirme Programı (Programmefor International StudentAssessment [PISA]) 2015'te kullanılan üç temel iş birliği yeterliliği göz önüne alınarak *yönlendirilmiş içerik analizi* yöntemiyle (HsiehveShannon, 2005) nitel olarak incelenmiştir (OECD, 2017a). Bu yeterlikler şunlardır: ortak anlayış oluşturma ve bunu sürdürme, verilen görevleri uygun eylemlerle tamamlama ve grup organizasyonunu oluşturma ve sürdürme.

Sonuç

Uygulama sonunda 24 katılımcıdan 11'inin vanHiele geometrik düşünme düzeylerini ikinci, birinin de üçüncü seviyeye çıktığı görülmüştür. 11 katılımcı için vanHiele geometrik düşünme düzeyi değişmezken, bir katılımcı birinci düzeyin altında kalmıştır. Aynı zamanda vanHiele Geometri Testi skorlarında etki büyüklüğü yüksek ve istatistiksel olarak anlamlı bir artış olmuştur (t (23) = 3.83, $p < .01$, d = .78.). Van Hiele geometrik düşünme düzeylerinde sağladıkları artışa göre başarılı ve başarısız olarak belirlenen iki grup öğrencinin VMT sohbet kayıtlarının nitel analizi yapılmıştır. Buna göre başarılı olarak değerlendirilen grubun VMT konuşmalarında OECD tarafından belirlenen iş birliği yeterliklerine dair davranışları gösterdiği, başarısız olan grubun ortak anlayış oluşturma ve bunu sürdürme ve grup organizasyonu oluşturma ve sürdürme bakımından daha az davranış gösterdiği belirlenmiştir. Buna göre iş birliği yeterliklerinin öğrencilerin geometrik düşünme seviyelerini geliştirmede önemli bir faktör olabileceği düşünülmektedir.

PISA 2015 sonuçlarına göre Türk öğrenciler 35 OECD ülkesi arasında iş birliğiyle problem çözme alanında sıralamada en sonda yer almışlardır (OECD, 2017b). İş birliği ile problem çözme önemli bir 21. yy becerisi olmasına rağmen okullarda özellikle öğretilen bir öğrenme çıktısı değildir. Bilgisayar destekli işbirliğiyle öğrenme ortamları bu becerilerin öğretilmesi için oldukça uygundur. Bu çalışmanın sonuçlarına göre CSCL kullanımına yönelik çabalar sayesinde öğrencilerin sadece iş birliği becerilerini değil geometrik düşünme becerilerini de geliştirmek mümkün olabilecektir.